Rare Disaster Risk and Asset Prices

Theoretical Considerations, Econometric Methodology, and Empirical Analyses

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CHAPTER 1

Introduction

The central paradigm in asset pricing theory asserts that positive expected excess returns result as a form of risk compensation – so according to the high U.S. equity premium after World War II, there must be considerable risk for which to compensate. A methodological lynchpin of this view is the basic pricing equation for a gross return R_{t+1} that is implied by Hansen and Singleton's (1982) canonical consumption-based asset pricing model (C-CAPM) with time-additive power utility:

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}\right] = 1, \tag{1.1}$$

where β is the subjective discount factor, γ is the coefficient of constant relative risk aversion and C_t denotes consumption in period t. As demonstrated by Mehra and Prescott (1985) though, the canonical C-CAPM cannot explain the U.S. equity premium with plausible values of β and γ, leading to a widespread belief that the model is strong in theory but weak in application.

This phenomenon – the equity premium puzzle – is frequently addressed in finance and many attempts have been made to solve it. A famous competitor amongst these attempts is the rare disaster hypothesis (RDH) by Rietz (1988) and Barro (2006), according to which the high U.S. postwar excess returns resulted from a sample selection effect: Investors ex ante demanded compensation for highly unlikely but possibly disastrous consumption contractions from which they ex post – due to the lucky path that U.S. history took – did not suffer. Considering the Cold War whose possible escalation loomed for decades, the prosperous consumption path that we observe today is indeed just one particularly pleasant string out of the multitude of outcomes that investors in the 1950s to 1980s had to consider. This intuitiveness makes the RDH an appealing explanation of the equity premium puzzle and the poor empirical performance of Hansen and Singleton's (1982) canonical consumption-based asset pricing model and its followup variants. However, as much as the RDH is appreciated in theory and supported by calibration studies, it is difficult to estimate the preference parameters of a C-CAPM that explicitly allows for disaster risk.

With this dissertation, I propose econometric estimation strategies for the preference parameters of such a disaster-including C-CAPM that facilitate testing of the

RDH. The empirical success of the hypothesis is evaluated based on the plausibility and precision of the parameter estimates as well as on the sensibility of fundamental model-implications, like the model-implied mean market and T-bill return, mean equity premium, and market Sharpe ratio. For this purpose, I consider three approaches that use simulated method of moments-type estimation strategies to account for the possibility of severe but rare contractions in consumption.

This introductory chapter provides an overview of the beginnings and recent advances in asset pricing with disasters. Section 1.1 explains the origin and reception of the RDH, whilst Section 1.2 reviews more recent disaster risk literature. Studies that criticize the RDH on theoretical or empirical grounds are discussed in Section 1.3 and Section 1.4 outlines the contribution and composition of this dissertation.

1.1 The equity premium puzzle and early RDH literature

Coining the term equity premium puzzle, Mehra and Prescott (1985) were the first to claim that the high U.S. equity premia, which have been observed for more than a century, could not be explained in an asset pricing framework that relies on time-additive power utility and economically plausible values for relative risk aversion and time preference. Their empirical assessment is based on annual data for the 1889-1978 period and the equity premium is computed from the Standard and Poor's 500 Index and short-term T-bill returns, resulting in an average annual equity premium of about 7%. Mehra and Prescott adapt Lucas's (1978) pure exchange model by assuming that the growth rate of consumption follows a two-state Markov process in which consumption growth is either slightly above or slightly below average and transition probabilities are symmetric. The parameters of the Markov process are calibrated to real consumption growth data. Deriving analytical expressions for the risk-free rate and the market return that solely depend on the parametrization of the Markov process and the choice of the two preference parameters, Mehra and Prescott (1985) evaluate the range of average risk premia that are feasible when restricting $0 < \gamma \leq 10$ and $0 \leq \beta \leq 1$. The chosen plausibility bounds of the preference parameters are generous: Mehra and Prescott themselves stress that an RRA value of 10 is very high compared to earlier theoretical and empirical findings (e.g., Arrow (1971) who claims on theoretical grounds that the RRA coefficient should be close to 1). Furthermore, studying time preference values between 0 and 1 implies that every degree of indifference towards future consumption is accounted for.[1] The largest premium that can be obtained using this set of preference parameters is 0.35%, which is accompanied by a mean annual risk-free rate of 4% that does not correspond to the empirically observed value of 0.8%. The framework proposed by Mehra and Prescott (1985) can thus neither explain the high U.S. excess returns, nor the low risk-free rate. The authors also consider a four-state Markov chain process and a wide range of transition probabilities. The results remain unchanged, leading to the conclusion that market frictions would be needed to reconcile plausible preference parameters with the data.

Rietz (1988) builds on Mehra and Prescott (1985) and proposes an alternative approach to solve the equity premium puzzle. He re-specifies their two-state Markov process to include a third state that only occurs with a very small probability but signifies a severe decline in consumption. Rietz argues that investors would demand compensation in form of excess returns for the possibility of such crashes and that the twentieth century in the U.S.A. is not representative of such disasters, in the sense that we only observe the high excess returns but no consumption contractions. In more recent literature, this idea is referred to as the rare disaster hypothesis.[2] By restricting the transition probability matrix accordingly, Rietz (1988) ensures that the crash state is followed by one of the two regular consumption growth specifications, thereby defining the consumption disasters to be single-period events. Using otherwise the same model specification as Mehra and Prescott (1985), these consumption contractions translate into expected equity and risk-free asset returns.[3] Using an annual crash probability of 0.09%, $\gamma = 6.90$, and $\beta = 0.995$, Rietz's (1988) model specification can account for an annual risk-free return of 0.77% and a corresponding risk premium of 6.38%. Consumption growth in the crash state is specified such that the level of consumption is about halved in the event of a disaster. If the level of consumption is reduced to 75% of its previous value in the case of a crash and if such disasters occur with an annual probability of at least 1%, risk aversion parameters in the range of 9-10 are necessary to replicate the empirical equity premium and the risk-free return. Analogously, a robustness check

[1] $\beta = 0$ implies that investors do not assign any value to future consumption when deciding on their consumption paths and $\beta = 1$ implies that investors are indifferent between current and future consumption.

[2] Although Rietz (1988) counts as the originator of the rare disaster hypothesis and the literature related to it, it is interesting to note that he actually never uses the term *disaster*, but refers to *crashes*, instead.

[3] The terminology *risk-free* is certainly misleading in this context, because if the asset was truly risk-free, its returns should not be affected by consumption disasters.

is performed in which the level of consumption contracts by more than 90% in the crash state. In this specification, $\gamma = 1.5$ suffices to match the observed risk-free return and risk premium values at an annual disaster probability of 0.01% to 0.1%.

In the same year, Mehra and Prescott (1988) counter Rietz's (1988) analysis by pointing out its possible shortcomings, such as the assumption that crashes appear as single-period contractions, that these contraction sizes are very large, and that the required RRA coefficients are frequently in the upper quarter of the plausible parameter range. Section 1.3 deals with these points of critique and outlines how they can be dealt with.

The seminal work by Mehra and Prescott (1985) induced a large literature that deals with different attempts to resurrect the consumption-based asset pricing model and solve the equity premium puzzle. The most famous of these second-generation C-CAPM advances are (a) the habit formation model by Campbell and Cochrane (1999), (b) Bansal and Yaron's (2004) long-run risk model, and (c) the rare disaster hypothesis revived by Barro (2006). In the habit formation model, representative agents maximize expected utility based on a power utility function, which considers the level of consumption in excess of an external habit level that itself depends on consumption. It is external in the sense that effects on the future habit level are not considered in the consumption optimization problem. Applying a variety of parameter restrictions and ensuring excess consumption to be non-negative, Campbell and Cochrane (1999) use a calibration study to show that such a model is able to explain the empirical equity premium and mean risk-free rate with $\gamma = 2$. Bansal and Yaron (2004) assume that there is a small but persistent component in consumption and dividend growth, whereas many other studies model consumption growth as an independent and identically distributed process. Using the available U.S. consumption growth series with their limited number of observations, differentiating between a process that features a small persistent component and one that has no memory, is difficult. The idea behind this long-run risk approach is that the long-term expected growth rates are directly affected by the persistent component and that asset prices will react to innovations in it. As long as the degree of persistence is high, the component as such may be small. Using recursive preferences and also a calibration approach, Bansal and Yaron (2004) are able to replicate the average equity premium and short-term interest rate with an intertemporal elasticity of substitution (IES) of 1.5 and $\gamma = 10$.

Apart from these three seminal second-generation consumption-based asset pricing models, many more studies were inspired by Mehra and Prescott (1985) to propose different attempts to vindicate the C-CAPM. However, these studies only partially succeed to explain the U.S. equity premium with plausible and precise estimates of the preference parameters. Yogo (2006), for example, proposes an asset pricing model that differentiates between the consumption of durable and nondurable goods. Most other studies solely rely on the consumption of nondurables and services, because only a small fraction of the stock of the durable good that investors own is actually consumed during the period lengths that are conventionally considered (e.g., months, quarters, years). Yogo's (2006) model can account for the cross-sectional variation in expected stock returns, but only with a very high level of risk aversion.[4] The smallest estimate of the RRA coefficient amounts to 174.5, while the estimated subjective discount factor implies a positive rate of time preference (0.88). Savov (2011) relies on waste data as a measure of consumption, calibrates time preference to 0.95, and obtains $\hat{\gamma} = 17.0$ with a large standard error (9.0). Similar results are reported by Kroencke (2017), who uses unfiltered NIPA consumption and receives RRA estimates between 19 and 23 in the postwar period. Again, standard errors are high (10.0). Julliard and Parker (2005) analyze the ultimate risk of consumption, defined as the covariance of returns and consumption growth aggregated over current and future periods. They assume indifference regarding the timing of consumption by calibrating the time preference parameter to 1 and obtain $\hat{\gamma} = 9.1$. However, this estimate has a relatively high standard error of 17.2.

1.2 Recent rare disaster literature: a review

After laying dormant for two decades, Barro (2006) revived the rare disaster literature with a calibration study in which he considers single-period disasters in a power utility context and finds that plausible values of the time preference and risk aversion parameters are compatible with high equity premia when allowing for rare but severe contractions in consumption. In 2009, Barro extended his base model with recursive preferences. Much thought has been given on how to estimate the size distribution of disasters. Whilst Barro (2006) uses GDP data on 35 countries to detect disastrous contractions in, Barro and Ursúa (2008) assembled consumption data for 41 countries and additionally created a GDP dataset for these countries. They find that the distribution of disasters that are detected in consumption data is

[4] Whilst Yogo's (2006) theoretical contribution is much appreciated and cited in related literature, his empirical assessment is currently rigorously criticized based on his readily available code. Borri and Ragusa (2017) find that the preference parameter estimates reported by Yogo (2006) do result from a failed optimization.

very close to that of disasters that are obtained from GDP data. The similarity is caused by the fact that the sharpest contractions, which are often related to one of the world wars, arise in GDP and consumption similarly. Barro and Ursúa's (2008) consumption dataset is also used by Barro and Jin (2011) who suggest to model (transformed) disaster sizes by means of power law distributions. They consider one-parameter power law distributions as well as double power law distributions, which result from merging two power laws at a certain threshold to allow for a different kurtosis in the lower and upper tail of the distribution. An alternative approach for estimating the distribution of disastrous contractions is proposed by Backus et al. (2011) who suggest to derive the distribution of consumption contractions from equity index options. They use these options to determine the risk-neutral distribution of stock returns and, assuming a time-additive power utility function and an RRA coefficient to match the empirical equity premium, obtain the physical distribution of consumption growth. This distribution, however, looks nothing like the the one that is implied by international macro data, because it implies that consumption contractions appear more frequently and are less disastrous than suggested by Barro (2006) or Barro and Ursúa (2008). Tsai and Wachter (2015), who provide a thorough survey of the RDH literature, argue that this option-implied consumption growth distribution is at odds with empirical data, because such frequent jumps should have been observed. A reconciliation of option prices and the consumption growth distribution that is implied by macro data is achieved by Seo and Wachter (2016) who allow for stochastic disaster probabilities and recursive preferences with the IES fixed at 1.

Seo and Wachter (2016) are not alone in assuming time-varying disaster probabilities. Whilst the early rare disaster literature focused on constant disaster risk, this approach has by now been discarded in favor of allowing for time-varying specifications of the disaster process. For example, Gourio (2012) introduces time-varying disaster risk into a business cycle model and Wachter (2013) shows that the volatility puzzle can be explained once a stochastic disaster intensity is accounted for. Nakamura et al. (2013) use the dataset assembled by Barro and Ursúa (2008) and conduct a Bayesian analysis to estimate the parameters of an elaborate disaster process that distinguishes contractions based on their origin and the longevity of their impact. They assume recursive preferences introduced by Epstein and Zin (1989), and Weil (1989) and, fixing the subjective discount factor and the IES at convenient values, find that the model-implied equity premium can be explained at plausible values of the RRA coefficient. Gabaix (2012) proposes another way of including time-variation in the disaster process by assuming that the probability of a disaster is constant, but that the sensitivity of cash flows to a disaster is stochastic, meaning that there is a time-varying resilience of assets, which in turn generates time-varying risk premia. Assuming time-additive utility and linearity-generating processes, Gabaix (2012) challenges 10 puzzles in finance.[5]

Gourio (2013) finds that allowing corporate debt to be affected by disasters accounts for important features of the credit spread. Bai et al. (2015) conclude that the RDH can be used to explain the value premium and Seo and Wachter (2016) argue that the volatility skew can be reconciled with the equity premium in models that include stochastic disaster probabilities. Gillman et al. (2015) shed light on various pricing phenomena in the equity and bond market by letting disasters affect the growth persistence of consumption and dividends. Assuming a time-varying probability of world disasters, Farhi and Gabaix (2016) explain an assortment of exchange rate puzzles. Tsai and Wachter (2016) propose a model that not only allows for rare disasters but also booms and find that the curious empirical simultaneity of growth stocks being riskier than value stocks and having lower returns becomes comprehensible. Building a bridge between two prominent attempts to resurrect the C-CAPM, Barro and Jin (2016) use Bayesian techniques to analyze a model that accounts for long-run and disaster risk. They find that the disaster component explains most of the equity premium.

1.3 Controversy about the rare disaster hypothesis

As indicated above, the empirical success of the RDH in calibrations was also accompanied by several studies that expressed doubt regarding allegedly unrealistic assumptions which are frequently invoked in the rare disaster literature. I will use this section to lay out the main points of criticism and what they imply.

For this purpose, let us return to Mehra and Prescott's (1988) note on Rietz (1988), which is the first documented critique of the RDH. The arguments of the authors are threefold. First, Mehra and Prescott stress that Rietz's calibrated crash sizes represent extremely large single-period contractions, which means that in some of the model settings, the level of consumption is reduced by more than 90% in the event of a disaster. Shocks like this have never been observed in U.S. data – not in aggregate size, and especially not as an instant jump. Second, the authors

[5] Linearity-generating processes are a class of stochastic processes that generates closed-form solutions for bond and stock prices. See Gabaix (2009) for details.

claim that the RRA coefficients which are needed to replicate the historical average equity premia are frequently in the upper quarter of the parameter range that was considered by Mehra and Prescott (1985) and that values so close to 10 may not be plausible in the first place. Third, they argue that there is not enough historical support for Rietz's (1988) RDH – a point of critique that is clearly linked to the first one which referred to the consumption contraction sizes.

Barro (2006) addresses this critique by being the first to actually consider the probability and size distribution of historical disasters. He defines a disaster as an aggregate decline of at least 14.5% and detects all large GDP contractions in a panel dataset that features 35 countries in the 1900-2003 period. As mentioned above, Barro and Ursúa (2008) extend this approach by assembling a cross-country consumption dataset that can be used for the same purpose. The idea behind this strategy is that the U.S. history was particularly lucky and that the GDP and consumption series of other countries may be more representative of the possible disasters that the U.S.A. sidestepped. Both studies also include data on the behavior of the respective stock market indexes and risk-free rate proxies during the detected disaster periods. These information provide the historical support that Rietz (1988), who uses a purely technical approach, does not offer. It turns out that these data-based disaster probabilities and sizes are closest to Rietz's (1988) setting in which the level of consumption is reduced to 75% of its previous value in the event of a disaster. With this Markov process, Rietz needs annual crash probabilities in the range of 1% to 1.4% and $\gamma = 10$ to replicate the observed average equity premium. Using his macro data approach, Barro (2006) obtains an annual disaster probability of 1.7% with an average disaster size of 29%. He is able to explain the equity premium with $\gamma = 4$, thus not only offering an answer to Mehra and Prescott's (1988) demand for a historical foundation, but also reconciling the empirical equity premium with a small RRA coefficient.

The first point of critique mentioned by Mehra and Prescott (1988) is not eradicated by Barro (2006), however. He, just as Rietz (1988), assumes that the consumption disasters evolve as single-period contractions and argues that this simplification is justified by the high correlation of consumption during disasters. In order to assess the possible consequences that may arise from this procedure and the criticism thereof, it is necessary to be specific about the differentiation of *single-period* and *multi-period* disasters in the RDH literature. Let us consider, for this purpose, U.S. GDP during the Great Depression, which declined by 31% in the four years between 1929 and 1933. Barro (2006) who derives a closed-form expression of the equity premium, performs his calibration on an annual frequency, meaning that parameters that represent mean consumption growth and variance are fitted to match annual consumption data. Disaster risk enters the expression through the disaster probability and through the distribution of contraction sizes. Whilst Barro (2006) calibrates the disaster probability to annual data, he neglects the time dimension when it comes to disaster sizes, instead just using the peak-to-trough contraction without regarding the number of periods over which this disaster accrued. This means that Barro's (2006) calibration features the 31% disaster instead of the 8.9% contraction which would result by splitting the disaster equally across four years. Many of the studies that were mentioned in the literature overview, amongst them Barro and Ursúa (2008), rely on the correlation argument and assume that disasters evolve in a single period, too.

By now, the question whether the simplification to single-period disasters is actually the driving force behind the hypothesis's success in calibrations has become the main controversy in the RDH literature. In a comment on Barro and Ursúa (2008), Constantinides (2008) argues that the use of single-period disasters causes a misspecification of the fundamental consumption-based asset pricing paradigm, which is founded on the contemporaneous dependence of consumption growth and (excess) returns. Modeling single-period disasters, according to Constantinides, actually implies considering consumption growth over multiple periods without accounting for returns in the same time span. Assuming the same model framework as Barro (2006) and using their modified dataset, Barro and Ursúa (2008) are able to replicate the empirically observed mean equity premium at plausible preference parameters. In a calibration akin to theirs, Constantinides replaces overall disaster sizes by annual contractions and shows that this causes the model-implied equity premium to shrink to less than 25% of the observed value.

Julliard and Ghosh (2012) perform an empirical application and choose four different angles of attack to argue in the same direction. The first part of their study is based on long annual consumption growth and return series assembled by Campbell (2003). The series span the range 1929-2009 and thus feature two of the GDP contractions detected by Barro (2006): the Great Depression (1929-1933; 31%) and a decline that followed World War II (1944-1947; 24%).[6] The average equity premium is about 6% in this sample. Using the empirical likelihood method,

[6] According to Barro (2006), the decline in GDP after World War II represents an aftermath of war, which is not related to a drop in consumption and thus excluded from his analysis. Indeed, when using the consumption data assembled by Barro and Ursúa (2008), no disastrous contraction is detected during these years.

which corresponds to a generalized method of moments (GMM) setting, in which the weighting of the respective observations can deviate from their relative frequency in the sample, Julliard and Ghosh (2012) find that the resulting RRA estimate is still implausibly large.

Allowing for the possibility that the consumption contractions in the data are just too few to explain the equity premium puzzle, the authors perform a second study in which they use Barro and Ursúa's (2009) identified disasters and equally split the aggregate contraction sizes over the number of periods over which the respective crash occurred, thus generating multi-period disasters. Using these contractions, and Campbell's (2003) data from which they discard the observations that are related to the Great Depression, they create counterfactual samples in which the disaster probability is varied. Julliard and Ghosh (2012) find that the annual disaster probability must be at least 9.6% to obtain an $\gamma \leq 10$. They argue that this result is comparable to Backus et al.'s (2011) distribution of option-implied consumption contractions.

For their third angle of attack, Julliard and Ghosh (2012) extend the previous analysis by studying how likely the occurrence of the equity premium puzzle would be if the RDH was indeed its solution. For this purpose, they choose the disaster probability such that the equity premium is matched and then simulate annual consumption growth and return series. The number of observations is chosen to accord with the length of the Campbell's (2003) series. Generating 10,000 of such counterfactual samples and computing the equity premia that would result from these series, Julliard and Ghosh (2012) find that the median equity premium is 0. Only about 2% of the simulated average equity premia reach the size of their empirically observed counterpart, implying that the equity premium puzzle itself would be a rare event. Finally, Julliard and Ghosh (2012) also show that allowing for rare disasters does not help explain the cross-sectional variation of asset returns. Considering the results depicted above, it does not come as a surprise that the authors conclude their analysis by labeling the RDH a very unlikely solution to the equity premium puzzle.

1.4 Contribution and composition of this thesis

My dissertation contributes to existing literature by proposing frequentist estimation approaches that facilitate empirically assessing and testing the RDH. Estimating the preference parameters of a disaster-including C-CAPM creates added value, because calibration studies do not allow answering questions related to estimation precision or model-implications of the empirical analysis – however, these issues constitute the benchmark by which all second-generation consumption-based asset pricing models are measured. The simulated method of moments (SMM) allows dealing with the extreme form of sample selection that is not only the RDH's legitimation, but also its great obstacle.

Besides proposing econometric techniques that allow estimation of the C-CAPM preference parameters, this dissertation also offers a contribution to the disaster duration controversy outlined above. The studies that are presented in Chapters 3 and 4 feature multi-period disasters and are still able to reconcile plausible preference parameters with high average equity premia. The great difference between my studies and the ones performed by Julliard and Ghosh (2012) and Constantinides (2008) is that they choose to work with a power utility function. This class of utility functions was also used by Barro (2006) and Barro and Ursúa (2008) and has the peculiarity of measuring risk aversion and the elasticity of substitution through the same parameter, as the IES is set to be the inverse of the RRA coefficient.

Is such a restriction plausible in the context of multi-period disasters? I think not. My reasoning is as follows: The IES captures an investor's willingness to substitute consumption over time. There is an ongoing discussion in the literature on whether its value should be larger or smaller than 1, but independent of this debate, a higher IES signifies a higher willingness to substitute. However, the size and duration of crashes constitute two different dimensions of disaster risk. The RRA coefficient relates to the contraction size and how harmful it is to an investor's utility. An investor who is more risk averse will suffer more from a given disaster than a less risk averse one. When assuming a time-additive power utility function, this is actually the only source of risk that matters. Accounting for multi-period disasters includes a second risk dimension: the length of a disaster. In these frameworks, the average contraction size per period is reduced, but these disaster periods materialize in clusters. The IES determines how well an investor can deal with such a clustering. With a higher willingness to substitute consumption over time, it will be easier to cope with multi-period disasters.

Accordingly, power utility does not suffice to account for these two different dimensions of risk. The IES is the inverse of the RRA coefficient and thus, there is no room for an investor that is plausibly risk averse *and* willing to substitute consumption over time. It is important to note that Julliard and Ghosh (2012) may refer to multi-period disasters, but their estimation approach cannot account for the clustering of consumption contractions. If they re-shuffled the observations in their counterfactual samples, such that the disasters were broken apart, this would not affect their RRA estimates in any way. For this reason, the multi-period disaster studies presented in Chapters 3 and 4 of this dissertation use recursive preferences which disentangle an investor's risk aversion from her willingness to substitute over time. The resulting RRA and IES estimates are both larger than 1; a parameter combination that cannot be accounted for by power utility.

I structure my thesis as follows: Assuming single-period disasters and a time-additive power utility function, Chapter 2 presents an SMM-based estimation approach that facilitates the estimation of the C-CAPM preference parameters. This study is based on the working paper *Consumption-Based Asset Pricing with Rare Disaster Risk: A Simulated Method of Moments Approach*, which is joint work with Joachim Grammig. Chapter 3 is based on my working paper *Empirical Asset Pricing with Multi-Period Disasters and Partial Government Defaults*, which extends the previous study to account for the duration of disasters. For reasons described above, I assume Epstein-Zin-Weil preferences to disentangle the RRA coefficient from the IES. This study no longer includes a risk-free rate, but computes excess returns over a T-bill return that can be subject to disasters. Chapter 4 modifies the pricing kernel by differentiating between consumption of durable and nondurable goods as proposed by Yogo (2006). It combines rare disaster and long-run risk literature to illustrate how multi-period consumption disasters translate into returns. This chapter is based on my working paper *The Taming of the Two: Simulation-Based Asset Pricing with Multi-Period Disasters and Two Consumption Goods.* Chapter 5 concludes with an assessment of how the results presented in this dissertation contribute to the RDH literature and what they imply for the link between the real economy and finance that is constituted by the C-CAPM.

CHAPTER 2

Consumption-Based Asset Pricing with Rare Disaster Risk*

2.1 Motivation

With this study, we provide an empirical assessment of the rare disaster hypothesis by estimating and testing a C-CAPM that allows for the possibility of disastrous consumption contractions. We propose and apply an SMM-type estimation strategy that accounts for the fact that no such disasters are actually observed in the postwar U.S. data. The empirical results vindicate both the consumption-based asset pricing paradigm and the rare disaster hypothesis.

Our work is inspired by Barro (2006), who draws on Rietz (1988) and proposes a model that allows to assess the effect of rare disasters on asset prices. He uses cross-country panel data to calibrate the probability and size of disastrous consumption contractions, assumes reasonable investor risk and time preferences, and shows that the model-implied equity premia are in the range of their empirically observed counterparts. While such calibrations are useful, they cannot answer the following questions: How do asset pricing models that account for rare disasters perform when econometric methods get applied to estimate (instead of calibrate) the preference parameters using empirical data? Are the resulting estimates economically plausible? What is the estimation precision that can be expected and how informative are the available data? Are model-implied key economic indicators like the equity premium, mean risk-free rate, and market Sharpe ratio comparable to their empirical counterparts? Addressing these questions is complicated by the rarity of extreme consumption contractions, such that they may not occur in any particular dataset. As John Cochrane puts it:

> We had no banking panics, and no depressions; no civil wars, no constitutional crises; we did not lose the Cold War, no missiles were fired over Berlin, Cuba, Korea, or Vietnam. If any of these things had happened, we might well have seen a calamitous decline in stock values, and I would not be writing about the equity premium puzzle. (Cochrane, 2005, p. 461)

* This chapter is based on Grammig and Sönksen (2016), available on ssrn: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2397065

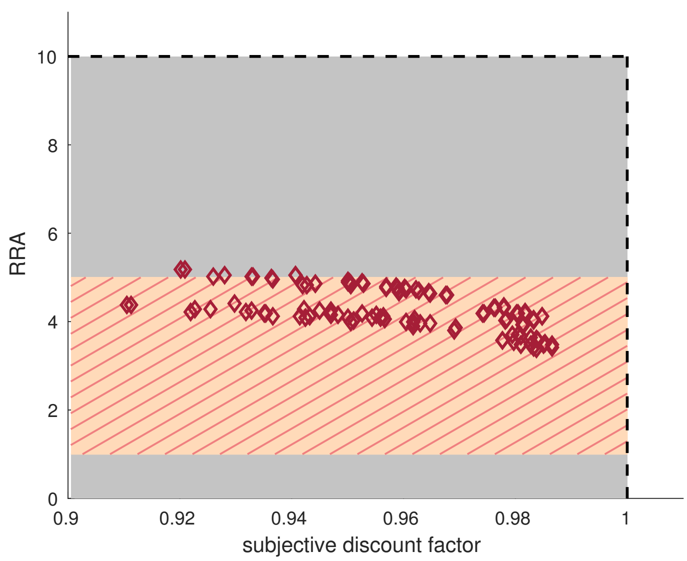
Accordingly, the empirical deficiencies of the C-CAPM may result from a sample selection effect, in that the consumption and return data that the U.S. economy produced over the past 65 years represent one lucky itinerary of all the various histories that could have been. If disastrous consumption contractions were possible but did not occur, then we should account for them by traveling the roads that the U.S. postwar economy did not take. We should consider histories marked by banking panics and depressions or in which the U.S.A. lost the Cold War – in short, alternative histories, in which we would not conduct this study in the first place. We use the simulated method of moments to facilitate such journeys, as advocated by Ken Singleton:

> More fully specified models allow experimentation with alternative formulations of economies and, perhaps, analysis of processes that are more representative of history for which data are not readily available. (Singleton, 2006, p. 254)

In the spirit of this view, we propose a two-step estimation strategy that entails simulating disaster-including consumption growth and return data. Conceiving a sequence of disaster events as a realization of a marked point process (MPP), we model the time durations between events ("points") using the autoregressive conditional hazard (ACH) framework introduced by Hamilton and Jorda (2002). The size of the consumption contraction (the "mark") is described by a double power law (DPL) distribution, as in Barro and Jin (2011). A Gaussian copula function links the marginal distributions of the return and consumption contractions. We rely on the disaster identification scheme used by Barro (2006) that is applied to the updated cross-country panel data collected by Barro and Ursúa (2008) and Bolt and van Zanden (2014), respectively. Using the estimated ACH-DPL model, we then generate disaster-including consumption and return series for the SMM estimation of the power utility preference parameters β and γ. A bootstrap simulation provides parameter standard errors and confidence intervals. To verify the robustness of the results, we use alternative ACH specifications, disaster-defining thresholds, test assets, and data simulation procedures.

As a preview of some results, Figure 1 shows that all the estimates of γ and β thus obtained lie within an area that confines values that are accepted as economically reasonable. Further analysis reveals that the boundaries of the 95% confidence intervals are also located in that region. Moreover, the model-implied market equity premium, mean risk-free rate, and market Sharpe ratio are also economically meaningful. Additionally, we find that the seemingly implausible preference parameter estimates reported in previous literature are not at all unlikely if the time series are as short as the postwar quarterly data used in empirical analyses of the C-CAPM. With the availability of longer, disaster-including time series, the estimates become perfectly reasonable.

Figure 1: Preview of estimation results
The figure shows the point estimates of the subjective discount factor β and the coefficient of relative risk aversion γ, using different specifications of a disaster-including C-CAPM and test assets. Each point represents a combination of one of two moment matching strategies, one of two ACH specifications assumed for the occurrence of a disaster, one of three methods to simulate disaster-including data, one of three disaster definitions, and one of three sets of test assets. The colored boxes indicate economically reasonable parameter values. Mehra and Prescott (1985) and Rietz (1988) consider a range of γ between 0 to 10 as plausible, whereas Cochrane (2005, Ch. 21) refers to 1 and 5 as the traditional bounds. A positive rate of time preference implies a β smaller than 1.



The remainder of the chapter is structured as follows: Section 2.2 outlines the empirical methodology. Section 2.3 describes the data. In Section 2.4, we present the empirical results. Section 2.5 concludes.

2.2 Methodology

To explain our empirical methodology, we proceed as follows: We first introduce a disaster-including consumption process in the spirit of Barro (2006) and work out the implications for the basic pricing equation (1.1). These considerations lead to moment matches for the SMM estimation of a disaster-including C-CAPM, which in turn necessitates the simulation of disaster-including consumption and return processes (Section 2.2.1). We then show how an MPP for the occurrence and size of disastrous consumption contractions can be used for this purpose. We employ a two-step strategy, that entails the estimation of the MPP parameters in the first and that of the C-CAPM preference parameters in the second step. Sections 2.2.2 and 2.2.3 explain the details. Section 2.2.4 outlines how to obtain bootstrap inference for the two-step estimation procedure.

2.2.1 Moment matches for a disaster-including C-CAPM

Barro (2006) considers a disaster-including consumption process, used to obtain closed-form solutions of equity premia, both conditional and unconditional on disaster periods. We draw on Barro's (2006) specification and assume that consumption evolves as

$$C_{t+1} = C_t e^{u_{t+1}} e^{v_{t+1}}, \tag{2.1}$$

where $u_{t+1} \sim (\tilde{\mu}, \sigma^2)$ and $v_{t+1} = \ln(1 - b_{t+1})d_{t+1}$. The indicator d_{t+1} is equal to 1 if a disaster occurs in period $t + 1$ and 0 otherwise. If $d_{t+1} = 1$, consumption contracts by a random factor $b_{t+1} \in [q, 1]$, where q is referred to as the disaster threshold, such that

$$\frac{C_{t+1}}{C_t} = e^{u_{t+1}}(1 - b_{t+1})^{d_{t+1}}. \tag{2.2}$$

Accordingly, $e^{u_{t+1}}$ denotes regular, non-disastrous consumption growth, and $(1 - b_{t+1})^{d_{t+1}}$ accounts for the effect of a disaster. Substituting the right-hand side of Equation (2.2) into Equation (1.1), we can write:

$$\begin{aligned}\mathbb{E}\left[\beta\left(e^{u_t}e^{v_t}\right)^{-\gamma} R_t\right] &= \mathbb{P}(d_t = 1)\mathbb{E}\left[\beta\left(e^{u_t}(1 - b_t)\right)^{-\gamma} R_t | d_t = 1\right] \\ &+ (1 - \mathbb{P}(d_t = 1))\mathbb{E}\left[\beta\left(e^{u_t}\right)^{-\gamma} R_t | d_t = 0\right] \\ &= 1.\end{aligned} \tag{2.3}$$

Rearranging terms, we obtain:

$$\mathbb{E}\left[\beta\left(e^{u_t}\right)^{-\gamma} R_t | d_t = 0\right] = \frac{1}{1 - \mathbb{P}(d_t = 1)}\left[1 - \mathbb{P}(d_t = 1)\mathbb{E}\left[\beta\left(e^{u_t}(1 - b_t)\right)^{-\gamma} R_t | d_t = 1\right]\right]. \tag{2.4}$$

For the pricing of an excess return R_t^e, the analogue of Equation (2.4) is:

$$\mathbb{E}\left[\left(e^{u_t}\right)^{-\gamma} R_t^e | d_t = 0\right] = -\frac{\mathbb{P}(d_t = 1)}{1 - \mathbb{P}(d_t = 1)} \left[\mathbb{E}\left[\left(e^{u_t}(1 - b_t)\right)^{-\gamma} R_t^e | d_t = 1\right]\right]. \tag{2.5}$$

If a sample with disaster observations were available, we could pursue a GMM estimation strategy, for which we would use the sample counterparts of the population moments in Equations (2.4)-(2.5) and rely on a uniform law of large numbers, such that

$$\frac{1}{T - D_T} \sum_{t=1}^{T} \beta c g_{nd,t}^{-\gamma} R_{nd,t}(1 - d_t) - \frac{1}{1 - \frac{D_T}{T}} \left[1 - \frac{D_T}{T} \left[\frac{1}{D_T} \sum_{t=1}^{T} \beta c g_{d,t}^{-\gamma} R_{d,t} d_t\right]\right] \xrightarrow[p]{} 0 \tag{2.6}$$

uniformly. Analogously,

$$\frac{1}{T - D_T} \sum_{t=1}^{T} \beta c g_{nd,t}^{-\gamma} R_{nd,t}^e(1 - d_t) + \frac{\frac{D_T}{T}}{1 - \frac{D_T}{T}} \left[\frac{1}{D_T} \left[\sum_{t=1}^{T} \beta c g_{d,t}^{-\gamma} R_{d,t}^e d_t\right]\right] \xrightarrow[p]{} 0 \tag{2.7}$$

uniformly. In Equations (2.6) and (2.7), $cg_{nd,t}$ and $R_{nd,t}$ denote regular consumption growth and return, and $cg_{d,t}$ and $R_{d,t}$ are consumption growth and gross return in a disaster period, respectively. Moreover, $D_T = \sum_{t=1}^{T} d_t$, and $\frac{D_T}{T} \xrightarrow[p]{} \mathbb{P}(d_t = 1)$. However, the GMM strategy is impeded, because even for long time series, the quality of the population and sample moment matches would be poor. Rare disasters are, well, rare, and T would have to be very large to ensure even moderate estimation precision.

Using U.S. postwar data – as in all of the studies mentioned in the introduction – the problem becomes aggravated. These data do not contain any disaster observations, such that $d_t = 0 \; \forall \; t$, and thus $D_T = 0$. To apply GMM, one has to rely on disaster-free consumption growth $cg_{nd,t}$ and returns $R_{nd,t}$ (with excess returns, $R_{nd,t}^e$), and match the left-hand side of Equation (2.4) (with excess returns, Equation (2.5)) with its sample counterpart $\frac{1}{T} \sum_{t=1}^{T} \beta c g_{nd,t}^{-\gamma} R_{nd,t}$ (with excess returns, $\frac{1}{T} \sum_{t=1}^{T} c g_{nd,t}^{-\gamma} R_{nd,t}^e$). However, the right-hand side of Equation (2.4) is sure to equal 1, and the right-hand side of Equation (2.5) is sure to equal 0, only if $\mathbb{P}(d_t = 1) = 0$. Therefore, the usual moment matches

$$\mathbb{G}_T(\beta, \gamma) \equiv \frac{1}{T} \sum_{t=1}^{T} \beta c g_{nd,t}^{-\gamma} \mathbf{R}_{nd,t} - 1, \tag{2.8}$$

using the gross returns of N test assets, $\mathbf{R}_{nd,t} = [R_{nd,t}^1, \ldots, R_{nd,t}^N]'$, and

$$\mathbb{G}_T(\gamma) \equiv \frac{1}{T} \sum_{t=1}^{T} c g_{nd,t}^{-\gamma} \mathbf{R}_{nd,t}^e, \tag{2.9}$$

using excess returns, $\mathbf{R}^e_{nd,t} = [R^{e1}_{nd,t}, \ldots, R^{eN}_{nd,t}]'$, are suitable only if disastrous consumption contractions are impossible.

We therefore propose an SMM estimation strategy that implies matching sample moments, obtained from a disaster-free sample of size T, with simulated theoretical moments that account for the possibility of consumption disasters. These moment matches are derived from Equations (2.4) and (2.5). Using time series of length T of regular, disaster-free consumption data and (excess) returns, the sample counterparts of the expected values on the left-hand sides of Equations (2.4) and (2.5) can be computed as $\frac{1}{T}\sum_{t=1}^{T}\beta cg^{-\gamma}_{nd,t}R_{nd,t}$ and $\frac{1}{T}\sum_{t=1}^{T}cg^{-\gamma}_{nd,t}R^e_{nd,t}$, respectively. In contrast, the right-hand side moments of Equations (2.4) and (2.5) can neither be expressed as functions of parameters nor can the sample counterparts be computed using disaster-free data. However, if it is possible to specify processes that are, in terms of Singleton's quote, "more representative of history," i.e., series that include disaster observations, these moments can be simulated:

$$\frac{1-\mathbb{P}(d_t=1)\mathbb{E}\left[\beta\left(e^{u_t}(1-b_t)\right)^{-\gamma}R_t|d_t=1\right]}{1-\mathbb{P}(d_t=1)} \approx \frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}}\left(1-\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\beta cg_s^{-\gamma}R_s d_s\right), \tag{2.10}$$

and

$$\frac{\mathbb{P}(d_t=1)}{1-\mathbb{P}(d_t=1)}\left[\mathbb{E}\left[\left(e^{u_t}(1-b_t)\right)^{-\gamma}R^e_t|d_t=1\right]\right] \approx \frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}}\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}cg_s^{-\gamma}R^e_s d_s, \tag{2.11}$$

where $D_{\mathcal{T}} = \sum_{s=1}^{\mathcal{T}} d_s$ denotes the number of disasters in a simulated sample of size $\mathcal{T}$. Using the gross risk-free rate R^f and a vector of excess returns $\mathbf{R}^e$ as test assets, we can then employ the following moment matches:

$$\mathbb{G}_T(\boldsymbol{\theta}) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\beta cg^{-\gamma}_{nd,t}R^f_{nd,t} - \frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}}\left(1-\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\beta cg_s^{-\gamma}R^f_s d_s\right) \\ \frac{1}{T}\sum_{t=1}^{T}\beta cg^{-\gamma}_{nd,t}\mathbf{R}^e_{nd,t} + \frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}}\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\beta cg_s^{-\gamma}\mathbf{R}^e_s d_s \end{bmatrix}, \tag{2.12}$$

where $\boldsymbol{\theta} = (\beta, \gamma)'$. To estimate only γ, we could use the following vector of moment matches, derived from Equation (2.7):

$$\mathbb{G}_T(\boldsymbol{\theta}) = \left[\frac{1}{T}\sum_{t=1}^{T}cg^{-\gamma}_{nd,t}\mathbf{R}^e_{nd,t} + \frac{1}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}}\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}cg_s^{-\gamma}\mathbf{R}^e_s d_s\right]. \tag{2.13}$$

Here, $\boldsymbol{\theta} = \gamma$, because β is not identified. One should use a large $\mathcal{T}$ to ensure that the simulated data contain enough disasters and the approximations in Equations (2.10) and (2.11) are sufficiently accurate.

Using either the moment matches from Equation (2.12) or Equation (2.13), SMM estimates then can be obtained by

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\arg\min}\ \mathbb{G}_T(\boldsymbol{\theta})' \mathbf{W}_T \mathbb{G}_T(\boldsymbol{\theta}), \tag{2.14}$$

where $\mathbf{W}_T$ is a symmetric and positive semi-definite distance matrix. The compact set $\boldsymbol{\Theta} \subset \mathbb{R}^p$ denotes the admissible parameter space, where p is the number of parameters.[7]

2.2.2 Modeling disasters as a marked point process

To use the proposed moment matches for the SMM estimation of the C-CAPM preference parameters, we have to simulate disaster-including consumption and return data. For that purpose, we split the parameter estimation into two steps. The first step consists of specifying and estimating a marked point process, with which we model the time durations between the occurrence of consumption disaster events ("points") and their size (the "mark"). The MPP parameters are estimated by maximum likelihood (ML) using chained cross-country panel data. Using the estimates, we can simulate disaster-including consumption and return processes and perform the SMM estimation of the C-CAPM preference parameters in the second step.

Hamilton and Jorda's (2002) autoregressive conditional hazard approach provides a framework to model the time duration between disaster events. The double power law distribution is used to describe the size of a disastrous consumption contraction. The initial choice for such an ACH-DPL model is the threshold q that defines the calendar time and size of a disaster, that is, the points and marks of the MPP.[8]

Suppose that the sequence of disaster events thus defined is observable in event time, that is, each consumption contraction $\geq q$ marks an event, and time is measured as the interval between two disaster events. To formalize the exposition, let $N(t)$

[7] References are Hansen (1982) for GMM and Duffie and Singleton (1993) for SMM; excellent synopses are provided by Hall (2005) and Singleton (2006).

[8] In the empirical analysis, we consider several threshold values proposed in previous literature. Our base setting is q=0.145, as in Barro (2006). The identification of a disaster event necessitates some additional assumptions and to account for the peculiarities of the data used for the empirical analysis. We use Barro's (2006) disaster identification procedure that is explained in Section 2.3. Further details are provided in Appendix A.1.

denote the number of disasters that have occurred as of calendar period $t = 1, \ldots, T$, and let τ_n denote the time duration, measured in quarters, between the nth and $(n+1)$th disaster event. Following Hamilton and Jorda (2002), the conditional expected duration $\psi_{N(t)} \equiv \mathbb{E}(\tau_{N(t)} | \tau_{N(t-1)}, \tau_{N(t-2)}, \ldots)$ is assumed to evolve as

$$\psi_{N(t)} = \alpha \tau_{N(t)-1} + \tilde{\beta} \psi_{N(t)-1}, \tag{2.15}$$

where α and $\tilde{\beta}$ are parameters.[9] The probability of a disaster occurring in period t, conditional on the information available in $t-1$, is the discrete-time hazard rate,

$$h_t = \mathbb{P}(N(t) \neq N(t-1) | \mathcal{F}_{t-1}). \tag{2.16}$$

If the information set $\mathcal{F}_{t-1}$ consists only of past durations, the hazard rate remains the same until the next disaster event occurs. Hamilton and Jorda (2002) show that in this case, the hazard rate and conditional expected durations are inversely related:

$$h_t = \frac{1}{\psi_{N(t-1)}}. \tag{2.17}$$

To allow for the impact of a constant and a predetermined variable x observed at $t-1$, Hamilton and Jorda (2002) propose to use a hazard rate that varies in calendar time, that is,

$$h_t = \frac{1}{\psi_{N(t-1)} + \mu + \delta x_{t-1}}, \tag{2.18}$$

where μ and δ are parameters. Equation (2.18) easily can be extended to include more than one predetermined variable.

To model the mark in this MPP, we follow Barro and Jin (2011) and assume that the distribution of transformed contraction sizes $z_c = (1-b)^{-1}$ can be described by a DPL density.[10] The joint probability density function of the disaster indicator d_t and the transformed contraction size $z_{c,t}$ can then be written as

$$\begin{aligned} f(d_t, z_{c,t} | \mathcal{F}_{t-1}; \boldsymbol{\theta}_{\text{ACH}}, \boldsymbol{\theta}_{\text{DPL}}) &= g(d_t | \mathcal{F}_{t-1}) \times q(z_{c,t} | d_t, \mathcal{F}_{t-1}) \\ &= [h_t(\boldsymbol{\theta}_{\text{ACH}})]^{d_t} [1 - h_t(\boldsymbol{\theta}_{\text{ACH}})]^{(1-d_t)} \times f_{\text{DPL}}(z_{c,t}; \boldsymbol{\theta}_{\text{DPL}})^{d_t}, \end{aligned} \tag{2.19}$$

where f_{DPL} denotes the DPL density function with parameters collected in the vector

[9] Equation (2.15) defines an ACH(1,1) model. The generalization is an ACH(p,q) specification that includes q lagged values of τ and p lagged values of ψ.
[10] Appendix A.2 collects useful information about the DPL distribution.

$\boldsymbol{\theta}_{\text{DPL}}$. The vector $\boldsymbol{\theta}_{\text{ACH}}$ contains the ACH parameters.

The conditional log-likelihood function of an ACH-DPL model is then given by:

$$\mathcal{L}(\boldsymbol{\theta}_{\text{ACH}}, \boldsymbol{\theta}_{\text{DPL}}) = \sum_{t=1}^{T} d_t \ln h_t(\boldsymbol{\theta}_{\text{ACH}}) + (1-d_t) \ln[1-h_t(\boldsymbol{\theta}_{\text{ACH}})] + \sum_{t=1}^{T} d_t \ln f_{\text{DPL}}(z_{c,t}; \boldsymbol{\theta}_{\text{DPL}}). \tag{2.20}$$

Assuming that the parameters $\boldsymbol{\theta}_{\text{ACH}}$ and $\boldsymbol{\theta}_{\text{DPL}}$ are variation-free, the parameter estimation can be split in two parts (for a similar approach, see Engle, 2000). To obtain estimates of the ACH parameters, we can maximize:

$$\mathcal{L}(\boldsymbol{\theta}_{\text{ACH}}) = \sum_{t=1}^{T} d_t \ln h_t(\boldsymbol{\theta}_{\text{ACH}}) + (1-d_t) \ln[1-h_t(\boldsymbol{\theta}_{\text{ACH}})], \tag{2.21}$$

whereas the DPL parameters can be estimated by maximizing:

$$\mathcal{L}(\boldsymbol{\theta}_{\text{DPL}}) = \sum_{t=1}^{T} d_t \ln f_{\text{DPL}}(z_{c,t}; \boldsymbol{\theta}_{\text{DPL}}). \tag{2.22}$$

The data used to estimate the ACH-DPL parameters are not be the same as those used for the second-step SMM estimation of the C-CAPM preference parameters. U.S. postwar data do not contain disastrous consumption contractions, at least not for conventional choices of disaster thresholds q, so they are not be useful. We therefore adopt Barro and Jin's (2011) idea and use cross-country panel consumption data to estimate the ACH-DPL model parameters. In particular, we use an updated version of the consumption dataset assembled by Barro and Ursúa (2008), as well as the GDP data collected by Bolt and van Zanden (2014). The data are described in greater detail in Section 2.3. Prior to the first-step estimation, these data – annual, unbalanced panels – must be represented as event time data that is, we have to identify disaster events and measure the time duration between them. When maximizing the log-likelihood function in Equation (2.21) to obtain $\hat{\boldsymbol{\theta}}_{\text{ACH}}$, the country-specific time series are chained, and $N(t)$, $\tau_{N(t)}$, and $\psi_{N(t)}$ get re-initialized whenever a country change occurs in the chained data. Details of the procedure are provided in Appendix A.1.[11] The estimates $\hat{\boldsymbol{\theta}}_{\text{DPL}}$ are obtained by maximizing the log-likelihood function

[11] This re-initialization procedure is adopted from Engle and Russell (1998), who introduce a dynamic duration model for the time interval between trading events, in which they must account for overnight interruptions of the trading process. To prevent previous-day observations from affecting the conditional expected durations of the next day, Engle and Russell (1998) re-initialize the conditional expected duration sequence at the start of every day when performing the maximum likelihood estimation.

in Equation (2.22), based on the observations of disaster sizes identified from the cross-country panel data. Once the estimates $\hat{\boldsymbol{\theta}}_{\text{ACH}}$ and $\hat{\boldsymbol{\theta}}_{\text{DPL}}$ are available, it is possible to simulate the disaster-including data required for the SMM estimation of the C-CAPM preference parameters.

2.2.3 Simulating disaster-including data

In this section, we describe how we simulate the sequences of the disaster indicator $\{d_s\}_{s=1}^{\mathcal{T}}$, consumption growth $\{cg_s\}_{s=1}^{\mathcal{T}}$, and asset returns $\{\mathbf{R}_s\}_{s=1}^{\mathcal{T}}$, which are needed to perform the SMM estimation as described in Section 2.2.1.

The first step of the simulation procedure is to obtain $\{cg_{nd,s}\}_{s=1}^{\mathcal{T}}$, $\{\mathbf{R}_{nd,s}\}_{s=1}^{\mathcal{T}}$, and $\left\{R_{nd,s}^{f}\right\}_{s=1}^{\mathcal{T}}$ by drawing with replacement from regular, disaster-free consumption and return data. To preserve the contemporaneous covariance structure of consumption growth and returns, these draws are performed simultaneously. Next, we simulate the ACH-DPL process, using the estimates $\hat{\boldsymbol{\theta}}_{\text{ACH}}$ and $\hat{\boldsymbol{\theta}}_{\text{DPL}}$, which yields a series of hazard rates/disaster probabilities $\{h_s(\hat{\boldsymbol{\theta}}_{\text{ACH}}, \hat{\boldsymbol{\theta}}_{\text{DPL}})\}_{s=1}^{\mathcal{T}}$ and disaster indicators $\{d_s\}_{s=1}^{\mathcal{T}}$. In the case of $d_s = 1$, we can obtain the size of the consumption contraction b_s by a draw from the DPL distribution with parameters $\hat{\boldsymbol{\theta}}_{\text{DPL}}$, which yields $cg_s = (1 - b_s)cg_{nd,s}$.

If financial return data corresponding to the consumption contractions were available, it would be possible to extend the ACH-DPL to account for more marks, in particular, the asset returns associated with a disastrous consumption contraction (disaster returns). However, the cross-country panel data collected by Barro and Ursúa (2008) and Bolt and van Zanden (2014) do not contain information about asset prices, such that we must devise an alternative way to simulate disaster returns. For that purpose, we transfer the notion of a disaster-including consumption growth process in Equation (2.2) to a gross return of an asset, viz:

$$R = (1 - \tilde{b})^{d} R_{nd}. \tag{2.23}$$

We consider three possible methods to obtain the return contraction factor $\tilde{b}$ that allow for different degrees of dependence between the consumption and return contractions. All variants are based on the assumption that the marginal distribution of the transformed return contractions $z_R = (1-\tilde{b})^{-1}$ is the same as that of $z_c = (1-b)^{-1}$,

$$f_{\text{DPL}}(z_c; \boldsymbol{\theta}_{\text{DPL}}) = f_{\text{DPL}}(z_R; \boldsymbol{\theta}_{\text{DPL}}), \tag{2.24}$$

and we use a copula function C to model the dependence between z_c and z_R. By

Sklar's theorem, their joint cumulative distribution function (cdf) can then be represented as:

$$F\left(z_c, z_R; \boldsymbol{\theta}_{\mathrm{DPL}}, \boldsymbol{\theta}_C\right) = C\left(F_{\mathrm{DPL}}(z_c; \boldsymbol{\theta}_{\mathrm{DPL}}), F_{\mathrm{DPL}}\left(z_R; \boldsymbol{\theta}_{\mathrm{DPL}}\right); \boldsymbol{\theta}_C\right), \tag{2.25}$$

where F_{DPL} refers to the cdf of the DPL distribution, and $\boldsymbol{\theta}_C$ contains the parameters of the copula function. Using the Gaussian copula function C_G, Equation (2.25) becomes:

$$F\left(z_c, z_R; \boldsymbol{\theta}_{\mathrm{DPL}}, \rho\right) = C_G\left(u_c, u_R; \rho\right), \tag{2.26}$$

where $u_c = F_{\mathrm{DPL}}(z_c; \boldsymbol{\theta}_{\mathrm{DPL}})$, and $u_R = F_{\mathrm{DPL}}(z_R; \boldsymbol{\theta}_{\mathrm{DPL}})$. The copula correlation coefficient ρ determines the dependence of z_c and z_R.

We focus on three potential choices for defining ρ. The first is to estimate ρ by the empirical correlation of regular consumption growth and the return of the respective test asset. The second choice would fix $\rho = 0.99$, motivated by empirical evidence that the correlations between financial returns increase in the tails of the joint distribution (Longin and Solnik, 2001). The third option is to set $\rho = 0$, which amounts to drawing z_c and z_R independently, but from the same DPL distribution. These three approaches are labeled *EmpCorr* (*Emp*irical *Corr*elation), *TailCorr* (*Tail Corr*elation), and *ZeroCorr* (*Zero Corr*elation), respectively. For an assessment of the robustness of the results, we perform a sensitivity analysis in which we vary ρ between 0 and 0.99.

To obtain the contraction factors b_s and $\tilde{b}_s$, we draw two standard normally distributed variables $y_{c,s}$ and $y_{R,s}$, which have correlation ρ, and compute $u_{c,s} = \Phi(y_{c,s})$ and $u_{R,s} = \Phi\left(y_{R,s}\right)$, where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. The simulated contraction factors b_s and $\tilde{b}_s$ then result from:

$$b_s = 1 - \frac{1}{F_{\mathrm{DPL}}^{-1}(u_{c,s}; \hat{\boldsymbol{\theta}}_{\mathrm{DPL}})} \qquad \text{and} \qquad \tilde{b}_s = 1 - \frac{1}{F_{\mathrm{DPL}}^{-1}\left(u_{R,s}; \hat{\boldsymbol{\theta}}_{\mathrm{DPL}}\right)}, \tag{2.27}$$

where F_{DPL}^{-1} is the quantile function of the DPL distribution. In turn, we can compute $cg_s = (1 - b_s)^{d_s} cg_{nd,s}$, $\mathbf{R}_s = (1 - \tilde{\mathbf{b}}_s)^{d_s} \mathbf{R}_{nd,s}$, $R_s^f = R_{nd,s}^f$ (assuming that the risk-free rate does not contract), and $\mathbf{R}_s^e = \mathbf{R}_s - R_{nd,s}^f$, for $s = 1, \ldots, \mathcal{T}$.

The simulated series can now be used to compute moment matches in Equations (2.12) and (2.13) and to estimate the C-CAPM parameters β and γ by minimizing the SMM objective function in Equation (2.14).

2.2.4 Bootstrap inference

The two-step estimation approach precludes the reliance on standard inference in the second estimation step. To circumvent this problem, we employ a bootstrap simulation. Using the first-step ML estimates $\hat{\boldsymbol{\theta}}_{\text{ACH}}$ and $\hat{\boldsymbol{\theta}}_{\text{DPL}}$, we simulate the estimated ACH-DPL model to obtain sequences of time durations between disaster events, and the associated contraction sizes. The length of the simulated series is equal to the number of country-quarters used to estimate the ACH-DPL parameters. With these simulated data, we can estimate the ACH-DPL parameters anew. When this procedure is replicated K times, we obtain an empirical distribution of first-step estimates, $\left\{\hat{\boldsymbol{\theta}}^{(k)}_{\text{DPL}}, \hat{\boldsymbol{\theta}}^{(k)}_{\text{ACH}}\right\}_{k=1}^{K}$, which provides the basis for bootstrap inference.

Within each of the K replications, we then estimate the preference parameters by SMM, based on bootstrapped data. For that purpose, we draw with replacement from the regular consumption growth and return data. Again, to preserve their contemporaneous dependence, consumption and return data are drawn simultaneously (for a similar approach, see Maio and Santa-Clara, 2012). The number of draws is identical to the number of observations in the original consumption/return time series. Using the bootstrapped consumption and return data from the kth replication and the parameter estimates $\hat{\boldsymbol{\theta}}^{(k)}_{\text{DPL}}$ and $\hat{\boldsymbol{\theta}}^{(k)}_{\text{ACH}}$, we perform the SMM estimation of the C-CAPM preference parameters as described in Section 2.2.1. Collecting the preference parameter estimates from the K bootstrap replications, $\left\{\hat{\beta}^{(k)}, \hat{\gamma}^{(k)}\right\}_{k=1}^{K}$, we can use the empirical distribution to compute parameter standard errors and confidence intervals using the percentile method as described by Efron and Tibshirani (1986).

2.2.5 Alternative Histories Bootstrap

We have argued previously that the moment matches in Equations (2.8) and (2.9) should not be used if disasters are possible but not observed. Because the simulated consumption growth and return series include disaster observations, these moment matches can be reconsidered and used for GMM estimation. We refer to this approach as *A*lternative *H*istories *B*ootstrap (AHB), a name that echoes Cochrane's quote from the introduction.

The input for the AHB procedure are H independent disaster-including simulated samples ("alternative histories") of size $\mathcal{T}$, which we generate as described in Section 2.2.3. Let $\left\{cg^{(h)}_s, R^{f(h)}_s, \mathbf{R}^{e(h)}_s\right\}_{s=1}^{\mathcal{T}}$ denote the simulated data from replication h. For each $h = 1, \ldots, H$, we estimate β and γ by GMM, using the moment matches:

$$\mathbb{G}_{\mathcal{T}}^{(h)}(\beta, \gamma) = \begin{bmatrix} \frac{1}{\mathcal{T}} \sum_{s=1}^{\mathcal{T}} \beta \left(cg_s^{(h)} \right)^{-\gamma} R_s^{f(h)} - 1 \\ \frac{1}{\mathcal{T}} \sum_{s=1}^{\mathcal{T}} \beta \left(cg_s^{(h)} \right)^{-\gamma} \mathbf{R}_s^{e(h)} \end{bmatrix}. \tag{2.28}$$

Then, we can estimate β and γ by averaging $\{\hat{\beta}^{(h)}\}_{h=1}^{H}$ and $\{\hat{\gamma}^{(h)}\}_{h=1}^{H}$ across ensembles:

$$\hat{\beta} = \frac{1}{H} \sum_{h=1}^{H} \hat{\beta}^{(h)} \qquad \text{and} \qquad \hat{\gamma} = \frac{1}{H} \sum_{h=1}^{H} \hat{\gamma}^{(h)}, \tag{2.29}$$

where $\hat{\gamma}^{(h)}$ and $\hat{\beta}^{(h)}$ refer to the estimates obtained in the h^{th} replication. We use the empirical distribution of $\hat{\gamma}^{(h)}$ and $\hat{\beta}^{(h)}$ to provide standard errors and confidence intervals. By varying $\mathcal{T}$, the AHB approach makes it possible to quantify the speculations about the relationships of sample size, frequency of disaster events, and estimation precision from Section 2.2.1.

2.3 Data

The first-step estimation of the ACH-DPL model is based on updated cross-country consumption data originally assembled by Barro and Ursúa (2008). These unbalanced panel data contain annual consumption information about 42 countries between 1800 and 2009, from which we select the same 35 countries that Barro (2006) used for his study.[12] Table 1 lists these countries and the time periods for which consumption data are available. To identify a disaster event for the baseline analysis, we follow Barro (2006) and set the disaster threshold to q=0.145. For robustness checks, we also consider q=0.095 and q=0.195, as in Barro and Jin's (2011) study.

Barro's (2006) disaster identification scheme provides the blueprint for our data preparation. A consumption contraction larger than q can accrue over more than one period and may contain single periods of intermittent positive growth, as long as (1) the positive growth intermezzo is smaller than the negative growth in the subsequent period, and (2) the size of the disaster does not decrease when we allow it to overlap the positive growth period.

As an alternative, we also perform the first-step estimation on annual cross-country panel GDP data beginning in 1900, as in Barro (2006) and Barro and Jin

[12] The data are available at http://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data accessed 04/24/2015.

Table 1: Cross-country panel data used for the first-step estimation
The table lists the 35 countries and time periods with available data that provide the basis for the ACH-DPL estimation. The second column reports the time periods for which consumption data assembled by Barro and Ursúa (2008) are available and the third column reports the time periods for which the GDP data provided by Bolt and van Zanden (2014) are available (beginning with 1900 onwards).

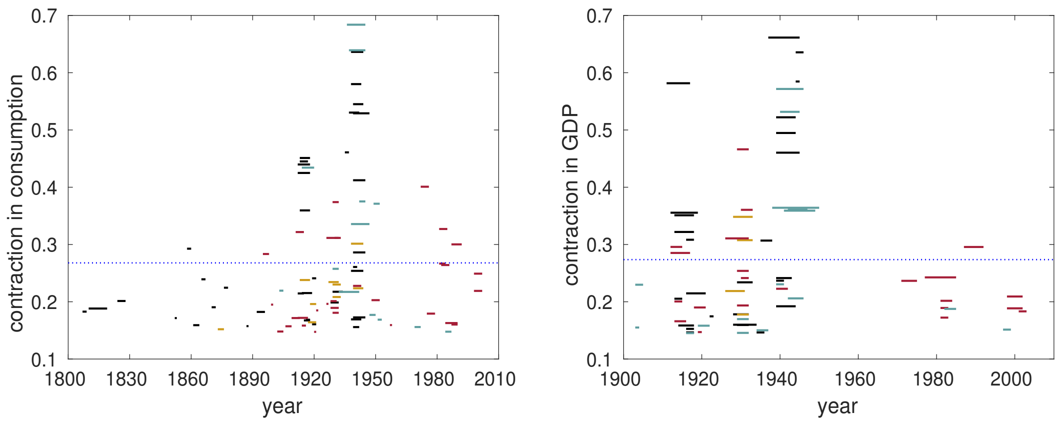
Country	Barro and Ursúa	Bolt and van Zanden
Argentina	1875 – 2009	1900 – 2010
Australia	1901 – 2009	1900 – 2010
Austria	1913 – 1918, 1924 – 1944, 1947 – 2009	1900 – 2010
Belgium	1913 – 2009	1900 – 2010
Brazil	1901 – 2009	1900 – 2010
Canada	1871 – 2009	1900 – 2010
Chile	1900 – 2009	1900 – 2010
Colombia	1925 – 2009	1900 – 2010
Denmark	1844 – 2009	1900 – 2010
Finland	1860 – 2009	1900 – 2010
France	1824 – 2009	1900 – 2010
Germany	1851 – 2009	1900 – 2010
Greece	1938 – 2009	1900 – 2010
India	1919 – 2009	1900 – 2010
Indonesia	1960 – 2009	1949 – 2010
Italy	1861 – 2009	1900 – 2010
Japan	1874 – 2009	1900 – 2010
Malaysia	1900 – 1939, 1947 – 2009	1911 – 1942, 1947 – 2010
Mexico	1900 – 2009	1900 – 2010
the Netherlands	1807 – 1809, 1814 – 2009	1900 – 2010
New Zealand	1878 – 2009	1900 – 2010
Norway	1830 – 2009	1900 – 2010
the Philippines	1946 – 2009	1902 – 1940, 1946 – 2010
Peru	1896 – 2009	1900 – 2010
Portugal	1910 – 2009	1900 – 2010
South Korea	1911 – 2009	1911 – 1940, 1950 – 2010
Spain	1850 – 2009	1900 – 2010
Sri Lanka	1960 – 2009	1900 – 2010
Sweden	1800 – 2009	1900 – 2010
Switzerland	1851 – 2009	1900 – 2010
Taiwan	1901 – 2009	1901 – 1940, 1950 – 2010
UK	1830 – 2009	1900 – 2010
U.S.A.	1834 – 2009	1900 – 2010
Uruguay	1960 – 2009	1900 – 2010
Venezuela	1923 – 2009	1900 – 2010

(2011). For that purpose, we use the data provided by Bolt and van Zanden (2014), who extend the database assembled by Angus Maddison and originally used by Barro (2006).[13] Table 1 lists the countries and time periods for which GDP data are available. Disastrous GDP contractions are identified as described previously and serve as proxies for consumption disasters. The advantage of using GDP data is that the GDP series of most countries start well before consumption data become available. We follow Barro (2006) and exclude disastrous GDP contractions that represent the aftermaths of war and that allegedly are not related to drops in consumption. Figure

[13] The data are available at http://www.ggdc.net/maddison/oriindex.net accessed 06/26/2014.

2 shows that the disaster events thus identified cluster during World War I, the Great Depression, World War II, and turmoil in South America during 1980–2000.

Figure 2: Disastrous consumption and GDP contractions
Panel (a) depicts the 89 consumption disasters identified from Barro and Ursúa's (2008) cross-country panel data (updated). The sampling period is 1800–2009. Panel (b) depicts the 68 GDP disasters identified from Bolt and van Zanden's (2014) cross-country panel data. The sampling period is 1900–2010. The disaster threshold q=0.145 in both cases. Black lines denote European countries, red lines South American countries and Mexico, golden lines Western offshores (Australia, Canada, New Zealand, U.S.A.), and blue lines represent Asian countries. The dotted horizontal lines depict the average contraction sizes.



(a) Barro and Ursúa data **(b)** Bolt and van Zanden data

For the second-step SMM estimation, we use quarterly U.S. real personal consumption expenditures per capita on services and non-durable goods in chained 2009 U.S. dollars, as provided by the Federal Reserve Bank of Saint Louis.[14] The data span the time period 1947:Q2–2014:Q4. The second-step estimation also uses financial data that come from CRSP and Kenneth French's financial data library.[15] We focus on the returns of the following three sets of test assets: (1) the market portfolio return calculated for the CRSP value-weighted market portfolio comprised of NYSE, AMEX, and NASDAQ traded stocks; (2) ten size-sorted portfolios; and (3) ten industry portfolios. We use the value-weighted variants of these portfolios. Nominal monthly returns are converted to real returns at a quarterly frequency, using

[14] For services: http://research.stlouisfed.org/fred2/series/A797RX0Q048SBEA. For non-durable goods: http://research.stlouisfed.org/fred2/series/A796RX0Q048SBEA. Both accessed 03/09/2016.

[15] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html, accessed 03/09/2016. Due to the frequent changes in the underlying CRSP data, newer or older downloads may results in different series.

Table 2: Descriptive statistics: Consumption and test asset returns 1947:Q2–2014:Q4 The table contains the descriptive statistics of consumption growth and gross returns of the three sets of test assets. Panel A: CRSP value-weighted market portfolio R^m and risk-free rate proxy R^f (*mkt*), Panel B: size-sorted portfolios and R^f (*size dec*), and Panel C: industry portfolios and R^f (*industry*). The data range is 1947:Q2–2014:Q4. In Panel B, 1^{st}, 2^{nd}, and so on refer to the deciles of the the ten size-sorted portfolios. The industry portfolios in Panel C are: nondurables (*NoDur*: food, textiles, tobacco, apparel, leather, toys), durables (*Durbl*: cars, TVs, furniture, household appliances), manufacturing (*Manuf*: machinery, trucks, planes, chemicals, paper, office furniture), energy (*Engry*: oil, gas, coal extraction and products), business equipment (*HiTec*: computers, software, and electronic equipment), telecommunication (*Telcm*: telephone and television transmission), shops (*Shops*: wholesale, retail, laundries, and repair shops), health (*Hlth*: healthcare, medical equipment, and drugs), utilities (*Utils*), and others (*Other*: transportation, entertainment, finance, and hotels). The column labeled *ac* gives information on the first order autocorrelation, and *std* is the standard deviation.

Panel A: mkt

	mean	std	ac	correlations	
				$\frac{C_{t+1}}{C_t}$	R^f
R^m	1.0211	0.0816	0.084	0.175	0.026
R^f	1.0017	0.0045	0.857	0.204	
$\frac{C_{t+1}}{C_t}$	1.0048	0.0051	0.311		

Panel B: size dec

	mean	std	ac	correlations										
				$\frac{C_{t+1}}{C_t}$	R^f	10^{th}	9^{th}	8^{th}	7^{th}	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}
1^{st}	1.0290	0.1251	0.061	0.178	-0.015	0.711	0.818	0.857	0.884	0.895	0.912	0.931	0.949	0.964
2^{nd}	1.0271	0.1177	-0.001	0.172	0.005	0.781	0.871	0.915	0.933	0.947	0.961	0.974	0.982	
3^{rd}	1.0287	0.1115	-0.024	0.165	-0.001	0.818	0.907	0.943	0.956	0.968	0.976	0.985		
4^{th}	1.0270	0.1072	-0.018	0.165	0.002	0.830	0.914	0.948	0.962	0.976	0.983			
5^{th}	1.0274	0.1036	0.013	0.167	0.019	0.855	0.936	0.967	0.972	0.982				
6^{th}	1.0262	0.0971	0.019	0.143	0.001	0.868	0.946	0.970	0.977					
7^{th}	1.0262	0.0964	0.042	0.157	0.009	0.892	0.965	0.982						
8^{th}	1.0249	0.0923	0.022	0.145	0.019	0.906	0.975							
9^{th}	1.0237	0.0841	0.068	0.148	0.021	0.935								
10^{th}	1.0198	0.0767	0.119	0.178	0.043									

Panel C: industry

	mean	std	ac	correlations										
				$\frac{C_{t+1}}{C_t}$	R^f	Other	Utils	Hlth	Shops	Telcm	HiTec	Engry	Manuf	Durbl
NoDur	1.0238	0.0811	0.047	0.090	0.105	0.838	0.674	0.800	0.871	0.656	0.642	0.445	0.829	0.685
Durbl	1.0236	0.1156	0.103	0.190	0.009	0.801	0.484	0.520	0.773	0.581	0.690	0.490	0.832	
Manuf	1.0229	0.0899	0.082	0.173	0.014	0.901	0.580	0.745	0.825	0.647	0.807	0.635		
Engry	1.0253	0.0888	0.041	0.163	-0.039	0.592	0.534	0.423	0.422	0.432	0.497			
HiTec	1.0258	0.1159	0.070	0.167	-0.000	0.758	0.470	0.663	0.733	0.659				
Telcm	1.0187	0.0805	0.148	0.099	0.104	0.695	0.627	0.568	0.668					
Shops	1.0238	0.0957	0.039	0.158	0.044	0.837	0.557	0.704						
Hlth	1.0267	0.0909	0.054	0.092	0.085	0.726	0.542							
Utils	1.0195	0.0711	0.080	0.069	0.071	0.655								
Other	1.0217	0.0982	0.078	0.159	0.034									

the growth of the consumer price index of all urban consumers.[16]

For the calculation of the risk-free rate proxy, we use the three-month nominal T-bill yield from the CRSP database. Following Beeler and Campbell (2012), we approximate the ex-ante risk-free rate by using a forecast for the ex-post real rate, where the predictors are the quarterly T-bill yield and the average of quarterly log inflation across the past year. Table 2 reports the descriptive statistics for these data.

2.4 Empirical results

2.4.1 First-step results: ACH-DPL parameter estimates

Table 3 presents the ML estimation results for four ACH-DPL specifications that differ with respect to the parsimony of the ACH part and that emerge as special cases of Equation (2.18). The baseline specification is an ACH(0,0) with $\alpha = \tilde{\beta} = \delta = 0$, henceforth referred to as ACH_0, for which the hazard rate is given by:

$$h_t = \frac{1}{\mu}. \tag{2.30}$$

The second specification is an ACH(0,1) with no predetermined variables, which we refer to as ACH_1, and which implies the hazard rate:

$$h_t = \frac{1}{\mu + \alpha \tau_{N(t-1)-1}}. \tag{2.31}$$

Extending Equation (2.31) to include the previous excess contraction $b^e_{N(t-1)}$ as a predetermined variable, we obtain the third specification, referred to as ACH_+, for which

$$h_t = \frac{1}{\mu + \alpha \tau_{N(t-1)-1} + \delta b^e_{N(t-1)}}. \tag{2.32}$$

Excess contractions are computed by subtracting the DPL-implied expected disaster size b from the previous disastrous consumption contraction. Finally, we consider a more restricted version of Equation (2.32) with $\alpha = 0$, such that

$$h_t = \frac{1}{\mu + \delta b^e_{N(t-1)}}, \tag{2.33}$$

[16] These data are provided by the Federal Reserve Bank of Saint Louis: http://research.stlouisfed.org/fred2/series/CPIAUCSL, accessed 03/09/2016.

which we refer to as ACH_X.[17] The ACH_0 model corresponds to Barro's (2006) method to estimate the unconditional disaster probability, while ACH_1, ACH_+, and ACH_X, imply time-varying conditional disaster probabilities.

Table 3: First-step estimation results: ACH-DPL parameters

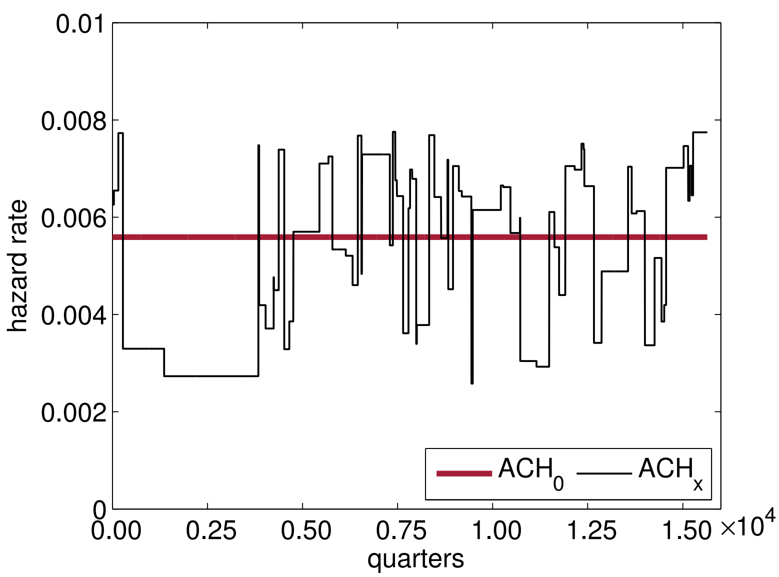
The table presents the maximum likelihood estimates for four ACH-DPL specifications. $\mathcal{L}$ is the log-likelihood value at the maximum, and $\mathcal{LR}$ gives the p-values (in percent) of the likelihood ratio tests of the null hypothesis that the parameter restrictions implied by the ACH_0 specification are correct. The alternative is the ACH_1, the ACH_X, or the ACH_+ specification, respectively. AIC = $2k - 2\ln(\mathcal{L})$ and SBC = $-2\ln(\mathcal{L}) + k\ln(T)$ denote the Akaike and Schwarz-Bayes information criteria; k is the number of model parameters. The estimation results in Panel A are based on the updated country panel consumption data assembled by Barro and Ursúa (2008) (consumption contractions, 1800–2009). The estimation results in Panel B are based on the country panel GDP data provided by Bolt and van Zanden (2014) (GDP contractions, 1900–2010). Standard errors are reported in parentheses.

					Panel A: Consumption 1800-2009					
	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\delta}$	μ	α	δ	$\mathcal{L}$	AIC	SBC	$\mathcal{LR}$
DPL	4.194 (0.720)	10.769 (2.173)	1.388 (0.043)				-40.7			
ACH_0				178.8 (19.2)			-538.5	1078.9	**1086.6**	
ACH_X				185.3 (21.3)		462.2 (229.8)	-535.4	**1074.8**	1090.1	1.3
ACH_1				196.5 (31.7)	-0.129 (0.165)		-538.2	1080.4	1095.7	47.0
ACH_+				187.0 (28.5)	-0.014 (0.154)	457.4 (235.5)	-535.4	1076.8	1099.8	4.7
					Panel B: GDP 1900-2010					
	$\tilde{\alpha}$	$\tilde{\theta}$	$\tilde{\delta}$	μ	α	δ	$\mathcal{L}$	AIC	SBC	$\mathcal{LR}$
DPL	3.938 (0.737)	11.840 (2.821)	1.364 (0.042)				-27.4			
ACH_0				210.4 (25.6)			-431.9	865.8	**873.4**	
ACH_X				215.0 (27.5)		649.9 (274.3)	-428.1	**860.2**	875.3	0.6
ACH_1				265.2 (82.8)	-0.319 (0.430)		-431.6	867.2	882.3	41.9
ACH_+				162.6 (48.2)	0.359 (0.331)	794.0 (331.2)	-427.7	861.3	884.0	1.4

[17] We also considered an ACH(1,1) specification, but the data are not sufficient to identify the autoregressive ACH parameter $\tilde{\beta}$. The maximization of the ACH(1,1) log-likelihood terminated at different points when starting with different values, such that it did not produce reliable estimates. No such problems occurred for the four ACH variants, for which we report the estimation results in Table 3. The optimizations are performed using either the pattern search algorithm or the Nelder-Mead simplex algorithm that are available in MATLAB's optimization toolbox.

Table 3 reports the ML estimates of the ACH parameters, their standard errors computed from numerical estimates of the Hessian of the log-likelihood function, as well as the Akaike (AIC) and Schwarz-Bayes (SBC) information criteria. The results reported in Panel A are based on the updated Barro and Ursúa (2008) consumption data, those in Panel B are based on Bolt and van Zanden's (2014) GDP data. The table also reports the p-values of likelihood-ratio statistics with which we test the more extensively parametrized ACH specifications against the parsimonious ACH_0. The likelihood-ratio tests reveal that the ACH_0 is not rejected at the 5% level against the ACH_1, but it is rejected against both the ACH_+ and ACH_X. Whereas the AIC suggests selecting the ACH_X, the SBC prefers the ACH_0. The standard errors indicate a reasonable estimation precision for both specifications. Figure 3 depicts the sequence of hazard rates implied by ACH_0 and ACH_X, using the estimates from Panel A of Table 3. We continue to focus on these two specifications and use them for the SMM estimation of the preference parameters in the second step. The ML

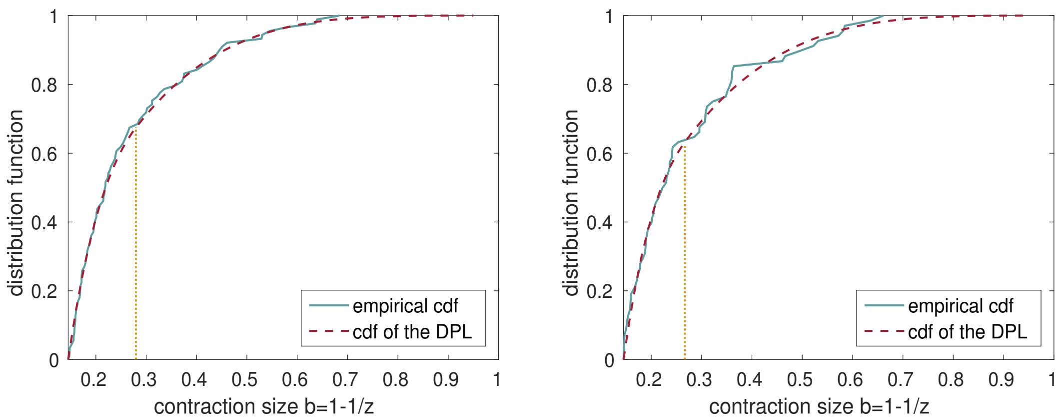
Figure 3: Hazard rates implied by ACH_0-DPL and ACH_X-DPL
The figure depicts the sequence of hazard rates/conditional disaster probabilities implied by the ACH_0-DPL and the ACH_X-DPL, respectively, using the estimates reported in Panel A of Table 3. The hazard sequences result from simulating data from the two ACH-DPL specifications. The simulated sample size $\mathcal{T}$=15,645, which is the number of chained country-quarters using the updated Barro and Ursúa (2008) consumption data.



estimates of the parameters of the DPL distribution also appear in Table 3, along with their standard errors, which indicate a reasonable estimation precision. Figure 4 depicts the empirical and fitted cdfs, revealing that the DPL fits the empirical distribution of disaster sizes well.

Figure 4: Fitted DPL and empirical cdfs for disaster sizes

Panel (a) illustrates the empirical distribution function (solid line) and the fitted cumulative distribution function (dotted line) of the disastrous contractions identified in Barro and Ursúa's (2008) consumption panel data using q=0.145. The vertical dashed line indicates the threshold at which one power law morphs into the other. Panel (b) shows the plot using Bolt and van Zanden's (2014) GDP panel data. The fitted cumulative distribution functions use the DPL parameter estimates from Table 3.



(a) Barro and Ursúa data **(b)** Bolt and van Zanden data

Comparing Panels A and B of Table 3, we observe that the results remain qualitatively the same whether we use the updated Barro and Ursúa (2008) cross-country consumption data from 1800-2009 or Bolt and van Zanden's (2014) GDP data ranging from 1900-2010 to obtain disaster information.

2.4.2 SMM preference parameter estimates and model-implied key financial indicators

The SMM estimation results in Table 4 are based on the moment matches in Equation (2.12) with $\mathcal{T}=10^7$. As a consistency check, we also report the estimation results obtained by applying the AHB method using $\mathcal{T}$=16k and H=1k. In all instances, the identity matrix serves as the distance matrix. The results reported in Table 4 are based on the *TailCorr* data simulation procedure.[18]

[18] The *EmpCorr* and *ZeroCorr* results are presented as parts of the robustness checks in Section 2.4.3 and Appendix A.3, respectively. The three data simulation procedures produce similar results and lead to the same conclusions.

Table 4: Second-step estimation results: C-CAPM preference parameters
The table presents the second-step estimates of the C-CAPM preference parameters β and γ. The SMM estimation is based on the moment matches in Equation (2.12) with $\mathcal{T}=10^7$, using the excess returns of the respective test assets and the risk-free rate proxy. The AHB estimates are based on $\mathcal{T}$=16k. All estimates rely on the *TailCorr* data simulation procedure. The numbers in parentheses are bootstrap standard errors and the numbers in brackets are the bounds of the 95% confidence intervals computed using the percentile method. The number of bootstrap replications is K=1k. The table also reports the p-values (in percent) of Hansen's J-statistic (see Equation (2.34)) and root mean squared errors (R), computed as explained in Equation (2.35). For the AHB method, R is obtained by averaging over the H=1k replications. Panels A-D break down the results by the MPP used to simulate the disaster-including data (Panels A and B: ACH_X-DPL, Panels C and D: ACH_0-DPL). In all cases, the disaster threshold is q=0.145. In Panels A and C, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panels B and D, the first-step estimation results are based on Bolt and van Zanden's (2014) GDP data. Each panel reports the results by set of test assets (*mkt*, *size dec*, *industry*).

Panel A: ACH_X/Consumption

	mkt		size dec				industry			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.985** (0.006)	**3.51** (0.61)	**0.979** (0.007)	**3.72** (0.65)	31.3	24	**0.983** (0.005)	**3.63** (0.61)	67.0	23
	[0.974 0.997]	[2.12 4.56]	[0.966 0.994]	[2.29 4.86]			[0.972 0.994]	[2.27 4.72]		
AHB	**0.983** (0.002)	**4.07** (0.74)	**0.976** (0.003)	**4.31** (0.78)		24	**0.980** (0.002)	**4.17** (0.75)		23
	[0.980 0.986]	[2.68 5.61]	[0.971 0.981]	[2.82 5.94]			[0.976 0.984]	[2.74 5.74]		

Panel B: ACH_X/GDP

	mkt		size dec				industry			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.986** (0.005)	**3.39** (0.62)	**0.981** (0.007)	**3.57** (0.65)	32.1	23	**0.984** (0.005)	**3.49** (0.62)	67.4	23
	[0.976 0.997]	[2.12 4.57]	[0.967 0.993]	[2.25 4.72]			[0.973 0.994]	[2.21 4.58]		
AHB	**0.984** (0.002)	**4.02** (0.77)	**0.978** (0.002)	**4.27** (0.81)		24	**0.981** (0.002)	**4.14** (0.78)		23
	[0.981 0.987]	[2.59 5.60]	[0.973 0.981]	[2.71 5.87]			[0.977 0.984]	[2.63 5.69]		

Panel C: ACH_0/Consumption

	mkt		size dec				industry			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.985** (0.005)	**3.47** (0.64)	**0.980** (0.007)	**3.56** (0.69)	31.1	24	**0.983** (0.006)	**3.47** (0.67)	68.7	23
	[0.974 0.996]	[1.98 4.59]	[0.966 0.993]	[2.13 4.87]			[0.972 0.993]	[2.09 4.75]		
AHB	**0.983** (0.002)	**4.03** (0.73)	**0.976** (0.003)	**4.32** (0.80)		24	**0.980** (0.002)	**4.18** (0.77)		23
	[0.980 0.986]	[2.67 5.60]	[0.971 0.981]	[2.85 6.07]			[0.976 0.984]	[2.77 5.86]		

Panel D: ACH_0/GDP

	mkt		size dec				industry			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.986** (0.006)	**3.35** (0.65)	**0.981** (0.007)	**3.51** (0.69)	31.0	24	**0.984** (0.005)	**3.43** (0.67)	67.2	23
	[0.975 0.997]	[2.05 4.67]	[0.968 0.993]	[2.08 4.87]			[0.973 0.993]	[2.06 4.72]		
AHB	**0.984** (0.002)	**4.05** (0.78)	**0.977** (0.002)	**4.30** (0.85)		24	**0.981** (0.002)	**4.17** (0.82)		23
	[0.981 0.987]	[2.62 5.70]	[0.972 0.981]	[2.74 6.08]			[0.977 0.984]	[2.67 5.89]		

Panels A-D in Table 4 break down the estimation results by the ACH-DPL specification and cross-country panel data on which the MPP parameters are estimated in the first step. Each panel shows the estimation results for the three sets of test assets: the excess returns of the CRSP value-weighted market portfolio (*mkt*), ten size-sorted portfolios (*size dec*), and ten industry portfolios (*industry*), each augmented by the risk-free rate proxy. We report the point estimates of β and γ, as well as their bootstrap standard errors and the associated 95% confidence intervals. The confidence bounds are obtained by the percentile method, that is, by computing the 0.025 and 0.975 quantiles of the bootstrap distribution.[19] We also report the p-values of Hansen's J-statistic,

$$J = \mathbb{G}_T(\hat{\beta}, \hat{\gamma})'[\widehat{\text{Avar}}(\mathbb{G}_T[\hat{\beta}, \hat{\gamma}])]^{+}\mathbb{G}_T(\hat{\beta}, \hat{\gamma}), \tag{2.34}$$

where $^{+}$ denotes the Moore-Penrose inverse. The reported root mean squared errors are computed as:

$$R = \sqrt{\frac{1}{M}\mathbb{G}_T(\hat{\beta}, \hat{\gamma})'\mathbb{G}_T(\hat{\beta}, \hat{\gamma})} \times 10^4, \tag{2.35}$$

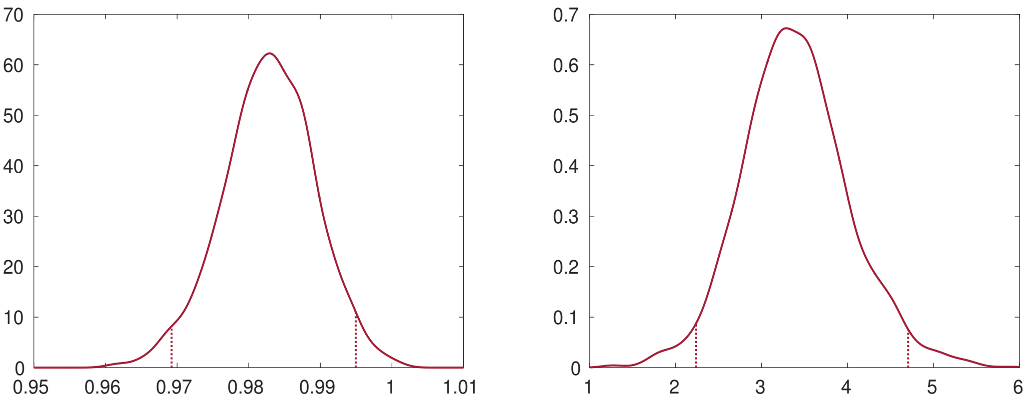
where M denotes the number of rows of $\mathbb{G}_T(\hat{\beta}, \hat{\gamma})$.

Table 4 shows that irrespective of the cross-country panel data, MPP specification, and set of test assets, all variants to estimate a disaster-including C-CAPM yield economically plausible results. The SMM estimates of the RRA coefficient γ are between 3.35 and 3.72, well within the canonical plausibility range of 1 to 10 suggested by Mehra and Prescott (1985) and Rietz (1988), and even smaller than the stricter traditional upper bound of γ=5 mentioned by Cochrane (2005). The estimates of the subjective discount factor range from 0.979 to 0.986, which implies positive and, in particular at the quarterly frequency, reasonable time preferences. The J-tests do not reject the disaster-including C-CAPM on conventional levels of significance. The estimation precision is good, as indicated by the small bootstrap standard errors and the narrow confidence intervals, the bounds of which also lie within the range that defines economically sensible parameter estimates. Figure 5 further illustrates these findings by means of kernel estimates. It shows that the distributions of the preference parameter estimates have their probability masses located at plausible values for β and γ. Table 4 also shows that the AHB estimates, which rely on simulated data only, and which should be seen as a consistency check, are close to

[19] Bias-corrected bootstrap confidence intervals and point estimates are reported as part of the battery of robustness checks, see Section 2.4.3.

the SMM estimates, both in terms of size and estimation precision.

Figure 5: Kernel estimates using pooled SMM bootstrap results
The figure depicts kernel densities using pooled bootstrap SMM estimates of β (Panel (a)) and γ (Panel (b)). The results of the K=1k bootstrap replications are pooled over three sets of test assets (*mkt*, *size dec*, *industry*), each of which is augmented by the risk-free rate. All estimations use the *TailCorr* simulation procedure, the ACH_X-DPL, and Barro and Ursúa's (2008) consumption data with q=0.145. The dotted lines indicate the 2.5% and 97.5% quantiles, respectively. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



(a) $\hat{\beta}$ **(b)** $\hat{\gamma}$

To perform another test of economic plausibility, we use the SMM results in Table 4 to estimate the mean risk-free rate and market portfolio return, equity premium, and market Sharpe ratio implied by the disaster-including C-CAPM, and compare them with their counterparts observed in the empirical data. Approximating population moments by averaging over the $\mathcal{T}$ simulated observations, we estimate the model-implied mean risk-free rate as:

$$\widehat{\mathbb{E}}(R^f) = \frac{1 - \text{cov}_\mathcal{T}(m(\hat{\beta},\hat{\gamma}), R^f)}{\mathbb{E}_\mathcal{T}(m(\hat{\beta},\hat{\gamma}))}, \tag{2.36}$$

and the model-implied expected mean market return as:

$$\widehat{\mathbb{E}}(R^m) = \frac{1 - \text{cov}_\mathcal{T}(m(\hat{\beta},\hat{\gamma}), R^m)}{\mathbb{E}_\mathcal{T}(m(\hat{\beta},\hat{\gamma}))}, \tag{2.37}$$

where $\mathbb{E}_\mathcal{T}(x) = \frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}} x_s$ and $\text{cov}_\mathcal{T}(x,y) = \mathbb{E}_\mathcal{T}(xy) - \mathbb{E}_\mathcal{T}(x)\mathbb{E}_\mathcal{T}(y)$.

The estimate of the model-implied equity premium is then $\widehat{\mathbb{E}}(R^m) - \widehat{\mathbb{E}}(R^f)$, and the model-implied Sharpe ratio of the market portfolio is estimated by

$$\frac{\widehat{\mathbb{E}}(R^m) - \widehat{\mathbb{E}}(R^f)}{\sigma_\mathcal{T}(R^m - R^f)}, \tag{2.38}$$

where $\sigma_{\mathcal{T}}(x) = \sqrt{\mathbb{E}_{\mathcal{T}}(x^2) - \mathbb{E}_{\mathcal{T}}(x)^2}$.

Table 5 reports the estimates of these model-implied key financial indicators and the associated 95% confidence intervals obtained by the percentile method.[20] As in Table 4, the four panels break down the results by MPP, country-panel data, and set of test assets. The SMM estimation is based on the *TailCorr* data simulation procedure.[21]

We observe that the point estimates of the model-implied key financial indicators are perfectly plausible and comparable to the values observed in the empirical data reported in Panel A of Table 5. Using the market portfolio and the risk free rate proxy as test assets (*mkt*) implies that the number of moment matches is identical to the number of parameters. However, it is worth noting that the SMM estimation strategy does not imply that the mean market portfolio return and mean risk-free rate observed in the empirical data are matched by the model-implied counterparts. Using the two other sets of test assets (*size dec* and *industry*), the market portfolio is not even among the test assets. Nevertheless, estimating the model-implied mean market return and the market Sharpe ratio based on these data yields economically sensible values, too, which can be seen as an out-of-sample plausibility test. Moreover, even the bounds of the 95% confidence intervals imply perfectly plausible values for the key financial indicators. In all instances, the confidence intervals overlap the empirically observed values reported in Panel A of Table 5.

The literature survey in Section 1.1 showed that previous tests of the canonical C-CAPM often yield implausibly large and imprecise RRA coefficient estimates, using in many cases calibrated subjective discount factors. In comparison with these studies, the overall appeal of the estimation results in terms of statistical and economic coherence is very good. They show that the canonical C-CAPM can explain the high market equity premium and the low risk-free rate with reasonable risk and time preferences, once rare disaster risk is accounted for. Barro (2006) came to a similar conclusion by using a calibrated theoretical model, and by showing that it can replicate the moments of financial indicators. By contrast, the strategy employed here is to use econometric techniques to estimate the parameters of a disaster-including C-CAPM on empirical data, which are then used to check whether the estimates and model implications are economically sensible and statistically useful. Unlike in

[20] For that purpose, we compute the respective financial indicator for each of the 1k bootstrap replications, and obtain the upper and lower bound by reading out the 0.025 and 0.975 quantile of the empirical distribution.

[21] The corresponding *EmpCorr* and *ZeroCorr* results are presented in the robustness checks section or in Appendix A.3.

Table 5: Key financial indicators implied by a disaster-including C-CAPM

The table presents estimates of the mean risk-free rate, mean market return, equity premium, and market Sharpe ratio implied by a disaster-including C-CAPM. These indicators are computed as given by Equations (2.36), (2.37), and (2.38). The estimates of the subjective discount factor and the RRA coefficient are taken from Table 4. The numbers in brackets are the bounds of the 95% confidence intervals based on K=1k bootstrap replications and computed using the percentile method. Panels A-D break down the results by the MPP used to simulate the disaster-including data (Panels A and B: ACH_X-DPL, Panels C and D: ACH_0-DPL). In Panels A and C, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panels B and D, the first-step estimation results are based on Bolt and van Zanden's (2014) cross-country panel GDP data. Each panel reports the results by set of test assets (*mkt*, *size dec*, *industry*). Panel A also contains the values of the indicators in the empirical data (1947:Q2–2014:Q4).

	Panel A: $\mathbf{ACH_X}$/Consumption			
	data	**mkt**	**size dec**	**industry**
mean risk-free rate (% per qtr)	*0.17*	0.17 [0.12 0.23]	0.18 [0.12 0.24]	0.18 [0.13 0.24]
equity premium (% per qtr)	*1.94*	1.79 [0.83 2.75]	2.27 [1.13 3.47]	2.00 [1.04 2.93]
mean market return (% per qtr)	*2.11*	1.96 [0.99 2.93]	2.45 [1.29 3.65]	2.18 [1.23 3.12]
Sharpe ratio (market)	*0.237*	0.212 [0.094 0.342]	0.270 [0.127 0.424]	0.237 [0.118 0.358]
		Panel B: $\mathbf{ACH_X}$/GDP		
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.18 [0.13 0.23]	0.18 [0.13 0.24]
equity premium (% per qtr)		1.81 [0.90 2.75]	2.29 [1.15 3.41]	2.02 [1.16 2.93]
mean market return (% per qtr)		1.98 [1.06 2.92]	2.47 [1.34 3.61]	2.20 [1.34 3.12]
Sharpe ratio (market)		0.215 [0.102 0.345]	0.273 [0.136 0.420]	0.240 [0.132 0.357]
		Panel C: $\mathbf{ACH_0}$/Consumption		
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.18 [0.13 0.24]	0.18 [0.13 0.24]
equity premium (% per qtr)		1.78 [0.83 2.71]	2.27 [1.13 3.40]	2.00 [1.06 2.95]
mean market return (% per qtr)		1.96 [1.01 2.89]	2.45 [1.31 3.58]	2.18 [1.23 3.13]
Sharpe ratio (market)		0.211 [0.096 0.339]	0.269 [0.129 0.423]	0.237 [0.124 0.365]
		Panel D: $\mathbf{ACH_0}$/GDP		
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.18 [0.13 0.23]	0.18 [0.13 0.24]
equity premium (% per qtr)		1.80 [0.79 2.76]	2.29 [1.23 3.39]	2.02 [1.17 2.90]
mean market return (% per qtr)		1.98 [0.95 2.93]	2.47 [1.41 3.57]	2.20 [1.34 3.09]
Sharpe ratio (market)		0.214 [0.092 0.346]	0.272 [0.142 0.410]	0.240 [0.134 0.357]

calibration exercises, it is a priori not obvious whether the model parameter estimates and model-implied key financial indicators will indeed be plausible and whether the available data will be sufficiently informative to allow precise assessments. Our results therefore provide new empirical evidence in favor of both the rare disaster hypothesis and the consumption-based asset pricing paradigm.

2.4.3 Robustness checks

The analyses presented in the previous section rely on the *TailCorr* data simulation procedure, but the results do not qualitatively change when we consider the alternative choices of the copula correlation ρ instead. The SMM estimates in Table 6 and the estimates of the key financial indicators in Table 7 are based on the *EmpCorr* data simulation procedure.[22] Comparing them with the *TailCorr* results in Tables 4 and 5, we arrive at the same result: We obtain economically plausible and precise estimates of the preference parameters and key financial indicators.

To further extend this robustness check, Figure 6 shows the effect of varying ρ between 0 and 0.99. We observe that $\hat{\gamma}$ decreases somewhat and $\hat{\beta}$ becomes bigger with increasing copula correlation, but the preference parameter estimates always remain economically plausible in size, and they exhibit small confidence bounds for all values of ρ. Figure 6 shows the results using the ACH_X-DPL model estimated on the updated Barro and Ursúa (2008) cross-country consumption data, and using the excess return of market portfolio and the risk-free rate as test assets; this version is representative for the other estimation variants.[23]

We obtain the estimation results in Table 8 when we use only the excess returns of the test assets but not the risk-free rate proxy – that is, when we use the alternative moment matches in Equation (2.13) instead of those in Equation (2.12). In this setup, the subjective discount factor β is not identified. Comparing the estimates in Table 8 with Panels A and B of Table 4, we observe that the pattern of γ estimates across data simulation procedures and test assets is very similar. Table 8 focuses on the results obtained using the estimated ACH_X-DPL model. The corresponding ACH_0-DPL-based results are, with respect to the pattern of the RRA coefficient estimates, akin those in Panels C and D of Table 4. We report them in Section A.3 of the appendix.

Moreover, we confirm that the results and conclusions do not depend on the disaster threshold size of q=0.145. Table 9 shows the estimation results obtained

[22] For the sake of brevity, the corresponding *ZeroCorr* results are deferred to Appendix A.3.
[23] Additional results can be found in Appendix A.3.

Table 6: Robustness check: C-CAPM preference parameter estimates using the *EmpCorr* data simulation procedure

The table presents the second-step estimates of the C-CAPM preference parameters β and γ based on the *EmpCorr* data simulation procedure. The other estimation settings (moment matches, $\mathcal{T}$=10^7 for SMM, $\mathcal{T}$=16k for AHB, q=0.145, K=H=1k), the table layout, and the reported statistics correspond to Table 4.

				Panel A: ACH_X/Consumption						
	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.965** (0.016)	**3.97** (0.87)	**0.945** (0.019)	**4.26** (0.75)	27.7	22	**0.953** (0.015)	**4.18** (0.70)	49.5	23
	[0.925 0.990]	[2.37 5.19]	[0.907 0.981]	[2.54 5.49]			[0.923 0.983]	[2.56 5.33]		
AHB	**0.959** (0.014)	**4.71** (0.78)	**0.950** (0.008)	**4.90** (0.83)		49	**0.957** (0.006)	**4.77** (0.80)		43
	[0.927 0.978]	[3.24 6.26]	[0.932 0.963]	[3.27 6.53]			[0.942 0.968]	[3.21 6.34]		
				Panel B: ACH_X/GDP						
	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.964** (0.016)	**3.83** (0.70)	**0.946** (0.019)	**4.07** (0.74)	28.9	21	**0.952** (0.015)	**4.01** (0.72)	36.7	27
	[0.927 0.988]	[2.38 5.20]	[0.907 0.982]	[2.50 5.34]			[0.921 0.982]	[2.47 5.24]		
AHB	**0.958** (0.018)	**4.67** (0.85)	**0.952** (0.008)	**4.84** (0.87)		51	**0.958** (0.006)	**4.72** (0.83)		45
	[0.919 0.980]	[3.07 6.32]	[0.935 0.965]	[3.04 6.49]			[0.943 0.969]	[2.99 6.31]		
				Panel C: ACH_0/Consumption						
	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.961** (0.016)	**3.99** (0.73)	**0.943** (0.018)	**4.07** (0.79)	23.1	36	**0.951** (0.015)	**4.00** (0.77)	60.5	24
	[0.922 0.988]	[2.27 5.23]	[0.911 0.980]	[2.31 5.51]			[0.921 0.982]	[2.30 5.38]		
AHB	**0.959** (0.016)	**4.67** (0.78)	**0.950** (0.008)	**4.91** (0.85)		47	**0.957** (0.006)	**4.78** (0.81)		42
	[0.924 0.978]	[3.16 6.22]	[0.934 0.963]	[3.28 6.69]			[0.944 0.968]	[3.20 6.51]		
				Panel D: ACH_0/GDP						
	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.964** (0.018)	**3.80** (0.74)	**0.951** (0.017)	**3.94** (0.78)	31.8	29	**0.958** (0.014)	**3.87** (0.76)	9.2	27
	[0.919 0.989]	[2.33 5.26]	[0.914 0.978]	[2.28 5.49]			[0.924 0.980]	[2.28 5.36]		
AHB	**0.960** (0.016)	**4.68** (0.85)	**0.952** (0.007)	**4.87** (0.91)		51	**0.958** (0.006)	**4.75** (0.87)		45
	[0.921 0.979]	[3.13 6.46]	[0.936 0.964]	[3.18 6.70]			[0.946 0.969]	[3.13 6.53]		

when using q=0.095 and q=0.195 instead. These disaster thresholds are in accordance with Barro and Jin's (2011) choices. Although there are 156 identified disasters for q=0.095, and only 56 for q=0.195, the estimation results remain qualitatively the same as in the base case. To conserve space, Table 9 only presents the ACH_X-DPL results, estimated using the updated Barro and Ursúa (2008) data. The other variants that involve the ACH_0-DPL specification are very similar, with respect to the pattern of the $\hat{\beta}$ and $\hat{\gamma}$ estimates, to those reported in Panels C-D of Table 4. These additional estimation results are available in Section A.3 of the appendix.

Table 7: Robustness check: Key financial indicators implied by a disaster-including C-CAPM using the *EmpCorr* data simulation procedure

The table presents estimates of the mean risk-free rate, mean market return, equity premium, and market Sharpe ratio implied by a disaster-including C-CAPM that uses the *EmpCorr* data simulation procedure. The table layout and the reported statistics correspond to Table 5.

Panel A: ACH_X/Consumption

	data	**mkt**	**size dec**	**industry**
mean risk-free rate (% per qtr)	*0.17*	0.17 [0.12 0.23]	0.18 [0.13 0.27]	0.18 [0.14 0.26]
equity premium (% per qtr)	*1.94*	1.79 [0.75 2.77]	2.63 [1.08 3.75]	2.33 [1.02 3.20]
mean market return (% per qtr)	*2.11*	1.96 [0.93 2.95]	2.81 [1.25 3.93]	2.51 [1.20 3.42]
Sharpe ratio (market)	*0.237*	0.212 [0.088 0.347]	0.312 [0.122 0.462]	0.276 [0.116 0.397]

Panel B: ACH_X/GDP

	mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)	0.17 [0.12 0.22]	0.18 [0.13 0.26]	0.18 [0.14 0.26]
equity premium (% per qtr)	1.81 [0.90 2.75]	2.62 [1.11 3.69]	2.35 [1.12 3.34]
mean market return (% per qtr)	1.98 [1.06 2.92]	2.80 [1.29 3.88]	2.53 [1.31 3.54]
Sharpe ratio (market)	0.215 [0.102 0.345]	0.312 [0.129 0.451]	0.280 [0.129 0.408]

Panel C: ACH_0/Consumption

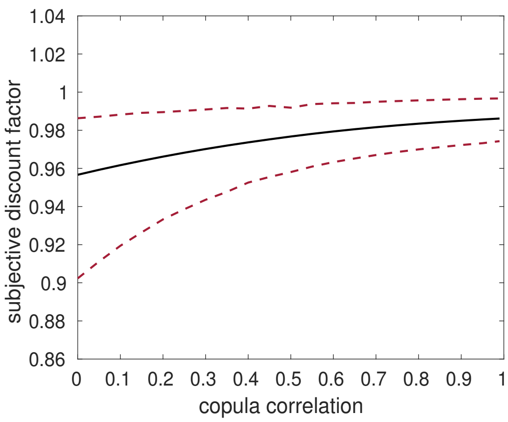
	mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)	0.17 [0.12 0.22]	0.19 [0.13 0.25]	0.18 [0.13 0.26]
equity premium (% per qtr)	1.78 [0.84 2.71]	2.77 [1.05 3.68]	2.44 [1.05 3.28]
mean market return (% per qtr)	1.96 [1.01 2.89]	2.96 [1.26 3.87]	2.62 [1.24 3.48]
Sharpe ratio (market)	0.211 [0.096 0.339]	0.328 [0.124 0.454]	0.289 [0.120 0.402]

Panel D: ACH_0/GDP

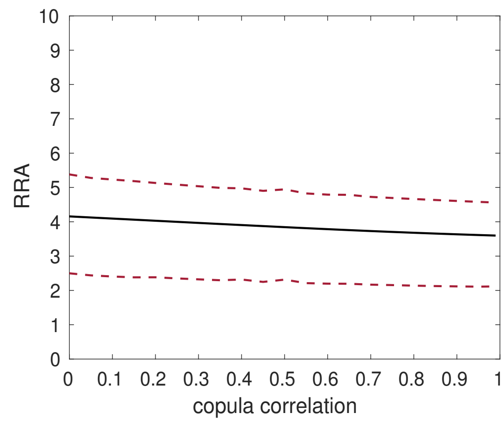
	mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)	0.17 [0.12 0.22]	0.19 [0.13 0.26]	0.18 [0.14 0.25]
equity premium (% per qtr)	1.80 [0.79 2.76]	2.78 [1.16 3.58]	2.49 [1.11 3.18]
mean market return (% per qtr)	1.98 [0.95 2.93]	2.97 [1.34 3.76]	2.67 [1.30 3.36]
Sharpe ratio (market)	0.214 [0.092 0.346]	0.331 [0.135 0.433]	0.297 [0.127 0.389]

Figure 6: Robustness check: Effect of a varying copula correlation

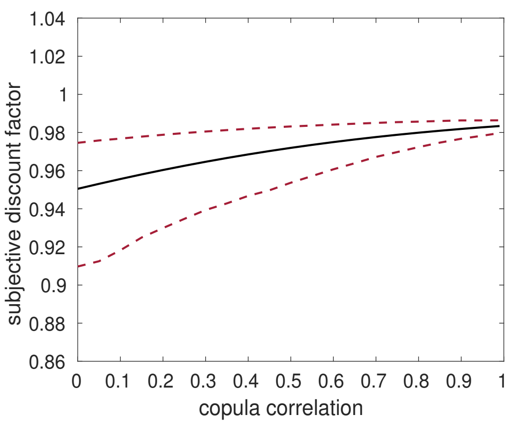
The figure depicts the estimates of the subjective discount factor β (Panels (a) and (c)) and the RRA coefficient γ (Panels (b) and (d)) using a varying copula correlation ρ. The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates using the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. Test assets are the excess return of the market portfolio (mkt) and the risk-free rate. Panels (a) and (b) refer to the SMM estimates, and Panels (c) and (d) pertain to the AHB estimates. The dashed (red) lines are the 95% confidence bounds.



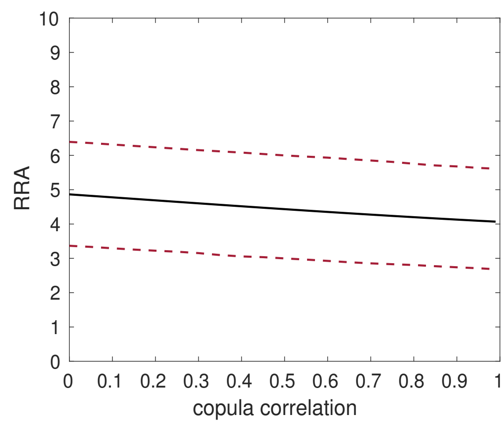
(a) SMM with ACH_X: $\hat{\beta}$



(b) SMM with ACH_X: $\hat{\gamma}$



(c) AHB with ACH_X: $\hat{\beta}$



(d) AHB with ACH_X: $\hat{\gamma}$

Table 8: Robustness check: Estimation results using only excess returns as test assets The table presents the second-step estimates of the RRA coefficient γ. The SMM estimation results are based on the moment matches in Equation (2.13) using the excess returns of the respective test assets. The other estimation settings ($\mathcal{T}$=10^7 for SMM, $\mathcal{T}$=16k for AHB, q=0.145, K=H=1k), and the reported statistics correspond to Table 4. The simulation of disaster-including data makes use of the first-step ACH_X-DPL estimates. In Panel A, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panel B, the first-step estimation results are based on Bolt and van Zanden's (2014) GDP data. Each panel breaks down the results by set of test assets (*mkt*, *size dec*, *industry*) and data simulation procedure (*TailCorr*, *EmpCorr*, *ZeroCorr*).

		Panel A: Consumption					
	TailCorr/mkt	**TailCorr/size dec**			**TailCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.51** (0.61)	**3.72** (0.65)	32.9	25	**3.63** (0.61)	67.2	24
	[2.12 4.56]	[2.29 4.86]			[2.27 4.72]		
AHB	**4.07** (0.74)	**4.31** (0.78)		26	**4.17** (0.75)		25
	[2.68 5.61]	[2.82 5.94]			[2.74 5.74]		
	EmpCorr/mkt	**EmpCorr/size dec**			**EmpCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.97** (0.87)	**4.26** (0.75)	33.0	25	**4.17** (0.70)	50.9	26
	[2.37 5.19]	[2.53 5.49]			[2.56 5.33]		
AHB	**4.71** (0.78)	**4.89** (0.83)		54	**4.77** (0.80)		47
	[3.24 6.26]	[3.26 6.52]			[3.20 6.34]		
	ZeroCorr/mkt	**ZeroCorr/size dec**			**ZeroCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**4.09** (0.88)	**4.40** (0.75)	35.2	27	**4.29** (0.72)	55.4	29
	[2.50 5.38]	[2.68 5.61]			[2.60 5.47]		
AHB	**4.86** (0.78)	**5.03** (0.84)		55	**4.88** (0.80)		48
	[3.37 6.39]	[3.38 6.67]			[3.35 6.45]		
		Panel B: GDP					
	TailCorr/mkt	**TailCorr/size dec**			**TailCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.39** (0.62)	**3.57** (0.65)	33.7	25	**3.49** (0.62)	67.6	24
	[2.12 4.57]	[2.25 4.72]			[2.21 4.58]		
AHB	**4.02** (0.77)	**4.27** (0.81)		25	**4.14** (0.78)		25
	[2.59 5.60]	[2.71 5.87]			[2.63 5.69]		
	EmpCorr/mkt	**EmpCorr/size dec**			**EmpCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.83** (0.70)	**4.07** (0.74)	34.6	23	**4.01** (0.72)	38.9	30
	[2.38 5.20]	[2.50 5.33]			[2.47 5.24]		
AHB	**4.67** (0.85)	**4.84** (0.87)		56	**4.72** (0.83)		49
	[3.07 6.32]	[3.04 6.49]			[2.99 6.31]		
	ZeroCorr/mkt	**ZeroCorr/size dec**			**ZeroCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.94** (0.71)	**4.21** (0.75)	37.6	24	**4.12** (0.73)	46.3	33
	[2.44 5.35]	[2.57 5.45]			[2.51 5.37]		
AHB	**4.82** (0.85)	**4.97** (0.87)		57	**4.83** (0.84)		51
	[3.21 6.47]	[3.15 6.63]			[3.08 6.41]		

Table 9: Robustness check: C-CAPM preference parameter estimates with varying q The table presents the second-step estimates of the preference parameters β and γ using alternative disaster thresholds. Panel A uses a disaster threshold of q=0.095, and Panel B uses q=0.195. The other estimation settings (moment matches, $\mathcal{T}=10^7$ for SMM, $\mathcal{T}$=16k for AHB, K=H=1k), and the reported statistics correspond to Table 4. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates. Each panel breaks down the results by set of test assets (*mkt*, *size dec*, *industry*) and data simulation procedure (*TailCorr*, *EmpCorr*, *ZeroCorr*).

				Panel A: q=0.095						
	TailCorr/mkt		**TailCorr/size dec**				**TailCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.984** (0.006)	**3.40** (0.62)	**0.978** (0.007)	**3.58** (0.65)	29.7	24	**0.981** (0.006)	**3.49** (0.63)	66.6	23
	[0.972 0.995]	[2.20 4.67]	[0.964 0.992]	[2.31 4.85]			[0.971 0.992]	[2.27 4.78]		
AHB	**0.981** (0.002)	**3.92** (0.68)	**0.974** (0.003)	**4.19** (0.75)		24	**0.978** (0.002)	**4.04** (0.71)		23
	[0.977 0.985]	[2.61 5.38]	[0.968 0.979]	[2.72 5.72]			[0.974 0.982]	[2.64 5.53]		
	EmpCorr/mkt		**EmpCorr/size dec**				**EmpCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.951** (0.023)	**4.02** (0.74)	**0.935** (0.024)	**4.19** (0.77)	19.6	36	**0.942** (0.019)	**4.13** (0.75)	28.3	24
	[0.892 0.986]	[2.63 5.43]	[0.878 0.975]	[2.65 5.71]			[0.902 0.976]	[2.61 5.64]		
AHB	**0.942** (0.030)	**4.82** (0.81)	**0.936** (0.011)	**4.98** (0.86)		55	**0.944** (0.009)	**4.86** (0.83)		48
	[0.875 0.974]	[3.22 6.43]	[0.910 0.955]	[3.25 6.69]			[0.924 0.960]	[3.19 6.48]		
	ZeroCorr/mkt		**ZeroCorr/size dec**				**ZeroCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.932** (0.031)	**4.21** (0.73)	**0.911** (0.030)	**4.37** (0.79)	16.6	38	**0.923** (0.023)	**4.29** (0.77)	34.5	24
	[0.862 0.978]	[2.75 5.56]	[0.847 0.963]	[2.75 5.93]			[0.877 0.967]	[2.68 5.82]		
AHB	**0.926** (0.040)	**5.03** (0.83)	**0.920** (0.018)	**5.18** (0.88)		55	**0.933** (0.013)	**5.02** (0.84)		49
	[0.831 0.967]	[3.39 6.70]	[0.875 0.947]	[3.42 6.93]			[0.905 0.953]	[3.32 6.65]		
				Panel B: q=0.195						
	TailCorr/mkt		**TailCorr/size dec**				**TailCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.987** (0.006)	**3.48** (0.65)	**0.981** (0.007)	**3.66** (0.63)	32.0	23	**0.984** (0.005)	**3.58** (0.61)	67.4	23
	[0.976 0.997]	[2.12 4.52]	[0.968 0.994]	[2.21 4.73]			[0.974 0.995]	[2.19 4.54]		
AHB	**0.985** (0.002)	**4.13** (0.78)	**0.978** (0.002)	**4.31** (0.84)		24	**0.982** (0.002)	**4.18** (0.81)		23
	[0.982 0.988]	[2.67 5.75]	[0.973 0.982]	[2.75 6.05]			[0.978 0.985]	[2.68 5.86]		
	EmpCorr/mkt		**EmpCorr/size dec**				**EmpCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.969** (0.014)	**3.86** (0.68)	**0.957** (0.015)	**4.06** (0.70)	32.1	17	**0.962** (0.013)	**4.00** (0.68)	47.0	25
	[0.937 0.993]	[2.36 4.96]	[0.922 0.984]	[2.48 5.11]			[0.936 0.985]	[2.43 5.05]		
AHB	**0.968** (0.011)	**4.61** (0.80)	**0.959** (0.006)	**4.76** (0.86)		44	**0.965** (0.005)	**4.64** (0.82)		39
	[0.941 0.983]	[3.04 6.27]	[0.947 0.969]	[3.11 6.45]			[0.955 0.973]	[3.04 6.25]		
	ZeroCorr/mkt		**ZeroCorr/size dec**				**ZeroCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.963** (0.017)	**3.95** (0.69)	**0.947** (0.018)	**4.16** (0.72)	36.6	16	**0.954** (0.014)	**4.09** (0.69)	52.8	25
	[0.922 0.991]	[2.40 5.11]	[0.910 0.979]	[2.53 5.27]			[0.924 0.981]	[2.46 5.12]		
AHB	**0.962** (0.013)	**4.72** (0.79)	**0.953** (0.008)	**4.86** (0.86)		44	**0.960** (0.006)	**4.72** (0.82)		40
	[0.933 0.980]	[3.13 6.34]	[0.936 0.965]	[3.21 6.56]			[0.948 0.971]	[3.13 6.33]		

Table 10: Robustness check: Bias-corrected estimates and confidence bounds
The table presents bias-corrected SMM estimates of the C-CAPM preference parameters β and γ. The numbers in brackets are the bounds of the 95% confidence intervals computed using the bias-correction method proposed by Efron and Tibshirani (1986). The table presents the results for each of the three data simulation procedures. In other respects, the table layout, estimation settings ($\mathcal{T}$=10^7, q=0.145, K=1k), and the reported statistics correspond to Table 4.

Panel A: ACH_X/Consumption

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
Tail	**0.985** (0.006)	**3.71** (0.61)	**0.979** (0.007)	**3.97** (0.65)	31.3	24	**0.982** (0.005)	**3.86** (0.61)	67.0	23
Corr	[0.974 0.996]	[2.62 5.28]	[0.966 0.993]	[2.86 5.47]			[0.971 0.992]	[2.79 5.43]		
Emp	**0.968** (0.016)	**4.19** (0.87)	**0.942** (0.019)	**4.59** (0.75)	27.7	22	**0.950** (0.015)	**4.49** (0.70)	49.5	23
Corr	[0.930 0.992]	[2.91 5.72]	[0.891 0.975]	[3.29 6.29]			[0.916 0.977]	[3.22 6.11]		
Zero	**0.963** (0.021)	**4.30** (0.88)	**0.925** (0.022)	**4.75** (0.76)	28.4	24	**0.940** (0.018)	**4.61** (0.72)	53.5	26
Corr	[0.918 0.992]	[2.97 5.86]	[0.874 0.965]	[3.44 6.57]			[0.901 0.973]	[3.35 6.36]		

Panel B: ACH_X/GDP

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
Tail	**0.986** (0.005)	**3.56** (0.62)	**0.981** (0.007)	**3.77** (0.65)	32.1	23	**0.984** (0.005)	**3.68** (0.62)	67.4	23
Corr	[0.976 0.996]	[2.45 5.08]	[0.967 0.992]	[2.61 5.70]			[0.972 0.993]	[2.57 5.50]		
Emp	**0.967** (0.016)	**4.00** (0.70)	**0.943** (0.019)	**4.35** (0.74)	28.9	21	**0.950** (0.015)	**4.28** (0.72)	36.7	27
Corr	[0.933 0.990]	[2.75 5.70]	[0.899 0.976]	[3.03 6.38]			[0.917 0.977]	[3.00 6.23]		
Zero	**0.961** (0.021)	**4.11** (0.71)	**0.925** (0.022)	**4.50** (0.75)	30.0	21	**0.938** (0.018)	**4.40** (0.73)	43.4	30
Corr	[0.916 0.989]	[2.85 5.83]	[0.876 0.967]	[3.15 6.59]			[0.903 0.970]	[3.09 6.40]		

Panel C: ACH_0/Consumption

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
Tail	**0.984** (0.005)	**3.66** (0.64)	**0.979** (0.007)	**3.66** (0.69)	31.1	24	**0.983** (0.006)	**3.57** (0.67)	68.7	23
Corr	[0.973 0.995]	[2.59 5.20]	[0.965 0.992]	[2.38 5.25]			[0.971 0.993]	[2.36 5.16]		
Emp	**0.960** (0.016)	**4.24** (0.73)	**0.937** (0.018)	**4.25** (0.79)	23.1	36	**0.947** (0.015)	**4.15** (0.77)	60.5	24
Corr	[0.912 0.985]	[2.98 6.00]	[0.892 0.971]	[2.85 6.13]			[0.912 0.974]	[2.71 5.91]		
Zero	**0.947** (0.021)	**4.41** (0.74)	**0.910** (0.022)	**4.44** (0.80)	24.2	39	**0.930** (0.018)	**4.30** (0.78)	71.9	24
Corr	[0.885 0.980]	[3.13 6.16]	[0.855 0.950]	[3.05 6.41]			[0.887 0.963]	[2.92 6.11]		

Panel D: ACH_0/GDP

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
Tail	**0.986** (0.006)	**3.46** (0.65)	**0.980** (0.007)	**3.65** (0.69)	31.0	24	**0.984** (0.005)	**3.56** (0.67)	67.2	23
Corr	[0.975 0.996]	[2.37 4.95]	[0.965 0.992]	[2.52 5.49]			[0.972 0.993]	[2.46 5.25]		
Emp	**0.967** (0.018)	**3.92** (0.74)	**0.952** (0.017)	**4.09** (0.78)	31.8	29	**0.960** (0.014)	**4.00** (0.76)	9.2	27
Corr	[0.926 0.990]	[2.66 5.63]	[0.915 0.979]	[2.77 6.11]			[0.929 0.983]	[2.73 5.89]		
Zero	**0.959** (0.023)	**4.06** (0.76)	**0.940** (0.020)	**4.20** (0.80)	36.8	34	**0.951** (0.016)	**4.10** (0.78)	10.2	26
Corr	[0.908 0.989]	[2.76 5.94]	[0.897 0.975]	[2.86 6.22]			[0.916 0.981]	[2.78 5.98]		

Finally, Table 10 presents bootstrap bias-corrected estimates and confidence bounds using the three data simulation procedures.[24] We note that the bias corrections are only moderate and do not alter the conclusions. The bias-corrected point estimates are slightly higher than the uncorrected estimates, and the confidence interval bounds shift slightly upwards.

2.4.4 AHB results

We argued in Section 2.2.1 that the quality of the usual C-CAPM moment matches is affected when using short time series that contain too few, if any, disaster observations to be representative of the possible paths of history that investors imagined. Using the AHB method, we can assess what sample size would be needed to achieve a reasonable estimation precision. We can also study the properties of the estimates of the subjective discount factor β and the RRA coefficient γ when the simulated sample size is as small as in the empirical data, but some simulated histories include disaster observations. A comparison with the estimation results using disaster-free empirical data serves as a check of the plausibility of the rare disaster hypothesis. In addition to $\mathcal{T}$=16k, we also perform AHB estimations with shorter simulated histories, namely, $\mathcal{T}$=271, 1k, and 5k.

For each $\mathcal{T}$, we perform separate AHB estimations using the alternative data simulation procedures and test assets. The results in Table 11 rely on the *Tail-Corr* procedure and the first-step ACH_X-DPL estimates based on the updated Barro and Ursúa (2008) data. These results are representative of the other data simulation variants.[25] Figure 7 illustrates the AHB estimation results using kernel densities of the bootstrapped estimates of β and γ.

The AHB estimates using $\mathcal{T}$=271, the number of observations in our 1947:Q2–2014:Q4 sample, reflect the notorious properties of their empirical counterparts: $\hat{\beta}$ is greater than 1 using size-sorted and industry portfolios as test assets. The estimated RRA coefficient $\hat{\gamma}$, and even more so the 95% quantiles of the bootstrap distribution, are far beyond the upper plausibility limit. Furthermore, the estimates are imprecise,

[24] Bias corrected estimates of a parameter θ are computed as $\hat{\theta}_{BC} = 2\hat{\theta} - \frac{1}{K}\sum_{k=1}^{K}\hat{\theta}^{(k)}$. Bias corrected confidence bounds are obtained as described by Efron and Tibshirani (1986). They propose to compute the lower and upper bound of the $1-\alpha$ confidence interval as $\theta^{l}_{BC}(\alpha) = \hat{G}^{-1}[\Phi(z_{\alpha/2} + 2\Phi^{-1}[\hat{G}(\hat{\theta})])]$ and $\theta^{u}_{BC}(\alpha) = \hat{G}^{-1}[\Phi(z_{1-\alpha/2} + 2\Phi^{-1}[\hat{G}(\hat{\theta})])]$, where Φ is the cdf, Φ^{-1} is the quantile function, and $z_{\tilde{\alpha}}$ is the $\tilde{\alpha}$ quantile of the standard normal distribution. Moreover, $\hat{G}(\hat{\theta}) = \frac{1}{K}\sum_{k=1}^{K}\mathbb{1}(\hat{\theta}^{(k)} < \hat{\theta})$, and $\hat{G}^{-1}(\tilde{\alpha})$ returns the $\tilde{\alpha}$-quantile of the bootstrap distribution of the estimator. According to this notation, the uncorrected confidence bounds using the percentile method are given by $\theta^{l}(\alpha) = \hat{G}^{-1}[\Phi(z_{\alpha/2})]$ and $\theta^{u}(\alpha) = \hat{G}^{-1}[\Phi(z_{1-\alpha/2})]$.

[25] Appendix A.3 reports the results for these alternative data simulation procedures and test assets.

Table 11: Effect of a varying $\mathcal{T}$ on AHB parameter estimates using the ACH_X model The table reports the AHB estimates of the subjective discount factor and the coefficient of relative risk aversion for a varying $\mathcal{T}$. The 95% quantiles of the parameter estimates from the H=1k simulated histories are underlined, and the standard deviations are reported in parentheses. Columns labeled p report the relative frequency for $\hat{\gamma}^{(h)} > 10$ (in percent). The last column reports the percentage of simulated histories for which no consumption disaster occurs. The estimations use the excess returns of the portfolios in the sets of test assets (*mkt*, *size dec*, *industry*), which in each case are augmented by the risk-free rate. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates and the *TailCorr* procedure. The disaster threshold is q=0.145.

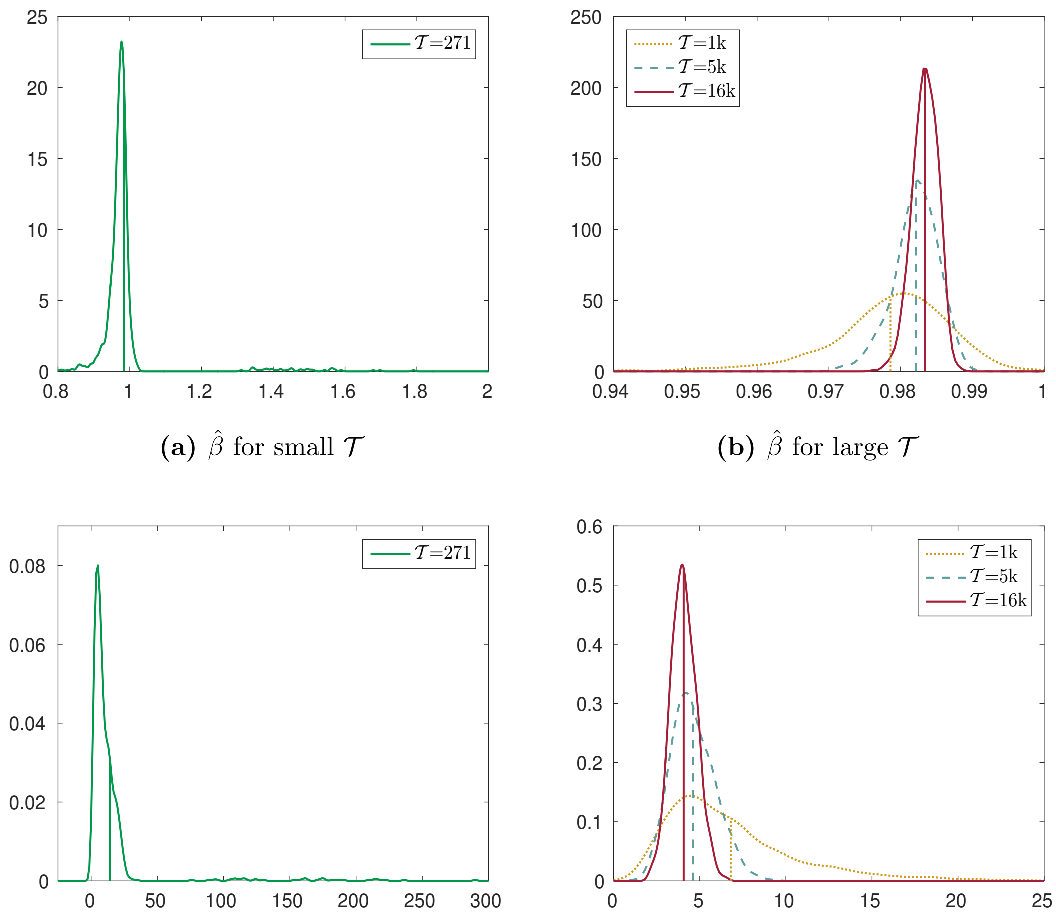
	mkt			**size dec**			**industry**			
$\mathcal{T}$	β	γ	p	β	γ	p	β	γ	p	no disaster
271	0.984	14.14	38.3	1.064	39.94	55.7	1.079	39.87	54.6	46.3
	(0.098)	(28.65)		(0.202)	(63.58)		(0.201)	(57.43)		
	1.007	23.83		1.473	175.45		1.494	163.85		
1k	0.979	6.80	16.1	0.974	9.04	19.0	0.981	8.82	16.9	2.3
	(0.010)	(3.87)		(0.056)	(17.48)		(0.055)	(17.90)		
	0.991	14.74		0.984	16.66		0.987	16.33		
5k	0.982	4.61	0.0	0.974	4.90	0.2	0.979	4.73	0.2	0.0
	(0.003)	(1.26)		(0.004)	(1.33)		(0.003)	(1.29)		
	0.987	6.84		0.981	7.31		0.984	7.04		
16k	0.983	4.07	0.0	0.976	4.31	0.0	0.980	4.17	0.0	0.0
	(0.002)	(0.74)		(0.003)	(0.78)		(0.002)	(0.75)		
	0.986	5.36		0.980	5.59		0.983	5.41		

as indicated by their large standard deviations, and the kernel densities on the left-hand side panels of Figure 7. Table 11 also reports the percentage of simulated histories that do not contain any disaster. For $\mathcal{T}$=271, we estimate that the odds of experiencing a disaster-free period like that from 1947 to 2015 are almost 1:1. We were lucky, but having been lucky was actually not an unlikely event.

Increasing the simulated sample size to $\mathcal{T}$=1k – roughly three investor generations – causes the AHB point estimates of β and γ to take on more plausible values. Now, only 2.3% of the simulated histories do not contain a disaster. The estimation precision increases but is still moderate, as indicated by the standard deviations and the shape of the kernel density estimates on the right-hand side panels of Figure 7. With $\mathcal{T}$=5k – about 15 investor generations – the estimates of the subjective discount factor and the RRA coefficient become economically reasonable, the standard deviations are small, and the kernel densities center more closely around the point estimates. There are no disaster-free histories anymore.

The AHB results suggest that the apparent empirical failure of the C-CAPM when applied to disaster-free postwar U.S. data comes as no surprise and is perfectly in line with the rare disaster hypothesis. If the rare disaster hypothesis were true, using standard econometric analysis, we would have to wait for a long time – with unpleasant intermezzos of consumption contractions – before we could expect more precise estimates. Our simulation-based methods thus provide a shortcut.

Figure 7: Effect of a varying $\mathcal{T}$ on AHB parameter estimates
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH$_\text{X}$-DPL estimates and the *TailCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



(a) $\hat{\beta}$ for small $\mathcal{T}$ **(b)** $\hat{\beta}$ for large $\mathcal{T}$

(c) $\hat{\gamma}$ for small $\mathcal{T}$ **(d)** $\hat{\gamma}$ for large $\mathcal{T}$

2.5 Discussion and conclusion

Adopting Barro's (2006) specification of a disaster-including consumption process, we consider revised moment matches that we use to estimate the C-CAPM preference parameters by SMM. To simulate the disaster-including consumption growth and return processes required for the SMM estimation, we specify a marked point process from which we obtain conditional disaster probabilities and calamitous contraction sizes. The MPP parameters are estimated using chained cross-country panel data. The SMM estimation relies on alternative ways to simulate disaster-including consumption growth and financial returns, as well as different MPP specifications and sets of test assets.

Whichever approach and data are used, the results remain qualitatively the same: The estimated preference parameters are economically plausible in size, and the estimation precision is much higher than in previous studies that have empirically tested the canonical C-CAPM. In particular, the estimates of the RRA coefficient are smaller than 5 for most specifications and always smaller than 10, that is, in a range consistent with reasonably risk-averse investors. Moreover, the estimates of the subjective discount factor β are smaller than 1, which implies a positive rate of time preference. The parameter standard errors are small, and the confidence bounds are narrow. Using the parameter estimates to calculate the model-implied market equity premium, risk-free rate, and market Sharpe ratio, these key financial indicators take on economically plausible values, with 95% confidence intervals that overlap the empirical counterparts. A comparable combination of plausibility and estimation precision has not been provided previously in related literature.

We also find that the size and precision of the parameter estimates reported in previous studies are realistic under the rare disaster hypothesis. Decades would have to pass before standard econometric techniques could yield precise estimation results with empirical data. The simulation-based estimation approaches that we apply in our study provide a shortcut to an empirical assessment of the effect of consumption disasters on asset prices. They come with the cost of assumptions, which may be questioned but also can be modified, and it is possible to study the sensitivity of the estimation results. In our study though, varying the assumptions did not change the results qualitatively.

A Appendix

A.1 Estimation of ACH-DPL model parameters using cross-country panel data: details

To estimate the ACH-DPL model parameters, the available cross-country data (annual, unbalanced panels) must be represented as event time data. For that purpose, we have to identify disaster events by setting the threshold value q that defines a disastrous consumption contraction and compute the time duration between these events. To match the frequency of the data used in the second estimation step, the time duration between disasters is measured in quarters. As described in the main text, we adopt the disaster identification scheme applied by Barro (2006). He focuses on the peak-to-trough effect of disasters, and ignores the length over which a consumption contraction $\geq q$ unfolds. As a result, there can be consumption contractions that unfold over multiple calendar periods. They are treated as a single disaster event. Each disaster event thus identified is associated with the calendar year at which the consumption contraction began, and we draw from a Bernoulli distribution with success probability 0.25 to determine in which quarter of the respective year the disaster started. Counting the number of quarters between the disaster events gives τ_n, the time duration between the nth and $(n+1)$th disaster. The resulting country-specific event time data series are then concatenated, which yields the chained cross-country data that are used for the estimation of the ACH parameters $\boldsymbol{\theta}_{\text{ACH}}$. The contraction sizes pertaining to the disaster events are saved in a separate vector and used for ML estimation of the DPL parameters $\boldsymbol{\theta}_{\text{DPL}}$. ACH parameter estimates $\hat{\boldsymbol{\theta}}_{\text{ACH}}$ result from maximizing the log-likelihood function in Equation (2.21).

As explained in the main text, we make sure not to use durations or predetermined variables x_{t-1} from another country when constructing the ACH log-likelihood function. Instead, we re-initialize at each country change in the chained series the last duration to $\tau_{N(1)} = \frac{1}{\hat{p}_q}$, where $\hat{p}_q$ is computed as the relative frequency of disasters in the chained cross-country series. In the updated Barro and Ursúa (2008) data (consumption, sample period 1800-2010), $\hat{p}_q = 0.0057$, such that $\tau_{N(1)} = 176$ quarters. In the Bolt and van Zanden (2014) data (GDP, sample period 1900-2010), $\hat{p}_q = 0.0047$ and $\tau_{N(1)} = 212$.

A.2 Using the DPL distribution to model the size of disasters

Following Barro and Jin (2011), we use a DPL distribution to model the size of the disastrous contractions b. For that purpose, we apply the transformation $z = (1-b)^{-1}$, for which we assume the density function:

$$f_{\text{DPL}}(z; \boldsymbol{\theta}_{DPL}) = \begin{cases} 0 & \text{if } z < z_0 \\ Bz^{-(1+\tilde{\theta})} & \text{if } z_0 \le z < \tilde{\delta} \\ Az^{-(1+\tilde{\alpha})} & \text{if } \tilde{\delta} \le z \end{cases}, \tag{A.1}$$

where $\boldsymbol{\theta}_{DPL} = (\tilde{\alpha}, \tilde{\delta}, \tilde{\theta},)'$, $B = A\tilde{\delta}^{(\tilde{\theta}-\tilde{\alpha})}$, and $A = \left[\frac{\tilde{\delta}^{(\tilde{\theta}-\tilde{\alpha})}}{\tilde{\theta}-1}\left(z_0^{(1-\tilde{\theta})} - \tilde{\delta}^{(1-\tilde{\theta})}\right) + \frac{\tilde{\delta}^{(1-\tilde{\alpha})}}{\tilde{\alpha}-1}\right]^{-1}$. In the present context, $z_0 = (1-q)^{-1}$.

A draw from the DPL distribution can be performed by drawing a standard uniform random variable ν and inserting it in the quantile function of the DPL distribution, given by:

$$F_{\text{DPL}}^{-1}(\nu; \boldsymbol{\theta}_{\text{DPL}}) = \begin{cases} \sqrt[-\tilde{\theta}]{z_0^{-\tilde{\theta}} - \frac{\tilde{\theta}}{B}\nu} & \text{if } \quad \nu \le F_{\text{DPL}}(\tilde{\delta}; \boldsymbol{\theta}_{\text{DPL}}) \\ \sqrt[-\tilde{\alpha}]{\tilde{\delta}^{-\tilde{\alpha}} - \frac{\tilde{\alpha}}{A}\left(\nu - \frac{B}{\tilde{\theta}}\left(z_0^{-\tilde{\theta}} - \tilde{\delta}^{-\tilde{\theta}}\right)\right)} & \text{if } \quad \nu > F_{\text{DPL}}(\tilde{\delta}; \boldsymbol{\theta}_{\text{DPL}}), \end{cases} \tag{A.2}$$

where F_{DPL} denotes the cdf of the DPL distribution.

The realizations of the random variables z drawn using the quantile function in Equation (A.2) must be re-transformed into contraction sizes by $b = 1 - \frac{1}{z}$. Applying the density transformation theorem, we can compute the expected value of the contraction size b by:

$$\mathbb{E}[b] = \mathbb{E}\left[1 - \frac{1}{z}\right] = 1 + A\tilde{\delta}^{-(\tilde{\alpha}+1)}\left(\frac{1}{\tilde{\theta}+1} - \frac{1}{\tilde{\alpha}+1}\right) - \frac{A}{(\tilde{\theta}+1)}\tilde{\delta}^{(\tilde{\theta}-\tilde{\alpha})} z_0^{-(\tilde{\theta}+1)}. \tag{A.3}$$

A.3 Additional results and robustness checks

In this section, we present additional results concerning the robustness checks in Section 2.4.3.

Table 12: Robustness check: C-CAPM preference parameter estimates using the *ZeroCorr* data simulation procedure

The table presents the second-step estimates of the C-CAPM preference parameters β and γ. The SMM estimation is based on the moment matches in Equation (2.12) with $\mathcal{T}$=10^7, using the excess returns of the respective test assets and the risk-free rate proxy. The AHB estimates are based on $\mathcal{T}$=16k. All estimates rely on the *ZeroCorr* data simulation procedure. The numbers in parentheses are bootstrap standard errors and the numbers in brackets are the bounds of the 95% confidence intervals computed using the percentile method. The number of bootstrap replications is K=1k. The table also reports the p-values (in percent) of Hansen's J-statistic (see Equation (2.34)) and root mean squared errors (R), computed as explained in Equation (2.35). For the AHB method, R is obtained by averaging over the H=1k replications. Panels A-D break down the results by the MPP used to simulate the disaster-including data (Panels A and B: ACH_X-DPL, Panels C and D: ACH_0-DPL). In all cases, the disaster threshold is q=0.145. In Panels A and C, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panels B and D, the first-step estimation results are based on Bolt and van Zanden's (2014) GDP data. Each panel reports the results by the set of test assets used for estimation (*mkt*, *size dec*, *industry*).

Panel A: ACH_x/Consumption

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.956** (0.021)	**4.09** (0.88)	**0.930** (0.022)	**4.40** (0.76)	28.4	24	**0.942** (0.018)	**4.29** (0.72)	53.5	26
	[0.902 0.986]	[2.50 5.38]	[0.888 0.974]	[2.69 5.62]			[0.907 0.978]	[2.60 5.48]		
AHB	**0.950** (0.018)	**4.86** (0.78)	**0.941** (0.012)	**5.04** (0.84)		49	**0.950** (0.009)	**4.89** (0.80)		43
	[0.910 0.975]	[3.37 6.39]	[0.913 0.958]	[3.41 6.68]			[0.928 0.964]	[3.35 6.45]		

Panel B: ACH_x/GDP

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.955** (0.021)	**3.94** (0.71)	**0.929** (0.022)	**4.21** (0.75)	30.0	21	**0.940** (0.018)	**4.12** (0.73)	43.4	30
	[0.907 0.983]	[2.44 5.35]	[0.886 0.974]	[2.58 5.46]			[0.907 0.975]	[2.51 5.37]		
AHB	**0.949** (0.026)	**4.82** (0.85)	**0.942** (0.011)	**4.98** (0.87)		52	**0.951** (0.009)	**4.84** (0.84)		46
	[0.899 0.976]	[3.21 6.47]	[0.915 0.959]	[3.16 6.63]			[0.931 0.965]	[3.08 6.42]		

Panel C: ACH_0/Consumption

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.948** (0.021)	**4.14** (0.74)	**0.922** (0.022)	**4.23** (0.80)	24.2	39	**0.937** (0.018)	**4.13** (0.78)	71.9	24
	[0.901 0.983]	[2.36 5.34]	[0.886 0.973]	[2.40 5.66]			[0.906 0.976]	[2.37 5.51]		
AHB	**0.951** (0.025)	**4.82** (0.78)	**0.941** (0.011)	**5.05** (0.85)		47	**0.950** (0.008)	**4.90** (0.81)		42
	[0.901 0.975]	[3.30 6.35]	[0.917 0.958]	[3.41 6.84]			[0.932 0.964]	[3.31 6.62]		

Panel D: ACH_0/GDP

	mkt		**size dec**				**industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.953** (0.023)	**3.92** (0.76)	**0.938** (0.020)	**4.06** (0.80)	36.8	34	**0.947** (0.016)	**3.98** (0.78)	10.2	26
	[0.901 0.985]	[2.36 5.37]	[0.893 0.970]	[2.33 5.64]			[0.911 0.974]	[2.35 5.48]		
AHB	**0.951** (0.022)	**4.83** (0.85)	**0.942** (0.011)	**5.01** (0.91)		51	**0.952** (0.008)	**4.86** (0.88)		46
	[0.900 0.976]	[3.26 6.61]	[0.919 0.959]	[3.29 6.86]			[0.933 0.965]	[3.24 6.64]		

Table 13: Robustness check: Key financial indicators implied by a disaster-including C-CAPM using the *ZeroCorr* data simulation procedure

The table presents estimates of the mean risk-free rate, mean market return, equity premium, and market Sharpe ratio implied by a disaster-including C-CAPM. These indicators are computed as given by Equations (2.36), (2.37), and (2.38). The estimates of the subjective discount factor and the RRA coefficient are taken from Table 12. The numbers in brackets are the bounds of the 95% confidence intervals based on K=1k bootstrap replications and computed using the percentile method. Panels A-D break down the results by the MPP used to simulate the disaster-including data (Panels A and B: ACH_X-DPL, Panels C and D: ACH_0-DPL). In Panels A and C, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panels B and D, the first-step estimation results are based on Bolt and van Zanden's (2014) cross-country panel GDP data. Each panel reports the results by set of test assets used for estimation (*mkt*, *size dec*, *industry*). Panel A also contains the values of the indicators in the empirical data (1947:Q2–2014:Q4).

	Panel A: ACH_x/Consumption			
	data	**mkt**	**size dec**	**industry**
mean risk-free rate (% per qtr)	*0.17*	0.17 [0.12 0.23]	0.18 [0.13 0.27]	0.18 [0.13 0.26]
equity premium (% per qtr)	*1.94*	1.79 [0.74 2.82]	2.57 [1.07 3.67]	2.19 [0.95 3.08]
mean market return (% per qtr)	*2.11*	1.96 [0.92 2.97]	2.75 [1.25 3.87]	2.37 [1.17 3.29]
Sharpe ratio (market)	*0.237*	0.212 [0.086 0.352]	0.305 [0.118 0.450]	0.260 [0.110 0.391]
	Panel B: ACH_x/GDP			
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.18 [0.13 0.27]	0.18 [0.14 0.27]
equity premium (% per qtr)		1.81 [0.90 2.75]	2.51 [1.12 3.88]	2.17 [1.06 3.12]
mean market return (% per qtr)		1.98 [1.06 2.92]	2.69 [1.30 4.10]	2.35 [1.27 3.33]
Sharpe ratio (market)		0.215 [0.102 0.345]	0.299 [0.129 0.466]	0.258 [0.123 0.388]
	Panel C: ACH_0/Consumption			
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.19 [0.13 0.26]	0.18 [0.13 0.26]
equity premium (% per qtr)		1.78 [0.84 2.71]	2.76 [1.03 3.60]	2.32 [1.00 3.12]
mean market return (% per qtr)		1.96 [1.01 2.89]	2.95 [1.24 3.78]	2.50 [1.18 3.31]
Sharpe ratio (market)		0.211 [0.096 0.339]	0.327 [0.122 0.440]	0.275 [0.115 0.385]
	Panel D: ACH_0/GDP			
		mkt	**size dec**	**industry**
mean risk-free rate (% per qtr)		0.17 [0.12 0.22]	0.19 [0.13 0.26]	0.18 [0.14 0.25]
equity premium (% per qtr)		1.80 [0.78 2.79]	2.76 [1.14 3.52]	2.38 [1.08 3.09]
mean market return (% per qtr)		1.98 [0.98 2.95]	2.94 [1.32 3.73]	2.56 [1.28 3.28]
Sharpe ratio (market)		0.214 [0.093 0.338]	0.328 [0.133 0.430]	0.283 [0.123 0.372]

Table 14: Robustness check: Estimation results using only excess returns as test assets (ACH_0)

The table presents the second-step estimates of the RRA coefficient γ. The SMM estimation results are based on the moment matches in Equation (2.13) using the excess returns of the respective test assets. The other estimation settings ($\mathcal{T}$=10^7 for SMM, $\mathcal{T}$=16k for AHB, q=0.145, K=H=1k), and the reported statistics correspond to Table 4. The simulation of disaster-including data makes use of the first-step ACH_0-DPL estimates. In Panel A, the first-step estimation results are based on the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). In Panel B, the first-step estimation results are based on Bolt and van Zanden's (2014) GDP data. Each panel breaks down the results by set of test assets (*mkt*, *size dec*, *industry*) and data simulation procedures (*TailCorr*, *EmpCorr*, *ZeroCorr*).

	Panel A: Consumption 1800-2009						
	TailCorr/mkt	**TailCorr/size dec**			**TailCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.47** (0.64)	**3.56** (0.69)	32.7	25	**3.47** (0.67)	68.9	24
	[1.98 4.59]	[2.13 4.87]			[2.09 4.75]		
AHB	**4.03** (0.73)	**4.32** (0.80)		26	**4.18** (0.77)		25
	[2.67 5.60]	[2.85 6.06]			[2.77 5.86]		
	EmpCorr/mkt	**EmpCorr/size dec**			**EmpCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.99** (0.73)	**4.07** (0.79)	29.9	40	**4.00** (0.77)	61.2	26
	[2.27 5.23]	[2.31 5.51]			[2.30 5.38]		
AHB	**4.67** (0.78)	**4.91** (0.85)		52	**4.78** (0.81)		46
	[3.16 6.22]	[3.28 6.67]			[3.20 6.50]		
	ZeroCorr/mkt	**ZeroCorr/size dec**			**ZeroCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**4.14** (0.74)	**4.23** (0.80)	34.5	45	**4.12** (0.78)	72.9	26
	[2.36 5.34]	[2.40 5.65]			[2.37 5.51]		
AHB	**4.82** (0.78)	**5.05** (0.85)		52	**4.89** (0.81)		47
	[3.30 6.35]	[3.40 6.84]			[3.31 6.61]		
	Panel B: GDP 1900-2010						
	TailCorr/mkt	**TailCorr/size dec**			**TailCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.35** (0.65)	**3.51** (0.69)	32.6	25	**3.43** (0.67)	67.3	24
	[2.05 4.67]	[2.08 4.87]			[2.06 4.72]		
AHB	**4.05** (0.78)	**4.30** (0.85)		25	**4.17** (0.82)		25
	[2.62 5.70]	[2.74 6.08]			[2.67 5.89]		
	EmpCorr/mkt	**EmpCorr/size dec**			**EmpCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.80** (0.74)	**3.94** (0.78)	36.2	32	**3.87** (0.76)	10.9	29
	[2.33 5.26]	[2.28 5.49]			[2.28 5.36]		
AHB	**4.68** (0.85)	**4.86** (0.91)		56	**4.74** (0.87)		49
	[3.13 6.46]	[3.17 6.69]			[3.13 6.52]		
	ZeroCorr/mkt	**ZeroCorr/size dec**			**ZeroCorr/industry**		
	γ	γ	J	R	γ	J	R
SMM	**3.92** (0.76)	**4.06** (0.80)	42.9	38	**3.98** (0.78)	12.7	29
	[2.36 5.37]	[2.33 5.63]			[2.34 5.48]		
AHB	**4.83** (0.85)	**5.00** (0.91)		57	**4.86** (0.88)		51
	[3.26 6.61]	[3.28 6.86]			[3.23 6.64]		

Table 15: Robustness check: C-CAPM preference parameter estimates with varying q (ACH_0)

The table presents the second-step estimates of the preference parameters β and γ using alternative disaster thresholds. Panel A uses a disaster threshold of q=0.095, and Panel B uses q=0.195. The other estimation settings (moment matches, $\mathcal{T}=10^7$ for SMM, $\mathcal{T}$=16k for AHB, K=H=1k), and the reported statistics correspond to Table 4. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The simulation of disaster-including data is based on the first-step ACH_0-DPL estimates. Each panel breaks down the results by set of test assets (*mkt*, *size dec*, *industry*) and data simulation procedures (*TailCorr*, *EmpCorr*, *ZeroCorr*).

					Panel A: q=0.095					
	TailCorr/mkt		**TailCorr/size dec**				**TailCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.983** (0.006)	**3.41** (0.64)	**0.978** (0.007)	**3.58** (0.65)	30.0	24	**0.981** (0.006)	**3.48** (0.62)	65.3	23
	[0.972 0.996]	[2.17 4.58]	[0.963 0.992]	[2.34 4.86]			[0.969 0.992]	[2.32 4.70]		
AHB	**0.981** (0.002)	**3.94** (0.72)	**0.974** (0.003)	**4.18** (0.81)		24	**0.978** (0.002)	**4.03** (0.77)		23
	[0.977 0.985]	[2.73 5.56]	[0.968 0.979]	[2.78 5.93]			[0.973 0.982]	[2.70 5.72]		
	EmpCorr/mkt		**EmpCorr/size dec**				**EmpCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.950** (0.023)	**4.08** (0.74)	**0.935** (0.022)	**4.20** (0.76)	25.1	30	**0.943** (0.019)	**4.12** (0.73)	5.8	30
	[0.898 0.987]	[2.50 5.34]	[0.888 0.977]	[2.65 5.66]			[0.902 0.977]	[2.66 5.53]		
AHB	**0.943** (0.033)	**4.83** (0.83)	**0.937** (0.010)	**4.97** (0.92)		55	**0.944** (0.009)	**4.84** (0.89)		48
	[0.872 0.972]	[3.28 6.63]	[0.914 0.955]	[3.27 6.88]			[0.927 0.960]	[3.21 6.67]		
	ZeroCorr/mkt		**ZeroCorr/size dec**				**ZeroCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.933** (0.028)	**4.26** (0.73)	**0.911** (0.029)	**4.38** (0.78)	23.0	33	**0.925** (0.023)	**4.28** (0.75)	7.2	31
	[0.873 0.979]	[2.69 5.49]	[0.857 0.967]	[2.81 5.88]			[0.881 0.969]	[2.80 5.73]		
AHB	**0.928** (0.038)	**5.04** (0.84)	**0.921** (0.016)	**5.17** (0.94)		56	**0.933** (0.012)	**5.01** (0.90)		50
	[0.846 0.966]	[3.44 6.87]	[0.885 0.947]	[3.40 7.08]			[0.908 0.953]	[3.34 6.84]		
					Panel B: q=0.195					
	TailCorr/mkt		**TailCorr/size dec**				**TailCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.987** (0.005)	**3.41** (0.66)	**0.981** (0.007)	**3.71** (0.70)	30.6	24	**0.984** (0.005)	**3.62** (0.67)	68.5	23
	[0.976 0.998]	[2.04 4.76]	[0.969 0.994]	[2.14 4.92]			[0.974 0.995]	[2.15 4.80]		
AHB	**0.985** (0.002)	**4.12** (0.79)	**0.978** (0.002)	**4.33** (0.84)		24	**0.982** (0.002)	**4.20** (0.81)		23
	[0.981 0.988]	[2.69 5.97]	[0.973 0.982]	[2.82 6.12]			[0.978 0.985]	[2.74 5.92]		
	EmpCorr/mkt		**EmpCorr/size dec**				**EmpCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.969** (0.014)	**3.79** (0.73)	**0.956** (0.015)	**4.13** (0.78)	15.4	25	**0.962** (0.012)	**4.06** (0.74)	68.5	21
	[0.938 0.992]	[2.27 5.27]	[0.926 0.984]	[2.35 5.46]			[0.936 0.984]	[2.34 5.32]		
AHB	**0.968** (0.010)	**4.60** (0.81)	**0.959** (0.006)	**4.79** (0.85)		43	**0.965** (0.005)	**4.66** (0.81)		39
	[0.942 0.983]	[3.07 6.32]	[0.945 0.969]	[3.18 6.53]			[0.953 0.974]	[3.12 6.35]		
	ZeroCorr/mkt		**ZeroCorr/size dec**				**ZeroCorr/industry**			
	β	γ	β	γ	J	R	β	γ	J	R
SMM	**0.962** (0.016)	**3.90** (0.74)	**0.947** (0.018)	**4.24** (0.79)	17.0	24	**0.955** (0.014)	**4.15** (0.75)	74.2	21
	[0.925 0.989]	[2.34 5.37]	[0.911 0.979]	[2.43 5.58]			[0.925 0.981]	[2.41 5.42]		
AHB	**0.963** (0.012)	**4.71** (0.81)	**0.952** (0.008)	**4.89** (0.85)		43	**0.960** (0.007)	**4.75** (0.81)		39
	[0.934 0.980]	[3.13 6.43]	[0.933 0.966]	[3.26 6.62]			[0.945 0.971]	[3.20 6.43]		

Table 16: Robustness Check: Effect of a varying $\mathcal{T}$ on AHB parameter estimates using the ACH_X model (*EmpCorr*/*ZeroCorr*)

The table reports the AHB estimates of the subjective discount factor and the coefficient of relative risk aversion for a varying $\mathcal{T}$. Panel A uses the *EmpCorr* data simulation procedure and Panel B uses *ZeroCorr*. The 95% quantiles of the parameter estimates from the H=1k simulated histories are underlined, and the standard deviations are reported in parentheses. Columns labeled p report the relative frequency for $\gamma^{(h)} > 10$ (in percent). The last column reports the percentage of simulated histories for which no consumption disaster occurs. The estimations use the excess returns of the portfolios in the sets of test assets (*mkt*, *size dec*, *industry*), which in each case are augmented by the risk-free rate. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates. The disaster threshold is q=0.145.

Panel A: *EmpCorr*

	mkt			**size dec**			**industry**			
$\mathcal{T}$	β	γ	p	β	γ	p	β	γ	p	no disaster
271	0.975	14.01	36.7	1.073	39.25	52.5	1.085	39.20	50.7	46.3
	(0.108)	(28.54)		(0.196)	(63.82)		(0.197)	(57.70)		
	1.021	22.50		1.473	175.45		1.494	163.85		
1k	0.961	7.22	16.7	0.967	9.14	18.2	0.974	8.94	16.2	2.3
	(0.032)	(3.47)		(0.058)	(17.32)		(0.057)	(17.76)		
	0.995	14.18		0.991	14.90		0.996	14.27		
5k	0.958	5.27	0.0	0.952	5.46	0.2	0.959	5.30	0.2	0.0
	(0.028)	(1.28)		(0.010)	(1.33)		(0.008)	(1.27)		
	0.981	7.44		0.967	7.72		0.971	7.48		
16k	0.959	4.71	0.0	0.950	4.90	0.0	0.957	4.77	0.0	0.0
	(0.014)	(0.78)		(0.008)	(0.83)		(0.006)	(0.80)		
	0.977	5.98		0.962	6.27		0.966	6.08		

Panel B: *ZeroCorr*

	mkt			**size dec**			**industry**			
$\mathcal{T}$	β	γ	p	β	γ	p	β	γ	p	no disaster
271	0.973	13.97	36.8	1.073	39.22	52.5	1.085	39.17	50.5	46.3
	(0.109)	(28.51)		(0.197)	(63.82)		(0.198)	(57.70)		
	1.024	21.94		1.473	175.45		1.494	163.85		
1k	0.956	7.31	16.8	0.962	9.23	18.3	0.971	9.02	16.2	2.3
	(0.038)	(3.39)		(0.060)	(17.29)		(0.058)	(17.74)		
	0.995	14.08		0.991	14.78		0.997	14.15		
5k	0.951	5.42	0.0	0.945	5.60	0.2	0.954	5.42	0.2	0.0
	(0.025)	(1.28)		(0.013)	(1.33)		(0.010)	(1.27)		
	0.979	7.58		0.963	7.90		0.968	7.61		
16k	0.950	4.86	0.0	0.941	5.04	0.0	0.950	4.89	0.0	0.0
	(0.018)	(0.78)		(0.012)	(0.84)		(0.009)	(0.80)		
	0.973	6.16		0.956	6.42		0.962	6.20		

Table 17: Robustness Check: Effect of a varying $\mathcal{T}$ on AHB parameter estimates using the ACH$_0$ model (*EmpCorr*/*ZeroCorr*)

The table reports the AHB estimates of the subjective discount factor and the coefficient of relative risk aversion for a varying $\mathcal{T}$. Panel A uses the *EmpCorr* data simulation procedure and Panel B uses *ZeroCorr*. The 95% quantiles of the parameter estimates from the H=1k simulated histories are underlined, and the standard deviations are reported in parentheses. Columns labeled p report the relative frequency for $\gamma^{(h)} > 10$ (in percent). The last column reports the percentage of simulated histories for which no consumption disaster occurs. The estimations use the excess returns of the portfolios in the sets of test assets (*mkt*, *size dec*, *industry*), which in each case are augmented by the risk-free rate. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The simulation of disaster-including data is based on the first-step ACH$_0$-DPL estimates. The disaster threshold is q=0.145.

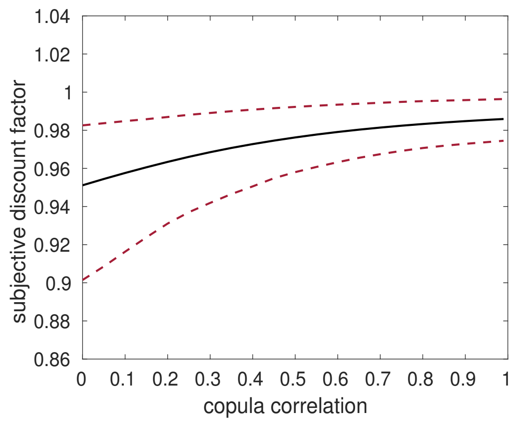
Panel A: *EmpCorr*

	mkt			size dec			industry			
$\mathcal{T}$	β	γ	p	β	γ	p	β	γ	p	no disaster
271	0.973	12.75	40.1	1.056	33.01	49.7	1.066	32.89	47.9	37.7
	(0.106)	(22.89)		(0.181)	(52.99)		(0.185)	(51.84)		
	1.023	21.86		1.474	162.09		1.475	160.50		
1k	0.962	7.57	20.8	0.961	7.93	22.6	0.969	7.65	20.5	0.2
	(0.032)	(3.85)		(0.024)	(5.16)		(0.023)	(5.46)		
	0.999	14.89		0.994	15.84		0.999	15.13		
5k	0.959	5.18	0.1	0.952	5.48	0.5	0.959	5.31	0.3	0.0
	(0.019)	(1.34)		(0.009)	(1.43)		(0.008)	(1.37)		
	0.981	7.46		0.966	8.01		0.972	7.81		
16k	0.959	4.67	0.0	0.950	4.91	0.0	0.957	4.78	0.0	0.0
	(0.016)	(0.78)		(0.008)	(0.85)		(0.006)	(0.81)		
	0.976	6.02		0.961	6.38		0.966	6.21		

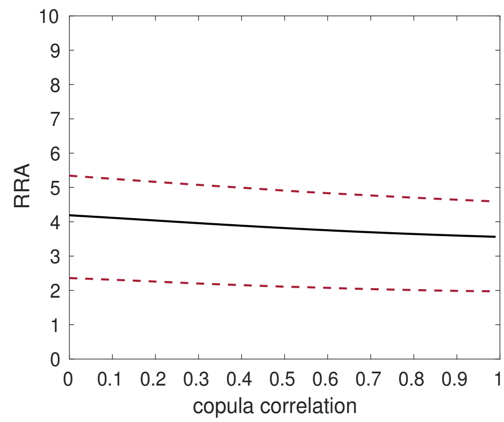
Panel B: *ZeroCorr*

	mkt			size dec			industry			
$\mathcal{T}$	β	γ	p	β	γ	p	β	γ	p	no disaster
271	0.971	12.70	39.0	1.055	32.96	49.7	1.065	32.86	47.5	37.7
	(0.109)	(22.86)		(0.182)	(53.00)		(0.185)	(51.84)		
	1.024	21.43		1.474	162.09		1.475	160.50		
1k	0.957	7.65	21.2	0.957	8.01	23.0	0.966	7.71	20.6	0.2
	(0.041)	(3.75)		(0.027)	(5.09)		(0.025)	(5.40)		
	1.000	14.65		0.995	15.75		1.000	15.07		
5k	0.952	5.33	0.1	0.945	5.62	0.7	0.954	5.43	0.3	0.0
	(0.023)	(1.33)		(0.013)	(1.43)		(0.010)	(1.37)		
	0.978	7.61		0.962	8.14		0.969	7.92		
16k	0.951	4.82	0.0	0.941	5.05	0.0	0.950	4.90	0.0	0.0
	(0.025)	(0.78)		(0.011)	(0.85)		(0.008)	(0.81)		
	0.972	6.17		0.955	6.52		0.962	6.34		

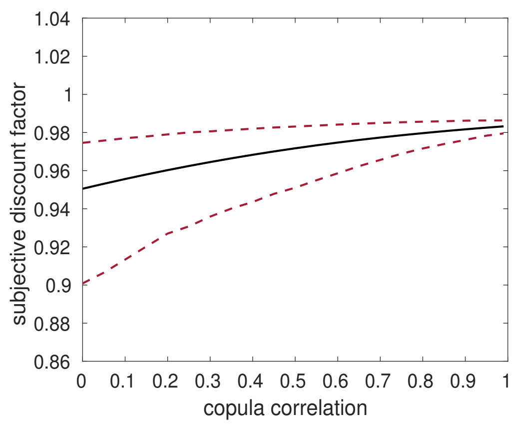
Figure 8: Robustness check: Effect of a varying copula correlation
The figure depicts the estimates of the subjective discount factor β (Panels (a) and (c)) and the RRA coefficient γ (Panels (b) and (d)) using a varying copula correlation ρ. The simulation of disaster-including data is based on the first-step ACH_0-DPL estimates using the updated cross-country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. Test assets are the excess return of the market portfolio (mkt) and the risk-free rate. Panels (a) and (b) refer to the SMM estimates, and Panels (c) and (d) pertain to the AHB estimates. The dashed (red) lines are the 95% confidence bounds.



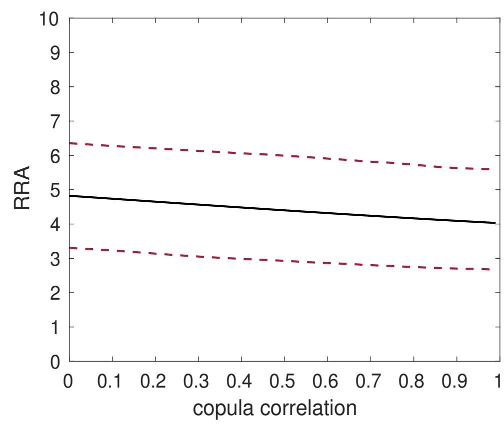
(a) SMM with $\mathbf{ACH_0}$: $\hat{\beta}$



(b) SMM with $\mathbf{ACH_0}$: $\hat{\gamma}$



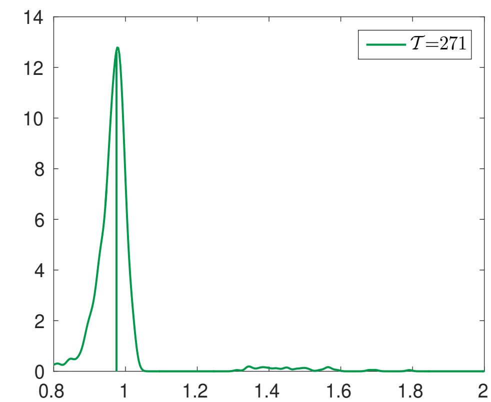
(c) AHB with $\mathbf{ACH_0}$: $\hat{\beta}$



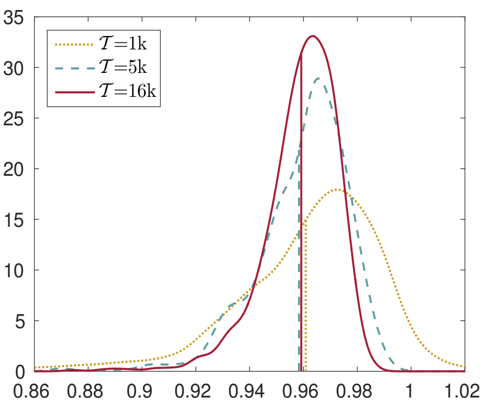
(d) AHB with $\mathbf{ACH_0}$: $\hat{\gamma}$

Figure 9: Robustness Check: Effect of varying $\mathcal{T}$ on AHB parameter estimates (*EmpCorr*/$\mathbf{ACH_X}$)

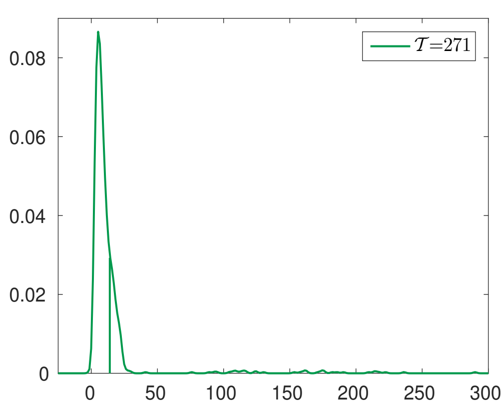
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates and the *EmpCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



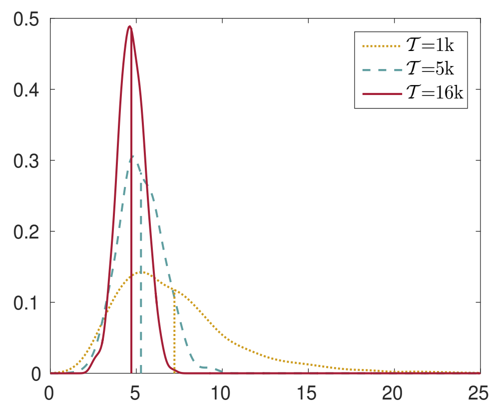
(a) $\hat{\beta}$ for small $\mathcal{T}$



(b) $\hat{\beta}$ for large $\mathcal{T}$



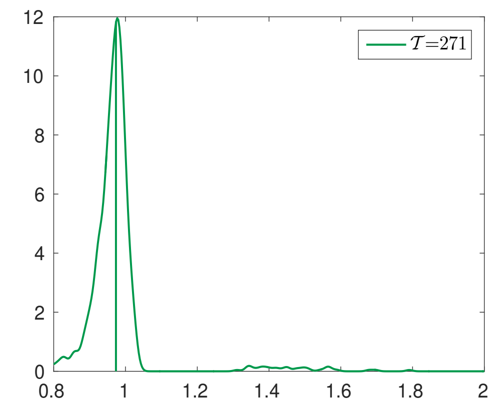
(c) $\hat{\gamma}$ for small $\mathcal{T}$



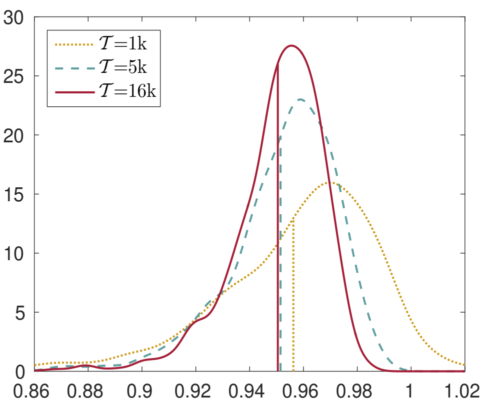
(d) $\hat{\gamma}$ for large $\mathcal{T}$

Figure 10: Robustness Check: Effect of varying $\mathcal{T}$ on AHB parameter estimates (*ZeroCorr*/$\mathbf{ACH_X}$)

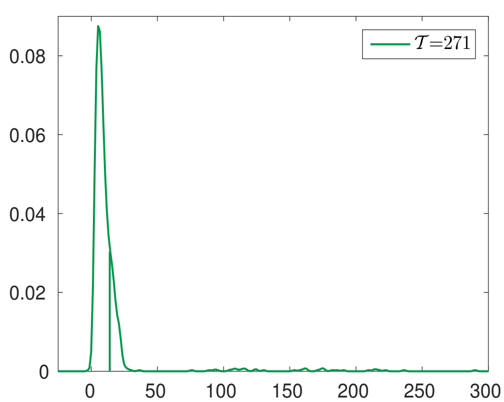
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH_X-DPL estimates and the *ZeroCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



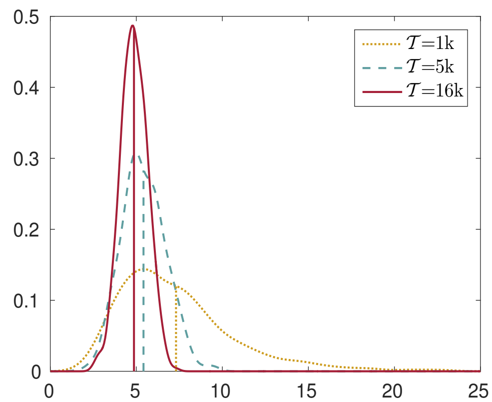
(a) $\hat{\beta}$ for small $\mathcal{T}$



(b) $\hat{\beta}$ for large $\mathcal{T}$



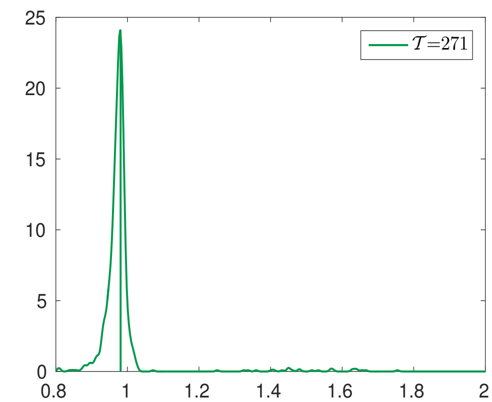
(c) $\hat{\gamma}$ for small $\mathcal{T}$



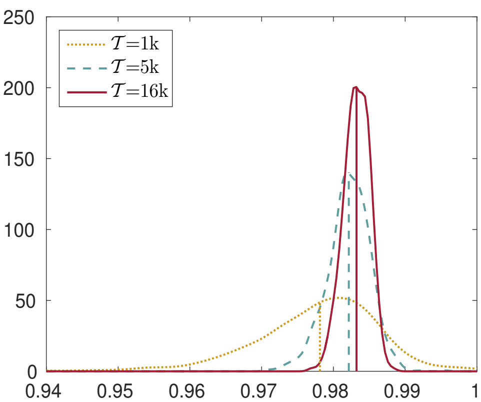
(d) $\hat{\gamma}$ for large $\mathcal{T}$

Figure 11: Robustness Check: Effect of varying $\mathcal{T}$ on AHB parameter estimates (*TailCorr*/ACH$_0$)

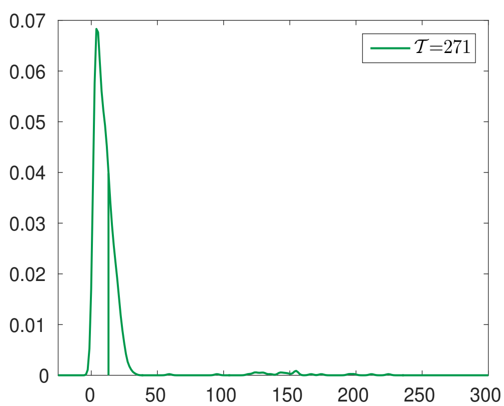
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH$_0$-DPL estimates and the *TailCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



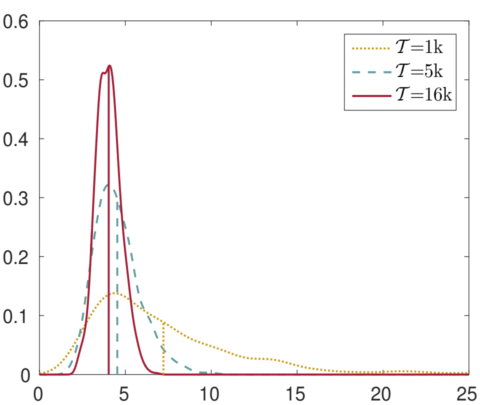
(a) $\hat{\beta}$ for small $\mathcal{T}$



(b) $\hat{\beta}$ for large $\mathcal{T}$



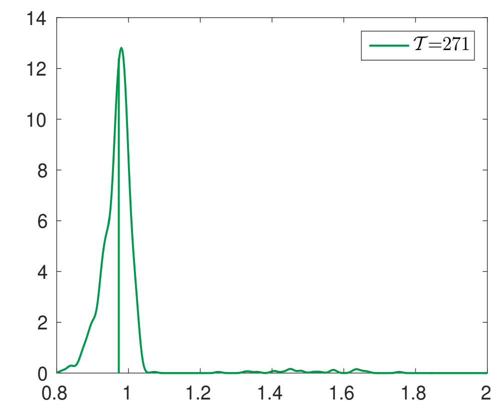
(c) $\hat{\gamma}$ for small $\mathcal{T}$



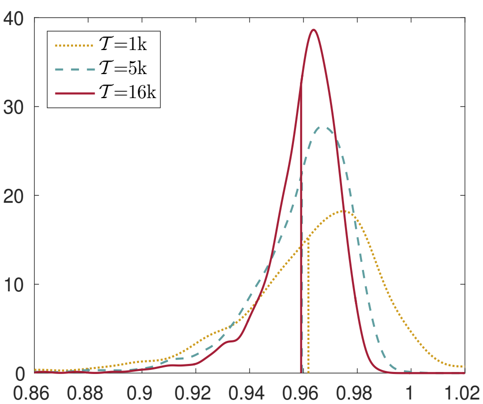
(d) $\hat{\gamma}$ for large $\mathcal{T}$

Figure 12: Robustness Check: Effect of varying $\mathcal{T}$ on AHB parameter estimates (*EmpCorr*/ACH$_0$)

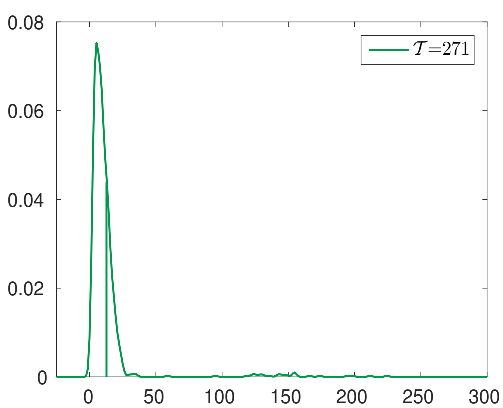
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH$_0$-DPL estimates and the *EmpCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



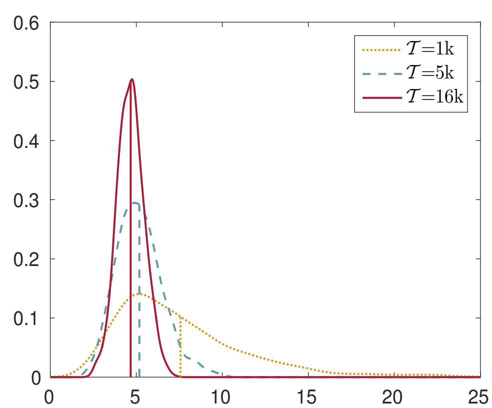
(a) $\hat{\beta}$ for small $\mathcal{T}$



(b) $\hat{\beta}$ for large $\mathcal{T}$



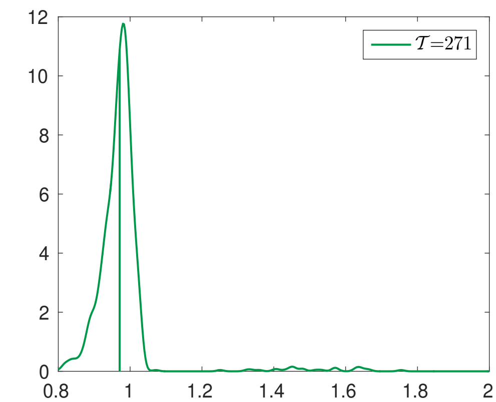
(c) $\hat{\gamma}$ for small $\mathcal{T}$



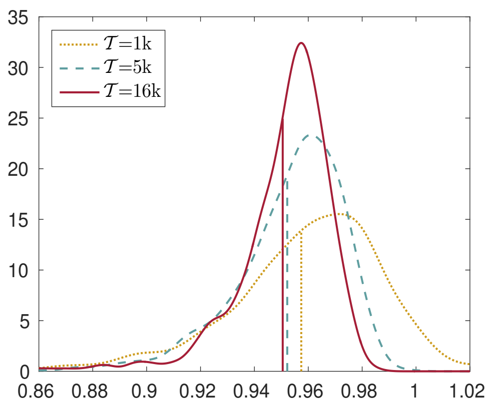
(d) $\hat{\gamma}$ for large $\mathcal{T}$

Figure 13: Robustness Check: Effect of varying $\mathcal{T}$ on AHB parameter estimates ($\textit{ZeroCorr}$/ACH$_0$)

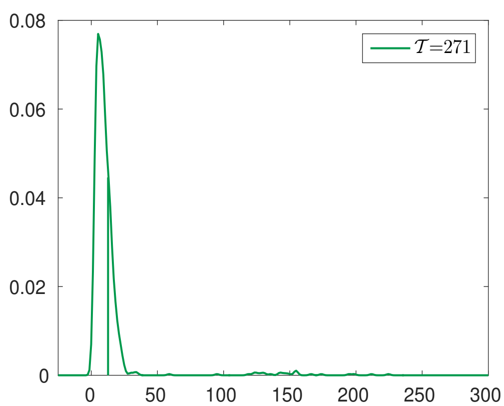
The four panels depict kernel densities of the AHB estimates of the subjective discount factor β (Panels (a) and (b)) and the RRA coefficient γ (Panels (c) and (d)). Test assets are the excess return of the market portfolio *mkt* and the risk-free rate proxy. The first-step estimation results are based on the updated country panel consumption data originally assembled by Barro and Ursúa (2008). The disaster threshold is q=0.145. The simulation of disaster-including data is based on the first-step ACH$_0$-DPL estimates and the *ZeroCorr* data simulation procedure. We use H=1k and vary $\mathcal{T}$ from 271 (Panels (a) and (c)), to 1k, 5k, and 16k (Panels (b) and (d)). The solid (green) densities in Panels (a) and (c) use $\mathcal{T}$=271. The dotted (golden) densities in Panels (b) and (d) use $\mathcal{T}$=1k, the dashed (cyan) densities reflect $\mathcal{T}$=5k, and the solid (red) densities use $\mathcal{T}$=16k. The AHB point estimates are indicated by vertical lines. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



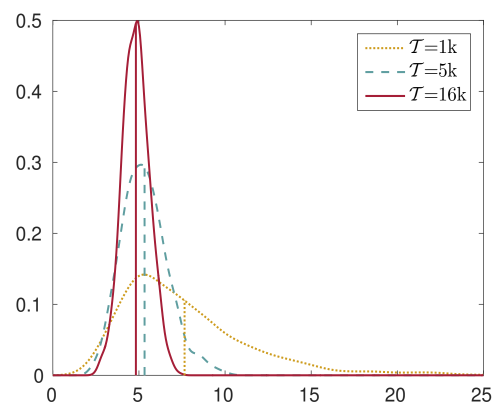
(a) $\hat{\beta}$ for small $\mathcal{T}$



(b) $\hat{\beta}$ for large $\mathcal{T}$



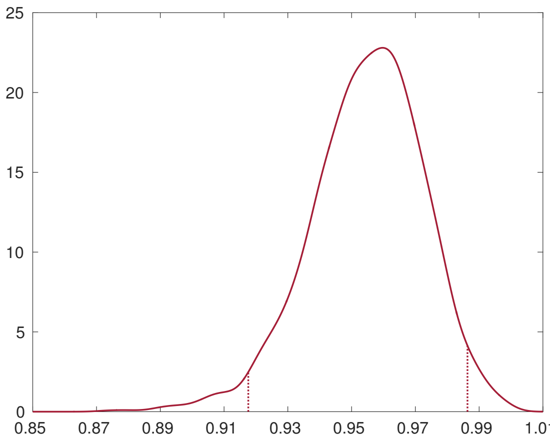
(c) $\hat{\gamma}$ for small $\mathcal{T}$



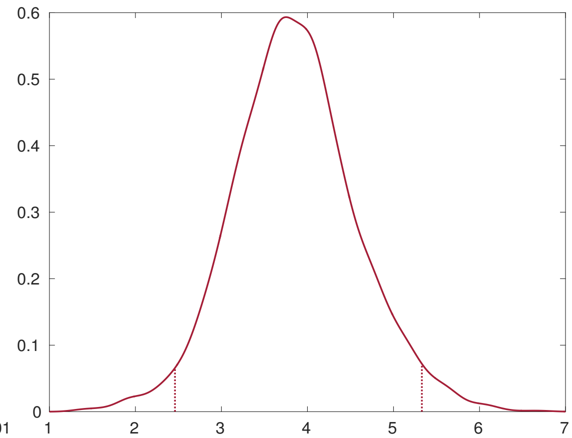
(d) $\hat{\gamma}$ for large $\mathcal{T}$

Figure 14: Robustness Check: Kernel estimates using pooled SMM bootstrap results (*EmpCorr*/*ZeroCorr*)

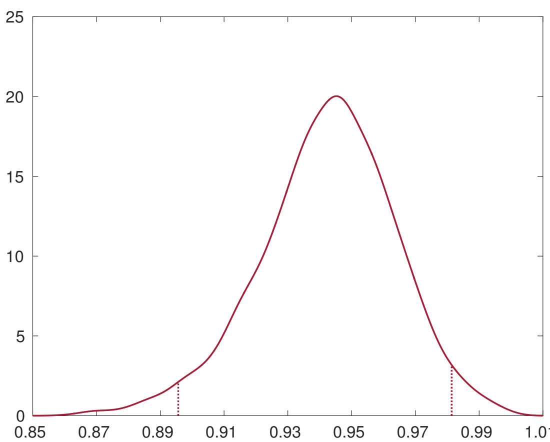
The figure depicts kernel densities using pooled bootstrap SMM estimates of β (Panels (a) and (c)) and γ (Panels (b) and (d)). The results of the K=1k bootstrap replications are pooled over three sets of test assets (*mkt*, *size dec*, *industry*), each of which is augmented by the risk-free rate. Panels (a) and (b) use the *EmpCorr* data simulation procedure and Panels (c) and (d) use *ZeroCorr*. All estimations use the ACH$_{\text{X}}$-DPL and Barro and Ursúa's (2008) consumption data with q=0.145. The dotted lines indicate the 2.5% and 97.5% quantiles, respectively. We use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



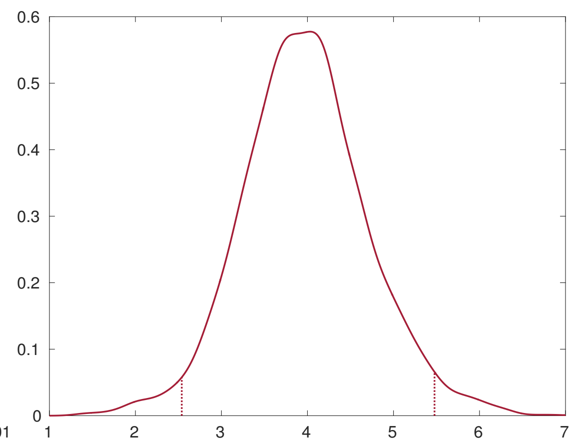
(a) *EmpCorr*: $\hat{\beta}$



(b) *EmpCorr*: $\hat{\gamma}$



(c) *ZeroCorr*: $\hat{\beta}$



(d) *ZeroCorr*: $\hat{\gamma}$

CHAPTER 3

Asset Pricing with Multi-Period Disasters and Partial Government Defaults†

3.1 Motivation

This chapter proposes a novel econometric strategy to resolve the inherent sample selection problem that is implied by the RDH, and to estimate and test an asset pricing model with recursive investor preferences that accounts for the possibility of rare and severe consumption contractions and partial government defaults. The moment restrictions implied by such a disaster-including C-CAPM are used for a simulation-based estimation of its structural parameters. By allowing for multi-period disasters, which are modeled as a marked point process (MPP), I can address the caveat that the success of the RDH may hinge on the assumption that a consumption disaster must unfold within a single period. The econometric analysis comprises two consecutive steps: maximum likelihood to estimate the MPP parameters using cross-country consumption data, and then a simulation-based estimation of the investor preference parameters based on U.S. macro and financial data. A bootstrap procedure gauges the estimation precision. To the best of my knowledge, this is the first study to estimate and test a C-CAPM that accounts for the possibility of multi-period disasters and partial government defaults.

The empirical analysis shows that the estimates of the investor preference parameters – relative risk aversion, the intertemporal elasticity of substitution, and the subjective discount factor – fall within a range that is economically meaningful, and they feature narrow bootstrap confidence bounds. Specifically, the estimates of the subjective discount factor are smaller than but close to unity, as would be expected of an investor with a reasonable positive rate of time preference. The RRA coefficient estimates range between 1.5 and 1.7; generally, RRA values <10 describe a reasonably risk averse investor (e.g., Mehra and Prescott (1985); Rietz (1988); Bansal and Yaron (2004)). Cochrane (2005) caps the interval of sensible relative risk aversion more strictly at 5, in line with results reported by Meyer and Meyer (2005). For the present study, the 95% confidence interval for the RRA estimate also lies within this strict plausibility range. In addition, the IES estimates are (significantly)

† This chapter is based on Sönksen (2017a), available on ssrn: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2789621

greater than unity and of a magnitude that is frequently assumed for calibrations. Moreover, the estimated RRA coefficient is (significantly) greater than the reciprocal of the IES estimate, which provides evidence that investors prefer early resolution of uncertainty. Several studies emphasize that an IES greater than 1, combined with a preference for early resolution of uncertainty, is necessary to obtain meaningful asset pricing implications from a C-CAPM (e.g., Bansal and Yaron (2004); Barro (2009); Nakamura et al. (2013)).

Accordingly, the model-implied key financial indicators – mean market return, T-bill return, equity premium, and market Sharpe ratio – exhibit meaningful magnitudes and are consistent with the empirically observed counterparts. These findings are robust with respect to alternative model specifications (e.g., first-step model, disaster definition, data simulation procedures). Compared with other prominent attempts to vindicate the C-CAPM paradigm, these results are encouraging. Empirical asset pricing studies often find implausible or imprecise parameter estimates that entail doubtful asset pricing implications, calling into question the explanatory power of the C-CAPM paradigm. The present results indicate instead that accounting for rare disasters in a consumption-based asset pricing framework helps restore the nexus between financial markets and the real economy.

The present study re-emphasizes the explanatory power of the RDH by showing that the equity premium can be explained with plausible preference parameters and assumptions regarding the disaster process. However, it is important to assume Epstein-Zin-Weil preferences instead of an additive power utility. As some related literature implies, it is crucial to allow for a preference for early resolution of uncertainty, and the IES and RRA both must be greater than unity. Accounting for the possibility of multi-period disasters and partial government default in an empirical C-CAPM yields conforming RRA and IES estimates and thus meaningful asset pricing implications.

The remainder of this chapter is structured as follows: Section 3.2 details the motivation for a multi-period disaster-including C-CAPM with recursive preferences and derives moment restrictions that provide the basis for the simulated method of moments-type estimation strategy. It also introduces a marked point process to explain the size and duration of and between disaster events. Section 3.3 contains the macroeconomic and financial data used in this study, and Section 3.4 describes the two-step estimation strategy. After a discussion of the estimation results and robustness tests in Section 3.5, Section 3.6 concludes.

3.2 Multi-period disasters in a C-CAPM

3.2.1 Asset pricing implications and moment restrictions

To formulate an empirically estimable asset pricing model that accounts for the possibility of multi-period disasters, I follow Barro (2006) and assume that consumption growth evolves as

$$\frac{C_{t+1}}{C_t} = e^{u_{t+1}} e^{v_{t+1}}, \tag{3.1}$$

where $u_{t+1} \sim (\tilde{\mu}, \sigma^2)$, $v_{t+1} = \ln(1 - b_{t+1})d_{t+1}$, and $e^{u_{t+1}}$ describes consumption growth in non-disastrous times. The term $\ln(1 - b_{t+1})$ comes into force only if the respective period is affected by a disaster, that is, if the binary disaster indicator d_{t+1} equals 1. In this case, the non-disastrous consumption growth component shrinks by the contraction factor b_{t+1}. Time is discrete, and the observation frequency is fixed (e.g., quarterly). In Barro's (2006) one-period disaster model, $b_{t+1} \in [q, 1]$, where q denotes the disaster threshold that differentiates regular bad times from disasters.

The definition of the contraction factor b_{t+1} must be adapted when accounting for multi-period disasters. Here, a disaster is defined as a succession of contractions that starts in period s_1 and lasts until period s_2, where $s_1 \leq t + 1 \leq s_2$, such that

$$1 - \prod_{j=s_1}^{s_2} (1 - b_j) \geq q. \tag{3.2}$$

In words, I refer to a *disaster event* as a severe decline in consumption at least of size q. The decline may accrue over multiple *disaster periods* or come in the form of one sharp contraction. Disaster periods are indicated by $d_t = 1$ and associated with a contraction factor $b_t \in (0, 1]$. If $d_t = 1$, asset returns will also contract. Adopting Barro's (2006) specification for returns on treasury bills, I assume, analogous to Equation 3.1, that for a gross return of an asset R_i:

$$R_{i,t+1} = (1 - \tilde{b}_{i,t+1})^{d_{t+1}} R_{i,nd,t+1}, \tag{3.3}$$

where $R_{i,nd}$ denotes the asset's gross return in non-disastrous periods, and $\tilde{b}_i$ is the return equivalent of the consumption contraction factor b.

A representative investor, who faces these consumption risks, has recursive preferences; as Epstein and Zin (1989) show, the basic asset pricing equations for a gross return R_i and an excess return $R_i^e = R_i - R_j$, respectively, are then given by:

$$\mathbb{E}_t \left[m_{t+1}(\beta, \gamma, \psi) R_{i,t+1} \right] = 1 \quad \text{and} \quad \mathbb{E}_t \left[m_{t+1}(\beta, \gamma, \psi) R_{i,t+1}^e \right] = 0, \tag{3.4}$$

where the stochastic discount factor (SDF) reads:

$$m_{t+1}(\beta,\gamma,\psi) = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{a,t+1}^{\theta-1}, \qquad \text{with} \qquad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{3.5}$$

In Equation (3.5), β denotes the subjective discount factor, ψ is the IES, and γ represents the coefficient of relative risk aversion; R_a is the return on aggregate wealth.

By conditioning down the basic asset pricing equation for a gross return, applying the law of total expectations, and using the consumption growth and return specifications from Equations (3.1) and (3.3), we can write:

$$\begin{aligned}\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}e^{v_t}\right)^{-\frac{\theta}{\psi}} R_{a,t}^{\theta-1}R_{i,t}\right] &= p\mathbb{E}\left[\beta^{\theta}\left((1-b_t)e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,d,t}^{\theta-1}R_{i,d,t}\middle| d_t=1\right]\\ &+ (1-p)\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1}R_{i,nd,t}\middle| d_t=0\right]\\ &= 1,\end{aligned} \tag{3.6}$$

where $p = \mathbb{P}(d_t = 1)$ is the unconditional disaster probability, and $R_{i,d,t} = R_{i,nd,t}(1-\tilde{b}_{i,t})$. Rearranging terms in Equation (3.6) yields the following moment restriction:

$$\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1}R_{i,nd,t}\middle| d_t=0\right] = \frac{1-p\mathbb{E}\left[\beta^{\theta}\left((1-b_t)e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,d,t}^{\theta-1}R_{i,d,t}\middle| d_t=1\right]}{1-p}. \tag{3.7}$$

The corresponding moment restriction for an excess return R_i^e reads:

$$\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1}R_{i,nd,t}^{e}\middle| d_t=0\right] = \frac{-p\mathbb{E}\left[\beta^{\theta}\left((1-b_t)e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,d,t}^{\theta-1}R_{i,d,t}^{e}\middle| d_t=1\right]}{1-p}, \tag{3.8}$$

where $R_{i,d}^e = R_{i,d} - R_{j,d}$ and $R_{i,nd}^e = R_{i,nd} - R_{j,nd}$.

Equations (3.7) and (3.8) are of particular interest, because they suggest how theoretical moments that can be approximated using the available non-disastrous data (left-hand sides) can be disentangled from expressions that rely on information about disasters (right-hand sides). In particular, using consumption growth and return data that do not include disasters, we can approximate the left-hand side of Equation (3.7) as follows:

$$\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1}R_{i,nd,t}\middle| d_t=0\right] \approx \frac{1}{T}\sum_{t=1}^{T}\beta^{\theta}cg_{nd,t}^{-\frac{\theta}{\psi}}R_{a,nd,t}^{\theta-1}R_{i,nd,t}, \tag{3.9}$$

where $cg_{nd,t}$ denotes observable, non-disastrous consumption growth. Similarly,

$$\mathbb{E}\left[\beta^{\theta}\left(e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{i,nd,t}^{e} \middle| d_t = 0\right] \approx \frac{1}{T}\sum_{t=1}^{T} \beta^{\theta} cg_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{i,nd,t}^{e}. \tag{3.10}$$

Because U.S. postwar data do not incorporate any disasters, attempting to approximate the right-hand side moments in Equations (3.7) and (3.8) using sample means of the available data would be futile. However, if it were possible to simulate consumption and return processes that account for the possibility of rare disasters, we could consider an approximation by simulated moments, such as:

$$\frac{1 - p\mathbb{E}\left[\beta^{\theta}\left((1-b_t)e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,d,t}^{\theta-1} R_{i,d,t} \middle| d_t = 1\right]}{1-p} \approx \frac{1 - \frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}} \beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_s d_s}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}}}, \tag{3.11}$$

and

$$\frac{-p\mathbb{E}\left[\beta^{\theta}\left((1-b_t)e^{u_t}\right)^{-\frac{\theta}{\psi}} R_{a,d,t}^{\theta-1} R_{i,d,t}^{e} \middle| d_t = 1\right]}{1-p} \approx \frac{-\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}} \beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_s^{e} d_s}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}}}, \tag{3.12}$$

where cg_s, $R_{a,s}$, R_s, and R_s^e denote simulated (disaster-including) consumption growth and (excess) returns, and $D_{\mathcal{T}} = \sum_{s=1}^{\mathcal{T}} d_s$. A large $\mathcal{T}$ ensures a good approximation of population moments by sample means, provided that a uniform law of large numbers holds. In the same spirit by which Singleton motivates the simulated method of moments, *"more fully specified models allow experimentation with alternative formulations of economies and, perhaps, analysis of processes that are more representative of history for which data are not readily available"* (Singleton, 2006, p. 254), the simulation should produce consumption and return data that are representative of history, assuming the RDH is true.

Equations (3.11) and (3.12) provide the basis for the SMM-type estimation of the preference parameters β, γ, and ψ. Before explaining the details of the estimation strategy, it is necessary to specify the stochastic process that generates the disastrous consumption contractions.

3.2.2 Multi-period disasters as a marked point process

I introduce an MPP to model the time duration between disastrous consumption contractions and their size, as well as to account for the duration of the multi-period disasters. In the present application, the disaster periods are the points of the MPP; the contraction sizes are the marks.

I draw on Hamilton and Jorda's (2002) autoregressive conditional hazard (ACH) framework to model the duration between disaster periods. Initially, this approach would set a threshold q to define a disaster event and thereby establish the respective disaster periods and their contraction sizes. Suppose that the sequence of consumption disaster events thus defined is observable at a quarterly frequency. Let $M(t)$ denote the number of disasters that occurred as of quarter t and let $N(t)$ refer to the respective number of disaster periods. The probability of quarter t being a disaster period, conditional on the information available in $t-1$, is the discrete-time hazard rate,

$$h_t = \mathbb{P}(N(t) \neq N(t-1) | \mathcal{F}_{t-1}). \tag{3.13}$$

Hamilton and Jorda's (2002) ACH framework also allows for flexible parametrization of the hazard rate in Equation (3.13). In a parsimonious specification, the hazard rate depends on just two parameters, μ and $\tilde{\mu}$:

$$h_t = \left[(\mu(1-d_{t-1}) + \tilde{\mu} d_{t-1})(1-d^+_{t-1}) + d^+_{t-1}\right]^{-1}, \tag{3.14}$$

where d^+_t is a binary indicator, such that

$$d^+_t = \mathbb{1}(d_t = 1) \cdot \mathbb{1}\left[[1 - \prod_{j=s_1}^{t-1}(1-b_j)] < q\right], \tag{3.15}$$

where $\mathbb{1}(\cdot)$ is the indicator function. That is, $d^+_t = 1$ if quarter t belongs to a disaster that commenced in period $s_1 \leq t$, and the accrued contractions up to t do not yet qualify as a disaster. In this case, quarter $t+1$ must be a disaster period too, such that $h_{t+1} = 1$. If $d^+_t = 0$ and $d_t = 1$, then $h_{t+1} = 1/\tilde{\mu}$. If $d_t = 0$, then $h_{t+1} = 1/\mu$.

More extensive parametrization of the hazard rate is possible too. For example, I could include the time durations of and between previous disaster events, the aggregate size of the previous disaster, and the size of the contraction of the last disaster period to explain the hazard rate:

$$\begin{aligned} h_t = \Big[&[(\mu + \alpha\tau_{M(t-1)-1} + \delta b^+_{M(t-1)})(1-d_{t-1}) \\ &+ (\tilde{\mu} + \tilde{\alpha}\tilde{\tau}_{M(t-1)-1} + \tilde{\delta} b_{N(t-1)}) d_{t-1}](1-d^+_{t-1}) + d^+_{t-1}\Big]^{-1}, \end{aligned} \tag{3.16}$$

where τ_m denotes the duration, measured in quarters, between the mth and $(m+1)$th disaster, and $\tilde{\tau}_m$ denotes the number of quarters that the mth disaster lasted.

Furthermore, b_n is the contraction size of the nth disaster period, and b_m^+ is the aggregate size of the mth disaster. For the empirical analysis, I consider several special cases of Equation (3.16). For example, the hazard rate specification in Equation (3.14) emerges when $\alpha = \delta = \tilde{\alpha} = \tilde{\delta} = 0$.

To model disaster size, I adopt an idea from Barro and Jin (2011) and employ a power law distribution (PL) to describe the transformed contraction size $z_c = \frac{1}{1-b}$.[26] I assume that contractions that contribute to reaching the disaster threshold q (when $d_t = 1$ and $d_t^+ = 1$) follow a different PL distribution than those that add to a disaster after q was reached (when $d_t = 1$, but $d_t^+ = 0$).

The joint conditional probability density function of the resulting marked point process, which I refer to as an ACH-PL model, can be written as:

$$\begin{aligned} f(d_t, d_t^+, z_{c,t}|\mathcal{F}_{t-1}; \boldsymbol{\theta}_{ACH}, \theta_{PL}^+, \theta_{PL}) &= f(d_t, d_t^+|\mathcal{F}_{t-1}) \times f(z_{c,t}|d_t, d_t^+, \mathcal{F}_{t-1}) \\ &= [h_t(\boldsymbol{\theta}_{ACH})]^{d_t} \times [1 - h_t(\boldsymbol{\theta}_{ACH})]^{1-d_t} \\ &\times \left(f_{PL}(z_{c,t}; \theta_{PL}^+)^{d_t^+} \times f_{PL}(z_{c,t}; \theta_{PL})^{1-d_t^+}\right)^{d_t}, \end{aligned} \tag{3.17}$$

where $\boldsymbol{\theta}_{ACH}$ contains the ACH parameters, f_{PL} denotes the power law density, and θ_{PL}^+ and θ_{PL} are the power law tail coefficients that describe the size of the contractions that contribute to reaching the disaster threshold and the size of contractions to add on top of q, respectively. The probability density function in Equation (3.17) is an essential ingredient for the estimation strategy, which entails drawing from that distribution to simulate disaster-including consumption data.

3.3 Data

The empirical analysis of the disaster-including C-CAPM relies on two data sources, which I use in two consecutive estimation steps. The estimation of the ACH-PL parameters relies on annual cross-country panel data about consumption that Barro and Ursúa (2008) assembled for 42 countries and that feature prominently in prior rare disaster literature.[27] From these data, I select the same 35 countries that Barro (2006) considered. Table 18 lists the countries and the years for which consumption data are available.

[26] Specifically, Barro and Jin (2011), who implicitly assume single-period disasters, use a double power law distribution that consists of two power law distributions that morph into each other at a certain threshold value. It turns out that the flexibility of the double power law distribution is not required when modeling multi-period disasters.

[27] These data are available at http://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data, accessed 04/24/2015.

Table 18: Country panel data used for the first-step estimation
This table lists the 35 countries and time periods with available data that provide the basis for the ACH-PL estimation. The second column reports the time periods for which consumption data assembled by Barro and Ursúa (2008) are available (beginning with 1800 onwards).

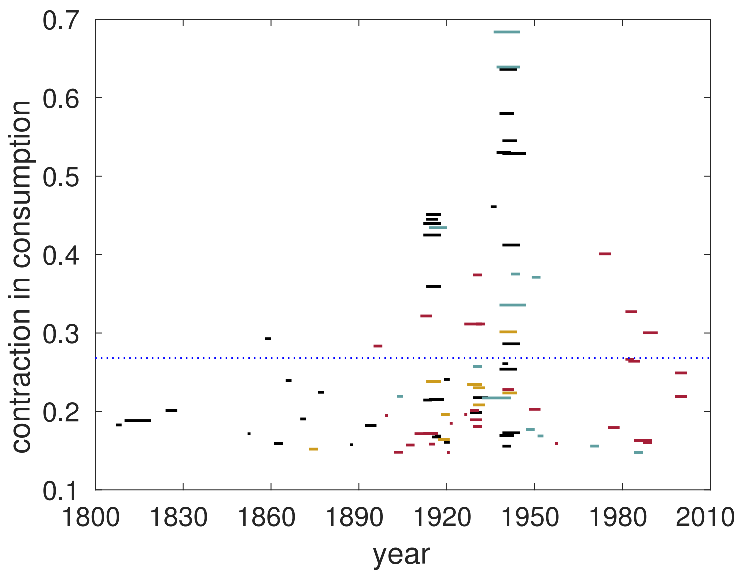
Country	Barro and Ursúa
Argentina	1875 – 2009
Australia	1901 – 2009
Austria	1913 – 1918, 1924 – 1944, 1947 – 2009
Belgium	1913 – 2009
Brazil	1901 – 2009
Canada	1871 – 2009
Chile	1900 – 2009
Colombia	1925 – 2009
Denmark	1844 – 2009
Finland	1860 – 2009
France	1824 – 2009
Germany	1851 – 2009
Greece	1938 – 2009
India	1919 – 2009
Indonesia	1960 – 2009
Italy	1861 – 2009
Japan	1874 – 2009
Malaysia	1900 – 1939, 1947 – 2009
Mexico	1900 – 2009
the Netherlands	1807 – 1809, 1814 – 2009
New Zealand	1878 – 2009
Norway	1830 – 2009
the Philippines	1946 – 2009
Peru	1896 – 2009
Portugal	1910 – 2009
South Korea	1911 – 2009
Spain	1850 – 2009
Sri Lanka	1960 – 2009
Sweden	1800 – 2009
Switzerland	1851 – 2009
Taiwan	1901 – 2009
UK	1830 – 2009
USA	1834 – 2009
Uruguay	1960 – 2009
Venezuela	1923 – 2009

To detect disaster events in these data, I rely on Barro's (2006) identification scheme, which implies that any sequence of downturns in consumption growth greater than or equal to $q = 0.145$ qualifies as a disaster. The same disaster threshold is used by Barro (2009) and Barro and Jin (2011). A disaster may pan out over multiple periods or occur as one sharp contraction. Positive intermezzos of consumption growth within a disaster are allowed if (1) this positive growth is smaller in absolute value than the negative growth in the following year and (2) the size of the disaster does not decrease by including the intermezzo. Using this disaster identification scheme, I detect 89 disaster events. Figure 15 depicts their size and the periods over which they accrue.

As previously mentioned, I assume that the ACH-PL process is observable at a quarterly frequency. However, Barro and Ursúa's (2008) data only permit the computation of annual contractions. I therefore generate quarterly observations by randomly distributing the annual contraction (see Appendix B.1 for details).

Figure 15: Consumption disasters

This figure depicts the 89 consumption disasters identified from Barro and Ursúa's (2008) country panel data (updated). The sampling period is 1800–2009. The disaster threshold q=0.145. Black lines denote European countries, red lines South American countries and Mexico, golden lines Western offshores (Australia, Canada, New Zealand, and U.S.A.), and blue lines represent Asian countries. The dotted horizontal line depicts the average contraction size.



The estimation of the preference parameters is based on quarterly U.S. real personal consumption expenditures per capita on services and nondurable goods in chained 2009 U.S. dollars, as provided by the Federal Reserve Bank of Saint Louis.[28] These data span the period 1947:Q2–2014:Q4. Financial data, at a monthly frequency, come from CRSP and Kenneth French's data library.[29] The data used for the empirical analysis are (1) the CRSP market portfolio, comprised of NYSE, AMEX, and NASDAQ traded stocks (*mkt*); (2) ten size-sorted portfolios (*size dec*); and (3) ten industry portfolios (*industry*). All portfolios are value-weighted. The gross return of the CRSP market portfolio serves as the proxy for R_a.[30]

[28] For services, see http://research.stlouisfed.org/fred2/series/A797RX0Q048SBEA. For nondurable goods, see http://research.stlouisfed.org/fred2/series/A796RX0Q048SBEA. Both accessed 03/09/2016.

[29] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html, accessed 03/09/2016. Due to the frequent changes in the underlying CRSP data, newer or older downloads may results in different series.

[30] The approximation of the return of the wealth portfolio by the return of the portfolio of financial assets is also employed by Weber (2000), Stock and Wright (2000), and Yogo (2006). Thimme and Völkert (2015) offer a critique of this approach, arguing that a large fraction of the wealth portfolio is comprised of non-financial wealth. They propose an alternative proxy based on Lettau and Ludvigson's (2001) *cay*-variable that accounts for the return on human capital.

Table 19: Descriptive statistics: Consumption and test asset returns 1947:Q2–2014:Q4 This table contains the descriptive statistics of consumption growth and gross returns of the three sets of test assets. Panel A: CRSP value-weighted market portfolio R_a and T-bill return R_b (*mkt*); Panel B: ten size-sorted portfolios and R_b (*size dec*); Panel C: ten industry portfolios and R_b (*industry*). The data range is 1947:Q2–2014:Q4. In Panel B, 1^{st}, 2^{nd}, and so on refer to the deciles of the the ten size-sorted portfolios. The ten industry portfolios in Panel C are: nondurables (*NoDur*: food, textiles, tobacco, apparel, leather, toys), durables (*Durbl*: cars, TVs, furniture, household appliances), manufacturing (*Manuf*: machinery, trucks, planes, chemicals, paper, office furniture), energy (*Engry*: oil, gas, coal extraction and products), business equipment (*HiTec*: computers, software, and electronic equipment), telecommunication (*Telcm*: telephone and television transmission), shops (*Shops*: wholesale, retail, laundries, and repair shops), health (*Hlth*: healthcare, medical equipment, and drugs), utilities (*Utils*), and others (*Other*: transportation, entertainment, finance, and hotels). The column labeled *ac* gives the first-order autocorrelation, and *std* is the standard deviation.

Panel A: mkt

	mean	std	ac	correlations	
				$\frac{C_{t+1}}{C_t}$	R_b
market	1.0211	0.0816	0.084	0.175	0.026
R_b	1.0017	0.0045	0.857	0.204	
$\frac{C_{t+1}}{C_t}$	1.0048	0.0051	0.311		

Panel B: size dec

	mean	std	ac	correlations										
				$\frac{C_{t+1}}{C_t}$	R_b	10^{th}	9^{th}	8^{th}	7^{th}	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}
1^{st}	1.0290	0.1251	0.061	0.178	-0.015	0.711	0.818	0.857	0.884	0.895	0.912	0.931	0.949	0.964
2^{nd}	1.0271	0.1177	-0.001	0.172	0.005	0.781	0.871	0.915	0.933	0.947	0.961	0.974	0.982	
3^{rd}	1.0287	0.1115	-0.024	0.165	-0.001	0.818	0.907	0.943	0.956	0.968	0.976	0.985		
4^{th}	1.0270	0.1072	-0.018	0.165	0.002	0.830	0.914	0.948	0.962	0.976	0.983			
5^{th}	1.0274	0.1036	0.013	0.167	0.019	0.855	0.936	0.967	0.972	0.982				
6^{th}	1.0262	0.0971	0.019	0.143	0.001	0.868	0.946	0.970	0.977					
7^{th}	1.0262	0.0964	0.042	0.157	0.009	0.892	0.965	0.982						
8^{th}	1.0249	0.0923	0.022	0.145	0.019	0.906	0.975							
9^{th}	1.0237	0.0841	0.068	0.148	0.021	0.935								
10^{th}	1.0198	0.0767	0.119	0.178	0.043									

Panel C: industry

	mean	std	ac	correlations										
				$\frac{C_{t+1}}{C_t}$	R_b	Other	Utils	Hlth	Shops	Telcm	HiTec	Engry	Manuf	Durbl
NoDur	1.0238	0.0811	0.047	0.090	0.105	0.838	0.674	0.800	0.871	0.656	0.642	0.445	0.829	0.685
Durbl	1.0236	0.1156	0.103	0.190	0.009	0.801	0.484	0.520	0.773	0.581	0.690	0.490	0.832	
Manuf	1.0229	0.0899	0.082	0.173	0.014	0.901	0.580	0.745	0.825	0.647	0.807	0.635		
Engry	1.0253	0.0888	0.041	0.163	-0.039	0.592	0.534	0.423	0.422	0.432	0.497			
HiTec	1.0258	0.1159	0.070	0.167	-0.000	0.758	0.470	0.663	0.733	0.659				
Telcm	1.0187	0.0805	0.148	0.099	0.104	0.695	0.627	0.568	0.668					
Shops	1.0238	0.0957	0.039	0.158	0.044	0.837	0.557	0.704						
Hlth	1.0271	0.0909	0.054	0.092	0.085	0.726	0.542							
Utils	1.0195	0.0711	0.080	0.069	0.071	0.655								
Other	1.0217	0.0982	0.078	0.159	0.034									

Nominal monthly returns are converted to real returns at a quarterly frequency, using the growth of the consumer price index of all urban consumers.[31] In line with Beeler and Campbell (2012), I approximate the ex ante non-disastrous T-bill return $R_{b,nd}$ (i.e., the "risk-free rate" proxy) by forecasting ex post $R_{b,nd}$ on the basis of the quarterly T-bill yield and the average of quarterly log inflation across the past year. The three-month nominal T-bill yield comes from the CRSP database. Table 19 contains the descriptive statistics for these data.

3.4 Estimation strategy

3.4.1 ACH-PL maximum likelihood estimation

The parameter estimation of the disaster-including C-CAPM involves two consecutive steps. I first compute maximum likelihood estimates of the ACH-PL parameters $\boldsymbol{\theta}_{ACH}$, θ_{PL}^{+}, and θ_{PL}. Using these estimates, it is possible to simulate disaster-including data, which are required for the simulation-based estimation of the preference parameters β, γ, and ψ in the second stage. Consider the maximum likelihood estimation step. Equation (3.17) implies the following conditional ACH-PL log-likelihood function:

$$\begin{aligned}\mathcal{L}(\boldsymbol{\theta}_{ACH}, \theta_{PL}^{+}, \theta_{PL}) = &\sum_{t=1}^{T} \left(d_t \ln h_t(\boldsymbol{\theta}_{ACH}) + (1 - d_t) \ln[1 - h_t(\boldsymbol{\theta}_{ACH})]\right) \\ &+ \sum_{t=1}^{T} d_t \left(d_t^{+} \ln f_{PL}(z_{c,t}; \theta_{PL}^{+}) + (1 - d_t^{+}) \ln f_{PL}(z_{c,t}; \theta_{PL})\right).\end{aligned} \tag{3.18}$$

The parameters in Equation (3.18) are variation-free, so it is possible to perform the estimation of $\hat{\boldsymbol{\theta}}_{ACH}$, θ_{PL}^{+}, and θ_{PL} separately. In particular, the maximization of

$$\mathcal{L}(\boldsymbol{\theta}_{ACH}) = \sum_{t=1}^{T} \left(d_t \ln h_t(\boldsymbol{\theta}_{ACH}) + (1 - d_t) \ln[1 - h_t(\boldsymbol{\theta}_{ACH})]\right) \tag{3.19}$$

yields $\hat{\boldsymbol{\theta}}_{ACH}$, whereas estimates of θ_{PL}^{+} and θ_{PL} can be obtained by maximizing

$$\mathcal{L}(\boldsymbol{\theta}_{PL}) = \sum_{t=1}^{T} d_t \left(d_t^{+} \ln f_{PL}(z_{c,t}; \theta_{PL}^{+}) + (1 - d_t^{+}) \ln f_{PL}(z_{c,t}; \theta_{PL})\right). \tag{3.20}$$

To perform the maximization of the log-likelihood function in Equation (3.19), the cross-country panel data are represented as event time data. For that purpose,

[31] These data are provided by the Federal Reserve Bank of Saint Louis: http:// research.stlouisfed.org/fred2/series/CPIAUCSL, accessed 03/09/2016.

sequences of the disaster indicators d_t and d_t^+ are computed for every country. Counting the number of quarters between disaster events gives τ_m, which equals the time duration between the mth and $(m+1)$th disaster. Moreover, $\tilde{\tau}_m$ is obtained by counting the number of quarters over which the respective disaster lasted. These data are needed to compute the hazard rate in Equation (3.16)

The maximum likelihood estimation of the ACH parameters $\boldsymbol{\theta}_{ACH}$ is then performed on the concatenated country-specific event time data series. During the maximization of the log-likelihood function in Equation (3.19), the disaster event and period counters $M(t)$ and $N(t)$ are reset to zero whenever a country change occurs in the concatenated data. If the hazard rate specification in Equation (3.16) is used, τ_0 must be re-initialized to the average duration between disasters (179.7 quarters), $\tilde{\tau}_0$ is reset to equal the average disaster length (13.1 quarters), and b_0^+ is reset to equal the average contraction size (0.268). These values are also the initial values for the maximum likelihood estimation. They correspond to $q = 0.145$; different disaster thresholds use different initial values. The re-initialization procedure is adopted from Engle and Russell (1998).[32]

3.4.2 Financial moment restrictions and data simulation

An SMM-type estimation of the preference parameters entails exploiting the moment restrictions in Equations (3.7) and (3.8). In particular, I rely on matching between empirical and simulated moments, as is implied by the moment restriction in Equation (3.7), that uses the sample moments in Equations (3.9) and (3.11). Applied to the T-bill return R_b

$$g^r(\boldsymbol{\vartheta}) = \left[\frac{1}{T} \sum_{t=1}^{T} \beta^\theta c g_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{b,nd,t} - \left[\frac{1 - \frac{1}{T} \sum\limits_{s=1}^{\mathcal{T}} \beta^\theta c g_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_{b,s} d_s}{1 - \frac{D_{\mathcal{T}}}{T}} \right] \right], \tag{3.21}$$

where $\boldsymbol{\vartheta} = (\beta, \gamma, \psi)'$. Similarly, I exploit the moment restriction in Equation (3.8) applied to an excess return $R_i^e = R_i - R_b$, which suggests the following matching of empirical and simulated moments:

$$g^e(\boldsymbol{\vartheta}) = \left[\frac{1}{T} \sum_{t=1}^{T} \beta^\theta c g_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{i,nd,t}^e - \left[\frac{- \frac{1}{T} \sum\limits_{s=1}^{\mathcal{T}} \beta^\theta c g_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_{i,s}^e d_s}{1 - \frac{D_{\mathcal{T}}}{T}} \right] \right]. \tag{3.22}$$

[32] They consider an ACH-like dynamic duration model for the time interval between intraday trading events. In this framework, the re-initialization accounts for overnight interruptions of the trading process.

Combining Equation (3.21) with Equation (3.22), and applied to the excess returns of N test assets, I obtain:

$$\mathbb{G}(\boldsymbol{\vartheta}) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T} \beta^{\theta} cg_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{b,nd,t} - \left[\frac{1-\frac{1}{\mathcal{T}}\sum\limits_{s=1}^{\mathcal{T}} \beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_{b,s} d_s}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}} \right] \\ \frac{1}{T}\sum_{t=1}^{T} \beta^{\theta} cg_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} \mathbf{R}_{nd,t}^{e} - \left[\frac{-\frac{1}{\mathcal{T}}\sum\limits_{s=1}^{\mathcal{T}} \beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} \mathbf{R}_s^{e} d_s}{1-\frac{D_{\mathcal{T}}}{\mathcal{T}}} \right] \end{bmatrix}, \tag{3.23}$$

where $\mathbf{R}^e = [R_1^e, \ldots, R_N^e]'$. Choosing $N \geq 2$, SMM-type estimation of the preference parameters can then be attempted by:

$$\hat{\boldsymbol{\vartheta}} = \underset{\boldsymbol{\vartheta} \in \boldsymbol{\Theta}}{\arg\min}\ \mathbb{G}(\boldsymbol{\vartheta})' \mathbf{W} \mathbb{G}(\boldsymbol{\vartheta}), \tag{3.24}$$

where $\boldsymbol{\Theta}$ denotes the admissible parameter space and $\mathbf{W}$ is a symmetric and positive semi-definite weighting matrix.

To evaluate $\mathbb{G}(\boldsymbol{\vartheta})$ within such an optimization, it is necessary to compute the moments of simulated disaster-including data. For that purpose, I use the first-step ACH-PL estimates $\hat{\boldsymbol{\theta}}_{ACH}$, $\hat{\theta}_{PL}^{+}$, and $\hat{\theta}_{PL}$ and simulate a series of hazard rates $\{h_s(\hat{\boldsymbol{\theta}}_{ACH}, \hat{\theta}_{PL}^{+}, \hat{\theta}_{PL})\}_{s=1}^{\mathcal{T}}$. The resulting conditional disaster probabilities then can generate a sequence of disaster indicators $\{d_s\}_{s=1}^{\mathcal{T}}$ and $\{d_s^{+}\}_{s=1}^{\mathcal{T}}$.

I obtain simulated series of non-disastrous consumption growth and returns, $\{cg_{nd,s}, R_{a,nd,s}, R_{b,nd,s}, R_{i,nd,s}\}_{s=1}^{\mathcal{T}}$ by block-bootstrapping from the non-disastrous U.S. postwar data. For that purpose, I rely on the automatic block-length selection procedure proposed by Politis and White (2004) and corrected by Politis et al. (2009), in combination with the stationary bootstrap of Politis and Romano (1994), in which the respective block-length gets drawn from a geometric distribution. The draws from the consumption and return data are simultaneous, to retain the contemporaneous covariance structure.

Because the cross-country consumption panel data collected by Barro and Ursúa (2008) do not include information on asset prices, further assumptions are needed to simulate disaster returns. In particular, I assume that the transformed contractions $z_c = 1/(1-b)$ and $z_R = 1/(1-\tilde{b})$ have the same marginal distribution,[33]

$$f(z_c; \theta_{PL}^{+}, \theta_{PL}) = f(z_R; \theta_{PL}^{+}, \theta_{PL}), \tag{3.25}$$

[33] The asset index i is omitted for brevity.

where

$$f(z;\theta_{PL}^{+},\theta_{PL}) = f_{PL}(z;\theta_{PL}^{+})^{d^{+}} \times f_{PL}(z;\theta_{PL})^{1-d^{+}}, \tag{3.26}$$

and write their joint cumulative distribution function (cdf) using a copula function that links the two marginal distributions:

$$F(z_c, z_R;\theta_{PL}^{+},\theta_{PL},\boldsymbol{\theta}_C) = C(F(z_c;\theta_{PL}^{+},\theta_{PL}), F(z_R;\theta_{PL}^{+},\theta_{PL});\boldsymbol{\theta}_C), \tag{3.27}$$

where $F(z_C;\theta_{PL}^{+},\theta_{PL})$ and $F(z_R;\theta_{PL}^{+},\theta_{PL})$ denote the marginal cdfs. The vector $\boldsymbol{\theta}_C$ collects the coefficients that determine the dependence of z_c and z_R. Using the Gaussian copula C_G, these dependencies can be measured by a single parameter, the copula correlation ρ. Equation (3.27) then becomes:

$$F(z_c, z_R;\theta_{PL}^{+},\theta_{PL},\rho) = C_G(u_c, u_R;\rho), \tag{3.28}$$

where $u_c = F(z_c;\theta_{PL}^{+},\theta_{PL})$ and $u_R = F(z_R;\theta_{PL}^{+},\theta_{PL})$.

I consider three choices for the copula correlation. First, ρ_i may be estimated by the empirical correlation between non-disastrous consumption growth and gross return. Second, I consider the extreme case that $\rho = 0.99$, motivated by the finding that the correlations between financial returns increase in the tails of their joint distribution (see Longin and Solnik (2001)). Third, I address the case when $\rho = 0$, which implies drawing b_s and $\tilde{b}_s$ independently from the same distribution.

Drawing b_s and $\tilde{b}_s$ in case of d_s=1 proceeds as follows: I draw $y_{c,s}$ and $y_{R,s}$ from a bivariate standard normal distribution with correlation ρ, then compute $u_{c,s} = \Phi(y_{c,s})$ and $u_{R,s} = \Phi(y_{R,s})$, where Φ denotes the standard normal cdf. Consumption growth and return contraction factors then can be obtained by

$$b_s = 1 - \frac{1}{F^{-1}(u_{c,s};\hat{\theta}_{PL}^{+},\hat{\theta}_{PL})} \qquad \text{and} \qquad \tilde{b}_s = 1 - \frac{1}{F^{-1}(u_{R,s};\hat{\theta}_{PL}^{+},\hat{\theta}_{PL})}, \tag{3.29}$$

where

$$F^{-1}(u;\theta_{PL}^{+},\theta_{PL}) = \left(F_{PL}^{-1}(u;\theta_{PL}^{+})\right)^{d^{+}} \times \left(F_{PL}^{-1}(u;\theta_{PL})\right)^{1-d^{+}}. \tag{3.30}$$

In this case, F_{PL}^{-1} denotes the quantile function of the PL distribution. The combination of the contraction factors with the bootstrapped non-disastrous series allows simulating disaster-including series for consumption growth, $cg_s = (1-b_s)^{d_s} cg_{nd,s}$; test asset returns, $R_{i,s} = (1-\tilde{b}_{i,s})^{d_s} R_{i,nd,s}$, $i = 1,\ldots,N$; and the return of the wealth portfolio proxy $R_{a,s} = (1-\tilde{b}_{a,s})^{d_s} R_{a,nd,s}$.

For the simulation of the T-bill return $R_{b,s}$, I draw on Barro (2006), who identifies partial government default in 42% of the disasters that he finds in the GDP series of 35 countries. Using this result, at the beginning of each disaster (that is, $d_s = 1$ but $d_{s-1} = 0$), I draw a government default indicator $d_{b,s}$ from a Bernoulli distribution with a success probability $\mathbb{P}(d_{b,s} = 1 | d_s = 1, d_{s-1} = 0) = 0.42$, which decides whether the T-bill return is affected by the disaster. If $d_{b,s} = 0$, the T-bill will not contract. If $d_{b,s} = 1$, a contraction factor $\tilde{b}_{b,s}$ is drawn in the same way as for the returns of the test assets, such that $R_{b,s} = (1 - \tilde{b}_{b,s})^{d_{b,s}} R_{b,nd,s}$. The simulated excess returns then can be computed as $R^e_{i,s} = R_{i,s} - R_{b,s}$, such that it becomes possible to evaluate $\mathbb{G}(\boldsymbol{\vartheta})$ in Equation (3.23).

3.4.3 Identifying the IES

Thimme (2017) points out that a joint estimation of the investor preference parameters that relies exclusively on moment restrictions obtained from conditioning down the basic asset pricing equations in (3.4) yields rather imprecise estimates of the IES. Although the moment restrictions used in the present study account for the possibility of disasters, they still conform to the basic asset pricing equation with an Epstein-Zin-Weil SDF, and the caveat applies. I therefore find it useful to identify and estimate the IES separately from β and γ, and through moment restrictions that can be derived from a (second-order) log-linearization of the Euler Equation (3.4) with the SDF in Equation (3.5). Yogo (2004) shows that this procedure leads to the following regression equation

$$r_{i,t+1} = \mu_i + \frac{1}{\psi} \Delta c_{t+1} + \eta_{i,t+1}, \tag{3.31}$$

where $r_{i,t+1} = \ln R_{i,t+1}$, and $\Delta c_{t+1} = \ln C_{t+1} - \ln C_t$. In addition, μ_i is a constant, and $\eta_{i,t+1}$ is a zero mean disturbance term. The derivation implies that $\eta_{i,t+1}$ is correlated with Δc_{t+1}, such that a linear projection of $r_{i,t+1}$ on Δc_{t+1} and a constant would not identify the IES. Instead, the IES is identified according to the orthogonality conditions,

$$\mathbb{E}\left((r_{i,t+1} - \mu_i - \frac{1}{\psi} \Delta c_{t+1}) \mathbf{z}_t \right) = \mathbf{0}, \tag{3.32}$$

where $\mathbf{z}_t$ consists of variables known at t (instrumental variables), which are correlated with Δc_{t+1}.[34]

[34] Estimation of the IES by GMM or two-stage least squares based on Equation (3.31) (or its reciprocal) and the moment restrictions in Equation (3.32) began with Hansen and Singleton (1983), was surveyed by Campbell (2003), and is critically discussed by Yogo (2004).

I adopt the instrumental variables approach to estimate the IES and use the log T-bill return $r_{b,t+1} = \ln R_{b,t+1}$ in Equation (3.31), the twice-lagged log T-bill return, log consumption growth, and a constant as instruments. The estimation is performed on the simulated disaster-including data. Using a linear GMM with an identity weighting matrix, the IES estimate $\hat{\psi}$ must fulfill the first-order conditions:

$$\begin{bmatrix} -1 & -\mathbb{E}_{\mathcal{T}}(\Delta c_s) & -\mathbb{E}_{\mathcal{T}}(r_{b,s}) \\ \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s)}{\hat{\psi}^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s \Delta c_{s-2})}{\hat{\psi}^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s r_{b,s-2})}{\hat{\psi}^2} \end{bmatrix} \begin{bmatrix} \mathbb{E}_{\mathcal{T}}(r_{b,s}) - \hat{\mu}_b - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta c_s) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s}\Delta c_{s-2}) - \hat{\mu}_b \mathbb{E}_{\mathcal{T}}(\Delta c_{s-2}) - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta c_s \Delta c_{s-2}) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s} r_{b,s-2}) - \hat{\mu}_b \mathbb{E}_{\mathcal{T}}(r_{b,s-2}) - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta c_s r_{b,s-2}) \end{bmatrix} = \mathbf{0}, \tag{3.33}$$

which reflect Hansen's (1982) notation $\mathbb{E}_{\mathcal{T}}(\cdot) = \frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}(\cdot)$. The estimation of the IES is appropriate when performed separately from that of the subjective discount factor and the RRA coefficient, which are estimated using Equation (3.24) with $\hat{\psi}$ held fixed, but it also is possible to augment Equation (3.23) with the IES-identifying moment matches of Equation (3.33) to obtain:

$$\mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}}) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T} \beta^{\theta} cg_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} R_{b,nd,t} - \left[\frac{1 - \mathbb{E}_{\mathcal{T}}\left(\beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} R_{b,s} d_s \right)}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}}} \right] \\ \frac{1}{T}\sum_{t=1}^{T} \beta^{\theta} cg_{nd,t}^{-\frac{\theta}{\psi}} R_{a,nd,t}^{\theta-1} \mathbf{R}_{nd,t}^{e} - \left[\frac{-\mathbb{E}_{\mathcal{T}}\left(\beta^{\theta} cg_s^{-\frac{\theta}{\psi}} R_{a,s}^{\theta-1} \mathbf{R}_s^{e} d_s \right)}{1 - \frac{D_{\mathcal{T}}}{\mathcal{T}}} \right] \\ \begin{bmatrix} -1 & -\mathbb{E}_{\mathcal{T}}(\Delta c_s) & -\mathbb{E}_{\mathcal{T}}(r_{b,s}) \\ \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s)}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s \Delta c_{s-2})}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta c_s r_{b,s-2})}{\psi^2} \end{bmatrix} \times \\ \begin{bmatrix} \mathbb{E}_{\mathcal{T}}(r_{b,s}) - \mu_b - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta c_s) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s}\Delta c_{s-2}) - \mu_b \mathbb{E}_{\mathcal{T}}(\Delta c_{s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta c_s \Delta c_{s-2}) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s} r_{b,s-2}) - \mu_b \mathbb{E}_{\mathcal{T}}(r_{b,s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta c_s r_{b,s-2}) \end{bmatrix} \end{bmatrix}, \tag{3.34}$$

where $\tilde{\boldsymbol{\vartheta}} = (\beta, \gamma, \psi, \mu_b)'$. The SMM-type estimates of the preference parameters are then obtained by:

$$\hat{\tilde{\boldsymbol{\vartheta}}} = \underset{\tilde{\boldsymbol{\vartheta}} \in \tilde{\boldsymbol{\Theta}}}{\arg\min}\ \mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}})' \mathbf{W} \mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}}). \tag{3.35}$$

Choosing $\mathbf{W}$ such that a large weight is placed on the last two moment matches in Equation (3.34) ensures that the IES will be identified by Equation (3.33). In particular, I use

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_{N+1} & 0 \\ 0 & 10^6 \times \mathbf{I}_2 \end{bmatrix}. \tag{3.36}$$

Because of the two-step approach, standard inference is not available for the second-step estimates, though I could rely on asymptotic maximum likelihood inference for the first-step ACH-PL estimates. Therefore, I combine a parametric and non-parametric bootstrap to obtain the standard errors and confidence intervals of the preference parameter estimates. The bootstrap procedure is detailed in Section B.2 of the appendix.

3.5 Empirical results

3.5.1 First-step estimation results

Table 20 reports the maximum likelihood estimates of the ACH-PL parameters and the Akaike (AIC) and Schwarz-Bayes (SBC) information criteria for various ACH specifications that emerge as special cases of the hazard rate specification in Equation (3.16). The most comprehensive alternative, referred to as ACH_1, estimates all

Table 20: Estimation results for the ACH-PL model

This table reports the ACH-PL maximum likelihood estimates. Here, $\mathcal{L}$ is the log-likelihood value at the maximum; AIC = $2k - 2\ln(\mathcal{L})$ and SBC = $-2\ln\mathcal{L} + k\ln(T)$, where k is the number of ACH model parameters, denote the Akaike and Schwarz-Bayes information criteria, respectively. Furthermore, $\mathcal{LR}$ gives the p-values (in percent) of the likelihood ratio tests of the null hypothesis that the parameter restrictions implied by the ACH_0 specification are correct. The respective alternative is the ACH_1, the ACH_2, the ACH_3, or the ACH_4 model. The estimation results are based on the updated country panel data originally assembled by Barro and Ursúa (2008), using the concatenated event data representation described in Section 3.3 and q = 0.145. Asymptotic standard errors are reported in parentheses.

	θ^+_{PL}	θ_{PL}	μ	$\tilde{\mu}$	α	$\tilde{\alpha}$	δ	$\tilde{\delta}$	$\mathcal{L}$	AIC	SBC	$\mathcal{LR}$
ACH_0			178.3	1.201					-790.3	1584.7	**1600.1**	
			(18.8)	(0.023)								
ACH_4			64.9	1.201			441.1		-787.0	**1580.0**	1603.2	<1.0
			(49.3)	(0.023)			(211.5)					
ACH_3			64.9	1.214			441.1	-0.375	-786.8	1581.5	1612.5	2.9
			(49.3)	(0.032)			(211.5)	(0.537)				
ACH_2			198.7	1.221	-0.145	-0.002			-789.9	1587.7	1618.7	63.5
			(30.9)	(0.052)	(0.153)	(0.004)						
ACH_1			71.4	1.237	-0.030	-0.002	431.0	-0.399	-786.6	1585.3	1631.7	11.8
			(55.0)	(0.058)	(0.161)	(0.004)	(120.4)	(0.542)				
PL	37.255	35.687										
	(1.478)	(1.696)										

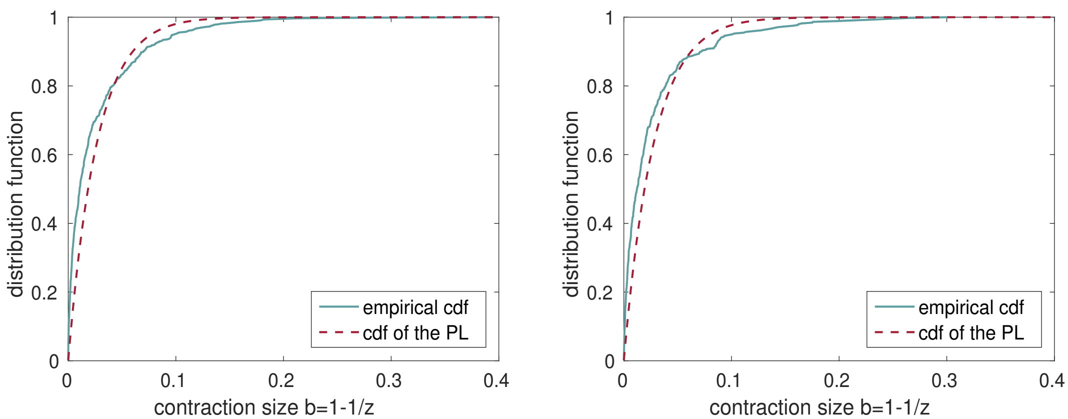
parameters in Equation (3.16). The most parsimonious parametrization, referred to as ACH_0, corresponds to the hazard rate in Equation (3.14), such that only the baseline hazard parameters μ and $\tilde{\mu}$ are estimated (while $\delta = \tilde{\delta} = \alpha = \tilde{\alpha} = 0$). The ACH_2 specification allows (only) for an effect of the durations between disasters and the disaster length on the hazard rate (while $\delta = \tilde{\delta} = 0$), and the ACH_3 allows (only) the magnitude of the previous disaster and the size of the contraction of the previous disaster period to affect the hazard rate (while $\alpha = \tilde{\alpha} = 0$). In the ACH_4 specification, the aggregate size of the previous disaster has an effect on the hazard rate, but the contraction of the previous disaster period does not (i.e., $\tilde{\delta} = \alpha = \tilde{\alpha} = 0$).

Table 20 shows that the AIC favors the ACH_4, but the SBC prefers the ACH_0, for which the baseline hazard parameter estimates $\hat{\mu}$ and $\hat{\tilde{\mu}}$ are highly significant. The estimates of $\tilde{\mu}$ and δ in the ACH_4 specification are significant at the 5% level, but the baseline hazard parameter μ is reduced in size and significance. Moreover, the likelihood-ratio statistics reported in Table 20 indicate that the constraints implied by the SBC-preferred ACH_0, at the 1% significance level, are only rejected in the case of the AIC-preferred ACH_4. Therefore, the subsequent analysis is confined to ACH_0 and ACH_4.

I obtain maximum likelihood estimates of the ACH_0 parameters equal to $\hat{\tilde{\mu}} = 178.3$ and $\hat{\mu} = 1.2$. These estimates imply a probability of entering a disaster from a non-disaster period of about 0.56%, and a probability of remaining in a disaster that is equal to 83%. Because I use these estimates as a foundation for the second estimation step, it is prudent to check their economic plausibility in advance. Accordingly, I use the ACH_0 and ACH_4 estimates to simulate disaster-including consumption time series with a number of observations that corresponds to the sample period, 1947:Q2-2014:Q4. The simulation is repeated 10k times, and I count the number of replications for which no disastrous consumption contraction occurs. The ACH_0 specification yields 21.9%, the ACH_4 14.1% disaster-free replications. The estimated disaster-including consumption process thus implies that U.S. postwar history represents a lucky but not unlikely path, and the model-implied disaster probabilities are not implausibly large.

Table 20 also shows that the estimates of the power law coefficients θ_{PL} and θ^{+}_{PL} are similar, so the distribution of contractions that occur before reaching the disaster threshold q is not very different from the distribution of contractions that occur after q is reached. The estimates $\hat{\theta}_{PL}$ and $\hat{\theta}^{+}_{PL}$ have encouragingly small standard errors. Figure 16 depicts the cdf of the power law distribution and the empirical cdf of quarterly contractions. Figure 16a uses the estimate $\hat{\theta}^{+}_{PL}$ and illustrates the fit for contractions that contribute to reaching the disaster threshold; Figure 16b uses $\hat{\theta}_{PL}$ and refers to contractions that add on top of the disaster threshold. In both cases, the fit is quite good.

Figure 16: Fitted power law vs. empirical cdf
This figure illustrates the empirical cdfs (solid lines) and the fitted cdf (dotted lines) of the contractions identified in Barro and Ursúa's (2008) data using a disaster threshold of q=0.145. Panel (a) captures the distribution of contractions that occur at the beginning of a disaster and contribute to reaching the disaster threshold. Panel (b) refers to contractions that add on top of the disaster threshold. The fitted cdfs use the PL parameter estimates from Table 20.



(a) contractions that help reaching q **(b)** contractions that add on top of q

3.5.2 Second-step estimation results

Table 21 reports the second-step estimation results based on the SBC-preferred ACH_0-PL and the AIC-preferred ACH_4-PL first-step estimates. The estimation uses different sets of test assets and copula correlation coefficients. It is based on the moment matches in Equation (3.34), using the weighting matrix in Equation (3.36), and $\mathcal{T}$=10^7. The table contains the point estimates of the preference parameters β, γ, and ψ and their bootstrap standard errors, as well as the associated 95% confidence bounds. These bounds are computed using the percentile method, meaning that they accord with the 0.025 and 0.975 quantiles of the respective bootstrap distribution.[35] Furthermore, Table 21 shows the p-values of Hansen's (1982) J-statistic,

$$J = \mathbb{G}(\hat{\boldsymbol{\vartheta}})'\widehat{\text{Avar}}(\mathbb{G}[\hat{\boldsymbol{\vartheta}}])^{+}\mathbb{G}(\hat{\boldsymbol{\vartheta}}), \tag{3.37}$$

where $^+$ denotes the Moore-Penrose inverse, which is approximately $\chi^2(N-1)$ under the null hypothesis that the financial moment restrictions are correct. The root mean squared errors (RMSEs; reported in Table 21) are computed as

[35] More formally, for a parameter ϑ, the α-quantile is computed as $\hat{G}^{-1}(\alpha)$, where $\hat{G}(\hat{\vartheta}) = \frac{1}{K}\sum_{k=1}^{K}\mathbb{1}(\hat{\vartheta}^{(k)} < \hat{\vartheta})$.

$$R = \sqrt{\frac{1}{N+1}\mathbb{G}(\hat{\boldsymbol{\vartheta}})'\mathbb{G}(\hat{\boldsymbol{\vartheta}})} \times 10^4. \tag{3.38}$$

When using only the market portfolio and the T-bill return as test assets, the number of moment restrictions is equal to the number of estimated parameters, so empirical and simulated moments are perfectly matched.[36] Table 21 shows that all variants for estimating a disaster-including C-CAPM yield economically plausible estimates for the preference parameters. The subjective discount factor estimates are smaller but close to 1, as would be expected of an investor with a plausible positive rate of time preference. The estimates of the subjective discount factor range between 0.9915 and 0.9948. The RRA estimates are between 1.50 and 1.65, well within the plausibility interval mentioned by Cochrane (2005). The estimated IES is larger than 1, ranging between 1.50 and 1.68. The inverse of the estimated IES is always smaller than the RRA estimate, which indicates a preference for an early resolution of uncertainty. Previous literature has pointed out that the inequality $\gamma > 1/\psi$ is crucial for obtaining meaningful asset pricing implications (as detailed subsequently).[37]

The choice of the test assets, the copula correlation, and the first-step ACH-PL specification exert only minor effects on the size of the preference parameter estimates. The IES estimates based on ACH_4-PL are slightly bigger than those implied by ACH_0-PL. Using only the market portfolio and the T-bill return as test assets, the RRA coefficient and IES estimates tend to be a bit smaller than the estimates based on industry and size-sorted portfolios. Using the ACH_0-PL first-step estimates yields a slightly smaller RMSE than using the ACH_4-PL estimates.

In all instances, the estimation precision is more than satisfactory, as indicated by the small bootstrap standard errors and the narrow confidence bounds. It is noteworthy that the confidence bounds for the RRA estimates also fall within the stricter plausibility range, and the lower bound of the 95% confidence interval for the IES is above unity too. Regarding the subjective discount factor estimate $\hat{\beta}$, the upper confidence bound is sometimes larger than 1, but given that quarterly time preferences should to be very close to 1, this finding is not surprising. The p-values of the J-statistic indicate that the disaster-including C-CAPM cannot be rejected at conventional significance levels.

[36] In this case, the RMSE is 0, and R and the J-statistic are not reported.

[37] It is worth noting that the estimation of ψ by reversing the regression in Equation (3.31) also yields an IES estimate greater than 1. As noted by Yogo (2004), such robustness cannot be expected when disaster-free data are used for IES estimation.

Table 21: SMM estimates of the C-CAPM preference parameters

This table reports the estimates of the subjective discount factor β, the coefficient of relative risk aversion γ, and the IES ψ using the moment matches in Equation (3.34), $\mathcal{T}$=10^7, and the weighting matrix in Equation (3.36). The second-step SMM-type estimates are based on the first-step ACH_4-PL and ACH_0-PL estimates, reported in Table 20. The numbers in parentheses are bootstrap standard errors. The numbers in brackets are the upper and lower bounds of the 95% confidence intervals computed as the α=0.025 and α=0.975 quantiles of the bootstrap distribution (percentile method). The table also reports the p-values (in percent) of Hansen's (1982) J-statistic (see Equation (3.37)) and root mean squared errors (R), computed according to Equation (3.38). Panels A-C break down the results by the copula correlation assumed in the data simulation procedure. Each panel reports the results by the set of test assets, namely, the excess returns of *mkt*, *size dec*, and *industry*, each augmented by the T-bill return.

	mkt			size dec					industry				
Panel A: ρ = Corr($cg_{nd,t}$, $R_{nd,t}$)													
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R
ACH_0	**0.9917** (0.0022)	**1.51** (0.30)	**1.50** (0.15)	**0.9939** (0.0047)	**1.60** (0.29)	**1.50** (0.15)	83.5	9	**0.9944** (0.0038)	**1.62** (0.32)	**1.50** (0.15)	11.7	39
	[0.9872 0.9957]	[1.10 2.29]	[1.31 1.88]	[0.9864 1.0052]	[1.24 2.34]	[1.29 1.88]			[0.9887 1.0032]	[1.20 2.44]	[1.29 1.88]		
ACH_4	**0.9920** (0.0023)	**1.54** (0.30)	**1.67** (0.15)	**0.9945** (0.0052)	**1.63** (0.29)	**1.65** (0.16)	68.7	11	**0.9947** (0.0071)	**1.64** (0.31)	**1.65** (0.16)	7.2	40
	[0.9872 0.9960]	[1.08 2.33]	[1.31 1.87]	[0.9862 1.0057]	[1.22 2.40]	[1.28 1.87]			[0.9891 1.0035]	[1.17 2.40]	[1.28 1.86]		
Panel B: ρ = 0.99													
	mkt			size dec					industry				
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R
ACH_0	**0.9915** (0.0022)	**1.51** (0.30)	**1.51** (0.15)	**0.9938** (0.0047)	**1.61** (0.29)	**1.51** (0.15)	83.3	9	**0.9942** (0.0038)	**1.62** (0.32)	**1.51** (0.15)	11.9	39
	[0.9870 0.9957]	[1.09 2.26]	[1.31 1.88]	[0.9861 1.0051]	[1.24 2.34]	[1.29 1.87]			[0.9885 1.0031]	[1.20 2.43]	[1.29 1.87]		
ACH_4	**0.9917** (0.0023)	**1.54** (0.31)	**1.68** (0.15)	**0.9942** (0.0067)	**1.64** (0.29)	**1.67** (0.15)	68.2	11	**0.9944** (0.0053)	**1.65** (0.32)	**1.67** (0.16)	7.6	40
	[0.9869 0.9959]	[1.05 2.32]	[1.30 1.87]	[0.9864 1.0061]	[1.19 2.33]	[1.29 1.87]			[0.9883 1.0035]	[1.17 2.46]	[1.28 1.87]		
Panel C: ρ = 0													
	mkt			size dec					industry				
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R
ACH_0	**0.9917** (0.0022)	**1.51** (0.30)	**1.50** (0.15)	**0.9939** (0.0047)	**1.60** (0.29)	**1.50** (0.15)	83.5	9	**0.9944** (0.0038)	**1.62** (0.32)	**1.50** (0.15)	11.7	39
	[0.9871 0.9959]	[1.10 2.28]	[1.31 1.88]	[0.9863 1.0052]	[1.24 2.34]	[1.29 1.88]			[0.9887 1.0032]	[1.20 2.44]	[1.29 1.88]		
ACH_4	**0.9920** (0.0024)	**1.54** (0.30)	**1.66** (0.15)	**0.9945** (0.0050)	**1.63** (0.28)	**1.64** (0.16)	68.7	11	**0.9948** (0.0069)	**1.64** (0.31)	**1.64** (0.15)	7.2	40
	[0.9872 0.9963]	[1.07 2.26]	[1.33 1.87]	[0.9863 1.0055]	[1.22 2.34]	[1.28 1.86]			[0.9889 1.0026]	[1.18 2.39]	[1.28 1.87]		

Compared with other prominent studies that assess empirical support for the C-CAPM paradigm, these results are certainly encouraging. Julliard and Parker (2005), for example, aggregate consumption over multiple periods and obtain an RRA estimate of plausible magnitude ($\hat{\gamma}$=9.1) but only moderate estimation precision (s.e.=17.2). By measuring consumption with waste, Savov (2011) obtains an RRA estimate of $\hat{\gamma}$=17.0 with a rather large standard error (s.e.=9.0). In both studies, the subjective discount factor is calibrated, with an assumption of additive power utility (such that $\gamma = 1/\psi$). Yogo (2006) splits consumption into a durable and a non-durable component and assumes Epstein-Zin-Weil preferences, as in the present study. His smallest RRA estimate is $\hat{\gamma}$=174.5 (s.e.=23.3), and the IES estimates reach $\hat{\psi}$=0.024 (s.e.=0.009) at most.

3.5.3 Asset pricing implications

When assessing whether an empirical C-CAPM has meaningful asset pricing implications, the magnitude and relative size of the subjective discount factor, relative risk aversion, and the IES all play important roles. The relative size of the RRA coefficient and the IES reflected in the parameter $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, which shows up in the Epstein-Zin-Weil SDF in Equation (3.5), is particularly important. If $\gamma = \frac{1}{\psi}$, then $\theta = 1$, the investor is indifferent to an early or late resolution of uncertainty, and the case of standard expected utility obtains. If $\gamma > \frac{1}{\psi}$, the agent has a preference for an early resolution of uncertainty, which is intuitively appealing, unless we were to resort to behavioral explanations (e.g., hope, fear).

The C-CAPM literature, and in particular the branch concerned with long-run risk, argues that an IES greater than unity combined with a preference for early resolution of uncertainty are necessary to explain the key features of asset prices (e.g., Bansal and Yaron (2004); Huang and Shaliastovich (2015)). When risk aversion is greater than unity, θ should be negative.[38] Therefore, calibration studies tend to combine moderate risk aversion with an IES>1 to illustrate the explanatory power of the asset pricing model (e.g., Bansal and Yaron (2004) assume γ=10 and ψ=1.5), yet none of the previously cited empirical C-CAPM studies reports conforming RRA and IES estimates. Rather, the IES point estimate in most empirical studies is smaller than 1 (see the meta-analysis by Havránek (2015); survey by Thimme (2017)).

Table 22 reports the ACH_0-PL-based, model-implied estimates of θ. We observe

[38] An alternative interpretation of θ is given by Hansen and Sargent (2010), where a $\theta < 0$ captures the agent's aversion to model mis-specification.

that for the alternative sets of test assets and choices of the copula correlation, $\hat{\theta}$ is always negative. Moreover, the confidence bounds reveal that the hypothesis that $\theta > 0$ can be rejected at conventional significance levels, so there is empirical evidence for early resolution of uncertainty, along with an IES greater than 1. According to the previous reasoning, the empirical disaster-including C-CAPM thus should yield meaningful asset pricing implications. I test whether the model-implied mean market portfolio and T-bill return, the equity premium, and the market Sharpe ratio are economically plausible. To estimate the model-implied mean T-bill return and mean market return, I approximate the population moments by averaging over the $\mathcal{T}$ simulated observations, such that

$$\widehat{\mathbb{E}}(R_b) = \frac{1 - \text{cov}_{\mathcal{T}}(m(\hat{\beta}, \hat{\gamma}, \hat{\psi}), R_b)}{\mathbb{E}_{\mathcal{T}}(m(\hat{\beta}, \hat{\gamma}, \hat{\psi}))}, \tag{3.39}$$

and

$$\widehat{\mathbb{E}}(R_a) = \frac{1 - \text{cov}_{\mathcal{T}}(m(\hat{\beta}, \hat{\gamma}, \hat{\psi}), R_a)}{\mathbb{E}_{\mathcal{T}}(m(\hat{\beta}, \hat{\gamma}, \hat{\psi}))}, \tag{3.40}$$

where $m(\hat{\beta}, \hat{\gamma}, \hat{\psi})$ is the Epstein-Zin-Weil SDF in Equation (3.5) evaluated according to the parameter estimates presented in Table 21, and $\text{cov}_{\mathcal{T}}(x, y) = \mathbb{E}_{\mathcal{T}}(xy) - \mathbb{E}_{\mathcal{T}}(x)\mathbb{E}_{\mathcal{T}}(y)$. The model-implied equity premium can be estimated by $\widehat{\mathbb{E}}(R_a) - \widehat{\mathbb{E}}(R_b)$, and the model-implied Sharpe ratio by

$$\frac{\widehat{\mathbb{E}}(R_a) - \widehat{\mathbb{E}}(R_b)}{\sigma_{\mathcal{T}}(R_a - R_b)}, \tag{3.41}$$

where $\sigma_{\mathcal{T}} = \sqrt{\mathbb{E}_{\mathcal{T}}(x^2) - \mathbb{E}_{\mathcal{T}}(x)^2}$. Performing the computation for each of the bootstrap replications accounts for parameter estimation uncertainty.

Table 22 contains the estimates of these model-implied financial indicators along with the 95% confidence interval bounds obtained by the percentile method. The panels break down the results by choice of the copula correlation parameter; each panel reports the estimates for the three sets of test assets. The column labeled *data* reports the values of the indicators in the sample period 1947:Q2-2014:Q4. The table shows that the magnitude of the model-implied equity premium, mean T-bill return, and the market Sharpe ratio are perfectly plausible and comparable to their sample equivalents. This finding is robust with respect to the choice of the copula correlation coefficient and the set of test assets. The model-implied $\hat{\mathbb{E}}(R_b)$ and $\hat{\mathbb{E}}(R_a)$ are somewhat smaller than the average T-bill return and the market return in the

Table 22: Model-implied key financial indicators

The table presents estimates of the mean T-bill return, mean market return, equity premium, and market Sharpe ratio implied by the disaster-including C-CAPM and computed according to Equations (3.39)-(3.41). The computation uses the SMM-type estimates of β, γ, and ψ based on the ACH_0 first-step estimates (see Table 21). The numbers in brackets are the lower and upper bounds of the 95% confidence intervals computed using the percentile method. Panels A-C break down the results by the copula correlation coefficient used in the data simulation procedure, and each panel reports the results by the set of test assets. The column labeled *data* reports the values of the indicators in the empirical data, 1947:Q2–2014:Q4.

		Panel A: $\rho = \text{Corr}(cg_{nd}, R_{nd})$		
	data	**mkt**	**size dec**	**industry**
$\hat{\theta} = (1-\hat{\gamma})/(1-\frac{1}{\hat{\psi}})$		-1.54	-1.81	-1.86
		[-3.55 -0.21]	[-3.77 -0.64]	[-4.07 -0.48]
mean T-bill return	*0.17*	0.10	0.12	0.14
(% per qtr)		[-0.13 0.29]	[-0.18 0.33]	[-0.17 0.36]
equity premium	*1.94*	1.85	2.06	2.11
(% per qtr)		[0.98 2.76]	[1.36 2.83]	[1.23 3.08]
mean market return	*2.11*	1.95	2.19	2.25
(% per qtr)		[1.13 2.80]	[1.51 2.89]	[1.38 3.09]
Sharpe ratio	*0.237*	0.226	0.252	0.257
(market)		[0.111 0.378]	[0.154 0.394]	[0.139 0.427]
		Panel B: $\rho = 0.99$		
		mkt	**size dec**	**industry**
$\hat{\theta} = (1-\hat{\gamma})/(1-\frac{1}{\hat{\psi}})$		-1.53	-1.80	-1.85
		[-3.51 -0.20]	[-3.75 -0.63]	[-4.05 -0.47]
mean T-bill return		0.10	0.13	0.14
(% per qtr)		[-0.12 0.29]	[-0.18 0.33]	[-0.16 0.36]
equity premium		1.85	2.06	2.11
(% per qtr)		[0.97 2.72]	[1.36 2.83]	[1.23 3.08]
mean market return		1.95	2.19	2.25
(% per qtr)		[1.13 2.78]	[1.50 2.89]	[1.38 3.09]
Sharpe ratio		0.226	0.252	0.257
(market)		[0.111 0.370]	[0.153 0.394]	[0.139 0.427]
		Panel C: $\rho = 0$		
		mkt	**size dec**	**industry**
$\hat{\theta} = (1-\hat{\gamma})/(1-\frac{1}{\hat{\psi}})$		-1.54	-1.80	-1.86
		[-3.50 -0.21]	[-3.76 -0.64]	[-4.07 -0.48]
mean T-bill return		0.10	0.13	0.14
(% per qtr)		[-0.12 0.29]	[-0.18 0.34]	[-0.16 0.36]
equity premium		1.84	2.05	2.09
(% per qtr)		[0.97 2.71]	[1.35 2.79]	[1.22 3.05]
mean market return		1.94	2.18	2.23
(% per qtr)		[1.12 2.76]	[1.50 2.87]	[1.37 3.07]
Sharpe ratio		0.225	0.251	0.256
(market)		[0.110 0.368]	[0.153 0.391]	[0.139 0.423]

empirical data, because the model-implied indicators account for the possibility of consumption disasters that affect the simulated moments, whereas the empirical data do not contain any disaster observation. However, the observed mean T-bill, mean market return, and equity premium lie within the 95% confidence interval bounds, which account for the first- and second-step estimation error.

When using only the market portfolio and the T-bill as test assets, the model is exactly identified, which seemingly could drive the favorable results. However, exact identification does not imply that the empirical mean market return and mean T-bill return must be matched by their model-implied counterparts. When using the *size dec* or *industry* portfolios, the market portfolio is not even among the set of test assets. These specifications serve as an out-of-sample plausibility test. In these instances, $\widehat{\mathbb{E}}(R_a)$ and the model-implied equity premium are still perfectly plausible and comparable to their empirical counterparts. In all instances, the confidence intervals overlap the empirically observed values.

The meaningful asset pricing implications of the estimated disaster-including C-CAPM show that the model can explain the considerable postwar equity premium and the relatively low T-bill return with plausible investor preferences. Unlike in previous studies of the rare disaster hypothesis, risk aversion, time preferences, and IES are not calibrated, i.e., conveniently chosen, but rather are obtained from the application of an econometric estimation strategy. These results thus provide new empirical evidence that the rare disaster hypothesis offers a solution to the equity premium puzzle.

3.5.4 Robustness checks

As robustness check, I perform bias corrections on the parameter estimates and confidence bounds, and report the results in Table 23. Following Efron and Tibshirani (1986), I compute bias-corrected estimates of a parameter ϑ as $\hat{\vartheta}_{BC} = 2\hat{\vartheta} - \frac{1}{K}\sum_{k=1}^{K}\hat{\vartheta}^{(k)}$. The lower and upper bounds of the bias-corrected $1-\alpha$ confidence interval are computed as $\vartheta^{l}_{BC}(\alpha) = \hat{G}^{-1}[\Phi(z_{\alpha/2} + 2\Phi^{-1}[\hat{G}(\hat{\vartheta})])]$ and $\vartheta^{u}_{BC}(\alpha) = \hat{G}^{-1}[\Phi(z_{1-\alpha/2} + 2\Phi^{-1}[\hat{G}(\hat{\vartheta})])]$, respectively, where Φ denotes the cdf, Φ^{-1} is the quantile function, and $z_{\tilde{\alpha}}$ is the $\tilde{\alpha}$-quantile of the standard normal distribution.[39] Comparing the results in Table 23 with those in Table 21, I find that in all instances, the corrections are rather benign. The similarity of the the bias-corrected estimates and confidence intervals to the uncorrected counterparts offers a sign of robustness.

[39] According to this notation, the uncorrected confidence bounds in Table 21 are computed as $\vartheta^{l}(\alpha) = \hat{G}^{-1}[\Phi(z_{\alpha/2})]$ and $\vartheta^{u}(\alpha) = \hat{G}^{-1}[\Phi(z_{1-\alpha/2})]$.

Table 23: Bias-corrected preference parameter estimates and confidence intervals
This table presents bias-corrected estimates (bold) and 95% confidence bounds (in brackets) of the subjective discount factor β, the coefficient of relative risk aversion γ, and the IES ψ. The bias correction of the point estimates and confidence bounds in Table 21 follows the method proposed by Efron and Tibshirani (1986).

				Panel A: ρ = Corr($\mathbf{cg_{nd}}, \mathbf{R_{nd}}$)					
	mkt			**size dec**			**industry**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$
ACH_0	**0.9918**	**1.44**	**1.40**	**0.9938**	**1.50**	**1.40**	**0.9942**	**1.52**	**1.40**
	[0.9877 0.9963]	[1.01 2.11]	[1.13 1.69]	[0.9871 1.0068]	[1.19 2.18]	[1.08 1.72]	[0.9893 1.0043]	[1.12 2.26]	[1.08 1.72]
ACH_4	**0.9924**	**1.49**	**1.73**	**0.9947**	**1.59**	**1.70**	**0.9948**	**1.61**	**1.69**
	[0.9881 0.9972]	[1.05 2.29]	[1.41 1.93]	[0.9871 1.0088]	[1.21 2.34]	[1.36 1.93]	[0.9894 1.0050]	[1.16 2.38]	[1.33 1.91]
				Panel B: ρ = 0.99					
	mkt			**size dec**			**industry**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$
ACH_0	**0.9916**	**1.46**	**1.41**	**0.9937**	**1.51**	**1.42**	**0.9940**	**1.53**	**1.42**
	[0.9875 0.9961]	[1.03 2.13]	[1.14 1.70]	[0.9869 1.0068]	[1.19 2.19]	[1.09 1.72]	[0.9891 1.0043]	[1.12 2.27]	[1.09 1.72]
ACH_4	**0.9918**	**1.50**	**1.75**	**0.9940**	**1.59**	**1.74**	**0.9944**	**1.60**	**1.74**
	[0.9873 0.9963]	[1.06 2.33]	[1.44 1.93]	[0.9876 1.0090]	[1.17 2.28]	[1.41 1.95]	[0.9887 1.0050]	[1.16 2.44]	[1.42 1.94]
				Panel C: ρ = 0					
	mkt			**size dec**			**industry**		
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$
ACH_0	**0.9918**	**1.45**	**1.39**	**0.9938**	**1.50**	**1.40**	**0.9942**	**1.51**	**1.40**
	[0.9877 0.9965]	[1.00 2.12]	[1.13 1.68]	[0.9871 1.0068]	[1.19 2.18]	[1.08 1.71]	[0.9894 1.0045]	[1.12 2.26]	[1.08 1.71]
ACH_4	**0.9923**	**1.50**	**1.73**	**0.9949**	**1.57**	**1.69**	**0.9950**	**1.59**	**1.69**
	[0.9878 0.9968]	[1.07 2.25]	[1.39 1.92]	[0.9877 1.0104]	[1.17 2.27]	[1.35 1.90]	[0.9896 1.0041]	[1.16 2.36]	[1.38 1.93]

A second robustness check investigates the effect of varying the disaster threshold q. Panel A of Table 24 uses q=0.095, and Panel B reports the results for q=0.195. These values are chosen in accordance with Barro and Jin (2011) and feature prominently in rare disaster literature. The results in Table 24 convey that the choice of q barely affects the parameter estimates; this finding may seem surprising at first, but it is a consequence of the multi-period character of the disasters. The effects of different choices of q enter the data simulation procedure through the ACH-PL estimates $\hat{\boldsymbol{\theta}}_{ACH}$ and $\theta^{+}_{PL}, \theta_{PL}$, obtained from quarterly (contraction) data that have been computed from annual (disaster) periods. Because θ^{+}_{PL} and θ_{PL} contain information about the distribution of quarterly contractions, they could vary strongly with q only if the distribution of the annual contraction sizes of disasters detected with a threshold of 0.095 were pronouncedly different from that of disasters that had been detected with q=0.195. This was not the case.

Therefore, the estimation results are robust with respect to alternative data simulation procedures, test assets, and disaster thresholds. The fact that they are also quite unbiased serves as a further recommendation.

Table 24: C-CAPM preference parameters with varying disaster thresholds

This table presents the SMM-type estimates of the preference parameters β, γ, and ψ using $\rho = \text{Corr}(cg_{nd}, R_{nd})$. Panel A relies on q=0.095, and Panel B contains results for q=0.195. Other estimation settings and the reported statistics correspond to Table 21.

	mkt			size dec					industry				
Panel A: q=0.095/ρ=Corr($\mathbf{cg_{nd}, R_{nd}}$)													
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R
ACH_0	**0.9918** (0.0047)	**1.49** (0.29)	**1.48** (0.14)	**0.9938** (0.0050)	**1.56** (0.26)	**1.49** (0.14)	78.9	10	**0.9942** (0.0090)	**1.57** (0.32)	**1.49** (0.14)	11.5	39
	[0.9878 0.9960]	[1.03 2.15]	[1.33 1.86]	[0.9864 1.0050]	[1.24 2.24]	[1.34 1.87]			[0.9891 1.0047]	[1.14 2.41]	[1.34 1.87]		
ACH_4	**0.9919** (0.0023)	**1.51** (0.30)	**1.58** (0.14)	**0.9941** (0.0051)	**1.56** (0.29)	**1.58** (0.14)	67.0	11	**0.9942** (0.0089)	**1.57** (0.32)	**1.58** (0.14)	8.9	39
	[0.9874 0.9962]	[1.07 2.23]	[1.34 1.87]	[0.9868 1.0053]	[1.22 2.33]	[1.31 1.86]			[0.9893 1.0037]	[1.14 2.39]	[1.31 1.86]		
Panel B: q=0.195/ρ=Corr($\mathbf{cg_{nd}, R_{nd}}$)													
	mkt			**size dec**					**industry**				
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\psi}$	J	R
ACH_0	**0.9917** (0.0023)	**1.51** (0.30)	**1.49** (0.16)	**0.9938** (0.0044)	**1.58** (0.27)	**1.47** (0.16)	84.9	9	**0.9943** (0.0034)	**1.60** (0.31)	**1.47** (0.16)	13.1	39
	[0.9869 0.9958]	[1.08 2.25]	[1.26 1.86]	[0.9863 1.0044]	[1.24 2.28]	[1.27 1.86]			[0.9893 1.0021]	[1.21 2.34]	[1.27 1.86]		
ACH_4	**0.9917** (0.0023)	**1.57** (0.30)	**1.63** (0.17)	**0.9940** (0.0061)	**1.66** (0.36)	**1.63** (0.19)	80.2	9	**0.9943** (0.0087)	**1.68** (0.48)	**1.63** (0.20)	11.4	39
	[0.9869 0.9959]	[1.08 2.22]	[1.26 1.89]	[0.9862 1.0063]	[1.15 2.32]	[1.19 1.88]			[0.9886 1.0040]	[1.08 2.44]	[1.17 1.88]		

3.6 Discussion and conclusion

This study adopts Barro's (2006) specification of a disaster-including consumption process and derives moment restrictions that facilitate the estimation of a disaster-including C-CAPM by an SMM-type strategy. The approach presented herein takes into account three main drawbacks of previous studies that aim to test the rare disaster hypothesis empirically. First, I allow for multi-period disasters. It has been argued that the success of the rare disaster hypothesis in calibration studies relies on the assumption that the entire disastrous contraction occurs in one period (see Julliard and Ghosh (2012); Constantinides (2008)). Second, I use Epstein-Zin-Weil preferences instead of a power utility to acknowledge preferences for an early resolution of uncertainty. Third, I allow for the possibility of a partial government default. Accounting for these three issues is crucial for finding empirical support for the RDH.

For an SMM-type estimation, I simulate disaster-including consumption growth and return series by means of a discrete-time marked point process that models the time duration of and between disasters, as well as the magnitude of contractions using a power law distribution. Parameter estimates of the MPP model are obtained through maximum likelihood, using chained country-panel data. Neither the choice of test assets nor the disaster thresholds change the results qualitatively: The magnitude of the estimated preference parameters is economically plausible, and the estimation precision is much higher than in previous C-CAPM studies. The subjective discount factor estimate is about 0.99 in all specifications; the RRA estimates (and 95% confidence bounds) fall within a strict plausibility range, and the IES parameter estimates are significantly greater than unity. The relative magnitude of the estimated IES and RRA coefficients indicate a preference for early resolution of uncertainty, which, in conjunction with an IES greater than unity, is an important condition for obtaining meaningful asset pricing implications. Computing the model-implied mean market return, T-bill rate, and market Sharpe ratio reveals that the disaster-including C-CAPM can explain these key financial indicators based on economically meaningful preference parameter estimates.

To the best of my knowledge, the present study is the first research to estimate all the preference parameters of a C-CAPM with Epstein-Zin-Weil preferences and multi-period disasters. It corroborates the notion that the rare disaster hypothesis can provide a solution to the equity premium puzzle, even when disasters do not shrink to one-period events.

B Appendix

B.1 Transformation of annual into quarterly consumption contractions

The ACH-PL model assumes a quarterly observation frequency. To obtain four quarterly contractions from an annual observation, I draw from a standard uniform distribution and determine the fraction of the annual contraction that is assigned to the first quarter. How much of the remaining contraction is allocated to the second quarter is determined by another standard uniform draw. The contraction assigned to the third quarter is determined the same way. The last quarter takes what is left. This procedure implies that the contraction in the first (last) quarter will be the largest (smallest), on average. To avoid such a seasonal pattern, I re-shuffle the four quarterly contractions randomly. This procedure applies to a year that is not the first or the last of a disaster. When dealing with the first (last) year of a disaster, or if the disaster consists of only one annual contraction, I determine the quarter when the contraction begins (ends) by a draw from a discrete uniform distribution, such that each quarter has a 1/4 probability of becoming the quarter when the disaster begins (ends). The annual contraction is then distributed across the disaster quarters in a way analogous to the method used for a "within" disaster year.

B.2 Bootstrap inference

Bootstrap inference for the second-step preference parameter estimates is based on a mix of parametric and non-parametric bootstraps. Using the first-step maximum likelihood estimates $\hat{\boldsymbol{\theta}}_{ACH}$, $\hat{\theta}_{PL}$, and $\hat{\theta}^{+}_{PL}$, I simulate a series of hazard rates, consumption contractions, and disaster indicators d_s and d^{+}_s as described in Section 3.4.2. The length of the simulated series is equal to the number of observations in the concatenated country data. Next, $\boldsymbol{\theta}_{ACH}$ and $\boldsymbol{\theta}_{PL}$ are re-estimated on the simulated series. These steps are repeated K times, and the estimates are collected in $\{\hat{\boldsymbol{\theta}}^{(k)}_{ACH}, \hat{\theta}^{(k)}_{PL}, \hat{\theta}^{+(k)}_{PL}\}^{K}_{k=1}$. Because I draw from the parametric ACH-PL distribution using the maximum likelihood estimates, this procedure can be characterized as a parametric bootstrap. It complements the asymptotic inference that is available for the first estimation step, but it is also crucial input for inference about the second-step SMM estimates of the preference parameters.

For each of the K replications, I perform a block-bootstrap to obtain series of non-disastrous consumption growth $\{cg^{(k)}_{nd,l}\}^{T}_{l=1}$, market and T-bill returns $\{R^{(k)}_{nd,a,l}\}^{T}_{l=1}$, $\{R^{(k)}_{nd,b,l}\}^{T}_{l=1}$, and test asset returns $\{R^{(k)}_{nd,i,l}\}^{T}_{l=1}$. As described previously, I determine the mean of the geometric distribution, from which the block-lengths are drawn using

Politis et al.'s (2009) automatic block-length selection algorithm. The length of the bootstrap data series (T) is the same as in the original financial and macro data. Draws from the series are exerted simultaneously to retain their contemporaneous dependence (see Maio and Santa-Clara (2012) for a similar approach).

To compute the simulated moments for each replication, I proceed as described in Section 3.4.2 and generate disaster-including data of length $\mathcal{T}$, $\{cg_s^{(k)}\}_{s=1}^{\mathcal{T}}$, $\{R_{i,s}^{(k)}\}_{s=1}^{\mathcal{T}}$, $\{R_{b,s}^{(k)}\}_{s=1}^{\mathcal{T}}$, and $\{R_{a,s}^{(k)}\}_{s=1}^{\mathcal{T}}$. For that purpose, I use the parametric bootstrap estimates $\hat{\boldsymbol{\theta}}_{ACH}^{(k)}$, $\hat{\theta}_{PL}^{(k)}$, and $\hat{\theta}_{PL}^{+(k)}$ obtained from the maximum likelihood estimation on the simulated data (instead of the original data). The block-bootstrap from non-disastrous data that is required to compute the simulated moments is performed on $\{cg_{nd,l}^{(k)}\}_{l=1}^{T}$, $\{R_{nd,a,l}^{(k)}\}_{l=1}^{T}$, $\{R_{nd,b,l}^{(k)}\}_{l=1}^{T}$, and $\{R_{nd,i,l}^{(k)}\}_{l=1}^{T}$ (instead of the original data). Then the SMM-type estimation of the preference parameters β, γ, and ψ proceeds as described in Section 3.2.1. Performing these steps for each of the K replications yields $\{\hat{\beta}^{(k)}, \hat{\gamma}^{(k)}, \hat{\psi}^{(k)}\}_{k=1}^{K}$, for which standard deviations and confidence intervals can be computed using the percentile method.

CHAPTER 4

Asset Pricing with Multi-Period Disasters and Two Consumption Goods‡

4.1 Motivation

In this chapter, I extend Yogo's (2006) study, which uses recursive preferences and assumes that utility is nonseparable in durable and nondurable consumption by allowing for disastrous contractions in both consumption goods. Whilst Yogo (2006) finds that his model can explain the cross-section of stock returns, the estimates of the coefficient of constant relative risk aversion and the intertemporal elasticity of substitution are implausible in size. Accounting for multi-period disasters in both consumption goods and allowing for partial government defaults, I use the simulated method of moments and obtain plausible estimates of these preference parameters. The simulation of the disaster-including series is inspired by Bansal and Yaron (2004) and Gabaix (2012), and a discrete-time marked point process (MPP) models the timing, length, and severity of the disasters; some of which are accompanied by a destruction of the stock of the durable consumption good. Whilst the estimation is generally built on U.S. data, the identification of the disaster process parameters is performed using cross-country panel data. A two-step bootstrap procedure is applied to evaluate the precision of the estimates. To the best of my knowledge, this is the first study that considers different types of consumption goods when estimating the preference parameters of a disaster-including C-CAPM.

The resulting RRA and IES estimates are economically sensible and qualitatively insensitive with respect to a battery of robustness checks that critically question the main assumptions that must be made throughout the analysis. Furthermore, the relation of the parameters is such that a preference for early resolution of uncertainty is implied; not only for the point estimates but also for the vast majority of the bootstrap replications performed to assess the estimation precision. The 95% confidence intervals of the IES are narrow. For the RRA coefficient, which features wider confidence intervals, more than 85% of the bootstrap replications result in estimates that lie in the (0,5] interval that is considered economically plausible.

‡ This chapter is based on Sönksen (2017b), available on ssrn: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2939039

The remainder of this chapter is structured as follows: Section 4.2 outlines the model specification that relies on recursive preferences and considers consumption of durable and nondurable goods. Furthermore, it illustrates through which channels multi-period disasters enter consumption growth and returns. Section 4.3 introduces a marked point process used to model the timing and size of multi-period disasters, and it explains how to link contraction sizes in overall consumption growth to those in the durable and nondurable good, respectively. Section 4.4 contains the macroeconomic and financial data used in this study. Section 4.5 explains the two-step estimation strategy and Section 4.6 addresses the caveats that must be dealt with when bringing the model to the data. Section 4.7 presents estimation results and robustness checks and Section 4.8 concludes.

4.2 Model outline

Yogo (2006) differentiates between consumption of durable and nondurable goods, and specifies an investor's intraperiod utility by the following function:

$$u(C,D) = \left[(1-\alpha)C^{\frac{\rho-1}{\rho}} + \alpha D^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{4.1}$$

which implies a constant elasticity of substitution $\rho \geq 0$ between the nondurable good C and the stock of the durable good D. For the weighting of the goods, it has to hold that $\alpha \in (0,1)$. Using the intraperiod utility specification from Equation (4.1) and assuming that the investor's interperiod utility follows recursive preferences introduced by Epstein and Zin (1989), and Weil (1989), Yogo (2006) derives the stochastic discount factor (SDF):

$$\begin{aligned} m_{t+1} &= \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)^{\frac{1}{\rho}-\frac{1}{\psi}} R_{a,t+1}^{\frac{\theta-1}{\theta}}\right]^{\theta}, \\ \text{where} \quad v\left(\frac{D}{C}\right) &= \left[1-\alpha+\alpha\left(\frac{D}{C}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \end{aligned} \tag{4.2}$$

In Equation (4.2), β denotes the subjective discount factor, which measures time preference and should be < 1, as it is generally assumed that investors prefer immediate over future consumption. γ is the coefficient of constant RRA and $0 < \gamma \leq 10$ is considered to describe reasonably risk-averse investors.[41] ψ denotes the intertemporal

[41] For example: Mehra and Prescott (1985), Rietz (1988), Bansal and Yaron (2004), and Barro (2006). Cochrane (2005) caps the interval at 5.

elasticity of substitution. Regarding the range of plausible IES values, there is no consensus in related literature: Whilst the vast majority of empirical studies points towards $\psi < 1$,[42] there are theoretical arguments in favor of $\psi > 1$. For instance, Nakamura et al. (2013) argue that ψ must exceed 1 to accord with the observed behavior of asset prices during consumption disasters, and Barro (2009) as well as Bansal and Yaron (2004) further claim that the observed dynamics of the price-dividend ratio cannot be replicated if $0 < \psi < 1$. Epstein and Zin (1989) argue that, apart from the specific parameter values, $\gamma > \frac{1}{\psi}$ implies a preference for early resolution of uncertainty, which is what we would expect from rational investors.[43]

Letting E_t denote the units of the durable consumption good purchased in period t, I consider log consumption growth $g_{C,t+1} = \ln(\frac{C_{t+1}}{C_t})$ and $g_{E,t+1} = \ln(\frac{E_{t+1}}{E_t})$ and claim that the respective processes evolve as:

$$\begin{aligned} x_{t+1} &= \varphi_e e_{t+1}, \\ g_{C,t+1} &= \mu_C + x_t + \ln(1 - b_{C,t+1}) d_{t+1} + \sigma_C \eta_{C,t+1}, \\ g_{E,t+1} &= \mu_E + x_t + \ln(1 - b_{E,t+1}) d_{t+1} + \sigma_E \eta_{E,t+1}, \\ & e_{t+1}, \eta_{C,t+1}, \eta_{E,t+1} \sim \mathcal{N}(0,1), \end{aligned} \tag{4.3}$$

where x_{t+1} denotes a fundamental growth component that is independent of the idiosyncratic innovations in the consumption growth processes. However, no restrictions are imposed on $\mathbb{E}[\eta_{C,t+1}\eta_{E,t+1}]$. d_{t+1} is a disaster indicator that turns 1 if period $t+1$ is a disaster period and 0 otherwise. If $d_{t+1} = 1$, log consumption growth of the nondurable good contracts by the random factor $\ln(1 - b_{C,t+1})$. For log consumption growth of the durable good, this factor is labeled $\ln(1 - b_{E,t+1})$, respectively. For both factors, it holds true that $b_{\cdot,t+1} \in (0,1]$.

$g_{C,t+1}$ and $g_{E,t+1}$ can be combined to the overall log consumption growth process:

$$\begin{aligned} g_{t+1} &= \ln\left(\frac{C_t}{C_t + E_t}\exp(g_{C,t+1}) + \frac{E_t}{C_t + E_t}\exp(g_{E,t+1})\right) \\ &= \mu + x_t + \ln(1 - b_{t+1}) d_{t+1} + \sigma \eta_{t+1}, \end{aligned} \tag{4.4}$$

[42] Havránek (2015) performs a meta-analysis of 169 published articles and concludes that IES values > 0.8 are inconsistent with empirical evidence.

[43] Equivalently, $\gamma < \frac{1}{\psi}$ indicates a preference for late resolution of uncertainty and $\gamma = \frac{1}{\psi}$ means indifference regarding the timing of the uncertainty resolution. From a behavioral perspective, a preference for late resolution of uncertainty can be explained by emotions like fear and hope. However, early resolution of uncertainty allows the inclusion of information on the outcome of the random experiment into considerations regarding the smoothing of the consumption path and is thus preferable from a rational perspective.

where b_{t+1} is the overall consumption contraction. The weighting in Equation (4.4) implies that $b_{t+1} \in (0,1]$, which means this multi-period disaster framework does not require the contractions of each respective disaster period to be severe. Rather, I define a disaster as a succession of contractions that starts in period s_1 and lasts until period s_2 with $s_1 \leq t+1 \leq s_2$ such that:

$$1 - \prod_{j=s_1}^{s_2} (1 - b_j) \geq q, \tag{4.5}$$

where q denotes the disaster threshold. Each period that contributes to shaping a disaster is denoted a *disaster period*.

The development of the stock of the durable consumption good is an important component of the SDF in Equation (4.2). Following Yogo (2006), I assume the stock of the durable consumption good D evolves as follows:

$$D_t = (1 - \delta_t) D_{t-1} + E_t, \tag{4.6}$$

where δ_t is a random depreciation factor:[44]

$$\delta_t = (\delta_t^* + b_{E,t}(1 - \delta_t^*) d_t)(1 - d_t^D) + d_t^D \qquad \text{with} \qquad \delta_t^* = a + \beta_\delta \delta_{t-1}^* + \varepsilon_t, \quad \text{and} \quad \varepsilon_t = \mathcal{N}(0, \sigma_\varepsilon^2). \tag{4.7}$$

The binary variable d_t^D indicates if a given disaster period is affected by the destruction of the stock of the durable good. If so, $d_t^D = 1$, otherwise $d_t^D = 0$. Considering the evolution of historic consumption disasters, such a differentiation is plausible: Wars and political or economic turmoils are the most frequent causes of the severe consumption contractions that we observed in the last 200 years in the Western world. Some of these disasters – and especially those linked to wars – also brought about a destruction of the stock of the durable good. I will refer to such cases as *destructive disasters*. By including the term $b_{E,t}(1-\delta_t^*)d_t$, Equation (4.7) furthermore implies that the stock of the durable good is subject to a stronger depreciation during disaster periods that do not belong to a destructive disaster. This extra depreciation should be understood as being caused by a lack of maintenance and repair due to financial constraints rather than a straightforward demolition of the good. During a destructive disaster, when the stock of the durable good is wiped out, the SDF reduces to the standard Epstein-Zin-Weil specification.

[44] In Yogo's (2006) study, this depreciation factor is deterministic at 6%.

The theoretical foundation of the log consumption growth specifications in Equation (4.3) results from combining Barro's (2006) model, in which non-disastrous consumption growth is scaled by a contraction factor in the event of a disaster, and Bansal and Yaron's (2004) log consumption growth specification, which is stripped from its long-run characteristic but still includes a joint fundamental component in the form of x_{t+1}. I rely on Barro's (2006) scaling approach to account for disastrous consumption contractions, because it features prominently in the rare disaster literature (e.g., Barro (2009); Nakamura et al. (2013); Gabaix (2012)), and I use Bansal and Yaron's (2004) joint fundamental growth component to model the dependencies between log consumption growth and the log dividend growth ($g_{d,i}$) and log price-dividend ratio (z_i) of a test asset i. These dependencies are of particular importance, because the $g_{d,i}$ and z_i series will be combined to obtain log returns of test asset i according to Campbell and Shiller (1988):

$$r_{i,t+1} = -\ln(\rho_i) + g_{i,d,t+1} + \rho_i(z_{i,t+1} - z_{i,t}), \tag{4.8}$$

where $\rho_i = \frac{\exp(\bar{z}_i)}{1+\exp(\bar{z}_i)}$.[45]

For this purpose, I assume that the log dividend growth process and the log price-dividend ratio can be expressed as:

$$\begin{aligned} g_{d,i,t+1} &= \mu_{d,i} + \phi_i x_t + \ln(1 - b_{t+1})d_{t+1} + \sigma_{d,i}u_{i,t+1}, \\ z_{i,t+1} &= \mu_{z,i} + \beta_{z,i}x_{t+1} + \ln(1 - b_{t+1})d_{t+1} + \rho_{z,i}z_{i,t}, \end{aligned} \tag{4.9}$$

where $u_{i,t+1} \sim \mathcal{N}(0,1)$ with $\mathbb{E}[u_{i,t+1}\varepsilon_{t+1}] = 0 \quad \forall i$. Again, no restrictions are imposed on the correlation between $u_{i,t+1}$ and the idiosyncratic consumption growth innovations.[46] The processes in Equation (4.9) are disaster-including extensions of Bansal and Yaron's (2004) log dividend growth and log price-dividend ratio. In the case of the latter, a further extension comes in the form of the lagged log price-dividend ratio. This term is needed to preserve the persistent pattern of z_i. Gabaix (2012) proposes to express disastrous dividend growth as a scaled version of non-disastrous dividend growth, which is in line with the $g_{d,i}$ specification in Equation (4.9). Regarding the size of z_i in case of a disaster, Barro (2009) argues the price-dividend ratio decreases if disasters are more severe and/or likely – a behavior also displayed by the process in Equation (4.9).

There are two returns for whose specification I do not rely on Equation (4.8): the

[45] A derivation of Equation (4.8) can be found in Section C.1 of the appendix.
[46] This assumption is in line with Bansal and Kiku (2011).

T-bill return and the return on the wealth portfolio. For the log T-bill return (r_b), I propose the following specification:

$$\begin{aligned} r_{b,t+1} &= \mu_b + \phi_b x_t + \delta_b (r_{b_t}(1 - d^b_{t+1}) + r^*_{b,t} d^b_{t+1}) + \sigma_b \eta_{b,t+1} + \ln(1 - b_{t+1}) d^b_{t+1}, \\ r^*_{b,t+1} &= \mu_b + \phi_b x_t + \delta_b r^*_{b,t} + \sigma_b \eta_{b,t+1}, \end{aligned} \tag{4.10}$$

where $\eta_{b,t+1} \sim \mathcal{N}(0,1)$ with $\mathbb{E}[\eta_{b,t+1}\varepsilon_{t+1}] = 0$. Dependencies between $\eta_{b,t+1}$ and the idiosyncratic consumption and dividend growth innovations are not restricted. d^b_{t+1} highlights those disaster periods in which there is also a partial government default, meaning a contraction of $r_{b,t+1}$. In these cases, $d^b_{t+1} = 1$, otherwise $d^b_{t+1} = 0$. It is important to note that either all periods of a disaster are affected by such a default or none, and that a government default can only occur if there is a consumption disaster ($d_{t+1} = 1$) in the first place. Whilst the $r_{b,t+1}$ specification in Equation (4.10) allows for disastrous contractions, $r^*_{b,t+1}$ is its non-disastrous counterpart. This differentiation in the autoregressive component ensures the overall contraction size cannot be grossly overstated due to the high persistence of the process.[47]

The return on aggregate wealth is frequently proxied by the return on the market portfolio (e.g., Yogo (2006)). However, Lustig et al. (2013) find that stock market wealth only accounts for roughly 1% of total household wealth, thus casting doubt on its suitability as a proxy. An alternative approach used by Thimme and Völkert (2015) is a decomposition of $R_{a,t+1}$ using the budget constraint $W_{t+1} = (W_t - K_t)R_{a,t+1}$, where $K_t = C_t + D_t$ denotes overall consumption:

$$\begin{aligned} R_{a,t+1} &= \frac{W_{t+1}}{W_t - K_t} = \frac{K_{t+1}}{K_t} \frac{K_t/W_t}{K_{t+1}/W_{t+1}} \left(1 - \frac{K_t}{W_t}\right)^{-1} \\ &= \exp(g_{t+1}) \exp((k_t - w_t) - (k_{t+1} - w_{t+1}))(1 - \exp(k_t - w_t))^{-1}. \end{aligned} \tag{4.11}$$

I combine the decomposition in Equation (4.11) and Campbell's (1993) definition of the log consumption-wealth ratio to study how consumption disasters diffuse into the return on aggregate wealth:

$$\begin{aligned} k_t - w_t &= \sum_{j=1}^{\infty} \lambda^j (r_{m,t+j} - g_{t+j}) + \frac{\lambda l}{1 - \lambda}, \\ \text{where} \qquad \lambda &= 1 - \exp(\overline{k_t - w_t}), \\ l &= \ln(\lambda) - \left(1 - \frac{1}{\lambda}\right)(\overline{k_t - w_t}). \end{aligned} \tag{4.12}$$

[47] Considering the high first-order autocorrelation of the T-bill return, it can be assumed that δ_b is smaller but very close to 1.

In Equation (4.12), r_m denotes the log return of the market portfolio and can be obtained from the general return specification in Equation (4.8). To model the consumption-wealth ratio using Equation (4.12), λ and l, which in turn depend on $k_t - w_t$, are required. Because the consumption-wealth ratio is unobserved, I follow Thimme and Völkert (2015) in using Lettau and Ludvigson's (2001) *cay*-variable as a proxy; that is, $\frac{K_t}{W_t} = \kappa \exp(cay_t)$, where κ is a scaling parameter.[48]

4.3 Specification of the disaster process

The consumption disaster process requires a model specification which can account for the length and size of disasters as well as for the duration between them. The RDH literature contains different suggestions for such processes. For example, Wachter (2013) suggests a recursive time-varying disaster intensity but does not allow for multi-period disasters. Nakamura et al. (2013) account for the time dimension of disasters by using a heavily parametrized process that differentiates between permanent and transitory effects of contractions and allows for different disaster types (world disasters, individual disasters). The estimation is performed using a cross-country consumption dataset assembled by Barro and Ursúa's (2008).[49] As I want to retain the multi-period character of disasters whilst allowing for a flexible parametrization, I turn to the class of discrete-time marked point processes, which provides a suitable framework in which the disaster periods represent the points and the respective contraction sizes the marks.

4.3.1 Timing and size of consumption disasters

The choice of the disaster threshold q determines the size and length of disasters as well as the time durations between them. Hamilton and Jorda's (2002) autoregressive conditional hazard (ACH) model constitutes a structure for modeling these durations. It implies the hazard rate:

$$h_t = \mathbb{P}(N(t) \neq N(t-1) | \mathcal{F}_{t-1}), \tag{4.13}$$

[48] Related literature considers different values for κ. I choose $\kappa = 0.05$, and decrease the time series mean of the *cay*-series by the average consumption contraction per period, when computing λ and l. Section 4.7.3 contains a robustness check in which the mean of the *cay*-variable is not adapted. This approach implies a consumption-wealth ratio of about 0.05, which is close to Lettau and Ludvigson's (2001) specification which suggests a consumption-wealth ratio of 0.04.

[49] The extensive parametrization in this approach limits the possible choices for the disaster threshold.

where – assuming that the sequence of disaster events is considered at a quarterly frequency – $N(t)$ is the number of disaster periods that occur as of period $t = 1, \ldots, T$. Due to the discrete-time set up, h_t can be understood as the conditional disaster probability in period t.

The hazard rate in Equation (4.13) is flexible regarding the information included to model the conditional disaster probabilities. A parsimonious specification is:

$$h_t = \left[(\pi(1 - d_{t-1}) + \tilde{\pi} d_{t-1})(1 - d_{t-1}^{+}) + d_{t-1}^{+} \right]^{-1}, \tag{4.14}$$

where the indicator d_t^+ is defined as $d_t^+ = \mathbb{1}(d_t = 1) \cdot \mathbb{1}([1 - \prod_{j=s_1}^{t-1}(1 - b_j)] < q)$, meaning that $d_t^+ = 1$ if period t is a disaster period but the disaster threshold q has not yet been reached, thereby setting $h_{t+1} = 1$.[50] π and $\tilde{\pi}$ are parameters.

A more general hazard rate specification also includes information on the durations of and between disasters, as well as on contraction sizes. Assuming that $M(t)$ gives the number of disasters that have occurred as of quarter t, I propose to use:

$$\begin{aligned} h_t = \Big[& \left[(\pi + \breve{\alpha}\tau_{M(t-1)-1} + \breve{\delta} b_{M(t-1)}^{+})(1 - d_{t-1}) \right. \\ & \left. + (\tilde{\pi} + \tilde{\alpha}\tilde{\tau}_{M(t-1)-1} + \tilde{\delta} b_{N(t-1)}) d_{t-1} \right] (1 - d_{t-1}^{+}) + d_{t-1}^{+} \Big]^{-1}, \end{aligned} \tag{4.15}$$

where τ_m refers to the duration (in quarters) between the mth and $(m+1)$th disaster, $\tilde{\tau}_m$ gives the number of quarters that the mth disaster lasted, b_n is the contraction factor of the nth disaster period, and b_m^+ denotes the aggregate size of the mth disaster. The parsimonious hazard rate in Equation (4.14) evolves as a special case of Equation (4.15) when setting $\breve{\alpha} = \breve{\delta} = \tilde{\alpha} = \tilde{\delta} = 0$. Both specifications – and others nested in Equation (4.15) – will be considered in the empirical analysis in Section 4.7.1.

Because of d_t^+, modeling the size distribution of contractions in disaster periods is already interwoven with specifying h_t. I fit a generalized pareto (GP) distribution to a series of transformed macroeconomic contractions $z = \frac{1}{1-b}$ in order to model b. This approach is inspired by Barro and Jin (2011), who suggest using a double power law (DPL) distribution that consists of two power laws that morph into each other at a certain threshold value. This approach is very elegant, because it allows for a heavier tail of the density whilst still capturing the fact that most of the observations are small. However, when estimating the DPL parameters with the quarterly contraction

[50] d_t^+ proves to be particularly useful in simulations of the disaster process in which it ensures that all the simulated contractions indeed form disasters.

data used in this study, numerical optimization fails as the threshold that links the two power laws cannot be identified. It turns out, however, that using the additional flexibility of a GP is preferable compared to a one-parameter pareto distribution. I allow the parametrization of contractions that went along with a destruction of the stock of the durable good ($d^D = 1$) to be different from the parametrization of contractions that did not coincide with such a blight ($d^D = 0$). The joint probability density function of the disaster period indicators d_t and d_t^D and the transformed contraction size z_t can then be written as:

$$\begin{aligned} f(d_t, d_t^D, z_t | \mathcal{F}_{t-1}; \boldsymbol{\theta}_{ACH}, \boldsymbol{\theta}_{GP}^D, \boldsymbol{\theta}_{GP}) &= f(d_t, d_t^D | \mathcal{F}_{t-1}) \times f(z_t | d_t, d_t^D, \mathcal{F}_{t-1}) \\ &= [h_t(\boldsymbol{\theta}_{ACH})]^{d_t} \times [1 - h_t(\boldsymbol{\theta}_{ACH})]^{1-d_t} \\ &\times \left(f_{GP}(z_t; \boldsymbol{\theta}_{GP}^D)^{d_t^D} \times f_{GP}(z_t; \boldsymbol{\theta}_{GP})^{1-d_t^D} \right)^{d_t}, \end{aligned} \tag{4.16}$$

where $\boldsymbol{\theta}_{ACH}$ contains the parameters of the hazard rate. f_{GP} denotes the GP density with parametrization $\boldsymbol{\theta}_{GP}^D$ ($d_t^D = 1$) and $\boldsymbol{\theta}_{GP}$ ($d_t^D = 0$), respectively.

The maximum likelihood estimation of the ACH and GP parameters is performed on consumption data for several countries that Barro and Ursúa (2008) assembled. As the parameters are variation-free, it is possible to estimate the GP and the ACH parameters separately.[51] Hereafter, I will refer to this MPP as ACH-GP.

4.3.2 Splitting b_t into $b_{E,t}$ and $b_{C,t}$

The ACH-GP process models the distribution of consumption contractions b_t, but it does not determine the relationship between b_t and the decrease in log consumption growth of the durable and nondurable good, $b_{E,t}$ and $b_{C,t}$, respectively.[52] To obtain $b_{E,t}$ and $b_{C,t}$ from b_t, I introduce ω_t, which denotes the fraction of the overall consumption contraction that is due to durable goods and follows the mixture distribution:

$$f_\omega(\omega_t) = \begin{cases} \mathbb{P}(\omega_t = 0) & \text{if } \omega_t = 0 \\ \frac{\Gamma(e+f)(1-\mathbb{P}(\omega_t=0)-\mathbb{P}(\omega_t=1))}{\Gamma(e)\Gamma(f)} \omega_t^{e-1}(1-\omega_t)^{f-1} & \text{if } 0 < \omega_t < 1 \\ \mathbb{P}(\omega_t = 1) & \text{if } \omega_t = 1 \end{cases}, \tag{4.17}$$

[51] See Engle (2000).

[52] This problem arises due to restrictions regarding data availability. The cross-country consumption dataset assembled by Barro and Ursúa (2008), which features prominently in the rare disaster literature and allows the estimation of the ACH-GP process, does not differentiate between durable and nondurable goods. Hence, using the available data, it is not possible to observe $b_{E,t}$ and $b_{C,t}$ directly and use Equation (4.4) to combine them to b_t. Instead, it is necessary to propose a way to derive $b_{E,t}$ and $b_{C,t}$ from b_t that is in line with Equation (4.4).

where Γ denotes a gamma distribution and e and f are parameters. Equation (4.17) allows for the fact that consumption contractions may result entirely from the durable or nondurable good – with probabilities $\mathbb{P}(\omega_t = 1)$ and $\mathbb{P}(\omega_t = 0)$, respectively – whilst using a beta distribution to model the density of $\omega_t \in (0, 1)$.[53]

The weighting factor ω_{t+1} then can be applied to the overall consumption contractions b_{t+1} to obtain $b_{E,t+1}$ and $b_{C,t+1}$, viz:

$$\begin{aligned} b_{E,t+1} &= \omega_{t+1} \left(1 - \frac{C_t}{E_t} \frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{C,t+1})}{\exp(g^*_{E,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{E,t+1})}\right) \\ b_{C,t+1} &= (1 - \omega_{t+1}) \left(1 - \frac{E_t}{C_t} \frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{E,t+1})}{\exp(g^*_{C,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{C,t+1})}\right), \end{aligned} \tag{4.18}$$

where $g^*_{E,t+1} = g_{E,t+1}|(d_{t+1}{=}0)$ and $g^*_{C,t+1} = g_{C,t+1}|(d_{t+1}{=}0)$ are non-disastrous counterparts of $g_{E,t+1}$ and $g_{C,t+1}$.[54]

4.4 Data

The estimation of the parameters that occur throughout the consumption, financial or disaster processes requires two different sources of data: Series that carry information on the behavior of consumption and financial series in non-disastrous times and data that is informative regarding the size distribution and time dimension of disasters.

4.4.1 Non-disastrous consumption and financial data

The estimation of the structural and preference parameters is based on quarterly U.S. data that span the period 1947:Q2–2014:Q4. Nondurable consumption is measured as the sum of real personal consumption expenditures per capita on services and nondurable goods (both in chained 2009 U.S. dollars), as provided by the Federal Reserve Bank of Saint Louis.[55] Data on the consumption of durable goods can be obtained from the same source.[56] The Bureau of Economic Analysis provides year-end estimates of the stock of the durable good.[57] I use this series in combination with

[53] The use of the beta distribution requires a rescaling with $1 - \mathbb{P}(\omega_t = 0) - \mathbb{P}(\omega_t = 1)$.
[54] Appendix C.2 provides details on the derivation of Equation (4.18).
[55] For services: http://research.stlouisfed.org/fred2/series/A797RX0Q048SBEA. For nondurable goods: http://research.stlouisfed.org/fred2/series/A796RX0Q048SBEA. Both accessed 06/22/2016.
[56] https://fred.stlouisfed.org/series/A795RX0Q048SBEA, accessed 06/22/2016.
[57] https://www.bea.gov/national/FA2004/Details/Index.htm, accessed 06/22/2016

the estimates of durable consumption to back out the depreciation rate recursively from Equation (4.7). Table 25 contains the descriptive statistics for these data.

Table 25: Descriptive statistics: consumption and financial data 1947:Q2–2014:Q4
Panel A of this table contains the descriptive statistics of log consumption growth, Lettau and Ludvigson's (2001) *cay*-variable, the log T-bill return r^b, as well as the log dividend growth and the log price-dividend ratio of the market portfolio. Panel B holds descriptive statistics of the log dividend growth and log price-dividend ratios of the ten size-sorted portfolios. The data range is 1947:Q2–2014:Q4, except for the *cay* series which starts in 1952:Q1. In Panel B, $g_{d,1}$ and z_1 refer to the log dividend growth and log price-dividend ratio of the first decile of the ten size-sorted portfolios. The column labeled *ac* gives the first-order autocorrelation, and *std* is the standard deviation.

Panel A: consumption growth and market portfolio

	mean	std	ac	correlations				
				cay	r^b	$g_{d,m}$	z_m	g_E
g_C	0.0047	0.0051	0.311	0.046	0.209	0.113	0.055	0.360
g_E	0.0095	0.0368	-0.086	0.031	0.084	0.206	0.115	
z_m	3.4932	0.4216	0.976	-0.195	0.116	0.064		
$g_{d,m}$	0.0056	0.0221	0.491	0.074	-0.090			
r^b	0.0018	0.0045	0.855	0.216				
cay	-0.0000	0.0200	0.906					

Panel B: size-sorted portfolios

	mean	std	ac	correlations										
				g_C	g_E	$g_{d,10}$	$g_{d,9}$	$g_{d,8}$	$g_{d,7}$	$g_{d,6}$	$g_{d,5}$	$g_{d,4}$	$g_{d,3}$	$g_{d,2}$
$g_{d,1}$	0.0111	0.0640	0.389	0.153	0.146	0.242	0.247	0.253	0.388	0.339	0.390	0.480	0.427	0.482
$g_{d,2}$	0.0069	0.0674	0.199	0.190	0.088	0.198	0.442	0.168	0.280	0.418	0.540	0.470	0.572	
$g_{d,3}$	0.0087	0.0649	0.251	0.101	0.104	0.185	0.355	0.312	0.288	0.366	0.495	0.267		
$g_{d,4}$	0.0075	0.0529	0.257	0.110	0.089	0.160	0.410	0.318	0.387	0.362	0.323			
$g_{d,5}$	0.0084	0.0550	0.316	0.143	0.133	0.289	0.409	0.233	0.290	0.468				
$g_{d,6}$	0.0080	0.0480	0.284	0.074	0.138	0.226	0.398	0.215	0.310					
$g_{d,7}$	0.0076	0.0358	0.299	0.069	0.052	0.142	0.384	0.190						
$g_{d,8}$	0.0073	0.0543	0.200	0.051	0.136	0.151	0.118							
$g_{d,9}$	0.0066	0.0330	0.349	0.102	0.041	0.268								
$g_{d,10}$	0.0045	0.0254	0.404	0.058	0.192									

	mean	std	ac	g_C	g_E	z_{10}	z_9	z_8	z_7	z_6	z_5	z_4	z_3	z_2
z_1	4.1793	0.6540	0.968	0.094	0.133	0.748	0.809	0.811	0.883	0.929	0.918	0.939	0.954	0.972
z_2	3.9604	0.7090	0.974	0.046	0.113	0.830	0.889	0.886	0.938	0.968	0.964	0.978	0.986	
z_3	3.8563	0.7062	0.975	0.007	0.100	0.860	0.916	0.919	0.961	0.981	0.977	0.986		
z_4	3.7769	0.6751	0.978	0.008	0.098	0.878	0.937	0.936	0.976	0.987	0.987			
z_5	3.7206	0.6805	0.978	-0.001	0.104	0.896	0.952	0.943	0.981	0.991				
z_6	3.6576	0.6126	0.978	0.005	0.110	0.896	0.953	0.946	0.984					
z_7	3.6306	0.5837	0.977	-0.006	0.097	0.919	0.975	0.963						
z_8	3.5592	0.5389	0.974	-0.023	0.095	0.929	0.967							
z_9	3.4777	0.4793	0.974	-0.015	0.096	0.952								
z_{10}	3.4702	0.4332	0.979	0.036	0.103									

i	1	2	3	4	5	6	7	8	9	10
Corr(z_m,z_i)	0.832	0.898	0.918	0.934	0.946	0.948	0.959	0.951	0.973	0.987
Corr($g_{d,m}$,$g_{d,i}$)	0.403	0.416	0.377	0.415	0.493	0.427	0.398	0.391	0.540	0.857
Corr(z_i,$g_{d,i}$)	0.145	0.150	0.135	0.091	0.140	0.150	0.119	0.131	0.128	0.038

Returns on the market portfolio, comprised of NYSE, AMEX, and NASDAQ traded stocks (*mkt*; including and excluding dividends) at a monthly frequency come from CRSP and can be used to extract the quarterly dividend growth and price-dividend ratio series.[58] The dividend growth thus obtained is very erratic and has a strong negative autocorrelation, which results because dividend payments occur very unevenly over time. I circumvent this problem by following Cochrane (1996), who smoothes a quarterly dividend series by computing means using the contemporaneous observation and the three preceding quarters. The ten size-sorted portfolios (*size dec*) serve as further test assets and returns (including and excluding dividends) can be obtained from Kenneth French's data library.[59] The same smoothing procedure is applied to the dividend growth series of the size-sorted portfolios. All portfolios are value-weighted.[60] In line with Beeler and Campbell (2012), I approximate the ex-ante non-disastrous T-bill return R_b (the "risk-free rate" proxy) by forecasting the ex-post R_b based on the quarterly T-bill yield, obtained from the CRSP database, and the average of quarterly log inflation across the past year.

The estimation of the elasticity of substitution between the two goods furthermore requires the price indexes for personal consumption expenditures on durable goods, nondurable goods, and services.[61]

As described in Section 4.2, I proxy the mean log consumption-wealth ratio in Equation (4.12) using Lettau and Ludvigson's (2001) *cay*-variable and a constant. The quarterly series are provided by the authors and continuously updated.[62]

4.4.2 Disaster-including cross-country data

Relying solely on U.S. data for the estimation of the ACH-GP parameters is of no use, because even when using long consumption series, there will not be enough disaster observations to identify the parameters reliably.[63] As frequently done in the

[58] As it is done in Beeler and Campbell (2012) and illustrated in their supplementary dataset DOI: 10.1561/104.00000004_data, accessed 06/22/2016.

[59] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html, accessed 03/09/2016. Due to the frequent changes in the underlying CRSP data, newer or older downloads may results in different series.

[60] These data are provided by the Federal Reserve Bank of Saint Louis: http:// research.stlouisfed.org/fred2/series/CPIAUCSL, accessed 03/09/2016.

[61] For services: https://fred.stlouisfed.org/series/DHCERG3Q086SBEA. For nondurable goods: https://fred.stlouisfed.org/series/DNDGRG3Q086SBEA. For durable goods: https://fred.stlouisfed.org/series/DDURRG3Q086SBEA. All accessed 08/01/2016.

[62] http://faculty.haas.berkeley.edu/lettau/data_cay.html, accessed 11/12/2016.

[63] Even Campbell's (2003) dataset, which starts in 1890 and serves as a robustness check in Julliard and Ghosh (2012), only includes two multi-period contractions that fulfill Barro's (2006) definition of a disaster.

RDH literature, I identify the disaster parameters from annual cross-country panel data on consumption that Barro and Ursúa (2008) assembled for 42 countries.[64] From these data, I select the same 35 countries as Barro (2006). To detect disaster events, I use Barro's (2006) identification scheme, according to which any sequence of downturns in consumption growth greater or equal to q=0.145 qualifies as a disaster.[65] A disaster may pan out over multiple periods or occur as one sharp contraction. Positive intermezzos of consumption growth within a disaster are allowed if (1) the positive growth is smaller in absolute value than the negative growth in the following year and (2) the size of the disaster does not decrease by including the intermezzo. Table 26 provides information on the consumption dataset and the identified disaster periods and sizes.

Table 26: Country panel data used for the first-step estimation
This table lists the 35 countries and time periods with available data that provide the basis for the MPP estimation. The second column reports the time periods for which consumption data assembled by Barro and Ursúa (2008) are available (beginning with 1800 onwards). The third column reports the disaster periods that result from q=0.145. The respective contraction sizes (in percent) are given in parentheses. The forth column lists the associated origins of these consumption contractions. Disasters that were allegedly related to a destruction of the stock of durable goods are underlined.

Country	Barro and Ursúa	Disasters	Origin
Argentina	1875–2009	1895–1898 (28.3)	aftermaths of Baring crisis
		1899–1900 (19.5)	aftermaths of Baring crisis
		1912–1917 (17.2)	World War I
		1928–1932 (18.9)	Great Depression
		1987–1990 (16.0)	Argentine Great Depression
Australia	1901–2009	1913–1918 (23.8)	World War I
		1927–1932 (23.4)	Great Depression
		1938–1944 (30.1)	World War II
Austria	1913–1918, 1924–1944, 1947–2009	1913–1918 (45.1)	World War I
		1929–1934 (21.8)	Great Depression
		1939–1947 (52.9)	World War II
Belgium	1913–2009	1913–1917 (44.5)	World War I
		1937–1942 (53.0)	World War II
Brazil	1901–2009	1902–1905 (14.8)	Vaccine revolt; depression; Revolt of the Lash
		1906–1909 (15.7)	Vaccine revolt; depression; Revolt of the Lash
		1920–1921 (14.7)	aftermaths of World War I
		1928–1931 (20.1)	Great Depression
		1984–1990 (16.3)	Stagnation after Mexican debt crisis
Canada	1871–2009	1873–1876 (15.2)	Canadian Confederation aftermaths
		1918–1921 (19.6)	World War I
		1929–1933 (23.0)	Great Depression
Chile	1900–2009	1911–1915 (32.2)	World War I
		1921–1922 (18.5)	political instabilities
		1926–1927 (19.6)	political instabilities
		1929–1932 (37.4)	Great Depression

Continued on next page

[64] The data are available at http://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data, accessed 04/24/2015.

[65] The same disaster threshold is used by Barro (2006, 2009) and Barro and Jin (2011). Other thresholds that feature prominently in RDH literature are q=0.195 and q=0.095, with the latter gaining popularity as the parametrization of the disaster process tends to increase.

Continued from previous page

Country	Barro and Ursúa	Disasters	Origin
		1972–1976 (40.1)	economic crisis and military coup
		1981–1985 (32.7)	economic collapse
Colombia	1925–2009	1929–1932 (18.1)	Great Depression
		1939–1943 (22.8)	World War I
Denmark	1844–2009	1919–1921 (24.1)	World War I
		1939–1941 (26.1)	World War II
Finland	1860–2009	1913–1918 (36.0)	World War I; civil war
		1928–1932 (19.9)	Great Depression
		1938–1944 (25.4)	World War II; Winter War
France	1824–2009	1824–1828 (20.1)	financial crisis
		1912–1915 (21.5)	World War I
		1938–1943 (58.0)	World War II
Germany	1851–2009	1912–1918 (42.5)	World War I
		1939–1945 (41.2)	World War II
Greece	1938–2009	1938–1944 (63.6)	World War II
India	1919–2009	1932–1942 (21.7)	World War II
		1947–1950 (17.7)	decolonization
Indonesia	1960–2009		
Italy	1861–2009	1939–1945 (28.6)	World War II
Japan	1874–2009	1937–1945 (63.9)	World War II
Malaysia	1900–1939, 1947–2009	1914–1920 (43.4)	World War I
		1929–1932 (25.8)	Great Depression
		1938–1947 (33.6)	World War II
		1951–1953 (16.9)	Malayan Emergency
		1984–1987 (14.8)	political instabilities
Mexico	1900–2009	1909–1913 (17.2)	Mexican Revolution
		1914–1916 (15.8)	World War I; Mexican Revolution
		1926–1932 (31.2)	Great Depression
the Netherlands	1807–1809, 1814–2009	1807–1809 (18.3)	reign of Napoleon Bonaparte
		1912–1918 (44.0)	World War I
		1939–1944 (54.5)	World War II
New Zealand	1878–2009	1939–1944 (22.4)	World War II
Norway	1830–2009	1916–1918 (16.9)	World War I
		1919–1921 (16.1)	economic aftermaths of World War I
the Philippines	1946–2009		
Peru	1896–2009	1975–1979 (17.9)	military coup
		1987–1992 (30.0)	chronic inflation; economic turbolences
Portugal	1910–2009	1914–1919 (21.5)	World War I
South Korea	1911–2009	1942–1945 (37.5)	World War II
		1949–1952 (37.1)	Korean War
Spain	1850–2009	1892–1896 (18.2)	Cuban Independence War; Philippine Revolution
		1935–1937 (46.1)	Spanish civil war
Sri Lanka	1960–2009	1969–1972 (15.6)	decolonization
Sweden	1800–2009	1810–1819 (18.8)	World War I
		1939–1942 (15.6)	World War II
Switzerland	1851–2009	1852–1853 (17.2)	fundamental political reorganization
		1858–1860 (29.3)	fundamental political reorganization
		1861–1864 (15.9)	fundamental political reorganization
		1865–1867 (23.9)	fundamental political reorganization
		1870–1872 (19.0)	fundamental political reorganization
		1876–1878 (22.5)	fundamental political reorganization
		1887–1888 (15.7)	fundamental political reorganization
		1939–1945 (17.3)	World War II
Taiwan	1901–2009	1903–1905 (21.9)	Guerrilla fighting under Japanese rule
		1936–1945 (68.4)	World War II
UK	1830–2009	1915–1918 (16.7)	World War I
		1938–1943 (16.9)	World War II
USA	1834–2009	1917–1921 (16.4)	World War I
		1929–1933 (20.8)	Great Depression
Uruguay	1960–2009	1981–1984 (26.7)	civil-military rule
		1998–2002 (21.9)	Uruguay Great Depression
Venezuela	1923–2009	1930–1933 (31.2)	Great Depression
		1948–1952 (20.3)	coup d'état
		1957–1958 (15.9)	coup d'état
		1982–1986 (26.4)	oil crisis

Thereafter, I check each disaster for its historical origin and evaluate whether it can be associated with a deletion of the stock of the durable good in the respective country. According to this approach, 22 of the 89 disaster events can be termed destructive disasters; they are highlighted as such in Table 26. In most of the cases, this applies to countries that suffered from military invasions during the world wars. Whilst this strategy is, admittedly, limited in precision, its results should be understood as benchmarks that allow studying the effects of a complete destruction of belongings.[66]

Figure 17: Consumption disasters

This figure depicts the 89 consumption disasters identified from Barro and Ursúa's (2008) country panel data (updated). The sampling period is 1800–2009. Red lines denote destructive disasters that allegedly were accompanied by an annihilation of the stock of the durable good and blue lines represent disasters for which this behavior is not assumed. The dotted horizontal line depicts the average contraction size.

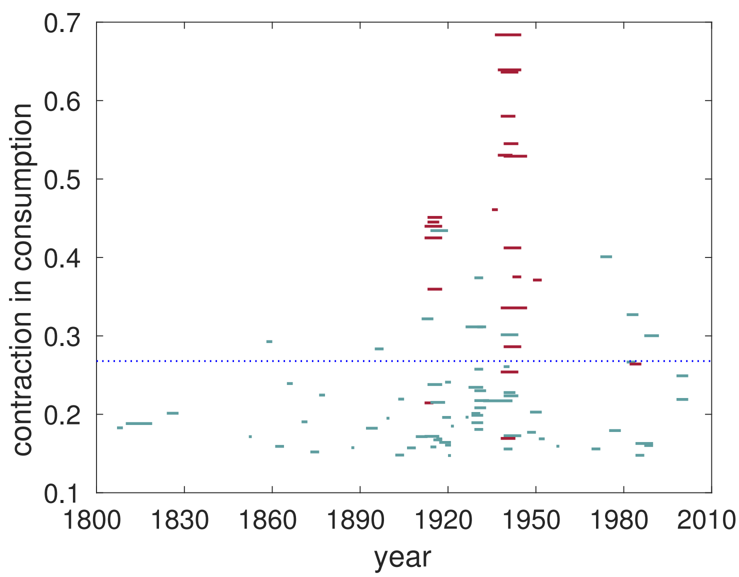


Figure 17 shows the size and the period over which the disasters accrue. Apparently, disasters that are assumed to be accompanied by a destruction of the stock of durable good are on average larger than non-destructive disasters. As previously mentioned, I assume the ACH-GP process is observable at the quarterly frequency. However, the Barro and Ursúa (2008) data only permit the computation of annual contractions. I therefore generate quarterly observations by randomly distributing the annual contraction. Appendix C.3 explains the details.

Whilst Barro and Ursúa's (2008) overall consumption data allow estimating the

[66] A robustness check that does not differentiate between destructive and non-destructive disasters is carried out in Section 4.7.3.

parameters of the ACH-GP process, they do not suffice to obtain the parameters of the (mixed) beta distribution that models the weighting factor ω_t. As can be seen from Figure 17, most consumption disasters took place before the middle of the 20^{th} century; detailed information on consumption of durable and nondurable goods, however, are not available until after World War II. To circumvent this problem, I assume that the distribution of ω_t can be estimated using consumption data on both goods during recession periods. As my study considers multi-period disasters, for which the contractions during the individual disaster periods are not necessarily sharp, this is not a bold assumption.

Information on the business cycles of various countries can be obtained from the Economic Cycle Research Institute.[67] The dataset contains business cycle dates for 21 countries. I select those countries for which data on the consumption of durable goods, nondurable goods, and services are available.[68] Recession periods do not necessarily coincide with negative consumption growth. I consider only those periods in which overall consumption growth was negative and compute the observed weighting factors as the fraction of the total consumption decline due to a reduced consumption of the durable good.

4.5 Estimation strategy

The estimation strategy follows a two-step approach. First, the fundamental macroeconomic and financial parameters that appear in Equations (4.3)-(4.12) must be estimated. In combination with the disaster process parameters, they allow for the simulation of consumption growth and return processes that are, in the second estimation step, used to obtain estimates of the preference parameters.

4.5.1 Estimating non-disastrous macro parameters

The U.S. postwar consumption growth, dividend growth, and price-dividend ratios do not include any disastrous observations. Hence, a GMM estimation of the funda-

[67] https://www.businesscycle.com/ecri-business-cycles/international-business-cycle-dates-chronologies, accessed 05/03/2016.

[68] Namely: Austria, Canada, France, Germany, Italy, Japan, South Korea, Mexico, Spain, Sweden, U.K., U.S.A. For Canada, consumption data come from Statistics Canada: http://www5.statcan.gc.ca/cansim/pick-choisir?lang=eng&p2=33&id=3800084, accessed 05/03/2016. For all other countries, consumption data are provided by the OECD: http://stats.oecd.org/index.aspx?queryid=218#, accessed 05/03/2016. The U.S. recession dates that are listed by the Economic Cycle Research Institute are identical to those provided by the National Bureau of Economic Research.

mental macroeconomic and financial parameters in Equations (4.3)-(4.12) cannot be performed using unconditional moments. However, as the disastrous contractions affect the log price-dividend ratios, as well as the consumption and dividend growth processes in an additive manner, it is possible to estimate these parameters using moments conditional on no disasters ($d_t = 0$).

I use an exact identification strategy to estimate the vector of fundamental parameters $\boldsymbol{\zeta}^a = (\mu_C, \mu_E, \mu_b, \varphi_e, \delta_b, \phi_b, \sigma_C, \sigma_E, \sigma_b, \boldsymbol{\mu}_d, \boldsymbol{\mu}_z, \boldsymbol{\sigma}_d, \boldsymbol{\rho}_z, \boldsymbol{\beta}_z, \boldsymbol{\phi}, \boldsymbol{\kappa})'$, where bold symbols denote vectors (e.g., $\boldsymbol{\mu}_d = (\mu_{d,m}, \mu_{d,1}, \ldots, \mu_{d,10})$) and the contemporaneous correlations between innovations in the log consumption, dividend growth series, and the log T-bill return are collected in $\boldsymbol{\kappa} = (\kappa_{g_C,g_E}, \kappa_{g_C,r_b^*}, \kappa_{g_E,r_b^*}, \boldsymbol{\kappa}_{g_C,g_{d,i}}, \boldsymbol{\kappa}_{g_E,g_{d,i}}, \boldsymbol{\kappa}_{r_b^*,g_{d,i}}, \boldsymbol{\kappa}_{g_{d,j},g_{d,i}})$. The estimation builds on the following set of moment matches:

$$\mathbb{G}_T^a(\boldsymbol{\zeta}^a) = \begin{bmatrix}
\mathbb{E}_T(g_{C,t+1}|d_{t+1}=0) & -\mu_C \\
\mathbb{E}_T(g_{E,t+1}|d_{t+1}=0) & -\mu_E \\
\mathbb{E}_T(z_{m,t+1}g_{C,t+1}|d_t=d_{t+1}=0) & -\frac{\mu_{z,m}\mu_C}{1-\rho_{z,m}} - \rho_{z,m}\beta_{z,m} \\
\mathbb{E}_T(z_{m,t+1}^2|d_t=d_{t+1}=0) & -\left(\frac{\mu_{z,m}}{1-\rho_{z,m}}\right)^2 - \frac{\beta_{z,m}^2\varphi_e^2}{1-\rho_{z,m}^2} \\
\mathbb{E}_T(g_{C,t+1}^2|d_{t+1}=0) & -\mu_C^2 - \varphi_e^2 - \sigma_C^2 \\
\mathbb{E}_T(g_{E,t+1}^2|d_{t+1}=0) & -\mu_E^2 - \varphi_e^2 - \sigma_E^2 \\
\mathbb{E}_T(r_{b,t+1}^* r_{b,t}^*) & -\left(\frac{\mu_b}{1-\delta_b}\right)^2 - \frac{\delta_b}{1-\delta_b^2}(\phi_b^2\varphi_e^2 + \sigma_b^2) \\
\mathbb{E}_T(r_{b,t+1}^*) & -\frac{\mu_b}{1-\delta_b} \\
\mathbb{E}_T(r_{b,t+1}^* z_{m,t+1}|d_t=d_{t+1}=0) & -\frac{\mu_b\mu_{z,m}}{(1-\delta_b)(1-\rho_{z,m})} - \frac{\phi_b\rho_{z,m}\varphi_e^2\beta_{z,m}}{1-\delta_b\rho_{z,m}} \\
\mathbb{E}_T((r_{b,t+1}^*)^2) & -\left(\frac{\mu_b}{1-\delta_b}\right)^2 - \frac{\phi_b^2\varphi_e^2+\sigma_b^2}{1-\delta_b^2} \\
\mathbb{E}_T(g_{C,t+1}g_{E,t+1}|d_{t+1}=0) & -\mu_C\mu_E - \varphi_e^2 - \kappa_{g_C,g_E}\sigma_C\sigma_E \\
\mathbb{E}_T(g_{C,t+1}r_{b,t+1}^*|d_{t+1}=0) & -\frac{\mu_b\mu_C}{1-\delta_b} - \phi_b\varphi_e^2 - \kappa_{g_C,r_b^*}\sigma_C\sigma_b \\
\mathbb{E}_T(g_{E,t+1}r_{b,t+1}^*|d_{t+1}=0) & -\frac{\mu_b\mu_E}{1-\delta_b} - \phi_b\varphi_e^2 - \kappa_{g_E,r_b^*}\sigma_E\sigma_b \\
\mathbb{E}_T(g_{d,i,t+1}|d_{t+1}=0) & -\mu_{d,i} \\
\mathbb{E}_T(z_{i,t}z_{i,t+1}|d_t=d_{t+1}=0) & -\left(\frac{\mu_{z,i}}{1-\rho_{z,i}}\right)^2 - \frac{\rho_{z,i}\beta_{z,i}^2\varphi_e^2}{1-\rho_{z,i}^2} \\
\mathbb{E}_T(z_{i,t+1}|d_t=d_{t+1}=0) & -\frac{\mu_{z,i}}{1-\rho_{z,i}} \\
\mathbb{E}_T(z_{i,t}g_{d,i,t+1}|d_t=d_{t+1}=0) & -\frac{\mu_{d,i}\mu_{z,i}}{1-\rho_{z,i}} - \phi_i\varphi_e^2\rho_{z,i}\beta_{z,i} \\
\mathbb{E}_T(g_{d,i,t+1}^2|d_{t+1}=0) & -\mu_{d,i}^2 - \phi_i^2\varphi_e^2 - \sigma_{d,i}^2 \\
\mathbb{E}_T(g_{C,t+1}g_{d,i,t+1}|d_{t+1}=0) & -\mu_C\mu_{d,i} - \phi_i\varphi_e^2 - \kappa_{g_C,g_{d,i}}\sigma_C\sigma_{d,i} \\
\mathbb{E}_T(g_{E,t+1}g_{d,i,t+1}|d_{t+1}=0) & -\mu_E\mu_{d,i} - \phi_i\varphi_e^2 - \kappa_{g_E,g_{d,i}}\sigma_E\sigma_{d,i} \\
\mathbb{E}_T(r_{b,t+1}^* g_{d,i,t+1}|d_{t+1}=0) & -\frac{\mu_b\mu_{d,i}}{1-\delta_b} - \phi_i\phi_b\varphi_e^2 - \kappa_{r_b^*,g_{d,i}}\sigma_b\sigma_{d,i} \\
\mathbb{E}_T(z_{j,t+1}z_{m,t+1}|d_t=d_{t+1}=0) & -\frac{\mu_{z,m}\mu_{z,j}}{(1-\rho_{z,m})(1-\rho_{z,j})} - \frac{\beta_{z,j}\varphi_e^2\beta_{z,m}}{1-\rho_{z,m}\rho_{z,j}} \\
\mathbb{E}_T(g_{d,i,t+1}g_{d,j,t+1}|d_{t+1}=0) & -\mu_{d,i}\mu_{d,j} - \phi_i\phi_j\varphi_e^2 - \kappa_{g_{d,i},g_{d,j}}\sigma_{d,i}\sigma_{d,j}
\end{bmatrix}, \tag{4.19}$$

where $i = m, 1, \ldots, 10$, $j = 1, \ldots, 10$, $i \neq j$, and $\mathbb{E}_T(\cdot) = \frac{1}{T}\sum_{t=1}^T(\cdot)$. If one was solely interested in using the market portfolio as a test asset, it would suffice to consider the first 13 moment conditions and moments 14-21 for $i = m$. For additionally obtaining the returns of the ten size-sorted portfolios, one must consider the entire set of moment matches, which amounts to 145 further moment restrictions and parameters to be estimated.

A second set of moment conditions is used to identify the three parameters of the non-disastrous depreciation factor that are collected in $\boldsymbol{\zeta}^b = (a, \beta_\delta, \sigma_\varepsilon)'$:

$$\mathbb{G}_T^b(\boldsymbol{\zeta}^b) = \begin{bmatrix} \mathbb{E}_T(\delta_{t+1}^*) & -\frac{a}{1-\beta_\delta} \\ \mathbb{E}_T((\delta_{t+1}^*)^2) & -\frac{\sigma_\varepsilon^2}{1-\beta_\delta^2} - \left(\frac{a}{1-\beta_\delta}\right)^2 \\ \mathbb{E}_T(\delta_t^* \delta_{t+1}^*) & -\frac{\beta_\delta \sigma_\varepsilon^2}{1-\beta_\delta^2} - \left(\frac{a}{1-\beta_\delta}\right)^2 \end{bmatrix}. \tag{4.20}$$

I compute parameter estimates of $\boldsymbol{\zeta} = (\boldsymbol{\zeta}^{a\prime}, \boldsymbol{\zeta}^{b\prime})'$ by stacking the vectors of moment conditions $\mathbb{G}_T(\boldsymbol{\zeta}) = (\mathbb{G}_T^a(\boldsymbol{\zeta}^a)', \mathbb{G}_T^b(\boldsymbol{\zeta}^b)')'$ and minimizing:

$$\hat{\tilde{\boldsymbol{\zeta}}} = \underset{\tilde{\boldsymbol{\zeta}} \in \tilde{\boldsymbol{\Xi}}}{\arg\min} \; \mathbb{G}(\tilde{\boldsymbol{\zeta}})' \mathbb{G}(\tilde{\boldsymbol{\zeta}}), \tag{4.21}$$

where – as the problem is exactly identified – the value of the GMM objective function should indeed be zero.

4.5.2 Estimating the asset pricing parameters

The SDF presented in Equation (4.2) relies on five preference parameters (α, β, γ, ψ, ρ), but some are extremely difficult to identify. For reasons outlined in Section 4.6.1, I decide to estimate the elasticity of substitution between the two goods, ρ, and their weighting factor, α, upfront and fix them to that estimated value. Furthermore, I will reduce the complexity of the problem by building the estimation strategy on excess returns – a framework in which the subjective discount factor β is not identified and can thus be set to any value. This approach allows to set the focus on the two preference parameters whose size is most discussed in recent literature: the RRA coefficient, γ, and the IES, ψ.

The starting point of my estimation is the basic asset pricing equation:

$$\mathbb{E}_t(m_{t+1}(\gamma, \psi, \beta^{fix}, \alpha^{fix}, \rho^{fix}) R_{i,t+1}^e) = 0, \tag{4.22}$$

where $R_{i,t+1}^e = R_{i,t+1} - R_{b,t+1}$ is the excess return of test asset i. We can rewrite Equation (4.22) as:

$$\mathbb{E}(R_{i,t}^e) = -\frac{\text{cov}(m_t(\gamma, \psi, \beta^{fix}, \alpha^{fix}, \rho^{fix}), R_{i,t}^e)}{\mathbb{E}(m_t(\gamma, \psi, \beta^{fix}, \alpha^{fix}, \rho^{fix}))}, \tag{4.23}$$

which facilitates an estimation of γ and ψ by the simulated method of moments. For this purpose, I suggest approximating the left-hand side expectation by a sample mean of the observed, non-disastrous excess return series:

$$\mathbb{E}(R^e_{i,t}) \approx \frac{1}{T}\sum_{t=1}^{T} R^e_{i,t}. \tag{4.24}$$

For the right-hand side moments, the approximation will be based on simulated, and thus possibly disaster-including, consumption and (excess) return series. Such a simulation is possible using the process specifications described in Section 4.2, once the fundamental macroeconomic and financial parameters have been estimated as detailed in Section 4.5.1 and the parametrization of the disaster process is obtained from the maximum likelihood approach outlined in Section 4.3. Whether a certain disaster is accompanied by a partial government default or a destruction of the stock of the durable good, is randomly determined at its onset by drawing from binary distributions, such that $d_{b,t} = Be(0.42)$ and $d^D_t = Be(\frac{22}{89})$.[69] Either all periods of a disaster are affected by the default/destruction – or none. The disaster-including consumption and excess return series are simulated of length $\mathcal{T}$, which must be large enough to achieve a good approximation of the moments:

$$\frac{\text{cov}(m_t(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix}),R^e_{i,t})}{\mathbb{E}(m_t(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix}))} \approx \frac{\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\left(R^e_{i,s}-\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}R^e_{i,s}\right)m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})}{\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})}. \tag{4.25}$$

The approximations in Equations (4.24)-(4.25) can be combined to the SMM moment matches:

$$\mathbb{G}(\boldsymbol{\vartheta}) = \left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{R}^e_t + \frac{\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\left(\mathbf{R}^e_s-\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}\mathbf{R}^e_s\right)m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})}{\frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})}\right], \tag{4.26}$$

where $\mathbf{R}^e$ denotes a vector of excess returns. Using the moment conditions in Equation (4.26) implies choosing the preference parameters in such a way that the simulated moments computed on series that are allegedly more representative of the

[69] The parameter of the first binary distribution is set to 0.42, because Barro (2006) finds there was a partial government default during 42% of the historical disasters he uses in his calibration study. For the second binary distribution, the success probability of $\frac{22}{89}$ results, because I claim in Section 4.4 that 22 of the 89 consumption disasters were accompanied by a destruction of the stock of the durable good.

possibly disaster-including consumption processes are maximally close to the means computed for actually observed excess returns. The idea is that these representative consumption and return processes must also account for the observed $\mathbf{R}_t^e$ series which coincidentally do not feature such disasters.

It turns out, however, that the moment conditions in Equation (4.26) do not suffice to identify the IES. For this reason, I estimate ψ separately by means of a simulation-based instrumental variables approach that relies on a second-order log-linearization of the basic asset pricing equation for returns, which leads to the following regression equation:

$$r_{i,t+1} = w_i + \frac{1}{\psi}\left(\ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right) + \nu_{i,t+1}, \tag{4.27}$$

where w_i is an asset-specific constant and $\nu_{i,t+1}$ denotes a zero-mean disturbance term.[70] Because $\nu_{i,t+1}$ is correlated with $\ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)$ by construction, it would not be possible to identify the IES with a linear projection of $r_{i,t+1}$ on $\ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)$ and a constant. Thus, I resort to an instrumental variables estimation strategy which implies the orthogonality constraints:

$$\mathbb{E}\left(\left(r_{i,t+1} - w_i - \frac{1}{\psi}\Delta cv_{t+1}\right)\mathbf{z}_t\right) = \mathbf{0}, \tag{4.28}$$

where $\Delta cv_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)$ and $\mathbf{z}_t$ denotes a vector of instrumental variables that are known at t.[71] Any log return can be used in Equation (4.28) to identify the IES; in this study, I choose the log T-bill return for this purpose. A constant as well as the twice-lagged log T-bill return and Δcv-variable serve as instruments. The estimation is performed on the simulated disaster-including data.

Applied to a GMM context with an identity weighting matrix, the IES estimate must fulfill the first-order conditions

$$\begin{bmatrix} -1 & -\mathbb{E}_{\mathcal{T}}(\Delta cv_s) & -\mathbb{E}_{\mathcal{T}}(r_{b,s}) \\ \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s)}{\hat{\psi}^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2})}{\hat{\psi}^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2})}{\hat{\psi}^2} \end{bmatrix} \begin{bmatrix} \mathbb{E}_{\mathcal{T}}(r_{b,s}) - \hat{w}_b - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta cv_s) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s}\Delta cv_{s-2}) - \hat{w}_b\mathbb{E}_{\mathcal{T}}(\Delta cv_{s-2}) - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2}) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s} r_{b,s-2}) - \hat{w}_b\mathbb{E}_{\mathcal{T}}(r_{b,s-2}) - \frac{1}{\hat{\psi}}\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2}) \end{bmatrix} = \mathbf{0}, \tag{4.29}$$

where $\mathbb{E}_{\mathcal{T}}(\cdot) = \frac{1}{\mathcal{T}}\sum_{s=1}^{\mathcal{T}}(\cdot)$. As the IES identification strategy is not entangled with any of the other preference parameters, it is possible to perform the estimation of

[70] Appendix C.4 details the derivation of Equation (4.27).

[71] Similar approaches that use standard Epstein-Zin-Weil preferences are well documented in related literature, e.g., Hansen and Singleton (1983), Campbell (2003), and Yogo (2006).

ψ and γ sequentially. Alternatively, the moment matches listed in Equations (4.26) and (4.29) can be combined to:

$$\mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}}) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t^e + \frac{\frac{1}{\mathcal{T}}\sum\limits_{s=1}^{\mathcal{T}}\left(\mathbf{R}_s^e - \frac{1}{\mathcal{T}}\sum\limits_{s=1}^{\mathcal{T}}\mathbf{R}_s^e\right)m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})}{\frac{1}{\mathcal{T}}\sum\limits_{s=1}^{\mathcal{T}} m_s(\gamma,\psi,\beta^{fix},\alpha^{fix},\rho^{fix})} \\ \begin{bmatrix} -1 & -\mathbb{E}_{\mathcal{T}}(\Delta cv_s) & -\mathbb{E}_{\mathcal{T}}(r_{b,s}) \\ \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s)}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2})}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2})}{\psi^2} \end{bmatrix} \times \\ \begin{bmatrix} \mathbb{E}_{\mathcal{T}}(r_{b,s}) - w_b - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s}\Delta cv_{s-2}) - w_b\mathbb{E}_{\mathcal{T}}(\Delta cv_{s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2}) \\ \mathbb{E}_{\mathcal{T}}(r_{b,s} r_{b,s-2}) - w_b\mathbb{E}_{\mathcal{T}}(r_{b,s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2}) \end{bmatrix} \end{bmatrix}, \tag{4.30}$$

where $\tilde{\boldsymbol{\vartheta}} = (\gamma, \psi, w_b)'$. The SMM-type estimates of the preference parameters are then obtained by:

$$\hat{\tilde{\boldsymbol{\vartheta}}} = \underset{\tilde{\boldsymbol{\vartheta}} \in \tilde{\boldsymbol{\Theta}}}{\arg\min}\ \mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}})'\mathbf{W}\mathbb{G}^{+}(\tilde{\boldsymbol{\vartheta}}). \tag{4.31}$$

I choose the weighting matrix $\mathbf{W}$, such that the identification of ψ is ensured to work through the IV-based moments in Equation (4.29):

$$\mathbf{W} = \begin{bmatrix} \mathbf{I}_N & 0 \\ 0 & 10^6 \times \mathbf{I}_2 \end{bmatrix}, \tag{4.32}$$

where N denotes the number of test assets.

Due to the non-standard character of the estimation approach, it is not possible to rely on standard asymptotic inference when trying to assess the precision of the preference parameter estimates. Whilst this would still be possible for the first-step estimates, meaning the fundamental macroeconomic and finance parameters, and the parameters of the ACH-GP process, the first-step parameter uncertainty must be accounted for when addressing the estimation precision of γ and ψ. For this purpose, I propose using a mixture of different parametric bootstraps that allows the computation of confidence intervals. This approach is detailed in Section C.5 of the appendix.

4.6 Caveats and how to deal with them

The estimation strategy proposed in this chapter of my dissertation faces some caveats that require special attention. Some of these challenges are of a theoretical nature and others of an econometric nature; some are disaster-related and other problems stem from Yogo's (2006) original model. In the following, I will briefly outline these caveats and how I deal with them.

4.6.1 Estimation of ρ and α on simulated data

The constant elasticity of substitution ρ between the two consumption goods and their weighting factor α are important features of the SDF in Equation (4.2). Because their identification is non-trivial, Yogo (2006) proposes further moment conditions for this specific purpose. In the case of ρ, he argues in favor of a cointegrating relation between $c_t - d_t$ and p_t, where lowercase letters denote logs and p_t is the (log) price of the durable good in units of the nondurable good, computed as the ratio of the price index for personal consumption expenditures on durable goods to the price index for personal consumption expenditures on nondurable goods and services. The elasticity of substitution then occurs in the normalized cointegrating vector $(1, -\rho)'$.

The identification of α requires at least one further moment condition and for this purpose, Yogo (2006) uses the equality between the marginal rate of substitution between the two consumption goods and the relative price of the durable good:

$$\frac{u_{Dt}}{u_{Ct}} = P_t - (1 - \delta_t)\mathbb{E}_t[m_{t+1}P_{t+1}] \qquad \text{with} \qquad \frac{u_D}{u_C} = \frac{\alpha}{1 - \alpha}\left(\frac{D}{C}\right)^{-1/\rho}. \tag{4.33}$$

The problem that arises from these two identification approaches is that an estimation of α and ρ in a disaster-including world would require the simulation of a P_t series that accounts for disaster risk – which is not straightforward and would need further assumptions. It may be possible to use the cointegrating relation for this purpose but then the estimation of ρ based on that property would follow from circular reasoning. Trying to identify these parameters from standard asset pricing moment conditions (i.e., the pricing of (excess) returns) fails.[72]

Due to the fact that the SDF reduces to the standard Epstein-Zin-Weil case during destructive disasters, it may be possible to argue that the α and ρ parameters that feature in the utility function when there are either no disasters at all or at least no destructive ones, can be estimated from non-disastrous data. In this case, the values could be estimated upfront and then be used throughout the different model specifications and in the bootstraps; meaning that α and ρ will be held constant. For ρ, this is easy as the identification strategy can be perfectly disentangled from the other preference parameters. The same cannot be said for the α-identifying Equation

[72] Meaning, there is a strong dependency on the starting values of the optimization.

(4.33), however, as all preference parameters are included through m_{t+1}.[73] It turns out this obstacle can be circumvented by rewriting the conditions in Equation (4.33), viz:

$$\begin{aligned}\frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{-1/\rho} &= P_t - (1-\delta_t)\mathbb{E}_t(m_{t+1}P_{t+1}) \\ &= P_t\left(1-(1-\delta_t)\mathbb{E}_t\left(m_{t+1}\frac{P_{t+1}}{P_t}\right)\right) \\ &= P_t\delta_t \\ P_t &= \frac{\alpha}{1-\alpha}\left(\frac{D_t}{C_t}\right)^{-1/\rho}\delta_t^{-1}.\end{aligned} \tag{4.34}$$

One moment condition that is implied by Equation (4.34) is:

$$\mathbb{E}\left(P_t\right) = \frac{\alpha}{1-\alpha}\mathbb{E}\left(\left(\frac{D_t}{C_t}\right)^{-1/\rho}\delta_t^{-1}\right), \tag{4.35}$$

from which $\hat{\alpha}$ can be obtained by replacing expectations by sample means of the non-disastrous data and using the $\hat{\rho}$ that results from the cointegration-based estimation.

The estimates obtained from this identification strategy are $\hat{\alpha} = 0.12$ and $\hat{\rho} = 0.75$, where $\hat{\alpha}$ differs strongly from Yogo's (2006) results, according to which it is in the range of 0.8. Considering that $\alpha \in (0, 1)$ and that both estimations were performed using non-disastrous data, this result is indeed striking. It is, however, in line with the findings of Borri and Ragusa (2017), who obtain values between 0.11 and 0.14.[74] The estimate of the elasticity of substitution between the two consumption goods is in the range of Yogo's (2006) values that vary – depending on the set of test assets used – between 0.52 and 0.87. The question of whether ρ is larger or smaller than 1 is of some importance as it determines how the ratio of durable and nondurable consumption goods is valued in the SDF. Because there are other studies that argue in favor of $\rho > 1$ (e.g., Ogaki and Reinhart (1998); Borri and Ragusa (2017)), I also calibrate ρ=1.25 in the empirical applications.

4.6.2 Potential non-stationarity of the stochastic discount factor

Another caveat arises from the ratio of the stock of the durable consumption good to consumption of the nondurable good and services that is part of the SDF presented

[73] Yogo (2006) uses the constraints implied by Equation (4.33) in the form of $\mathbb{E}\left(\left(1 - \frac{u_{Dt}}{P_t u_{Ct}} - (1-\delta)m_{t+1}\frac{P_{t+1}}{P_t}\right)\mathbf{z}_t\right) = 0$, where $\mathbf{z}_t$ denotes a vector of instrumental variables.

[74] The authors try to replicate the results from Yogo's (2006) study and argue that the values reported in the original paper were caused by a failed optimization.

in Equation (4.2): If the long-run growth rates of C and D are not identical, this will cause a non-stationary SDF. To outline the mechanisms behind this problem, reconsider the way in which the D-C ratio affects m_{t+1}

$$\begin{aligned} m_{t+1} &= \left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)^{\frac{1}{\rho}-\frac{1}{\psi}} R_{a,t+1}^{\frac{\theta-1}{\theta}}\right]^{\theta}, \\ \text{where} \qquad v\left(\frac{D}{C}\right) &= \left[1-\alpha+\alpha\left(\frac{D}{C}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \end{aligned} \tag{4.36}$$

and assume that the average long-run growth rate of D (π_D) is larger than that of C (π_C), such that $\lim_{j\to+\infty}\left(\frac{D_j}{C_j}\right) = +\infty$.

If we further assume that the elasticity of substitution between the two goods is smaller than unity, this implies:

$$\begin{aligned} \lim_{j\to+\infty} v\left(\frac{D_j}{C_j}\right) &= [1-\alpha]^{\frac{\rho}{\rho-1}} \\ \text{and thus} \qquad \lim_{j\to+\infty}\left(\frac{v(D_{j+1}/C_{j+1})}{v(D_j/C_j)}\right)^{\frac{1}{\rho}-\frac{1}{\psi}} &= 1, \end{aligned} \tag{4.37}$$

meaning that Yogo's (2006) SDF converges towards the standard Epstein-Zin-Weil case.

If the elasticity of substitution between the two goods is larger than unity, a similar picture arises:

$$\begin{aligned} \lim_{j\to+\infty} v\left(\frac{D_j}{C_j}\right) &= +\infty \\ \text{and} \qquad \lim_{j\to+\infty}\left(\frac{v(D_{j+1}/C_{j+1})}{v(D_j/C_j)}\right)^{\frac{1}{\rho}-\frac{1}{\psi}} &= \left(\frac{\pi_D}{\pi_C}\right)^{\frac{1}{\rho}-\frac{1}{\psi}}. \end{aligned} \tag{4.38}$$

Again, this signifies the SDF reduces to a scaled version of standard recursive preferences.

Consequently, the moments of the SDF depend on t and the process is not stationary.[75] This caveat is caused by the model specification and is not a result of allowing for rare disaster risk. In fact, the assumption of destructive disasters,

[75] Consider the variance of the SDF: For small j and $\rho < 1$, we have $\mathrm{var}(m_{s+1}) = \mathrm{var}\left(\beta^{\theta}\left(\frac{C_{s+1}}{C_s}\right)^{-\frac{\theta}{\psi}}\left(\frac{v(D_{s+1}/C_{s+1})}{v(D_s/C_s)}\right)^{\frac{\theta}{\rho}-\frac{\theta}{\psi}} R_{a,s+1}^{\theta-1}\right)$, but $\lim_{j\to+\infty}\mathrm{var}(m_{s+1})=\mathrm{var}\left(\beta^{\theta}\left(\frac{C_{s+1}}{C_s}\right)^{-\frac{\theta}{\psi}} R_{a,s+1}^{\theta-1}\right)$.

during which the stock of the durable consumption good is wiped out, helps resolve the problem as the D-C ratio is temporarily returned to zero. There may still be long periods during which the D-C ratio increases and the volatility of the SDF is reduced but when allowing for destructive disasters, the timing, frequency, and length of these periods is random and does not depend on t.

4.6.3 b_C and $b_E \in [0, 1]$ are not ensured

Different growth rates of C and E (and thus D) also can hamper the splitting of b_{t+1} into $b_{C,t+1}$ and $b_{E,t+1}$, as described in Equation (4.18). The reasoning is as follows: Assume that period $t + 1$ is a disaster period and that $E_{t+1} \gg C_{t+1}$ and $\omega_{t+1} = 0$, meaning $b_{E,t+1} = 0$ and the overall consumption contraction b_{t+1} is entirely caused by $b_{C,t+1} > 0$; in such a scenario, it would be likely for Equation (4.18) to result in $b_{C,t+1} > 1$. Such a contraction factor is, however, not meaningful from theoretical considerations: It cannot get worse than consuming nothing ($b_{C,t+1} = 1$).

To circumvent this problem, I only split b_{t+1} using ω_{t+1} if it is ensured that any value of ω_{t+1} would yield $b_{C,t+1}$ and $b_{E,t+1} \in [0, 1]$. The constraints on b_{t+1} implied by this approach, are:[76]

$$\begin{aligned} & b_{t+1} \leq 1 - \frac{E_t}{E_t + C_t} \frac{\exp(g^*_{E,t+1})}{\exp(g^*_{t+1})} \\ \text{and} \qquad & b_{t+1} \leq 1 - \frac{C_t}{E_t + C_t} \frac{\exp(g^*_{C,t+1})}{\exp(g^*_{t+1})}. \end{aligned} \tag{4.39}$$

If the inequalities in Equation (4.39) hold, I draw an ω_{t+1} and proceed as described in Section 4.3.2; if they are violated, I resort to $b_{t+1} = b_{C,t+1} = b_{E,t+1}$, instead.

4.6.4 Estimated correlation matrix may not be positive semidefinite

The moment matches listed in Equation (4.19) identify the correlations between the innovations in the log consumption growth, dividend growth, and log T-bill return series. However, it is not ensured that combining these thus identified individual estimates to the estimated correlation matrix indeed results in a positive semidefinite object. In order to overcome this caveat, I choose to take the estimated (potentially) not positive semidefinite matrix $\hat{\mathbf{A}}$ and find the closest proper correlation matrix $\mathbf{B}$.

There is a substantial branch of literature in mathematics that deals with finding the closest correlation matrix to some arbitrary square matrix. However, the definition

[76] See Appendix C.2 for a detailed derivation of Equation (4.39).

of closeness is not a trivial issue and there are multiple approaches that differ in their manner of weighting the distances between the single elements of $\hat{\mathbf{A}}$ and $\mathbf{B}$. I propose to choose $\mathbf{B}$ as the $(n \times n)$ correlation matrix that minimizes the Frobenius norm of $\hat{\mathbf{A}} - \mathbf{B}$:

$$\left\|\hat{\mathbf{A}} - \mathbf{B}\right\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} |\hat{a}_{ij} - b_{ij}|^2}. \tag{4.40}$$

Both $\hat{\mathbf{A}}$ and $\mathbf{B}$ are real matrices, so $\mathbf{B}$ can be understood as the proper correlation matrix whose individual entries are closest to those of $\hat{\mathbf{A}}$ in the least squares sense. As the minimization of the sum of squared residuals is an often-applied concept in economics, the Frobenius norm is a natural choice for the problem at hand. However, the computation of $\mathbf{B}$ is challenging. For its purpose, I use an algorithm developed by Qi and Sun (2006), who propose a quadratically convergent Newton method based on theoretical considerations by Higham (1988).[77]

4.7 Estimation results

Using the estimation approach outlined in Sections 4.3 and 4.5.1 on the data presented in Section 4.4 allows the simulation of disaster-including consumption and return series, which facilitates the SMM estimation proposed in Section 4.5.2. Section 4.7.1 presents the first-step fundamental macroeconomic and financial parameter estimates used for this purpose, as well as the estimation results for various ACH-GP specifications. Section 4.7.2 contains the estimates of the preference parameters and Section 4.7.3 provides the results for a variety of robustness checks.

4.7.1 First-step estimation results

Table 27 contains the estimates of the fundamental macroeconomic and financial parameters that result from combining the data presented in Section 4.4 with the moment conditions in Equation (4.19). Standard errors are reported in parentheses.

The parameter estimates in Panels A and B are plausible with respect to their signs and magnitude. However, the estimation precision is mixed. Whilst some parameters feature small standard errors, other estimates are quite imprecise (e.g., $\hat{\phi}_i$ and $\hat{\beta}_{z,i}$). There may be alternative identification approaches that would allow improvement of the overall first-step estimation precision; however, one should be realistic regarding the limited informational content of the available data. The series

[77] I thank the authors for making their code available.

are not particularly long and – in the case of the dividend growth processes – even had to be transformed to make the estimation feasible in the first place. So, yes, some of the estimates are rather imprecise, but at least that imprecision is accounted for when assessing the precision of the second-step preference parameter estimates.

Table 27: First-step estimates: fundamental macroeconomic and financial data
This table contains the estimates of the fundamental macroeconomic and financial parameters. Panel A reports the parametrization of the log consumption growth, T-bill return, and depreciation rate processes. Panel B contains asset-specific parameter estimates that describe the log price-dividend ratio and dividend growth processes for the return on the market portfolio ($i = m$) and the returns on the ten size-sorted portfolios ($i = 1, \dots, 10$). The estimated correlations between the idiosyncratic innovations in log consumption growth, dividend growth, and the T-bill return are presented in Panel C. These are the individual correlations for which the closest correlation matrix is computed using Qi and Sun's (2006) algorithm. Correlations that are statistically significantly different from 0 on a 5% significance level are reported in bold.

Panel A: Basic macroeconomic and financial parameters

$\hat{\mu}_C$	$\hat{\mu}_E$	$\hat{\sigma}_C$	$\hat{\sigma}_E$	$\hat{\varphi}_e$	$\hat{\mu}_b$	$\hat{\delta}_b$	$\hat{\phi}_b$	$\hat{\sigma}_b$	$\hat{a}$	$\hat{\beta}_\delta$	$\hat{\sigma}_\varepsilon$
0.0048	0.0095	0.0049	0.0366	0.0010	0.0002	0.9039	0.0501	0.0016	0.0158	0.6995	0.0016
(0.0003)	(0.0022)	(0.0008)	(0.0033)	(0.0029)	(0.0002)	(0.1042)	(0.1870)	(0.0008)	(0.0154)	(0.2932)	(0.0007)

Panel B: Asset-specific parameters

i	m	1	2	3	4	5	6	7	8	9	10
$\hat{\mu}_{d,i}$	0.0056	0.0111	0.0069	0.0087	0.0075	0.0084	0.0080	0.0076	0.0073	0.0066	0.0045
	(0.0014)	(0.0039)	(0.0041)	(0.0040)	(0.0032)	(0.0034)	(0.0029)	(0.0022)	(0.0033)	(0.0020)	(0.0016)
$\hat{\phi}_i$	1.32	53.23	55.46	46.04	30.38	44.46	33.90	26.66	23.36	20.14	3.30
	(8.94)	(142.79)	(130.02)	(109.16)	(69.15)	(98.90)	(78.08)	(62.11)	(59.33)	(49.42)	(11.84)
$\hat{\sigma}_{d,i}$	0.0220	0.0377	0.0405	0.0470	0.0439	0.0341	0.0349	0.0247	0.0491	0.0265	0.0252
	(0.0020)	(0.1118)	(0.1236)	(0.0781)	(0.0458)	(0.1300)	(0.0669)	(0.0560)	(0.0216)	(0.0247)	(0.0032)
$\hat{\mu}_{z,i}$	0.0606	0.0849	0.0704	0.0651	0.0520	0.0488	0.0527	0.0555	0.0694	0.0665	0.0558
	(0.2599)	(0.2519)	(0.2256)	(0.2204)	(0.2242)	(0.2277)	(0.2153)	(0.2147)	(0.2250)	(0.2051)	(0.2834)
$\hat{\rho}_{z,i}$	0.9827	0.9798	0.9824	0.9832	0.9863	0.9870	0.9857	0.9848	0.9806	0.9810	0.9840
	(0.0742)	(0.0599)	(0.0566)	(0.0568)	(0.0590)	(0.0609)	(0.0585)	(0.0588)	(0.0630)	(0.0588)	(0.0814)
$\hat{\beta}_{z,i}$	79.66	109.48	121.02	120.65	106.95	106.38	100.40	99.81	103.48	93.25	78.20
	(382.90)	(422.34)	(498.84)	(509.59)	(498.59)	(511.64)	(457.29)	(443.15)	(425.75)	(379.65)	(398.72)

Panel C: Correlations

	u_{10}	u_9	u_8	u_7	u_6	u_5	u_4	u_3	u_2	u_1	u_m	η_b	η_E
η_C	0.09	-0.05	0.03	-0.15	-0.19	-0.28	-0.05	-0.12	-0.08	-0.23	0.10	0.42	**0.39**
η_E	**0.20**	0.02	0.06	0.03	-0.11	-0.02	0.08	-0.03	0.03	-0.03	0.16	0.27	
η_b	-0.13	-0.15	-0.08	-0.13	-0.23	-0.23	-0.08	-0.21	0.05	-0.56	-0.15		
u_m	**0.86**	0.63	0.40	0.52	0.53	0.72	0.46	0.46	0.62	0.61			
u_1	0.24	-0.49	-0.16	-0.48	-0.50	-0.67	0.06	-0.30	-0.46				
u_2	0.16	-0.07	-0.31	-0.72	-0.30	-0.24	0.05	0.05					
u_3	0.14	-0.09	0.03	-0.42	-0.20	-0.10	-0.19						
u_4	0.11	0.12	0.11	-0.03	-0.03	-0.22							
u_5	0.31	-0.12	-0.17	-0.65	-0.16								
u_6	0.19	-0.02	-0.11	-0.38									
u_7	0.07	-0.08	-0.19										
u_8	0.11	-0.18											
u_9	0.24												

The correlations between the innovations of the log consumption growth, dividend growth, and log T-bill return processes often display unexpected signs and generally are not statistically significant. However, it turns out it is important to include them in the estimation, because setting them to zero and thus enforcing that all innovations are independent, imposes a severe restriction on the model and has negative effects on the plausibility of the parameter estimates depicted in Panels A and B.

Table 28 reports the parameter estimates of the GP distribution (separately for destructive and non-destructive disasters) and estimation results for different ACH model specifications that vary in terms of their parsimony. The ACH_1-GP model corresponds to the specification in Equation (4.15). It is the most heavily parametrized of the considered models and accounts for the effects of past durations between and the lengths of disasters, as well as the aggregate size of the last disaster and the magnitude of the contraction in the preceding disaster period. All other disaster processes that I consider are nested in the ACH_1-GP model. For example,

Table 28: Estimation results for the ACH-GP model

This table reports the ACH-GP maximum likelihood estimates. Here, $\mathcal{L}$ is the log-likelihood value at the maximum; AIC = $2k - 2\ln(\mathcal{L})$ and SBC = $-2\ln\mathcal{L} + k\ln(T)$, where k is the number of ACH model parameters, denote the Akaike and Schwarz-Bayes information criteria, respectively. Furthermore, $\mathcal{LR}$ gives the p-values (in percent) of the likelihood ratio tests of the null hypothesis that the parameter restrictions implied by the ACH_0 specification are correct. The respective alternative is the ACH_1, the ACH_2, the ACH_3, or the ACH_4 model. σ_{GP} denotes the scale parameter of a generalized pareto distribution and ξ is its shape parameter. Parameter estimates are reported for GP distributions estimated on contractions that belong to destructive disasters ($d^D = 1$) and on those that belong to non-destructive disasters ($d^D = 0$). The estimation results are based on the updated country panel data originally assembled by Barro and Ursúa (2008), using the concatenated event data representation described in Section 4.4 and $q = 0.145$. Asymptotic standard errors are reported in parentheses.

	σ_{GP}	ξ	π	$\tilde{\pi}$	$\breve{\alpha}$	$\tilde{\alpha}$	$\breve{\delta}$	$\tilde{\delta}$	$\mathcal{L}$	AIC	SBC	$\mathcal{LR}$
ACH_0			178.3	1.201					-790.3	1584.7	**1600.1**	
			(18.8)	(0.023)								
ACH_4			64.9	1.201			441.1		-787.0	**1580.0**	1603.2	<1.0
			(49.3)	(0.023)			(211.5)					
ACH_3			64.9	1.214			441.1	-0.375	-786.8	1581.5	1612.5	2.9
			(49.3)	(0.032)			(211.5)	(0.537)				
ACH_2			198.7	1.221	-0.145	-0.002			-789.9	1587.7	1618.7	63.5
			(30.9)	(0.052)	(0.153)	(0.004)						
ACH_1			71.4	1.237	-0.030	-0.002	431.0	-0.399	-786.6	1585.3	1631.7	11.8
			(55.0)	(0.058)	(0.161)	(0.004)	(120.4)	(0.542)				
GP (d^D=1)	0.015	0.708										
	(0.002)	(0.097)										
GP (d^D=0)	0.010	0.736										
	(0.001)	(0.081)										

the ACH_2-GP model sets $\breve{\delta} = \tilde{\delta} = 0$ and thus focuses on the duration of and between disasters, whilst the ACH_3-GP model only allows for the aggregate size of the last disaster and the preceding disaster period to affect the hazard rate by setting $\breve{\alpha} = \tilde{\alpha} = 0$. In the ACH_4-GP model, $\breve{\alpha} = \tilde{\alpha} = \tilde{\delta} = 0$ and only the aggregate size of the last disaster is included in the estimation. The most parsimonious specification is given by the ACH_0-GP model, in which $\breve{\alpha} = \tilde{\alpha} = \breve{\delta} = \tilde{\delta} = 0$ and the hazard rate is solely determined by the constants π and $\tilde{\pi}$, meaning the conditional disaster probability of the next period only depends on whether the current period is a disaster period or not.

To compare the empirical performance of the proposed ACH specifications, Table 28 also reports the Akaike (AIC) and Schwarz-Bayes (SBC) information criteria, and the p-values of a likelihood-ratio statistic that tests whether the constraints implied by the ACH_0 model can be rejected when compared to any of the other specifications. I find that ACH_0 is preferred by the SBC, whilst ACH_4 is the best choice according to AIC. Furthermore, the ACH_4 is the only specification that rejects the ACH_0-implied constraints on a significance level of 1%. For these reasons, I will focus on the ACH_0-GP in the second estimation step. Its estimates $\hat{\pi} = 178.3$ and $\hat{\bar{\pi}} = 1.2$ imply that the quarterly probability of entering a disaster is 0.56%, with a quarterly probability of remaining in a disaster of 83%. Section 4.7.3 also contains a robustness check which considers the ACH_4-GP instead.

Figure 18: Distribution of weighting factor ω_t
This figure depicts the estimated distribution of the weighting factor ω_t that is described in Equation (4.17). The probabilities of the corner solutions $\omega_t = 0$ and $\omega_t = 1$ are indicated in red. The estimates and their (bootstrapped) standard errors are: $\hat{\mathbb{P}}(\omega_t{=}0) = 0.19$ (0.03), $\hat{\mathbb{P}}(\omega_t{=}1) = 0.16$ (0.03), $\hat{e} = 1.15$ (0.16), and $\hat{f} = 1.68$ (0.24).

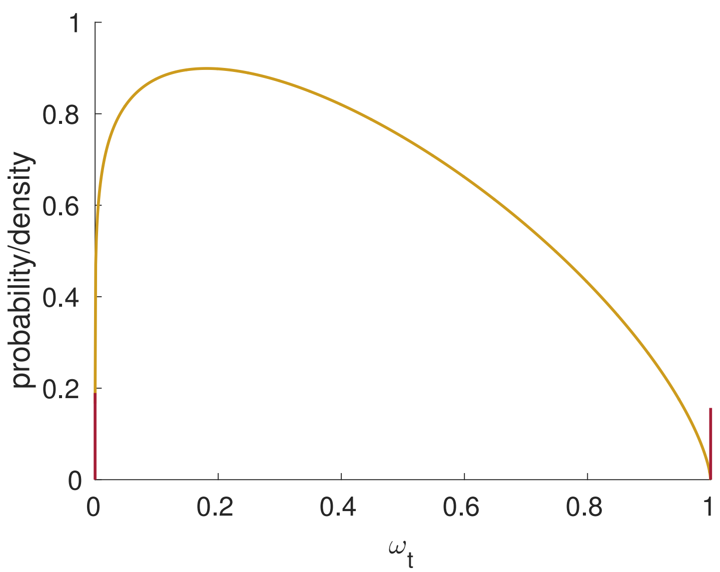


Figure 18 depicts the estimated distribution of the disaster weighting factor ω_t. According to this distribution, there is a 19% probability of the consumption contraction being entirely caused by a drop in consumption of the nondurable good; the probability of a pure durable consumption disaster is 16%.

The fitted beta distribution accounts for the distribution of the weighting factor for those cases in which both the durable and the nondurable good contract during the disaster. The shape of the density reveals that most of the probability mass is assigned to smaller weighting factors, which (keeping all else equal) implies an increased importance of the nondurable good in the decomposition of the consumption disaster.

4.7.2 Second-step estimation results

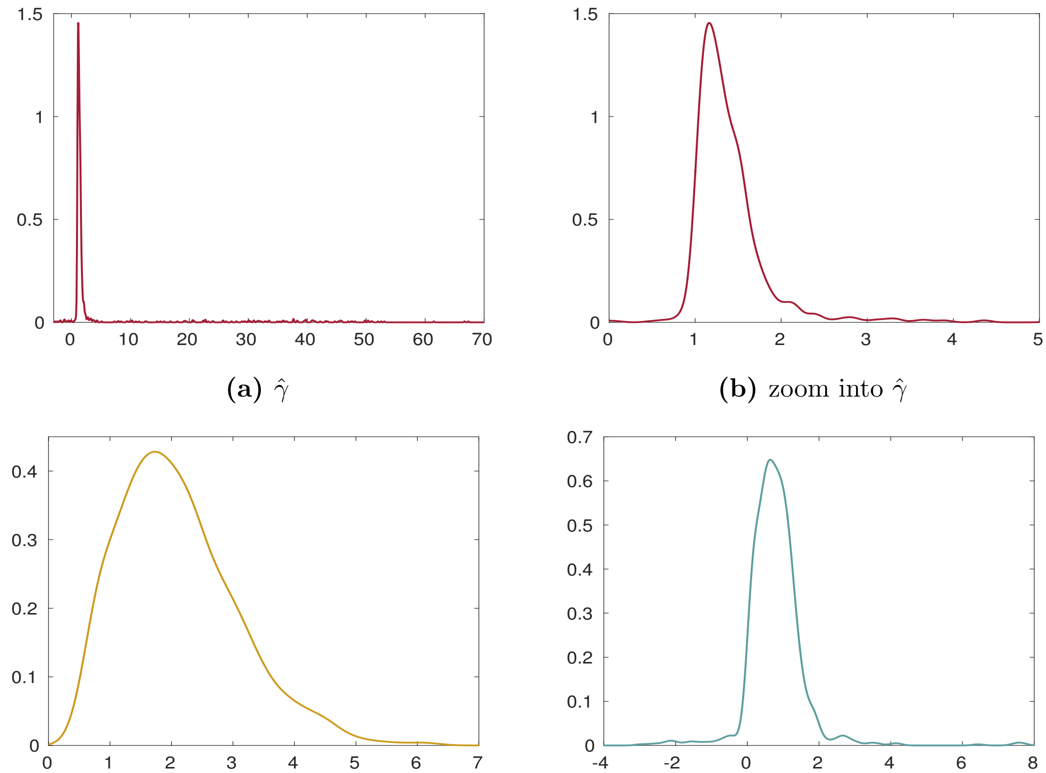
Table 29 presents the SMM-estimates of the C-CAPM preference parameters. The estimations are performed using disaster-including consumption growth and return series that are simulated based on the ACH_0-GP disaster process, a series length $\mathcal{T}$=10^7, and K=1k bootstrap replications.

Table 29: Second-step: SMM estimates of the C-CAPM preference parameters
This table reports the estimates of the RRA coefficient γ and the IES ψ using the moment matches in Equation (4.30), $\mathcal{T} = 10^7$, and the weighting matrix in Equation (4.32). The second-step SMM-type estimates are based on the first-step ACH_0-PL estimates, reported in Table 28. The numbers in brackets are the upper and lower bounds of the 95% confidence intervals computed as the 0.025 and 0.975 quantiles of the bootstrap distribution (percentile method). The table also reports the p-values (in percent) of Hansen's (1982) J-statistic (see Equation (4.41)) and root mean squared errors (R), computed according to Equation (4.42). $\hat{p} = \frac{1}{K}\sum_{k=1}^{K} \mathbb{1}(\hat{\gamma}^{(k)} > 1/\hat{\psi}^{(k)})$ denotes the fraction of the bootstrap replications in which a preference for early resolution of uncertainty is implied. Panels A sets the elasticity of substitution between the two goods to 0.75; in Panel B, $\rho = 1.25$. Both panels use $\alpha = 0.12$ and break down the results by the set of test assets, namely, the excess returns of the market portfolio (*mkt*) and the ten size-sorted portfolios (*size dec*).

Panel A: $\alpha = 0.12$ **and** $\rho = 0.75$							
mkt			**size dec**				
$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{p}$
1.31	**2.03**	0.97	**1.06**	**1.40**	1.7	94	0.99
[0.87 42.63]	[0.72 4.41]		[1.00 12.84]	[0.65 4.41]			
Panel B: $\alpha = 0.12$ **and** $\rho = 1.25$							
mkt			**size dec**				
$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	$\hat{\gamma}$	$\hat{\psi}$	J	R	$\hat{p}$
1.31	**2.00**	0.93	**1.04**	**1.20**	1.7	94	0.95
[-0.70 31.71]	[0.64 4.96]		[1.00 9.88]	[0.41 5.21]			

Table 29 contains the estimates of the RRA coefficients and the IES, together with their respective 95% confidence bounds, which are computed according to the percentile method as the 0.025 and 0.975 quantiles of the bootstrap distribution. It can be seen from Figure 19 that the distribution of $\hat{\gamma}$ is strongly skewed to the right. In light of this finding, I choose not to report bootstrap standard errors, because they do not qualify as an appropriate measure of estimation precision.

Figure 19: Kernel density estimates of bootstrapped preference parameters
This figure illustrates the kernel density estimates of the bootstrapped RRA and IES estimates that result from using the model specification in Panel A of Table 29 and the excess return of the market portfolio as test assets. Panel (a) depicts the kernel density estimate for $\hat{\gamma}$ and Panel (b) zooms into the interval that is generally assumed to make up the range of economically plausible RRA values. Panel (c) refers to $\hat{\psi}$. Panel (d) contains the kernel density estimate of $\hat{\gamma} - \hat{\psi}^{-1}$. The number of bootstrap replications is K = 1k and I use a Gaussian kernel with a bandwidth as suggested by Silverman's (1986) rule of thumb.



(a) $\hat{\gamma}$ **(b)** zoom into $\hat{\gamma}$

(c) $\hat{\psi}$ **(d)** $\hat{\gamma} - \hat{\psi}^{-1}$

Furthermore, Table 29 reports p-values of Hansen's (1982) J-statistic,

$$J = \mathbb{G}(\hat{\boldsymbol{\vartheta}})' \widehat{\text{Avar}}(\mathbb{G}[\hat{\boldsymbol{\vartheta}}])^{+} \mathbb{G}(\hat{\boldsymbol{\vartheta}}), \tag{4.41}$$

which – under the null hypothesis that the financial moment restrictions are correct – is approximately $\chi^2(N-1)$, where N is the number of test assets used for the estimation and $^+$ denotes the Moore-Penrose inverse. The root mean squared errors (RMSEs) are computed as:

$$R = \sqrt{\frac{1}{N} \mathbb{G}(\hat{\boldsymbol{\vartheta}})' \mathbb{G}(\hat{\boldsymbol{\vartheta}})} \times 10^4. \tag{4.42}$$

When the estimation of γ is exclusively based on the excess return of the market portfolio ($N = 1$), the moments in Equation (4.30) are exactly matched, thus setting $R = 0$ and leaving no room for testing overidentifying restrictions using the J-statistic.

Using the bootstrap replications, Table 29 also reports:

$$\hat{p} = \frac{1}{K} \sum_{k=1}^{K} \mathbb{1} \left(\hat{\gamma}^{(k)} > \frac{1}{\hat{\psi}^{(k)}} \right), \tag{4.43}$$

which is the relative frequency with which $\hat{\gamma}$ exceeds $\hat{\psi}^{-1}$ such that preference for early resolution of uncertainty is implied.

Table 29 shows that both choices for the elasticity of substitution between the two goods, ρ, and both sets of test assets yield economically plausible point estimates of the RRA coefficient and the IES. The point estimates for γ range between 1.04 and 1.31 and the IES estimates lie between 1.20 and 2.03. The confidence interval of ψ is reasonably narrow and covers values of a sensible size.

For the RRA coefficient γ, the confidence interval is rather wide and its bounds are sometimes implausible. The kernel density estimates in Figure 19 reveal, however, that the vast majority of the estimates lies in the interval $(0, 5]$, which is considered economically sensible. When using $\rho = 0.75$ and the market portfolio as the test asset, the fraction of estimates that fall in this range is 87.5%; it is even 94.9% when using the ten size-sorted portfolios. Choosing 10 to be the upper plausibility bound, the fractions change to 88.3% and 96.9%, respectively. For all choices of ρ and test assets, a preference for early resolution of uncertainty is implied in more than 90% of the replications.

When using the size-sorted portfolios as test assets, the p-value of the J-statistic and the RMSE are virtually unaffected by the choice of ρ as can be seen by comparing the results in Panels A and B. In both cases, the restrictions imposed by the asset pricing moment matches cannot be rejected on the 1% significance level, but could be on 5%.

The parameter estimates obtained in this C-CAPM that explicitly accounts for rare disaster risk and two different consumption goods are strikingly different from those reported by Yogo (2006). In his study, $\hat{\psi} \leq 0.0024$ and $\hat{\gamma}$ exceeds 170 for all considered sets of test assets. These parameter estimates are far away from the ranges considered economically plausible; and also at odds with a preference for early resolution of uncertainty.

4.7.3 Robustness checks

The estimation results reported in the previous section indicate that plausible estimates of the preference parameters can be obtained once rare disaster risk is accounted for in Yogo's (2006) model. Due to the fact that a variety of assumptions had to be made to facilitate such an estimation, it should be checked how crucial these assumptions are for the results. In this section, robustness is checked with respect to (1) the ACH-GP specification, (2) synchronicity of consumption and return disasters, (3) computation of the mean *cay*-variable in Equation (4.12), (4) the moment matches used to identify the preference parameters, (5) the choice of the SDF (Yogo (2006) versus standard Epstein-Zin-Weil), and (6) the differentiation between destructive and non-destructive disasters. Table 30 contains the preference parameter estimates and 95% confidence bounds that result from performing these robustness checks. All panels labeled A use the market portfolio as test asset, while those labeled B refer to the size-sorted portfolios, instead.

Chosen ACH-GP specification

Panels A1 and B1 contain the parameter estimates of the RRA coefficient and the IES that are obtained when using the ACH_4-GP to model the disaster process. The plausibility of the preference parameter point estimates is not affected by changing the MPP specification. Compared to the results reported in Table 29, the estimates vary slightly, but there is no clear indication for a systematic direction in which they change: When considering the market portfolio as test asset, the point estimates of ψ and γ decrease slightly though there is barely a difference regarding the confidence bounds. When considering the size-sorted portfolios, estimates increase slightly and so do the upper bounds of the confidence interval. Furthermore, the reported $\hat{p}$-statistics, and thus the implication regarding the preferred timing of uncertainty resolution, is robust with respect to the ACH specification. Using the ACH_4-GP, the computed RMSEs are more than twice the size of their ACH_0 counterparts, meaning the additional variation that enters through the more variable disaster probability poses a challenge to the model fit. This can also be seen from the p-value of the J-statistic, which no longer allows not rejecting the hypothesis that the constraints implied by the financial moments are correct on a 1% significance level.

Synchronicity of consumption and return disasters

The model setup defined in Section 4.2 assumes that every consumption disaster is accompanied by a return contraction. This synchronicity follows from the $z_{i,t+1}$ and

Table 30: Robustness checks: SMM estimates of the C-CAPM preference parameters This table reports the estimates of the RRA coefficient γ and the IES ψ that result from the robustness checks that are described in Section 4.7.3. The estimates in Panel A are obtained when using the excess return of the market portfolio as test asset; Panel B uses the excess returns of the ten size-sorted portfolios. Panels A1 and B1 rely on the first-step ACH_4-GP estimates; in Panels A2 and B2, returns do not contract during every consumption disaster. The mean of the *cay*-variable is not adapted by the average contraction size in Panels A3 and B3. Panels A4 and B4 use the conventional representation of the basic asset pricing equation and entirely simulated consumption and return series. The estimates in Panels A5 and B5 are obtained from using standard Epstein-Zin-Weil preferences without explicitly accounting for consumption of the durable good. Panels A6 and B6 do not allow for a destruction of the stock of the durable good during some disasters. Other estimation settings and the reported statistics correspond to Table 29.

	Panel A: excess return of the market portfolio (mkt)									
	Panel A1: ACH_4					**Panel A2: partial contractions**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$			$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$		
ρ = 0.75	**1.25**	**1.52**	0.97			**1.51**	**1.76**	0.94		
	[0.85 42.96]	[0.78 4.64]				[-1.35 50.42]	[0.74 4.45]			
ρ = 1.25	**1.20**	**1.36**	0.94			**1.31**	**1.35**	0.91		
	[-0.53 30.22]	[0.63 5.39]				[-1.35 33.61]	[0.61 4.96]			
	Panel A3: *cay* w/o disasters					**Panel A4: standard pricing equation**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$			$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$		
ρ = 0.75	**1.31**	**2.03**	0.95			**1.43**	**2.03**	0.96		
	[0.74 43.57]	[0.70 4.30]				[0.94 1.82]	[0.69 4.37]			
ρ = 1.25	**1.31**	**2.00**	0.94			**1.42**	**2.00**	0.91		
	[0.30 32.25]	[0.64 4.88]				[0.80 2.02]	[0.61 4.30]			
	Panel A5: standard EZW-preferences					**Panel A6: no destruction of D**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$			$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$		
	2.60	**1.65**	0.68			**2.08**	**0.43**	0.00		
	[-3.33 9.50]	[0.30 4.25]				[2.56 4.46]	[0.01 0.05]			
	Panel B: excess returns of the ten size-sorted portfolios (size dec)									
	Panel B1: ACH_4					**Panel B2: partial contractions**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R
ρ = 0.75	**1.29**	**1.52**	1.00	0.5	215	**1.00**	**1.40**	0.98	1.3	244
	[1.01 18.65]	[0.72 4.78]				[1.00 16.80]	[0.64 4.45]			
ρ = 1.25	**1.23**	**1.36**	0.97	0.4	216	**1.00**	**1.20**	0.95	1.3	244
	[1.01 11.28]	[0.42 5.39]				[1.00 17.02]	[0.41 5.02]			
	Panel B3: *cay* w/o disasters					**Panel B4: standard pricing equation**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R
ρ = 0.75	**1.06**	**1.40**	0.99	1.7	94	**1.07**	**1.40**	0.99	0.0	261
	[1.00 19.05]	[0.67 4.57]				[1.00 1.73]	[0.66 4.47]			
ρ = 1.25	**1.04**	**1.20**	0.95	1.7	94	**1.04**	**1.20**	0.94	0.0	261
	[1.01 13.97]	[0.41 4.94]				[1.00 2.29]	[0.40 4.97]			
	Panel B5: standard EZW-preferences					**Panel B6: no destruction of D**				
	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R	$\hat{\gamma}$	$\hat{\psi}$	$\hat{p}$	J	R
	1.42	**1.02**	0.66	0.2	508	**1.80**	**0.42**	0.01	0.2	26
	[-3.76 5.65]	[0.29 4.34]				[-12.41 6.21]	[0.01 0.05]			

$g_{d,i,t+1}$ specifications in Equation (4.9) and it could be argued the correlation between consumption and returns is thus artificially increased. In order to check whether the second-stage parameter estimates are driven by this assumption, I perform a robustness check in which log dividend growth and the log price-dividend ratio are only affected by the consumption disaster if the respective disaster entails a partial government default:

$$
\begin{aligned}
g_{d,i,t+1} &= \mu_{d,i} + \phi_i x_t + \ln(1 - b_{t+1}) d_{b,t+1} + \sigma_{d,i} u_{i,t+1}, \\
z_{i,t+1} &= \mu_{z,i} + \beta_{z,i} x_{t+1} + \ln(1 - b_{t+1}) d_{b,t+1} + \rho_{z,i} z_{i,t}.
\end{aligned} \tag{4.44}
$$

Panels A2 and B2 contain the parameter estimates that result from this robustness check. Again, the point estimates remain plausible and are not much affected by the reduced synchronicity of consumption and return disasters; the same holds true for the fraction of the bootstrap replications that implies a preference for early resolution of uncertainty. However, the 95% confidence interval of γ widens compared to the results of the base study and the RMSEs increase. The p-values of the J-statistic are slightly reduced but still lead to a non-rejection of the model-implied constraints on a 1% significance level.

Computation of the mean cay-variable

Panels A3 and B3 provide the estimation results for a robustness check that computes the time series mean of the *cay*-variable entirely on the non-disaster-including series that are provided by Lettau and Ludvigson (2001). As this is only a minor adjustment, one would not expect to see a strong reaction of the second-step estimation results, and indeed, the parameter estimates, confidence bounds, and test statistics are robust with respect to this change in the mean of the *cay*-variable.

Second-step moment matches

A further robustness check entails estimating the preference parameters using the basic asset pricing equation and entirely simulated series that potentially include disasters:

$$
\mathbb{E}\left[m_s(\gamma, \psi, \beta^{fix}, \alpha^{fix}, \rho^{fix}) \mathbf{R}_s^e\right] = \mathbf{0}, \tag{4.45}
$$

where the theoretical moment will be approximated by sample means of the simulated series for the estimation purpose. Using the constraints in Equation (4.45) in combination with the IES-identifying moment conditions in Equation (4.29), yields

$$
\mathbb{G}^+(\tilde{\boldsymbol{\vartheta}}) = \begin{bmatrix}
\frac{1}{\mathcal{T}} \sum_{s=1}^{\mathcal{T}} m_s(\gamma, \psi, \beta^{fix}, \alpha^{fix}, \rho^{fix}) \mathbf{R}_s^e \\
\begin{bmatrix}
-1 & -\mathbb{E}_{\mathcal{T}}(\Delta cv_s) & -\mathbb{E}_{\mathcal{T}}(r_{b,s}) \\
\frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s)}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2})}{\psi^2} & \frac{\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2})}{\psi^2}
\end{bmatrix} \times \\
\begin{bmatrix}
\mathbb{E}_{\mathcal{T}}(r_{b,s}) - w_b - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s) \\
\mathbb{E}_{\mathcal{T}}(r_{b,s}\Delta cv_{s-2}) - w_b \mathbb{E}_{\mathcal{T}}(\Delta cv_{s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s \Delta cv_{s-2}) \\
\mathbb{E}_{\mathcal{T}}(r_{b,s} r_{b,s-2}) - w_b \mathbb{E}_{\mathcal{T}}(r_{b,s-2}) - \frac{1}{\psi}\mathbb{E}_{\mathcal{T}}(\Delta cv_s r_{b,s-2})
\end{bmatrix}
\end{bmatrix}, \tag{4.46}
$$

which can be used in the objective function in Equation (4.32). Panels A4 and B4 in Table 30 contain the results of this estimation strategy. It turns out the point estimates are close to their counterparts from the base analysis, but the confidence interval of γ is pronouncedly more narrow. The $\hat{p}$-statistics are basically not affected by the change in the estimation strategy. However, the RMSEs increase and the p-value of the J-statistic decreases, such that the null hypothesis can be rejected on any conventional significance level.

Stochastic discount factor

The results in Panels A5 and B5 of Table 30 belong to robustness checks that used the standard Epstein-Zin-Weil SDF without allowing for a differentiation between durable and nondurable goods, hence:

$$m_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{a,t+1}^{\theta-1}, \qquad \text{where} \qquad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{4.47}$$

The point estimates of γ and ψ reveal the results do not crucially depend on differentiating between the two goods in the SDF. The confidence interval of γ gets more narrow, however, now its lower bound is decidedly smaller than 0. Furthermore, the p-value of the J-statistic shrinks and allows the rejection of the null hypothesis on all conventional significance levels, and the RMSE is reduced. This is the first robustness check, in which results in a pronounced reaction of $\hat{p}$: In this model specification, a preference for early resolution of uncertainty is only implied in about 68% of the bootstrap replications.

Destructive and non-destructive disasters

The final robustness check, the results of which are reported in Panels A6 and B6, considers an economy in which there are no destructive disasters. As described in Section 4.6.2, it follows that the D/C-ratio increases. Estimates are reported for ρ=0.75. This is the only robustness check that results in point estimates of the IES that are smaller than 1. It is furthermore striking that the bootstrapped confidence bounds are even smaller than these point estimates (for the market portfolio: $\hat{\psi} = 0.43$ with 95% confidence bounds 0.01 and 0.05). The RRA estimate is plausible and precise when using the market portfolio as test asset, but the lower confidence bound is pronouncedly negative when using the size-sorted portfolios, instead. A preference for early resolution of uncertainty is implied in maximally 1% of the bootstrap replications and the p-value of the J-statistic allows the rejection of the null hypothesis on all conventional significance levels, whilst the small RMSE indicates a good model fit.

Overall, the results presented in Table 29 appear to be robust with respect to the assumptions made regarding the disaster process, the behavior of consumption and financial series during disasters, as well as with respect to the SDF and the moment matches. The assumption of destructive disasters is apparently important for the outcome of the estimation. However, even if a complete destruction of the stock of the durable good is not accounted for by the model, the RRA estimates still remain in a plausible range, and, though $\hat{\psi}$ is decidedly smaller than unity, this is not an uncommon result in other studies that try to estimate the IES.

4.8 Discussion and conclusion

This is the first study that accounts for two consumption goods in the context of asset pricing with rare disaster risk. I propose a framework that draws on different strands of asset pricing literature, such as the seminal contributions by Barro (2006), Bansal and Yaron (2004), Gabaix (2012), and Yogo (2006), to link multi-period consumption disasters transparently to return contractions, and thus to facilitate a simulation-based estimation of a disaster-including C-CAPM. For this purpose, I assume the disaster process can be modeled using a discrete-time MPP, in which the duration of and between disasters is determined by autoregressive conditional hazard models, and the size of the contractions follows a generalized pareto distribution. Some disasters are accompanied by a destruction of the stock of the durable consumption good or a partial government default. Different datasets must be used to estimate the parameters of the disaster process: a cross-country consumption dataset facilitates the estimation of the MPP parameters and an assortment of international business cycle dates allows the study of how contractions in the durable and nondurable good contribute to overall consumption disasters.

The results show that asset pricing models that account for different consumption goods, such as Yogo's (2006) model, can explain the high U.S. equity premia with plausible values of the IES and RRA preference parameters once the possibility of unlikely but severe consumption contractions is accounted for. The estimates of the RRA coefficient and the IES are plausible and so is the implication that investors have a preference for an early resolution of uncertainty. As indicated by the 95% bootstrap confidence bounds, the IES estimates are even quite precise. The bootstrap distribution of the RRA coefficient is spread more widely; however, closer analysis reveals the vast majority of bootstrap estimates lie indeed in the (0,5] interval, which is generally considered to describe the range of plausible risk aversion estimates.

Of course, assumptions must be made in order to account for the risk of rare disasters. However, far-reaching robustness checks are performed to question these assumptions and assess their importance for the estimation results. These robustness checks entail the use of an alternative specification of the disaster process, a reduced correlation between consumption and return disasters, different moment conditions to identify the preference parameters, the application of standard recursive preferences that do not differentiate between different types of goods, and the assumption that the stock of the durable good cannot be entirely destroyed during consumption disasters. It turns out that only the last assumption has a pronounced effect on the estimation results – but even then, the plausibility of the RRA estimates is not affected.

Consequently, this chapter extends previous literature from two angles. It contributes to the set of rare disaster studies by proposing a simulation-based strategy that enables the estimation of the preference parameters of a two consumption good C-CAPM. Thereby, it not only considers a new SDF specification but furthermore addresses the question how consumption of the durable and the nondurable good are affected by disaster risk. The study also adds to the literature that contemplates asset pricing models that differentiate between different types of consumption by showing that this class of models can explain the high U.S. excess returns at plausible preference parameters once rare disaster risk is adequately accounted for.

C Appendix

C.1 Campbell and Shiller's (1988) return representation

Equation (1) in Campbell and Shiller (1988) states that the realized log gross return on a portfolio i that was held from the beginning of t to the beginning of $t+1$ can be written as:

$$r_{i,t} = \ln(\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}) - \ln(\mathcal{P}_{i,t}), \tag{C.1}$$

where $\mathcal{P}_{i,t}$ denotes the price of the portfolio at the beginning of period t and $\mathcal{D}_{i,t}$ refers to the real dividend paid on the portfolio during period t. Rewriting Equation (C.1) as:

$$r_{i,t} = \ln(\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}) + \Delta\ln(\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}) - \ln(\mathcal{P}_{i,t}) \tag{C.2}$$

allows a first-order Taylor expansion around $\Delta\ln(\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t})$, evaluated at $\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t} = \mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}$:

$$\begin{aligned}
\Delta\ln(\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}) &= \ln\left(\frac{\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}}{\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}}\right) \\
&\approx \ln\left(\frac{\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}}{\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}}\right)\Bigg|_{\mathcal{P}_{i,t+1}+\mathcal{D}_{i,t}=\mathcal{P}_{i,t}+\mathcal{D}_{i,t-1}} \\
&+ \frac{1}{\mathcal{P}_{i,t+1} + \mathcal{D}_{t}}\Bigg|_{\mathcal{P}_{i,t+1}+\mathcal{D}_{i,t}=\mathcal{P}_{i,t}+\mathcal{D}_{i,t-1}} (\mathcal{P}_{i,t+1} - \mathcal{P}_{i,t} + \mathcal{D}_{i,t} - \mathcal{D}_{i,t-1}) \\
&= \frac{1}{\mathcal{P}_{t} + \mathcal{D}_{i,t-1}}(\mathcal{P}_{i,t+1} - \mathcal{P}_{i,t} + \mathcal{D}_{i,t} - \mathcal{D}_{i,t-1}) \\
&= \frac{\mathcal{P}_{i,t+1} - \mathcal{P}_{i,t}}{\mathcal{P}_{t} + \mathcal{D}_{i,t-1}} + \frac{\mathcal{D}_{i,t} - \mathcal{D}_{i,t-1}}{\mathcal{P}_{t} + \mathcal{D}_{i,t-1}}.
\end{aligned} \tag{C.3}$$

Furthermore, Campbell and Shiller (1988) assume there exists a time-invariant constant ρ_i that approximates the ratio of $\mathcal{P}_{i,t}$ and $\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}$, such that $\mathcal{P}_{i,t} \approx \rho_i(\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1})$ and $\mathcal{D}_{i,t-1} \approx (1-\rho_i)(\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1})$. This approximation can be used to rewrite Equation (C.3) viz:

$$\begin{aligned}
\Delta\ln(\mathcal{P}_{i,t+1} + \mathcal{D}_{i,t}) &\approx \rho_i\left(\frac{\mathcal{P}_{i,t+1} - \mathcal{P}_{i,t}}{\mathcal{P}_{i,t}}\right) + (1-\rho_i)\left(\frac{\mathcal{D}_{i,t} - \mathcal{D}_{i,t-1}}{\mathcal{D}_{i,t-1}}\right) \\
&\approx \rho_i\Delta\ln(\mathcal{P}_{i,t+1}) + (1-\rho_i)\Delta\ln(\mathcal{D}_{i,t}).
\end{aligned} \tag{C.4}$$

Going back to Equation (C.2) finally yields Equation (4.8):

$$
\begin{aligned}
r_{i,t} &\approx \ln(\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}) + \rho_i \Delta \ln(\mathcal{P}_{i,t+1}) + (1-\rho_i)\Delta \ln(\mathcal{D}_{i,t}) - \ln(\mathcal{P}_{i,t}) \\
&= \ln\left(\frac{\mathcal{P}_{i,t} + \mathcal{D}_{i,t-1}}{\mathcal{P}_{i,t}}\right) + \rho_i \Delta \ln(\mathcal{P}_{i,t+1}) + (1-\rho_i)\Delta \ln(\mathcal{D}_{i,t}) \\
&= -\ln(\rho_i) + \rho_i(\ln(\mathcal{P}_{i,t+1}) - \ln(\mathcal{P}_{i,t})) - \rho_i(\ln(\mathcal{D}_{i,t}) - \ln(\mathcal{D}_{i,t-1})) + g_{d,i,t} \\
&= -\ln(\rho_i) + \rho_i(\ln(\mathcal{P}_{i,t+1}) - \ln(\mathcal{D}_{i,t})) - \rho_i(\ln(\mathcal{P}_{i,t}) - \ln(\mathcal{D}_{i,t-1})) + g_{d,i,t} \\
&= -\ln(\rho_i) + \rho_i z_{i,t} - \rho_i z_{i,t-1} + g_{d,i,t}.
\end{aligned} \tag{C.5}
$$

C.2 Derivation of $b_{E,t+1}$ and $b_{C,t+1}$

The derivation of $b_{E,t+1}$ and $b_{C,t+1}$ starts from dividing the consumption growth specifications into their disastrous and non-disastrous components:

$$
\begin{aligned}
\frac{K_{t+1}}{K_t} &= \exp(g^*_{t+1})(1-b_{t+1}) \qquad \text{with} \qquad g^*_{t+1} = \mu + x_t + \sigma\eta_{t+1}, \\
\frac{E_{t+1}}{E_t} &= \exp(g^*_{E,t+1})(1-b_{E,t+1}) \qquad \text{with} \qquad g^*_{E,t+1} = \mu + x_t + \sigma_E\eta_{E,t+1}, \\
\frac{C_{t+1}}{C_t} &= \exp(g^*_{C,t+1})(1-b_{C,t+1}) \qquad \text{with} \qquad g^*_{C,t+1} = \mu + x_t + \sigma_C\eta_{C,t+1}.
\end{aligned} \tag{C.6}
$$

Using the notation in Equation (C.6), the equality $K_{t+1} = E_{t+1} + C_{t+1}$ can be put as:

$$
(C_t + E_t)\exp(g^*_{t+1})(1-b_{t+1}) = C_t \exp(g^*_{C,t+1})(1-b_{C,t+1}) + E_t \exp(g^*_{E,t+1})(1-b_{E,t+1}), \tag{C.7}
$$

which links $b_{E,t+1}$ and $b_{C,t+1}$ to b_{t+1}. Let us now consider the specific values that the weighting factor ω_{t+1} can take.

Assuming that $\boldsymbol{\omega}_{t+1} = \mathbf{1}$:
In this scenario, the entire consumption contraction b_{t+1} arises solely from the durable good, meaning that $b_{E,t+1} > 0$ and $b_{C,t+1} = 0$. Using these restrictions in Equation (C.7) thus allows determining $b_{E,t+1}$:

$$
\begin{aligned}
(C_t + E_t)\exp(g^*_{t+1})(1-b_{t+1}) &= C_t \exp(g^*_{C,t+1}) + E_t \exp(g^*_{E,t+1})(1-b_{E,t+1}) \\
b_{E,t+1} &= 1 - \frac{C_t}{E_t}\frac{\exp(g^*_{t+1})(1-b_{t+1}) - \exp(g^*_{C,t+1})}{\exp(g^*_{E,t+1})} - \frac{\exp(g^*_{t+1})(1-b_{t+1})}{\exp(g^*_{E,t+1})}.
\end{aligned} \tag{C.8}
$$

Assuming that $\boldsymbol{\omega}_{t+1} = \mathbf{0}$:
With this ω_{t+1} specification, the entire consumption contraction comes from the nondurable good, meaning that $b_{E,t+1} = 0$ and $b_{C,t+1} > 0$. Applying this setting to Equation (C.7) results in:

$$\begin{aligned}(C_t + E_t)\exp(g^*_{t+1})(1 - b_{t+1}) &= C_t \exp(g^*_{C,t+1})(1 - b_{C,t+1}) + E_t \exp(g^*_{E,t+1}) \\ b_{C,t+1} &= 1 - \frac{E_t}{C_t}\frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{E,t+1})}{\exp(g^*_{C,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{C,t+1})}.\end{aligned} \tag{C.9}$$

Assuming that $\mathbf{0} < \boldsymbol{\omega}_{t+1} < \mathbf{1}$:
When $0 < \omega_{t+1} < 1$, $b_{C,t+1}$ and $b_{E,t+1}$ are obtained by weighting the expressions in Equations (C.8) and (C.9) appropriately:

$$\begin{aligned}b_{E,t+1} &= \omega_{t+1}\left(1 - \frac{C_t}{E_t}\frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{C,t+1})}{\exp(g^*_{E,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{E,t+1})}\right) \\ b_{C,t+1} &= (1 - \omega_{t+1})\left(1 - \frac{E_t}{C_t}\frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{E,t+1})}{\exp(g^*_{C,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{C,t+1})}\right).\end{aligned} \tag{C.10}$$

As mentioned in Section 4.6.3, the expressions in Equation (C.10) do not yet ensure that $0 \leq b_{E,t+1}, b_{C,t+1} \leq 1$, with $b_{C,t+1} \leq 1$ being the most critical condition. To obtain the restrictions in Equation (4.39), which must be imposed on b_{t+1} to ensure the plausibility of $b_{C,t+1}$ and $b_{E,t+1}$, I consider the corner solutions that $\omega_{t+1} = 0$ and $\omega_{t+1} = 1$, meaning that either $b_{C,t+1}$ or $b_{E,t+1}$ must account for the entire consumption contraction. If these cases would yield plausible contraction factors for the durable and the nondurable good, then so would any $\omega_{t+1} \in (0, 1)$.

First, I consider $\omega_t = 0$:

$$\begin{aligned}1 &\geq b_{C,t+1} \\ 1 &\geq 1 - \frac{E_t}{C_t}\frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{E,t+1})}{\exp(g^*_{C,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{C,t+1})} \\ 0 &\geq -\left(\frac{E_t}{C_t} + 1\right)\frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{C,t+1})} + \frac{E_t}{C_t}\frac{\exp(g^*_{E,t+1})}{\exp(g^*_{C,t+1})} \\ \left(\frac{E_t + C_t}{C_t}\right)(1 - b_{t+1}) &\geq \frac{E_t}{C_t}\frac{\exp(g^*_{E,t+1})}{\exp(g^*_{t+1})} \\ b_{t+1} &\leq 1 - \left(\frac{E_t}{E_t + C_t}\right)\frac{\exp(g^*_{E,t+1})}{\exp(g^*_{t+1})}.\end{aligned} \tag{C.11}$$

Second, I proceed analogously for $\omega_{t+1} = 1$:

$$
\begin{aligned}
1 &\geq b_{E,t+1} \\
1 &\geq 1 - \frac{C_t}{E_t} \frac{\exp(g^*_{t+1})(1 - b_{t+1}) - \exp(g^*_{C,t+1})}{\exp(g^*_{E,t+1})} - \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{E,t+1})} \\
0 &\geq - \left(\frac{C_t}{E_t} + 1 \right) \frac{\exp(g^*_{t+1})(1 - b_{t+1})}{\exp(g^*_{E,t+1})} + \frac{C_t}{E_t} \frac{\exp(g^*_{C,t+1})}{\exp(g^*_{E,t+1})} \\
\left(\frac{E_t + C_t}{E_t} \right) (1 - b_{t+1}) &\geq \frac{C_t}{E_t} \frac{\exp(g^*_{C,t+1})}{\exp(g^*_{t+1})} \\
b_{t+1} &\leq 1 - \left(\frac{C_t}{E_t + C_t} \right) \frac{\exp(g^*_{C,t+1})}{\exp(g^*_{t+1})}.
\end{aligned}
\tag{C.12}
$$

C.3 Transform annual into quarterly consumption contractions

The estimation of the ACH-GP model is performed on quarterly data, but the consumption dataset assembled by Barro and Ursúa (2008) is of an annual frequency. To obtain quarterly contractions from the annual data, I proceed as follows: For a year that is neither the first nor the last of a disaster, I draw from a standard uniform distribution to determine which fraction of the annual contraction is assigned to the first quarter. A second draw from a standard uniform distribution determines the fraction of the remaining contraction that is assigned to the second quarter; the same technique is applied for the third quarter and the last quarter takes what is left of the contraction. Applying this procedure implies that the contraction in the first (last) quarter will be the largest (smallest), on average. Thus, I re-shuffle the four quarterly contractions randomly to avoid such a seasonal pattern. When dealing with the first (last) year of a disaster, or if the disaster consists of only one annual contraction, I determine the quarter when the contraction begins (ends) by a draw from a discrete uniform distribution, such that each quarter has a 1/4 probability of becoming the quarter when the disaster begins (ends). The annual contraction is then distributed across the disaster quarters in a way analogous to the method used for a "within" disaster year.

C.4 Derive IES-identifying regression

The derivation of the IES-identifying regression starts from the basic asset pricing equation to which a law of total expectations was applied to get rid of the time conditioning:

$$\begin{aligned}\mathbb{E}[m_{t+1}R_{t+1}] &= 1 \\ \ln\left(\mathbb{E}[m_{t+1}R_{t+1}]\right) &= 0.\end{aligned} \tag{C.13}$$

In line with related literature (e.g., Yogo (2004)), I proceed by assuming $m_{t+1}R_{t+1}$ to be log-normally distributed. This assumption would certainly be discarded in the context of single period disasters, but when considering a multi-period disaster framework, the contractions in the respective disaster periods are far less extreme. Using the properties of the log-normal distribution, it is possible to rewrite Equation (C.13) as:

$$0 = \mathbb{E}\left[\theta\ln(\beta) - \frac{\theta}{\psi}\ln\left(\frac{C_{t+1}}{C_t}\right) + \left(\frac{\theta}{\rho} - \frac{\theta}{\psi}\right)\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right) + (\theta-1)r_{a,t+1} + r_{i,t+1}\right] + 0.5\sigma_t^2, \tag{C.14}$$

where σ_t^2 denotes the variance of $m_{t+1}R_{t+1}$, which I assume to be time-invariant. Rewriting Equation (C.14) yields:

$$\begin{aligned}0 = {} & \theta\ln(\beta) + 0.5\sigma^2 - \frac{\theta}{\psi}\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right)\right] + \left(\frac{\theta}{\rho} - \frac{\theta}{\psi}\right)\mathbb{E}\left[\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right] \\ & + (\theta-1)\mathbb{E}[r_{a,t+1}] + \mathbb{E}[r_{i,t+1}].\end{aligned} \tag{C.15}$$

Next, an expression for $\mathbb{E}[r_{a,t+1}]$ can be found by pricing the return of aggregate wealth:

$$\mathbb{E}[r_{a,t+1}] = \ln(\beta) + \frac{\sigma^2}{2\theta} - \frac{1}{\psi}\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right)\right] + \left(\frac{1}{\rho} - \frac{1}{\psi}\right)\mathbb{E}\left[\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right]. \tag{C.16}$$

Substituting the expression in Equation (C.16) into Equation (C.15) gives:

$$\begin{aligned}0 = {} & \theta\ln(\beta) + 0.5\sigma^2 - \frac{\theta}{\psi}\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right)\right] + \left(\frac{\theta}{\rho} - \frac{\theta}{\psi}\right)\mathbb{E}\left[\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right] \\ & + (1-\theta)\left(\ln(\beta) + \frac{\sigma^2}{2\theta} - \frac{1}{\psi}\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right)\right] + \left(\frac{1}{\rho} - \frac{1}{\psi}\right)\mathbb{E}\left[\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right]\right) + \mathbb{E}[r_{i,t+1}],\end{aligned} \tag{C.17}$$

and thus:

$$\begin{aligned}\mathbb{E}[r_{i,t+1}] = {} & \frac{1}{\psi}\left(\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right]\right) \\ & - \frac{1}{\rho}\mathbb{E}\left[\ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right] - \ln(\beta) - \frac{\sigma^2}{2\theta}.\end{aligned} \tag{C.18}$$

Finally, I assemble invariant components of Equation (C.18) in the asset specific constant w_i:

$$\mathbb{E}[r_{i,t+1}] = w_i + \frac{1}{\psi}\left(\mathbb{E}\left[\ln\left(\frac{C_{t+1}}{C_t}\right) - \ln\left(\frac{v(D_{t+1}/C_{t+1})}{v(D_t/C_t)}\right)\right]\right). \tag{C.19}$$

C.5 Bootstrap inference

The parametric bootstrap consists of several components, the first of which refers to the parametrization of the mixed beta distribution from which the ω_t weights are obtained. In this part of the bootstrap, I draw from the mixed beta distribution with the number of draws equaling the number of observations in the original sample. Next, the parameters of the distribution are re-estimated on the bootstrapped sample, which gives $\hat{\mathbb{P}}(\omega_t = 0)^{(k)}$, $\hat{\mathbb{P}}(\omega_t = 1)^{(k)}$, $\hat{e}^{(k)}$, and $\hat{f}^{(k)}$. Repeating this procedure K times thus yields $\{\hat{\mathbb{P}}(\omega_t = 0)^{(k)}, \hat{\mathbb{P}}(\omega_t = 1)^{(k)}, \hat{e}^{(k)}, \hat{f}^{(k)}\}_{k=1}^{K}$.

In the second part of the bootstrap, I use the disaster process parameters $\hat{\boldsymbol{\theta}}_{ACH}$, $\hat{\boldsymbol{\theta}}_{GP}$, and $\hat{\boldsymbol{\theta}}_{GP}^{D}$ to simulate a series of hazard rates, consumption contractions, disaster dummies, and dummies that determine whether the respective disaster is a destructive disaster. Again, the length of the simulated series is identical to the number of observations in the concatenated cross-country data. Then, the ACH-GP parameters are re-estimated using the simulated series. These steps are repeated K times and thus result in $\left\{\hat{\boldsymbol{\theta}}_{ACH}^{(k)}, \hat{\boldsymbol{\theta}}_{GP}^{(k)}, \hat{\boldsymbol{\theta}}_{GP}^{D,(k)}\right\}_{k=1}^{K}$.

In the third part of the bootstrap, I fit an AR(2) process to Lettau and Ludvigson's (2001) *cay* series and perform a residual bootstrap where the length of the bootstrapped series is identical to that of the original *cay* series.[78] Using the thus obtained bootstrapped *cay* series, I resort to the relationship between the *cay*-variable and the consumption-wealth ratio $\frac{K_t}{W_t} = \kappa \exp(cay_t)$, where κ=0.05. This procedure yields bootstrapped values of the λ and l parameters in Equation (4.12) and after K replications: $\{\hat{\lambda}^{(k)}, \hat{l}^{(k)}\}_{k=1}^{K}$.

The last component of the bootstrap uses the estimated fundamental macroeconomic and financial parameters presented in Table 27 to simulate non-disaster-including log consumption and dividend growth series, as well as the log price-dividend ratio and T-bill return. The number of simulated periods equals the length of the original series. The fundamental macroeconomic and financial parameters are then

[78] According to a Ljung-Box 1978 test, the null hypothesis of zero serial correlation could not be rejected at conventional significance levels when allowing for two lags in the specification of the autoregressive process.

re-estimated on the simulated series as outlined in Section 4.5.1. Under certain circumstances, the set of estimates $\hat{\boldsymbol{\zeta}}^{(k)}$ thus obtained is discarded and the simulation and re-estimation is performed anew. This is done, when the estimated parameters are evidently unsound, meaning that (a) the means of the processes have an implausible sign, or (b) some of the resulting estimates have a non-zero imaginary component (e.g., standard deviations computed from negative estimates of variances). Again, this procedure is repeated K times and yields $\left\{\hat{\boldsymbol{\zeta}}^{(k)}\right\}_{k=1}^{K}$, as well as the non-disaster-including series $\left\{g_{C,l}^{(k)}\right\}_{l=1}^{T}$, $\left\{g_{E,l}^{(k)}\right\}_{l=1}^{T}$, $\left\{g_{d,i,l}^{(k)}\right\}_{l=1}^{T}$, $\left\{z_{i,l}^{(k)}\right\}_{l=1}^{T}$, $\left\{\left(\frac{D_l}{C_l}\right)^{(k)}\right\}_{l=1}^{T}$, $\left\{R_{a,l}^{(k)}\right\}_{l=1}^{T}$, $\left\{R_{b,l}^{(k)}\right\}_{l=1}^{T}$, and $\left\{\mathbf{R}_l^{e,(k)}\right\}_{l=1}^{T}$.

Finally, for each of the K replications, I combine the respective bootstrapped parameters from these four components to repeat the simulation and estimation procedure outlined in Section 4.5.2, where the simulation of the disaster-including SDF and excess return series of length $\mathcal{T}$ is now based on the bootstrapped first-stage parameters and the non-disastrous $\mathbf{R}_t^e$ in Equation (4.30) is replaced by $\mathbf{R}_l^{e,(k)}$. Each of these K replications results in estimates of the IES and the RRA coefficient. The precision of the point estimates can then be measured by computing confidence intervals of γ and ψ using the percentile method and $\left\{\hat{\gamma}^{(k)}, \hat{\psi}^{(k)}\right\}_{k=1}^{K}$.

CHAPTER 5

Conclusion

Empirical tests of Hansen and Singleton's (1982) canonical C-CAPM have been notoriously disappointing. Yet the model approach cannot be easily discarded, because it represents a rational link between the real economy and financial markets, such that many attempts have been made to vindicate the C-CAPM paradigm. Within the canonical C-CAPM, scaled factors have been constructed to account for time-varying risk aversion (Lettau and Ludvigson (2001)) and alternative measures for the errors-in-variables-prone consumption data have been employed (e.g., Julliard and Parker (2005); Yogo (2006); Savov (2011)). The main theoretical extensions of the canonical C-CAPM focus on habit formation (Campbell and Cochrane (1999)), investor heterogeneity (Constantinides and Duffie (1996)), and long-run-risks (Bansal and Yaron (2004)). Although these efforts can claim some empirical success, the problem of implausible and imprecise preference parameter estimates and problematic asset pricing implications of the estimated model (e.g., too low model-implied equity premium, too high risk-free rate) has been mitigated at best.

Rietz (1988) has offered another explanation for the model's poor empirical performance: the rare disaster hypothesis, according to which the apparent failure of the C-CAPM is a consequence of the positive path that the U.S. economy took after World War II. However, this path may not be representative of the potentially disastrous future consumption that investors in the 1950s to 1980s had in mind. In the middle of the Cold War, the benign U.S. consumption path was just one among multiple more unfavorable histories.

A variety of calibration studies supports the RDH, in the sense that the high U.S. excess returns and plausible preference parameters can be reconciled in a C-CAPM that explicitly allows for disasters. However, there are also critics, such as Constantinides (2008) and Julliard and Ghosh (2012), who suspect the frequently used simplification to model disasters as single-period events to be the driving force behind the hypothesis's empirical success.

My dissertation contributes to previous literature by proposing SMM-type estimation strategies that facilitate econometrically assessing and testing the RDH. The studies are performed using single- (Chapter 2) and multi-period disasters (Chapters 3 and 4) and consider different SDF specifications, simulation approaches, and test assets. Further robustness checks are conducted regarding the specification of the disaster process, disaster threshold, the choice of test assets and moment conditions. The results are unambiguous: The preference parameter estimates of a disaster-including C-CAPM are plausible and precise for all model-specifications and the same holds true for the model-implied key financial indicators which are reported in Chapters 2 and 3. This thesis thus also contributes to the ongoing debate on accounting for the duration of disasters.

I argue that Barro's (2006) reasoning regarding the high correlation of consumption during disasters holds some value, however, I agree with Julliard and Ghosh (2012) and Constantinides (2008) that multi-period disasters are certainly more realistic. If one assumes that disasters appear as single-period events, they only exhibit one risk dimension, which is their size, and a power utility function suffices to deal with that. Accounting for multi-period disasters – contrarily to what Julliard and Ghosh's (2012) approach implies – does not simply decrease per-period contraction sizes, but rather adds a second risk dimension, which must be accounted for. This can be done by recursive preferences, such as the utility function by Epstein and Zin (1989), and Weil (1989), which allows disentangling the IES from the RRA coefficient and which is used in Chapters 3 and 4. Interestingly, the estimation results in both chapters indicate that investors are plausibly risk averse *and* willing to substitute consumption over time (with IES and RRA coefficient both > 1). Additionally, this relation of the RRA and IES estimates implies a preference for early resolution of uncertainty – a characteristic that cannot be accounted for by a power utility function.

The results reported in this thesis should encourage those who believe that rational investor behavior prevails in financial markets. What was suggested by calibration exercises is also supported by empirical evidence using econometric analysis: The canonical C-CAPM can explain the high market equity premium and the low risk-free rate with plausible risk and time preferences, once rare disaster risk is accounted for. The nexus between finance and the real economy postulated by the consumption-based asset pricing model is, after all, empirically not refuted.

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