Establishing Optimal Core Sampling Strategies: Theory, Simulation and Practical Implications

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Abstract

We describe a new method by which ceramics are classified and compared in terms of the curvature functions which are associated with each profile (cross section). The theoretical and practical aspects of the method are illustrated in terms of a test case of archaeological interest, where curvature analysis was successfully applied.

Introduction

Pottery is the most frequent find in typical excavations. Archaeologists who study ceramic assemblages seek to identify in them distinctive patterns that can be used for a better understanding of the material culture. Amongst the several attributes, which comprise the typological vocabulary, shape attributes are the most commonly used. Traditional shape descriptions and classifications, however, rely on intuitive, often vague characterizations, which are hard to quantify. Several attempts to harness computers and mathematical tools in pottery analysis have appeared in the proceedings of the CAA conferences (for instance: Hall and Laflin 1985, Goodson 1989, Liming et al. 1989, Durham et al. 1995 and 1996, Schurmans et al. 2001, Kampel et al. 2002), or books e.g., Orton (1980), Orton et al. (1993) and Whallon and Brown (1982). Here, we classify ceramic profiles by their curvature function - which gives the curvature as a function of the arc length along the profile (Mokhtarian and Boder 2003 for previous use of the same function). This function contains the entire information about the profile, and therefore it is a convenient basis for a detailed, objective and comprehensive typology of ceramics, which can be implemented in computer algorithms. The new aspects of our method and its advantages will be pointed out in the sequel.

Theoretical Considerations

We summarize here the main definitions and relations which are necessary for the subsequent presentation. Its implementation in practice will be described in the next section. The profile of a ceramic artifact is a curve in R^2 which is not necessarily closed, since we treat pot-shreds and whole vessels on the same footing. Let s be the arc length measured along the profile from the point which defines the top of the rim, and let $\theta(s)$ denote the direction of the tangent at s, so that $\theta(0)=\pi$. The curvature $\kappa(s)$ is the rate of change of $\theta(s)$ at s: $\kappa(s)=d\theta(s)/ds$. Given the curvature function $\kappa(s)$, the curve is *completely and uniquely* specified and can be reconstructed by integration. The expression of the two-dimensional profile by a single function, which emphasizes the features of archaeological relevance (highly curved sections such as the vessels rim or base appear as peaks, see (Fig. 1.d)), is one of the main advantage of the representation of curves in terms of their curvature functions.

In order to classify and compare profiles of vessels in a given assemblage, we standardize the description of the assemblage by scaling the arc length and the curvature of each profile by the rim radius, which is assumed to be provided. We define the distance $d_{\alpha\beta}$ between two curvature functions $\kappa_\alpha(s)$ and $\kappa_\beta(s)$ as $d^2_{\alpha\beta}=\int(\kappa_\alpha(s)-\kappa_\beta(s))^2\omega(s)ds$, where $\omega(s)$ is a non-negative, finitely supported, normalized *weight function*. In the present application we take $\omega(s)$ to be the characteristic function of the interval $[s_{\min},s_{\max}]$ which focuses the attention to the rim of the vessels. Alternative choices of $\omega(s)$ allow the archaeologist to introduce her/his judgment concerning the relative importance of the various features in the compared potsherds. The *scalar product* is defined by $\langle\alpha|\beta\rangle = \int \kappa_\alpha(s)\kappa_\beta(s)\omega(s)ds$, and the profiles correlation is $C_{\alpha,\beta} \equiv \langle\alpha|\beta\rangle / \sqrt{\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle}$. These definitions allow us to treat the space of curvature functions as a Hilbert space, which is of great help in the analysis to be described below.

The profiles are classified by their affinity to some prototypes prescribed by the Archaeologist. Often, this is done by selecting a few representative subgroups of profiles which manifest in the best way the attributes that distinguish each of the prototypes. Taking the average of the curvature functions in each of the defining sets, we obtain the curvature functions of virtual profiles which represent the prototypes in the subsequent analysis (Fig.2). Let us assume that *D* predetermined prototypes were selected, and are represented by the curvatures functions $\kappa_{A_1}(s), \kappa_{A_2}(s), \dots, \kappa_{A_D}(s)$. We must require that the prototype curvature functions are linearly independent (but not *necessarily* orthogonal), so that the *D x D* matrix $A_{r,s} = \langle A_r | A_s \rangle$ is regular ($\det A \neq 0$).

Assume that a given assemblage, which consists of *M* profiles (with curvature functions $\kappa_m(s)$, *m=1,...,M*) are to be sorted according to their affinity to the set of *D* prototypes defined above. The coordinates $x_1(m), x_2(m), \dots, x_D(m)$ which express $\kappa_m(s)$ in the basis provided by the prototypes, are computed for each fragment in the assemblage. Thus, $\kappa_m(s) = \sum_{j=1}^{D} x_i(m) \cdot \kappa_{A_i}(s) + \lambda_m(s)$, where $\lambda_m(s)$ stands for that component of $\kappa_m(s)$ which is *orthogonal* to the subspace spanned by the prototypes. $\lambda_m(s)$ must be included, since the prototypes do not span the entire space of curvature functions. To obtain the coordinates $x_i(m)$ we take the scalar product of $\kappa_m(s)$ as written above with each of the prototype curvatures. Since by definition, $\lambda_m(s)$ is orthogonal to the $\kappa_m(s)$, we obtain a set of *D* linear equations, with the explicit solution.

$$x_i(m) = \sum_{j=1}^{D} (A^{-1})_{i,j} \langle A_j | m \rangle$$

Each profile is now represented by a point *x(m)* whose distribution in the *D* dimensional space gives a direct measure of the partition of the assemblage to types. Once the coordinate vectors *x(m)* are known, we can easily compute the magnitude of the vectors $\lambda_m(s)$ which we denote by $\|\lambda_m(s)\|$. The assumption that the assemblage can be sorted by the prototypes $\kappa_{A_1}, \kappa_{A_2}, \dots, \kappa_{A_D}$ is justified only if the "*quality factors*" defined by $\varsigma_m \equiv \frac{\|\lambda_m\|^2}{\langle m|m\rangle} = 1 - \sum_{i=1}^{D} x_i(m) \frac{\langle m|A_i\rangle}{\langle m|m\rangle}$, are small (less than 1/2). Clearly, ς_m vanishes if $\kappa_m(s)$ is in the space spanned by the prototypes, and it equals one (which is its maximal value) if $\kappa_m(s)$ is orthogonal to all the prototypes. In summary, the quantities which we derived give us information on the affinity of the studied profiles to the prototypes, and, at the same time, they test the degree by which the prototypes account for all the features encountered in the assemblage.

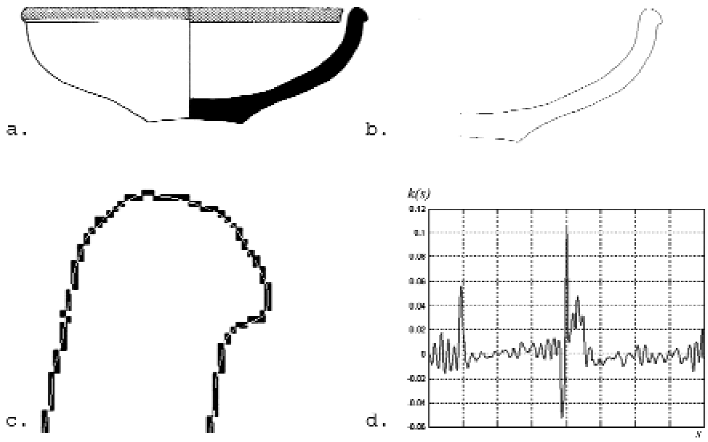

Figure 1 The steps leading from a scanned drawing to the curvature function. a. A scanned drawing of a bowl. b. The pixelized profile. c. Enlarged detail of the pixelized profile and the interpolated curve. d. The curvature function

The above construction can be applied in the frequently met situations, where one likes to quantify the observation that 'types' gradually change over time, space, or some other variable of archaeological interest. In such cases, it is convenient to demonstrate the transition between the types along a linear axis. For this purpose, one can try to analyze the assemblage in terms of only two prototypes *(D=2)*, and associate to each profile an interpolation parameter $\mu(m)$ which indicates the transition between the prototypes. We define the interpolating parameter by $\mu(m) = x_2(m)/[x_1(m) + x_2(m)]$, which takes the values 0 or 1 when the profile is identical to one of the prototypes. Such analysis is justified only if the corresponding quality factors ς_m are small.

Finally, an important advantage of the formal construction which was proposed above is that its application is independent of the form by which the curvature functions are represented. The curvature functions can be expressed as continuous or discrete functions in coordinate or in Fourier space. This freedom will be used in the subsequent description of the implementation of the method in practice.

Practical Implementation

[1] Data acquisition.

The data analyzed in the present work consists of linear drawings printed in archaeological reports. The various steps taken to extract the information and transform it to digital form are shown in (Fig. 1.a-c). The drawings are scanned, and the boundary of the cross section is identified and stored as a discrete set of *(x,y)* coordinates. The distance s traversed along the curve (measured with respect to the highest point) is computed, and the coordinate values at equal s intervals are obtained by interpolation, and are stored. In this form, the data still contains irrelevant 'noise', which is due to the finite resolution of the scanning device, imperfections of the origi-

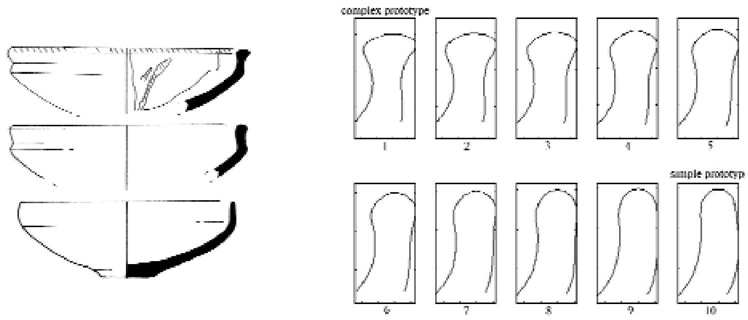

Figure 2 Three Dor bowls illustrating the typological transition. From top to bottom - the sub-types dominant in Iron Ia, Iron Ib and Iron II (left). The same transition is modeled by virtual rim profiles, constructed by interpolation between the two extreme prototypes (right)

nal drawings, etc. These features are filtered out to yield the smooth, yet accurate presentation of the original boundary as describe below. The whole procedure does not take more then a few seconds using MATLAB software.

[2] Smoothing, and the computation of the curvature function

The smoothing of the data alluded to above, is most conveniently performed in Fourier space: $\hat{X}(n)=\sum_{m=1}^{N} X(m)\cdot e^{-i\frac{2\pi}{N}(m-1)(n-1)}$ where *X(m)* stands for the original discrete coordinate vector, and *N* is the number of points along the curve.

The smoothing is implemented by attenuating the irrelevant high-frequency fluctuations of the coordinated vectors. This is achieved by a filter function which suppresses the n'th Fourier component by a factor $(1+\exp[(n-n_0)/d])^{-1}$ where n_0 is the filter range, and *d* is the transition width.

Transforming back to real space, we get the smooth coordinate functions, which are the basis of the curvature computations and the subsequent analysis. We found it necessary to suppress features which have a wavelength smaller than 8 pixels, however some times one can suppress even lower frequencies, without deteriorating the quality of the curves.

The first and second derivatives, which are needed for the curvature calculations, are also computed in Fourier space, and then transformed back to real space so the curvature is expressed as a function of s. Using the explicit formula $\kappa(s)=(x'(s)\cdot y''(s)-y'(s)\cdot x''(s))/([x'(s)]^2+[y'(s)]^2)$ the curvature is finally computed at a discrete set of equidistant points on the boundary (Fig.1d).

[3] Prototypes and Typology

To illustrate our method in action, we show here a few results obtained in the analysis of an assemblage consisting of 86 Iron-age bowls from the Tel-Dor (Israel). The bowls are all of the same general type (simple carinated bowls) and span a chronological range of one century or less. Manual analysis (Gilboa 2001; Gilboa and Sharon in press) suggested a typological development (albeit morphologically and quantitatively gradual) from "complex" rim type (dominant in the Iron-Ia stratum) to the "simple" rim type (abundant in the Iron I/II and Iron-IIa strata), as shown in (Fig.2). To test this hypothesis, we defined "complex" and "simple" prototypes (number 1 and 10 in (Fig.2), respectively), and analyzed the assemblage in the plane spanned by these profiles. The distribution of the bowls in the 2-dimensional complex-simple plane is shown in (Fig.3b,c), where the points representing profiles from different strata. In spite of the large (expected) overlaps between the distributions from the various strata, the increase of the mean values of μ is clear (Fig.3a). Notice that the μ values are increasing over time and that the ς values are consistently low, justifying *a-posteriori* the analysis of the assemblage in terms of two prototypes.

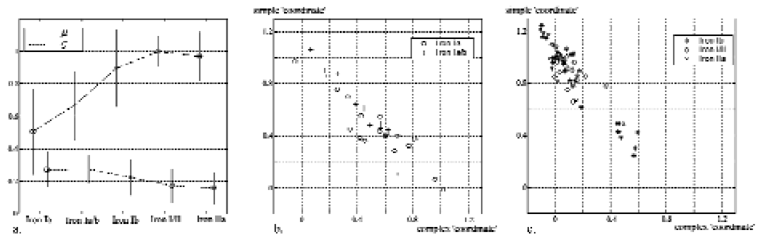

Figure 3 Representative results from the analysis of Dor bowls. a. The mean values and the standard deviations of and by periods. The lines through the points are to distinguish between the two functions which are shown on the same plot. b. The distribution in the complex-simple plane of the points corresponding to bowls from the Iron Ia and Iron Ia/b strata. c. Same as b. for the Iron Ib, Iron I/II and Iron IIa strata.

Detailed reports of typological analysis of assemblages from other Iron Age sites in the Southern Levant are in preparation. To summarize, the curvature analysis and the Hilbert space approach, enables a classification of ceramics based on rather minute stylistic details at a level and scope which was rarely attempted before. Its application is rather straight forward and it provided archaeologically relevant information.

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