THE LAYOUT OF WOODHENGE

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Abstract

Woodhenge consists of 6 concentric egg-shaped rings of post holes. A single ring can be laid out equally well using either Thom's 4 peg construction or Angell's 3 peg construction. Both methods assume the need to construct a right angle but it is shown that an error of 10° or even 20° in the angle is not detectable. Three of the rings can be fitted with common centres using Thom's method but only two by Angell's method. The axes of symmetry of the inner 4 rings point some 7° north of the direction of midsummer sunrise, that of the outer 2 about 17° north.

Introduction

This monument consists of 6 concentric egg-shaped rings of postholes surrounded by the remains of a bank and ditch. For laying out a single ring of this type Thom proposed a procedure involving the construction of a right angled triangle. In practical terms this means using 4 fixed pegs and a rope of fixed length (Cowan 1970 fig. 1 or Heggie 1981 fig 6) to produce 4 circular arcs. Angell (1977) suggested a loop of rope round 3 pegs, producing 6 elliptical arcs. Both methods give equally good fits. However we get a more sensitive comparison of the two methods if we use them to lay out a set of concentric rings.

Both methods assume that the builders could (a) set out lengths in units of a megalithic yard and (b) construct a right angle. Before starting on the main problem we shall show that the second assumption is unnecessary.

In this paper all lengths are expressed in units of the Thom's megalithic yard = 2.72 feet.

If we look at Thom's construction (1967 fig 6.1b) we see that point A is the centre of 6 concentric semicircles while B is the centre of 6 concentric 140° arcs, of radii 1 yard less than that of the corresponding semicircle. If this construction were achieved by fixing one end of the rope to C then the only restriction on the position of C would be that it must be 1 yard nearer A than B i.e. could be anywhere on one branch of an hyperbola. If we follow Thom in making angle BAC = 90° then this does fix C but as we shall see later, the centres of the short arcs joining the semicircles to the 140°arcs and much closer to A than this. To complete the rings, Thom chooses a fourth point D which is the mirror image of C in AB.

In Angell's contruction, 3 pegs are placed at the apices of an isosceles triangle, whose base and perpendicular height are integral multiples of a yard. This implies constructing a right angle at the mid point of the base, so what happens if this angle is in error? As an experiment we took the isosceles triangle with the base, height and rope length that best fitted Woodhenge ring F, then keeping the slant height of the triangle fixed, rotated the base through 20°. The ring this produced was of course, asymmetrical but rotating the whole figure through 5.4° produced a ring almost identical with the correctly drawn one - the small differences could have disappeared if the base length had been slightly increased in the oblique case.

Thom's fit to Woodhenge

If we look closely at Thom's figure we see that he achieved a rather poor fit; measurements on that figure showed the s. d. of the errors to be about 0.36 yards and in fact a lot of the error is systematic for example, 13 of the C ring points are outside the fitted ring, while for the outermost ring 11 consecutive points at the southern end and 11 at the northern end are also outside.

Thom did not use Cunnington's (1929) plan but surveyed the concrete pillars placed in the postholes after the excavation. Superimposing the two plans showed the latter were displaced by up to 6" or 0.2 yards from the centres of the postholes, enough to be infuriating but not enough to account for the errors in Thom's fit. Thom chose as his axis of symmetry the direction of midsummer sunrise in 1900 BC, which is 1.3° nearer north than the 1929 AD value drawn on Cunnington's plan.

One of us (Williams 1983, unpublished undergraduate dissertation) tried fitting the Cunnington plan using Angell's 3 peg construction based on Cunnington's axis (midsummer sunrise). Using a mainframe computer with a slow turn-round, optimum fits were not obtained (r.m.s. error 0.46 yards). However the work was sufficient to show that (a) the rings could not be all fitted with the same three peg positions and (b) the fits could be improved considerably by changing the axis of symmetry. Conclusion (a) was sufficient to make it very unlikely that Angell's construction was used.

Deducing the model from the data

The poor fits obtained by both methods (s.d. $>$ post diameter) are caused by trying to force the data to fit preconceived models. The first casualty is the axis of symmetry which is obviously not the sunrise line. Fitting an Angell's egg inside each ring in turn and rotating to obtain the best fit by visual inspection the axis of symmetry was found to be rotated anticlockwise from Cunnington's axis by 8, 8, 7 and 7 degrees for rings F, E, D and C, and by 23 and 12 degrees for rings B and A. Principal components gave a rotation of 7.7° for the inner 4 rings combined and 17.7° for the outer 2. The 2 outer rings are so nearly circular (ratio of diameters 1:12) that their axes of symmetry are not defined very precisely, but are clearly diffe-ent from that for the inner 4.

Next we fitted circular arcs to the ends of each ring. The fits are shown in figure 1 and are necessarily much closer than Thom's. The horizontal line passes through the centroid of the points and is parallel to Cunnington's axis. We assume that in each case a perfect circular arc was intended but that errors in execution led to radial variations in positions of holes with standard deviation D. Since we use up 3 d. of f. in finding the centre and radius our best estimate of D is given by

$$D^2 = \Sigma d^2/(n - 3)$$

where n is the number of points fitted and d the distance from the centre of each hole to the fitted arc. The results are given at the top of table 1. X, Y are the coordinates of each centre (relative to an arbitrary origin), R the radius and arc the angle is degrees covered by each fitted arc. Our best overall estimate of D is .232 yards and there is no significant difference in accuracy of layout from one ring to another.

For fitting complete circles the s.d. of R would be $D/\sqrt{n}$ and of X or Y, $D\sqrt{(2/n)}$. For the arcs the s.d of Y is little changed for the same n, those of X and R increased considerably. Some of the factors calculated from numerical experiments are given in table 2. These calculations ignore angular variations which other calculations showed to be insignificant for the small variations observed.

For D = .232 and the appropriate corrections we get the s.ds given in the middle of table 1. For rings C, D and E all values of X and Y are within 1.3 standard deviations of the mean. Hence these three rings have common centres but all the others are different. We now have the amended results for these three rings given at the bottom of table 1.

Using Angell's 3 peg construction with fixed peg positions we can't get as good a fit to rings C, D and E as with Thom's 4 peg. The reason is that our only variable is rope length and a change in rope length changes the width of the figure more than the length, so we cannot get the uniform annular widths required.

Finally we lookat the short arcs joining the left hand and right hand arc. The 4 inner rings have only 3 postholes on each so we cannot fit them. For rings

A and B the results are shown in table 3. For the short arcs the s.d. of the fitted radii (estimated from numerical experiments) are very large. Nevertheless it is clear that these radii are much less than Thom's expected values (41.5 and 38.3 yards), and hence that the points C and D were much closer to the axis than Thom's very precise construction requires.

Figure 2 shows all 4 arcs fitted to rings A and B. The small circles are the peg positions for ring A, the small rectangles for ring B.

Conclusions

All radii are integral multiples of a yard to within 1.5 standard deviations. C, D and E used the same peg positions and hence were laid out at the same time. For F the same orientation as for C, D and E was achieved but with the pegs displaced by about a yard, presumably when the original positions had been obliterated (if F were built first then it was no longer there when C, D and E were laid out).

Rings A and B are much more interesting. To avoid confusion let us use lower case letters a, b, c, d for the four pegs, c being below the axis and d above. Take ring A first. If we write cb for the distance from c to b and so on then from table 3 we calculate cb - ca = 1.94 but db -da = 2.79 and hence the rope was tethered at c. Further,

$$cb + Rb = 29.0$$

$$ca + Ra = 29.0$$

where Rb is the radius of the arc centred on b. The corresponding values for ring B are 29.1 and 29.0.

Our best fits to both rings require a rope length of 29 yards as compared with 41½ yards for Thom's and about 70 or 80 for Angell's. Displacing peg c away from the axis ab by one s.d. adds 2 yards to the length of rope required.

If we look at figure 2 we see a hiatus at about 120 degrees, where the short arc misses the semicircle by about 2 feet, possibly because peg d had got in the way! Ring B, laid out after ring A (because it is inside) is much neater although our standard deviations are much larger. If we move point c down by one s.d. we must add 5½ yards to the rope length. However the rope was certainly much shorter than Thom required because we can see the discontinuities in slope where the arcs meet.

If one looks for astronomical orientation, the axis of symmetry of the inner 4 rings is 6.4° north of the midsummer sunrise line for 1900 B.C. or 3.7° south of the midwinter moonrise at maximum declination.

Arguments about sophisticated geometrical techniques blind us to the real achievements of these late Neolithic people. For people unable to write, integers are the only possible numbers, plus possibly the simplest of fractions, such as $\frac{1}{2}$. From being able to set out a measured length it is only a simple step to lay out a circle, but an ellipse is a much more difficult concept. Obviously these people wanted to set out elongated circles with a given axis of symmetry and they started well by setting out semicircles with centres on that axis. They then found by trial centres for short arcs to join the semicircles, but could not see from ground level that the radii used were not long enough to give continuity of slope at the joins.

References

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TABLE 1

Data derived by fitting circular arcs. X axis is parallel to Cunnington's sunrise line but origin is arbitrary.

	Left Hand arc						Right Hand arc					
Ring	n	arc	X	Y	R	d	n	arc	X	Y	R	d
F	5	140	1.58	.43	5.23	.28	5	151	6.38	1.32	4.43	.28
E	8	167	.90	.13	7.85	.35	7	160	7.56	.71	7.22	.21
D	8	165	.96	.09	10.83	.28	7	145	7.43	.73	10.35	.17
C	7	153	1.24	.07	14.77	.28	6	133	8.00	1.07	14.03	.03
B	13	147	2.24	-.10	20.90	.26	12	139	7.48	1.30	20.04	.20
A	22	129	3.03	-.21	25.10	.23	21	146	8.40	1.08	23.08	.17

Overall r.m.s. value d = .233. Using this we estimate:-

Ring	σX	σY	σR		σX	σY	σR
F	.37	.16	.26		.34	.15	.23
E	.23	.11	.17		.28	.12	.19
D	.25	.11	.17		.33	.13	.24
C	.30	.12	.22		.40	.15	.30
B	.25	.10	.19		.29	.10	.22
A	.28	.08	.22		.24	.08	.18

Derived data assuming common centres for C, D and E

Ring		X	Y	R	d		X	Y	R	d
E		1.02	.10	7.91	.30		7.66	.83	7.17	.20
D		1.02	.10	10.87	.24		7.66	.83	10.21	.18
C		1.02	.10	14.63	.24		7.66	.83	14.29	.18

TABLE 2

Circular arc extending from 180 − t/2 to 180 + t/2 degrees. n points spaced evenly round arc but with small, normally distributed variations in radius. CX, CY and CR are amounts by which s.d. of X, Y or R is increased above value for same number of points spread round a complete circle.

n	t	CX	CY	CR
5	180	1.7	1	1.4
5	140	2.5	1.1	2.5
5	33	44	4	60
8	180	1.9	1	1.8
8	140	3.0	1	3.2
8	42	30	6	40
10	180	2.0	1	1.9
10	140	3.0	1	3.3
10	46	22	4	30
11	180	2.0	1	1.9
11	140	3.1	1	3.3
11	49	21	3.7	29

Each number is mean of 400 experiments for long arcs, 800 for short arcs.

TABLE 3

Fits of all 4 arcs to rings A and B

No. of points	Arc from	to	degree	X	σX	Y	σY	R	σR
Ring A									
10	74	120	46	4.9	.4	-3.7	2.4	28.3	2.3
22	121	249	128	3.03	.25	-.21	.08	25.1	.22
11	254	303	49	2.1	.4	3.7	1.9	28.5	1.9
21	294	79	146	8.40	.22	1.08	.09	23.1	.17
Ring B									
5	75	108	33	5.6	.6	-7.5	5.8	29.0	5.7
13	106	253	147	2.24	.28	- .10	.11	20.9	.21
6	259	301	42	1.6	.7	6.6	3.5	27.3	3.5
12	294	73	139	7.43	.33	1.30	.11	20.0	.23

Fig. 1