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A bitter choice turned sweet: How acknowledging individuals' concern at having a low relative income serves to align utilitarianism and egalitarianism

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A bitter choice turned sweet: How acknowledging individuals' concern at having a low relative income serves to align utilitarianism and egalitarianism

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Abstract

When individuals' utility is a convex combination of their income and their concern at having a low relative income (the weights attached to income and to the concern at having a low relative income sum up to one), the maximization of aggregate utility yields an equal income distribution. This alignment of utilitarianism and egalitarianism is obtained for any number of individuals, and for general utility functions that are convex combinations of a power function of income and the concern at having a low relative income. The alignment can also hold when the weights sum up to a number different than one.

Keywords: Utilitarianism; Egalitarianism; Social welfare maximization; Low relative income

JEL classification: H0, I0, I30, I31

1. Introduction

This paper addresses the tension between utilitarianism, which conceptualizes social welfare as the sum of the individuals' utilities, and egalitarianism, which cherishes equality between individuals. We show that when individuals' utility is a convex combination of their income and their concern at having a low relative income, the maximization of aggregate utility yields an equal income distribution. This alignment of utilitarianism and egalitarianism is obtained for any number of individuals, and for general utility functions that are convex combinations of a power function of the individuals' income and of a measure of their concern at having a low relative income. Moreover, with respect to linear utility functions this alignment is not restricted to convex combinations of income and concern at having a low relative income.

Utilitarianism postulates that the collective choice should maximize the sum of the individuals' utilities. According to Bentham (1823), utility is equivalent to pleasure, therefore it is rational for a social planner to maximize the sum of utilities just as it is rational for an individual to maximize utility. It follows that the "good" from which individuals derive utility should be distributed in such a way that more of the good is given to the individuals who stand to benefit more from having the good. Mill (1863) refers to utilitarianism as the "Greatest Happiness Principle."

Egalitarianism requires a collective decision to distribute the available "good" (say, income) in such a way that all the individuals end up enjoying equal benefits. This requirement is one of the most important concepts of justice in social and political thought (Hare, 1981; Scanlon, 1998; Dworkin, 2000). Theories of egalitarianism differ from one another depending on what it is that is to be equalized: utility, resources, income, rights, capabilities, opportunities, or access.

Equalizing utilities is tantamount to making individuals equal in terms of achieved welfare. Rawlsian equality (Rawls, 1999) is the equality of "primary goods," namely "things that every rational man is presumed to want," namely "rights, liberties and opportunities, income and wealth, and the social bases of self-respect" (Rawls, 1999, pp. 60-65). Rawls condemns inequalities by invoking the "Difference Principle" according to which priority is given to the worst-off individuals. There can be no tradeoffs between basic liberties and gains in terms of utility. Sen (1980) notes that Rawlsian equality is incomplete in the sense that it is too concerned with primary goods instead of with what these goods bring to people, and intimates the need for utility to take into account "pleasures," regardless of their source. Thus, Sen defines "basic capability equality," which "can be seen as a natural extension of Rawls's concern with primary goods, shifting attention from goods to what goods do to human beings" (Sen, 1980, pp. 218-219). Sen simply acknowledges the fact that human beings differ, and "that the conversion of goods to capabilities varies from person to person substantially" (Sen, 1980, p. 219). Opportunities are the object of equalization in the theory of equality of opportunity (Roemer, 1998). According to this theory, individuals differ with respect to characteristics beyond their control (circumstances), and with respect to characteristics within their control (responsibility parameters). Circumstances are the illegitimate source of inequalities and, thus, the goal of a society is to eliminate such inequalities without distorting inequalities that arise due to responsibility parameters.

In social and political thought, egalitarianism is a meta-theory with the specifics depending on the equalizandum in question. In this regard, even utilitarianism can be interpreted as an egalitarian ethic for which treating individuals as equals entails equalization of marginal utilities (Harsanyi, 1977). Nevertheless, in public discourse in modern democratic societies, "an egalitarian" is typically a person who supports greater income and wealth equality. It is this type of egalitarianism that Sen (1973, p. 18) appears to have in mind when he states that "[i]t seems fairly clear that fundamentally utilitarianism is very far from an egalitarian approach."

Taking this statement as our starting point, in this paper we attend to "income egalitarianism," although our findings can apply to the more general context of "resource egalitarianism." The "resources" can be external material goods (such as land), or Rawlsian primary goods. Resource egalitarianism is essentially a non-welfarist ethic, that is, it is not concerned with the utility / wellbeing that individuals derive from resources, whereas utilitarianism is the best-known welfaristic criterion. Our contribution is to show that under different sets of conditions, when inter-personal comparisons of the type modelled by us are taken into account, utilitarianism and egalitarianism are equivalent.

Indeed, the utilitarian and egalitarian stands typically entail different distributions of a given aggregate income. To a utilitarian social planner, issues such as equality of incomes or of utilities are immaterial. A utilitarian social planner directs the available income to those who have "superior efficiency in producing utility" (Sen, 1980, p. 203). To illustrate this principle, consider the utility functions $u_1(x_1) = x_1$ and $u_2(x_2) = 100x_2$, where x_i is the income of individual i, $i = 1,2$. To maximize social welfare, a utilitarian social planner will allocate the entire available income to individual 2. As put vividly by Sen (1980), the (say) cripple (individual 1) is doubly worse off compared to the pleasure-wizard (individual 2): first, because he receives less income and second, because he derives lower utility from a given level of income. When individuals' utilities depend on their income, then equalizing marginal utilities will usually mandate unequal division of a given total income. In short, in most cases, an optimal income distribution under a utilitarian rule will diverge from the income distribution under an egalitarian rule.

Interestingly, and along with many others, in a spirited debate Tullock (1975) and Sen (1982) have already grappled with the assumptions or conditions necessary to render equal division the optimal distributional rule for a given total income, and were pondering whether the utilitarian approach to the maximization of social welfare can be made compatible with egalitarian principles. However, neither of them used individuals' concern at having a low relative income as a conciliator. Modern day evidence from econometric studies, experimental economics, social psychology, and neuroscience indicates that people routinely compare themselves with others who constitute their "comparison" or "reference" group, and that the negative outcome of upward comparisons (cf., for example, Andolfatto, 2002; Frey and Stutzer, 2002), impinges on their sense of wellbeing (Fliessbach et al., 2007; Takahashi et al., 2009; Clark and Senik, 2010). People are unhappy when their consumption, income, or social standing fall below those of others with whom they naturally compare themselves (those who constitute their "reference group"). Consequently, economic processes are impacted, and economic realizations differ from what they would have been if comparisons with others did not matter (see, for example, Stark and Taylor, 1991; Zizzo and Oswald, 2001; Luttmer, 2005; Blanchflower and Oswald, 2008; Stark and Hyll, 2011).

Taking total income as given, the example presented above shows starkly that when individuals care only about their absolute income, the maximization of a social welfare function that sums up the individuals' utilities mandates allocating the available income such that the individual who values income most highly ends up receiving the entire income. This result is obvious and, of course, is well-known. However, when individuals care also about trailing behind others in the income hierarchy (exhibit a concern at relative deprivation), we show that maximization of the social welfare function mandates income equalization. This result is somewhat surprising. Had individuals' utilities been strictly positive under income equality and equal to zero otherwise, then it would not have been surprising for utilitarianism to mandate income equalization. However, this paper provides conditions under which *any* concern at one's income falling behind the incomes of others suffices to nudge the utilitarian social planner to distribute incomes equally.

In Section 2 we show that when individuals feel concern at having a low relative income, a utilitarian social planner will divide the available income equally. In Section 3 we show that the alignment of the optimal utilitarian income distribution with the optimal egalitarian income distribution is not confined to utility functions in which the preferences concerning absolute income are linear. In Section 4 we offer our conclusion.

2. The tension between utilitarianism and income equality forgone: linear utility functions

We study the maximization of a utilitarian social welfare function (the sum of the individuals' utility functions). Here, the social planner is impartial in the sense that in the construction of the social welfare function, the utility function of each individual is accorded the same weight. Individual utility functions depend on both absolute income and "relative deprivation," our chosen measure of the concern at having a low relative income. When the weights attached to absolute income and to relative deprivation sum up to one, we show that the maximization of a utilitarian objective entails income equalization.[1]

To begin with, consider two individuals, $i = 1,2$, who are not concerned about relative deprivation, who attach a weight $\alpha_i \in (0,1)$ to absolute income x_i, and whose utility function is $u_i(x_i) = \alpha_i x_i$. It is clear that the utilitarian social planner will give all the available income to the individual whose α_i is higher. Only in the special case of $\alpha_1 = \alpha_2$ the equal division of incomes is optimal for the utilitarian but then, any division is also optimal.

Acknowledging the concern at relative deprivation, let the utility functions take the form

$$u_i(x_1, x_2) = \alpha_i x_i - (1-\alpha_i) RD_i(x_1, x_2),$$

such that $\alpha_i \in (0,1)$, $RD_i(x_1, x_2) = \frac{1}{2}\max\{x_j - x_i, 0\}$ is the index of relative deprivation of individual *i*, where $j \neq i$. Let the available income be $x_1 + x_2 = A$. Then, we have that $RD_i(x_1, x_2) = \frac{1}{2}\max\{A - 2x_i, 0\}$. For $x_i < x_j$, we get that $RD_i(x_1, x_2) = \frac{1}{2}(A - 2x_i) > 0$, and that $RD_j(x_1, x_2) = 0$. When income is transferred from individual j to individual i, without changing the hierarchy of the two income earners, the marginal increase in i's utility is $\alpha_i - \frac{1-\alpha_i}{2}(-2) = 1$, whereas the marginal decrease in j's utility is α_j. Because $\alpha_j < 1$, transferring income from j to i increases the sum of the individuals' utilities, and is in accord with the goal of the utilitarian social planner. Consequently, $x_1^* = x_2^* = \frac{A}{2}$ (where a star indicates optimal value) is the unique optimum of the utilitarian social

[1] The inference that utilitarianism aligns with egalitarianism draws upon, but is not contingent on, the weights in the individuals' utility functions necessarily summing up to 1. Under such an assumption, the utility functions have the characteristic that an individual's weak taste for absolute income correlates with a strong distaste for low relative income (and vice versa). However, incorporation of the individuals' distaste for low relative income can sustain the alignment of utilitarianism with egalitarianism even when the weights in the individuals' utility functions do not sum up to 1, though additional conditions then need to be imposed. In Appendix B we provide the analysis of such case of preferences' specification.

welfare function. When a concern at having a low relative income is acknowledged by the utilitarian social planner, moving to an equal division causes less pain to the individual from whom income is taken away than it brings joy to the individual whose income is increased. As we show next, this intuition carries through in the case of any number of individuals.

In a population consisting of $n \geq 2$ individuals, let the income of individual *i* be x_i. The income distribution of the population is represented by a vector $x \in \mathbf{R}^n_{\geq 0}$. We define the relative deprivation of individual *i*, $RD_i(x)$, as[2]

$$RD_i(x) \equiv \frac{1}{n}\sum_{j=1}^{n} \max\left\{x_j - x_i, 0\right\}. \tag{1}$$

Let the utility function of individual *i* be

$$u_i(x) = \alpha_i x_i - (1-\alpha_i) RD_i(x), \tag{2}$$

where $\alpha_i \in (0,1)$. Let the social welfare be measured by the utilitarian function

$$SWF(x) = \sum_{i=1}^{n} u_i(x).$$

To find the optimal income distribution under a budget constraint $A > 0$, we search for the solution to the maximization problem

$$\begin{aligned} &\max SWF(x) \\ &\text{for } x \in \mathbf{R}^n_{\geq 0} \text{ s.t. } \sum_{i=1}^{n} x_i = A \,. \end{aligned} \tag{3}$$

The form of the individuals' utility function (2), in which the coefficients sum up to one, is equivalent to the social planner "giving" to an individual 100 percent of weight that he can assign to income and relative deprivation in any way that he wants. Then, we can ascertain that each individual's preferences enter the maximization problem with equal "importance:" the sum of the coefficients is constant for all individuals, and in the

[2] In an Appendix, Stark (2013) attends to the origins of the concept of relative deprivation, elaborates on the measure, and provides more evidence that upward interpersonal comparisons affect significantly subjective wellbeing.

process of maximizing aggregated utility the social planner is impartial.

In the following proposition we state and prove the main result of this paper: maximization of (3) yields as a solution equal distribution of incomes. The policy implication of this result is quite stark: the utilitarian social planner does not need to collect information on the magnitudes of the individuals' α_i's, nor can any individual affect the allocation of the utilitarian social planner by voluntarily reporting a particular high weight accorded, say, to his concern at having a low relative income. As long as $\alpha_i \in (0,1)$ for all $i \in \{1,\ldots,n\}$, such reporting is inconsequential for the utilitarian and the egalitarian social planners reaching unanimity.

Proposition 1: The unique solution to (3) is

$$x^* = \left(\frac{A}{n}, \frac{A}{n}, \ldots, \frac{A}{n}\right).$$

Proof: The proof is in Appendix A.

Proposition 1 states that a consensus between the utilitarian and the egalitarian social planners does not hinge on the size of the population concerned being limited to just two individuals. When the coefficients in the utility function of the individuals that are assigned to absolute income, α_i, and to relative deprivation, $1-\alpha_i$, sum up to 1, the unique solution to (3) is the *egalitarian* income distribution.

It is of interest to note that the inequality (A1) obtained in the course of proving Proposition 1 has an appealing interpretation: it says that, starting from an egalitarian distribution x^*, increasing the income of any k individuals at the expense of the remaining $n-k$ individuals would yield a bigger loss of utility for those from whom income is taken away than the gain of utility for those whose income is raised, just as in the case of the two-person example.

We next show that our result does not hinge on a linear specification of preferences for absolute income.

3. The tension between utilitarianism and income equality forgone: nonlinear utility functions

Hitherto we have assumed that the preferences for absolute income are linear. We now show that our result that concern at having a low relative income leads a utilitarian social planner to allocate incomes equally does not hinge on such a class of functions.

Let the budget constraint be $A \geq n/e$. This condition is tantamount to a requirement that the income to be given to each individual under equal income distribution, A/n, cannot be "too low." (To put it more bluntly, for our result to hold it suffices that the egalitarian level of the income of an individual is not less than 1/2.) The rationale behind this condition is related to the decreasing marginal utility from absolute income in the case of the particular specification of preferences discussed here (cf. (4) below). A high enough budget constraint ensures that under a perfectly equal income distribution, the marginal utility from "shifting" income towards individuals with high α's will not override the marginal disutility from low relative income experienced by individuals with low α's.

Let the utility function of individual *i*, $i = 1,...,n$, be

$$u_i(x) = \alpha_i x_i^{\gamma} - (1-\alpha_i) RD_i(x), \tag{4}$$

where $\alpha_i, \gamma \in (0,1)$ and where $RD_i(x)$ is defined as in (1). We obtain the following result.

Proposition 2: Let the individual's utility function be of the form given by (4). Then, if $A \geq n/e$, maximization of utilitarian social welfare results in a perfectly equal income distribution x^*.

Proof: The proof is in Appendix C.

Proposition 2 accords robustness to the result that when the utilitarian social planner acknowledges the individuals' concern at having a low relative income, he will choose an egalitarian income distribution. We reiterate that as long as $\alpha_i \in (0,1)\ \forall i \in \{1,...,n\}$, this result does not hinge on the particular magnitudes of the individuals' α_i's, nor even on knowing them.

4. Conclusion

It is fairly widely recognized in economics that the utility of individuals is shaped via a process of relatedness and comparison with others. We have followed this perspective in research stretching over the past two decades, incorporating perceptions as diverse as altruism and social stigma (for example, Stark, 1993, and Fan and Stark, 2011, respectively). Here, we acknowledge that an individual cares about his income trailing behind the incomes of others in his "comparison group." In such a setting, when assigned the task of dividing the available income to maximize social welfare, a utilitarian social planner who, without such an acknowledgement will choose a "corner solution," will act exactly the same as an egalitarian social planner; a long-lived tension in social choice and welfare economics is resolved. This conclusion is robust to alternative characterizations (linear and non-linear) of the preference for absolute income.

That individuals' preferences incorporate a relative deprivation term changes substantially a good many implications of the standard economic models. For example, in structuring optimal taxation, a desire for status calls for higher marginal taxes (Boskin and Sheshinski, 1978; Oswald, 1983; Blomquist, 1993; Aronsson and Johansson-Stenman, 2008; Wendner and Goulder, 2008). Concerns about status and relative deprivation bear on labor supply (Neumark and Postlewaite, 1998), economic growth (Easterlin, 1974; Clark et al., 2008), poverty measurement (Sen, 1973), migration decisions (Stark and Taylor, 1991), and risk-taking behavior (Callan et al., 2008; Haisley et al., 2008). In a similar vein we show in this paper that such concerns are also important for designing welfare maximizing income policies.

When considering the welfare effects of alternative policies, egalitarian distribution should be studied in and by itself. This is particularly true when individual utility functions depend only on absolute and relative income (or consumption). It is not all that clear, however, how relative deprivation "fares" against other concerns such as labor supply / leisure, which are also important for the behavior of economic aggregates. One recent attempt to merge decisions how much effort to exert, relative deprivation, and inequality outcomes as measured by the Gini index is Sorger and Stark (2013). There is little doubt in our mind that the roles of concern at relative deprivation and low status in the design of welfare maximizing income policies, and the characteristic social welfare functions are subjects ripe for additional inquiry.

Appendix A: Proof of Proposition 1

With n individuals, there are $n!$ possible orderings of incomes (permutations of the set $\{1,2,...,n\}$). Assuming a specific ordering $x_1 \le x_2 \le \ldots \le x_n$, we proceed to show that a corresponding result holds independently of the values of $\alpha_1,...,\alpha_n$. By symmetry of the maximized function with respect to incomes and income weights, the result obtained below holds for other orderings of incomes.

Without loss of generality, and normalizing the sum of incomes if necessary, we assume that this sum is $\sum_{i=1}^{n} x_i = 1$. The set of admissible distributions $D \equiv \left\{ x \in \mathbf{R}^n_{\ge 0} \colon x_1 \le x_2 \le \ldots \le x_n, \sum_{i=1}^{n} x_i = 1 \right\}$ is convex (in fact, it is a convex polytope) and has a finite number of extremal points of the form

$$x^k = (\underbrace{0,0,...,0}_{n-k},\underbrace{1/k,1/k,...,1/k}_{k}),\; k \in \{1,2,...,n\},$$

where we have that $x^n = (1/n,1/n,...,1/n) = x^*$.

Over the closed set D, the function SWF is linear and therefore its maximum on D is attained in one of the extremal points x^k. We have that

$$SWF(x^k) = \underbrace{\frac{1}{k}\sum_{i=n-k+1}^{n} \alpha_i}_{income} - \underbrace{\frac{1}{n}\sum_{i=1}^{n-k}(1-\alpha_i)}_{relative\ \ deprivation},$$

namely, compared to x^n, x^k for $k \ne n$ represents taking away income from k individuals and distributing that income equally between the remainder $n-k$ individuals.

A distribution x^n constitutes a unique maximum if and only if $SWF(x^n) > SWF(x^k)$ for any $k \ne n$. To see this, note that

$$\frac{n-k}{n} = \left(\frac{1}{k}-\frac{1}{n}\right)k > \left(\frac{1}{k}-\frac{1}{n}\right)\sum_{i=n-k+1}^{n} \alpha_i\,,$$

$$\frac{1}{n}\sum_{i=n-k+1}^{n}\alpha_i > \frac{1}{k}\sum_{i=n-k+1}^{n}\alpha_i - \frac{n-k}{n},$$

$$\frac{1}{n}\sum_{i=n-k+1}^{n}\alpha_i + \frac{1}{n}\sum_{i=1}^{n-k}\alpha_i > \frac{1}{k}\sum_{i=n-k+1}^{n}\alpha_i - \frac{n-k}{n} + \frac{1}{n}\sum_{i=1}^{n-k}\alpha_i ,$$

$$\frac{1}{n}\sum_{i=1}^{n}\alpha_i > \frac{1}{k}\sum_{i=n-k+1}^{n}\alpha_i + \frac{1}{n}\left[\sum_{i=1}^{n-k}\alpha_i - (n-k)\right].$$

We have

$$\sum_{i=1}^{n-k}\alpha_i - (n-k) = \sum_{i=1}^{n-k}(\alpha_i - 1) = -\sum_{i=1}^{n-k}(1-\alpha_i) .$$

Consequently, we get

$$SWF(x^n) = \frac{1}{n}\sum_{i=1}^{n}\alpha_i > \frac{1}{k}\sum_{i=n-k+1}^{n}\alpha_i - \frac{1}{n}\sum_{i=1}^{n-k}(1-\alpha_i) = SWF(x^k) \qquad \text{(A1)}$$

for any $k \neq n$, which completes the proof that $x^n = x^*$ is the maximum. □

Appendix B: A loosened assumption on the weights in the individuals' preferences

In this Appendix we attend to a model of preferences in which there are two (positive) parameters in the individual's utility function rather than one: $\gamma_i > 0$ weight to be placed on individual's i own income, and $\beta_i > 0$ weight to be placed on low relative income, that is, the individual's utility function is

$$u_i(x) = \gamma_i x_i - \beta_i RD_i(x) . \qquad \text{(B1)}$$

When this extra degree of freedom is allowed, we obtain the result that, as we increase the β weights, loosely speaking, the optimal utilitarian solution moves toward egalitarianism or, more correctly, social welfare decreases as we move away from egalitarianism.

Taking first as an example two individuals, and using the same notation as in the beginning of Section 2, for the case of $x_i < x_j$, when income is transferred from individual j to individual i, without changing the hierarchy of the two income earners, the marginal increase in i's utility is $\gamma_i + \beta_i$, whereas the marginal decrease in j's utility is γ_j. Unlike in the case of two parameters α_1, α_2 in which the marginal gain of individual *i*, which is equal to 1, is greater than the marginal loss of individual *j*, which is equal to $\alpha_j \in (0,1)$, here the marginal gain of *i* will be greater than the marginal loss of *j* if $\gamma_i + \beta_i > \gamma_j$, which translates into the condition that the marginal disutility of low relative income of the poorer individual, β_i, has to be larger than the difference between the two individuals' marginal utilities of own income, $\gamma_j - \gamma_i$. From analyzing in a similar manner the case of $x_i > x_j$, we get an analogous condition on the marginal increase in social welfare, namely $\beta_j > \gamma_i - \gamma_j$.

Summing up: when the weights in the utility function of an individual do not sum up to 1, then for equality of incomes to be the optimal utilitarian solution, that is, for $x_1^* = x_2^* = \frac{A}{2}$ to obtain, we need to have that $\beta_1 > \gamma_2 - \gamma_1$ and that $\beta_2 > \gamma_1 - \gamma_2$.

This extension to a setting of the weights not summing up to one can be carried further and be applied to the case of more than two individuals. Let the utility of the *i*-th, $i \in \{1, ..., n\}$, individual in a population with income vector $x = (x_1, ..., x_n)$ be described by (B1). In the following proposition we state and prove a condition on the weights γ_i, β_i in the population which, if satisfied, yields that the egalitarian income distribution maximizes utilitarian social welfare.

Proposition B1: If

$$\gamma_j + \beta_j > \gamma_l + \frac{n-2}{n}\beta_l \tag{B2}$$

for all $j \neq l$, then the egalitarian income distribution maximizes social welfare.

Proof: The problem of the utilitarian social planner under a budget constraint $A > 0$ is

$$\max SWF(x)$$
$$\text{for } x \in \mathbf{R}^n_{\geq 0} \text{ s.t. } \sum_{i=1}^{n} x_i = A, \tag{B3}$$

where $SWF(x) = \sum_{i=1}^{n} u_i(x)$. With n individuals, there are $n!$ possible orderings of incomes (permutations of the set $\{1, 2, ..., n\}$). In what follows, we divide the maximization problem (B3) into $n!$ sub-problems corresponding each to a given ordering of incomes, and we show that in each sub-problem the egalitarian income distribution $x^* = (A/n, A/n, \dots, A/n)$ is optimal.

Specifically, we divide the set $X = \left\{ x \in \mathbf{R}^n_{\geq 0} : \sum_{i=1}^{n} x_i = A \right\}$ into sets X_P, $X = \bigcup_P X_P$, where the summation is over all possible orderings of incomes. For $x \in X_P$, the income of individual $P(i)$ is the *i*-th lowest in the population, namely $x_{P(1)} \leq x_{P(2)} \leq ... \leq x_{P(n)}$; and $X_P = \{x \in X : x_{P(k)} < x_{P(l)} \Rightarrow k < l\}$.[3] Obviously, $x^* \in X_P$ for every P. For the ordering P, the maximization problem is

$$\max_{x \in X_P} SWF(x). \tag{B4}$$

For $x \in X_P$ the social welfare function can be rewritten as

$$SWF(x) = \sum_{i=1}^{n} a_{P(i)} x_{P(i)},$$

where

$$a_{P(i)} = \gamma_{P(i)} + \frac{(n-i)\beta_{P(i)}}{n} - \frac{1}{n} \sum_{j=1}^{i-1} \beta_{P(j)}. \tag{B5}$$

Consider the income distribution $x \in X_P$ which is not egalitarian, namely $x \neq x^*$,

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[3] To illustrate our notation, consider $n = 3$ and $x = (x_1, x_2, x_3) = (2,1,1)$. For such an x, we have that $x_2 = x_3 < x_1$. Then, x belongs to X_Q, where Q is a permutation of the set $\{1,2,3\}$ such that $Q(1) = 2$, $Q(2) = 3$, and $Q(3) = 1$; and it also belongs to X_R, where R is a permutation of the set $\{1,2,3\}$ such that $R(1) = 3$, $R(2) = 2$, and $R(3) = 1$.

and a set of individuals whose income is the lowest in the population under distribution x. Obviously, individual $P(1)$ belongs to this set. We introduce $i_x = \max\{i : x_{P(1)} = x_{P(i)}\}$. The individuals $P(1), \ldots, P(i_x)$ have the same income, the lowest in the population. Given that $x \neq x^*$, we have that $P(i_x) < n$, and that $x_{P(i_x)} < x_{P(i_x+1)}$. We construct an income distribution $x' \in X_P$ such that $x' \neq x$ in the following way: for $i \leq i_x + 1$, $x'_{P(i)} = \frac{1}{i_x+1}\sum_{j=1}^{i_x+1} x_{P(j)} = \frac{i_x x_{P(i_x)} + x_{P(i_x+1)}}{i_x+1}$, and for $i > i_x + 1$, $x'_{P(i)} = x_{P(i)}$. We thus obtain that

$$\begin{aligned} &SWF(x') - SWF(x) \\ &= \sum_{j=1}^{i_x} a_{P(j)} \left(\frac{i_x x_{P(i_x)} + x_{P(i_x+1)}}{i_x+1} - x_{P(i_x)} \right) + a_{P(i_x+1)} \left(\frac{i_x x_{P(i_x)} + x_{P(i_x+1)}}{i_x+1} - x_{P(i_x+1)} \right) \\ &= \frac{x_{P(i_x+1)} - x_{P(i_x)}}{i_x+1} \left(\sum_{j=1}^{i_x} a_{P(j)} - i_x a_{P(i_x+1)} \right). \end{aligned} \tag{B6}$$

Next, from (B2) applied to $j \in \{1,...,i\}$ and to $l = i+1$, we obtain by summation that

$$\sum_{j=1}^{i} (\gamma_{P(j)} + \beta_{P(j)}) > i\gamma_{P(i+1)} + \frac{i(n-2)\beta_{P(i+1)}}{n},$$

which implies

$$\sum_{j=1}^{i} (\gamma_{P(j)} + \beta_{P(j)}) > i\gamma_{P(i+1)} + \frac{i(n-i-1)\beta_{P(i+1)}}{n}. \tag{B7}$$

Recalling (B5), because

$$\sum_{j=1}^{i} a_{P(j)} = \sum_{j=1}^{i} \gamma_{P(j)} + \frac{n-i}{n} \sum_{j=1}^{i} \beta_{P(j)},$$

and because

$$i a_{P(i+1)} = i\gamma_{P(i+1)} + \frac{i(n-i-1)\beta_{P(i+1)}}{n} - \frac{i}{n} \sum_{j=1}^{i} \beta_{P(j)},$$

we get from (B7) that

$$\sum_{j=1}^{i} a_{P(j)} > i a_{P(i+1)} \qquad \text{(B8)}$$

for all $i \in \{1,...,n-1\}$.

Thus, by joining (B6) and (B8), we have that $SWF(x') - SWF(x) > 0$. Consequently, distribution x such that $x \neq x^*$ cannot be a solution to (B4). Because the set X_P is compact, $SWF(x)$ attains its maximum over this set. Therefore, distribution x^* is the solution to (B4) for any ordering P, and x^* is the unique maximum of the social welfare function over the set X. □

Appendix C: Proof of Proposition 2

Akin to the proof of Proposition 1, and again without loss of generality, we assume a specific ordering of the incomes, $x_1 \le x_2 \le \ldots \le x_n$, and show that the result holds independently of the values of $\alpha_1,...,\alpha_n$. The symmetry of the maximized function with respect to incomes and income weights implies that the result obtained below holds for other orderings of incomes. For the assumed ordering, the maximization problem of the utilitarian social planner is

$$\max_{x\in\Omega}\left\{\sum_{i=1}^{n}\left[\alpha_i x_i^{\gamma} - \frac{1-\alpha_i}{n}\sum_{j=i+1}^{n}\left(x_j - x_i\right)\right]\right\}$$
$$\text{s.t. } \Omega=\left\{\sum_{i=1}^{n} x_i = A;\ \ x_1 \ge 0;\ \ x_i \ge x_{i-1} \text{ for } i = 2,...,n\right\}.$$

We denote $F(x) = -\sum_{i=1}^{n}\left[\alpha_i x_i^{\gamma} - \frac{1-\alpha_i}{n}\sum_{j=i+1}^{n}\left(x_j - x_i\right)\right]$, $h(x) = \sum_{i=1}^{n} x_i - A$, and $g_i(x) = x_{i-1} - x_i$ for all $i \in \{2,...,n\}$, $g_1(x) = -x_1$. Transforming the maximization problem to a minimization problem,

$$\min F(x)$$
$$\text{s.t. } h(x) = 0,\ g_i(x) \le 0 \text{ for } i \in \{1,...,n\},$$

we obtain the Lagrangian

$$L = F(x) + \sum_{i=1}^{n} \mu_i g_i(x) + \lambda h(x)$$
$$= -\sum_{i=1}^{n}\left[\alpha_i x_i^{\gamma} - \frac{1-\alpha_i}{n}\sum_{j=i+1}^{n}\left(x_j - x_i\right)\right] + \sum_{i=1}^{n}\mu_i g_i(x) + \lambda h(x).$$

We have that

$$\frac{\partial L}{\partial x_1} = -\alpha_1 \gamma x_1^{\gamma-1} - (n-1)\frac{1-\alpha_1}{n} - \mu_1 + \mu_2 + \lambda,$$

that

$$\frac{\partial L}{\partial x_n} = -\alpha_n \gamma x_n^{\gamma-1} + \sum_{j=1}^{n-1}\frac{1-\alpha_j}{n} - \mu_n + \lambda,$$

and that

$$\frac{\partial L}{\partial x_l} = -\alpha_l \gamma x_l^{\gamma-1} + \sum_{j=1}^{l-1}\frac{1-\alpha_j}{n} - (n-l)\frac{1-\alpha_l}{n} - \mu_l + \mu_{l+1} + \lambda$$

for $l \in \{2,...,n-1\}$.

Because the maximized function F is convex,[4] and the feasible set D is described by affine constraints, the following set of the Karush-Kuhn-Tucker conditions is both sufficient and necessary for x to be a unique minimum:

$$\frac{\partial L}{\partial x_l}(x) = 0,\ l \in \{1,...,n\}\ ,$$
$$\mu_l g_l(x) = 0,\ \mu_l \geq 0,\ g_l(x) \leq 0,\ l \in \{1,...,n\}\ ,$$
$$h(x) = 0\ .$$

We now show that point $x^* = (A/n, A/n, ..., A/n)$ satisfies the preceding conditions and that, therefore, it constitutes the global minimum of this problem.

[4] Note that given $\gamma \in (0,1)$, we have that $\frac{\partial^2 F}{\partial x_l^2} = -\alpha_l \gamma(\gamma-1)x_l^{\gamma-2} > 0$ for $l \in \{1,...,n\}$ and that $\frac{\partial^2 F}{\partial x_l \partial x_k} = 0$ for $k \neq l$, which implies that the Hessian matrix of F is positive definite.

We note that because $x_1^* = A/n > 0$, we set $\mu_1 = 0$, and hence the considered problem reduces to a set of linear equations in $\mu_2,...,\mu_n,\lambda$. We denote

$$C_l = \begin{cases} -\alpha_1 \gamma A^{\gamma-1} n^{1-\gamma} - (n-1)\dfrac{1-\alpha_1}{n} & l = 1, \\ -\alpha_n \gamma A^{\gamma-1} n^{1-\gamma} + \displaystyle\sum_{j=1}^{n-1} \dfrac{1-\alpha_j}{n} & l = n, \\ -\alpha_l \gamma A^{\gamma-1} n^{1-\gamma} + \displaystyle\sum_{j=1}^{l-1} \dfrac{1-\alpha_j}{n} - (n-l)\dfrac{1-\alpha_l}{n} & \text{otherwise.} \end{cases}$$

Thus, we obtain

$$\begin{cases} C_1 = -\mu_2 - \lambda & l = 1, \\ C_n = \mu_n - \lambda & l = n, \\ C_l = \mu_l - \mu_{l+1} - \lambda & \text{otherwise,} \end{cases}$$

which can be written as

$$C = \mathrm{M} v,$$

where $C = (C_l)_{l=1}^n$, $v = (\lambda, \mu_2, ..., \mu_n)$, and

$$\mathrm{M} = \begin{bmatrix} -1 & -1 & 0 & ... & ... & ... & 0 \\ -1 & 1 & -1 & 0 & ... & ... & 0 \\ -1 & 0 & 1 & -1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ -1 & 0 & ... & ... & 0 & 1 & -1 \\ -1 & 0 & ... & ... & ... & 0 & 1 \end{bmatrix}.$$

It can easily be verified that

$$
\mathrm{M}^{-1} = \begin{bmatrix} -\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & \dots & -\frac{1}{n} & -\frac{1}{n} \\ -\frac{n-1}{n} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} & \frac{1}{n} \\ -\frac{n-2}{n} & -\frac{n-2}{n} & \frac{2}{n} & \frac{2}{n} & \dots & \frac{2}{n} & \frac{2}{n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ -\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & -\frac{1}{n} & \dots & -\frac{1}{n} & \frac{n-1}{n} \end{bmatrix}.
$$

Then,

$$
\lambda = -\frac{1}{n}\sum_{l=1}^{n} C_l ,
$$

and for $i \in \{1, 2, ..., n-1\}$

$$
\mu_{i+1} = -\frac{n-i}{n}\sum_{l=1}^{i} C_l + \frac{i}{n}\sum_{l=i+1}^{n} C_l \equiv f_{i+1}(\alpha_1, ..., \alpha_n).
$$

Obviously the functions f_{i+1} are linear and, moreover, for $i \in \{1, 2, ..., n-1\}$, we have that $f_{i+1}(1, ..., 1) = 0$. We calculate for $j = l$

$$
\frac{\partial C_l}{\partial \alpha_l} = -\gamma A^{\gamma-1} n^{1-\gamma} + \frac{n-l}{n},
$$

for $j < l$

$$
\frac{\partial C_l}{\partial \alpha_j} = -\frac{1}{n},
$$

and for $j > l$

$$
\frac{\partial C_l}{\partial \alpha_j} = 0.
$$

Therefore, for $j > i$ we obtain

$$
\frac{\partial f_{i+1}}{\partial \alpha_j} = \frac{i}{n}\sum_{l=j}^{n}\frac{\partial C_l}{\partial \alpha_j} = \frac{i}{n}\left(\frac{\partial C_j}{\partial \alpha_j} + \sum_{l=j+1}^{n}\frac{\partial C_l}{\partial \alpha_j}\right) = -i\gamma A^{\gamma-1} n^{-\gamma} < 0,
$$

whereas for $j \le i$, we have

$$\frac{\partial f_{i+1}}{\partial \alpha_j} = -\frac{n-i}{n}\left(\frac{\partial C_j}{\partial \alpha_j} + \sum_{l=j+1}^{i}\frac{\partial C_l}{\partial \alpha_j}\right) + \frac{i}{n}\sum_{l=i+1}^{n}\frac{\partial C_l}{\partial \alpha_j}$$
$$= -\frac{n-i}{n}\left(-\gamma A^{\gamma-1}n^{1-\gamma} + \frac{n-j}{n} - \frac{i-j}{n}\right) - \frac{i(n-i)}{n^2}$$
$$= \frac{n-i}{n}\left(\gamma A^{\gamma-1}n^{1-\gamma} - 1\right).$$

From the assumption that $A \ge n/e$ we get that $A \ge n\gamma^{\frac{1}{1-\gamma}}$, where this last inequality is due to the fact that $\lim_{\gamma\to 1}\gamma^{\frac{1}{1-\gamma}} = 1/e$,[5] and that $\gamma^{\frac{1}{1-\gamma}}$ as a function of γ is non-decreasing.[6] Thus, we have that $\left(\gamma A^{\gamma-1}n^{1-\gamma} - 1\right) \le \left[\gamma\left(n\gamma^{\frac{1}{1-\gamma}}\right)^{\gamma-1} n^{1-\gamma} - 1\right] = 0$. Therefore, $\frac{\partial f_{i+1}}{\partial \alpha_j} \le 0$ for any $i \in \{1,2,...,n-1\}$ and $j = \{1,...,n\}$. Thus, because $f_{i+1}(1,...,1) = 0$, for any set of weights $(\alpha_1,...,\alpha_n) \in (0,1)^n$ we obtain that $f_{i+1}(\alpha_1,...,\alpha_n) \ge 0$ for $i \in \{1,2,...,n-1\}$.

In sum, we obtained a set of non-negative multipliers $\mu_2,...,\mu_n$ for inequality constraints $g_2,...,g_n$ which are satisfied by x^* with equality. This completes the proof that x^* is the global maximum of the considered problem. □

---

[5] We have that $\lim_{\gamma\to 1}\gamma^{\frac{1}{1-\gamma}} = \lim_{\gamma\to 1} e^{\frac{\ln\gamma}{1-\gamma}} = 1/e$ because from l'Hospital's rule we obtain that $\lim_{\gamma\to 1}\frac{\ln\gamma}{1-\gamma} = -1$.

[6] For $f(\gamma) = \gamma^{\frac{1}{1-\gamma}}$ we have that $f'(\gamma) = \frac{\gamma^{\frac{\gamma}{1-\gamma}}}{(1-\gamma)^2}\left(\ln\gamma - \frac{\gamma-1}{\gamma}\right) \ge 0$ due to the fact that $\ln x \ge \frac{x-1}{x}$ for any $x > 0$.

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