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The Use of Monte-Carlo Techniques for the Estimation of Visibility

K. Nackaerts, G. Govers, L. Loots

Abstract

In archaeological research, visibility analysis is usually based on binary viewshed maps. However, Digital Elevation Models inherently contain errors due to inaccuracies in the original topographic data and the data structure used. Therefore the binary classification will be subject to an unknown amount of error. It would be more appropriate to use a method for visibility calculation which allows us to take the uncertainty on the elevation data into account. In this paper a further development of a method to calculate fuzzy viewsheds is proposed. The method is based on Monte-Carlo simulation of the DEM-error. From a large number of simulations, the visibility probability of a given cell can then be calculated. First experiments with this new methodology for our study area indicate that the hidden error in classical binary viewsheds is considerable. The deterministic use of such binary viewsheds for visibility analysis may therefore lead to erroneous conclusions. Furthermore, binary viewshed calculations appear to be rather sensitive to the algorithms used for DEM construction and visibility calculation. Fuzzy viewsheds appear to be much more robust. The application of fuzzy viewsheds requires information on the mean elevation error as well as an estimation of the number of Monte Carlo simulations needed. The first can be obtained by comparing interpolated elevation data with manually measured values. For the estimation of the required number of simulations two methodologies are proposed.

1 Introduction

Many commercial GIS packages offer easy to use raster-DEM based viewshed algorithms which calculate the visibility of each cell of the study area in a deterministic way: a cell is either classified as visible or invisible. By this, the analysis of visual relationships between elements in a landscape (e.g. ancient watchtowers) might look simple and will result in deterministic information.

However, one must not forget that DEMs are inherently liable to a wide range of inaccuracies due to the map-making process, digitalisation of analogue maps, interpolation, vector to raster conversion errors, etc . Limitations in the algorithms that analyse the visibility between cells in a raster-DEM -referred to as data structure induced errors (Sorensen and Lanter 1993)- can also have a major impact on the result. All these errors can sum up and may interact with each other, resulting in inaccuracies of unknown extent in the binary viewshed (Sorensen and Lanter 1993, 1158; Fisher 1991, 1327) and the observed visual relationships.

To solve this problem, Fisher (1991, 1322; 1992, 345; 1993, 344) proposed a method for calculating fuzzy viewsheds. In these viewsheds, the probability of a cell lying in the viewshed area is calculated by means of a Monte Carlo simulation of the DEM error. Sorensen and Lanter (1993, 1150-1158) presented other algorithms to deal with DEM inaccuracies in the visibility determination, but they only take data structure induced errors into account. Because their algorithms neglect all other sources of inaccuracies (see above) an approach as proposed by Fisher seemed the most appropriate for this study.

In this paper, we focus on the estimation of a minimum number of simulations needed for the calculation of fuzzy viewsheds. Two new methodologies are proposed. First, an exploratory method is described, resulting in qualitative information. Second, a quantitative method is proposed, based on the binomial distribution function, by which it is possible to calculate confidence intervals for the calculated fuzzy visibilities. The results clearly show that the use of too few simulations will lead to unreliable documents.

2 Fuzzy visibility

To account for the problem of inaccuracies in the DEM, a Monte Carlo simulation of the DEM (Fisher 1992, 345) was performed, in which a simulated random error-matrix was added to the original DEM resulting in new DEMs. These new simulated DEMs represent samples of a population that describes the real DEM. A classic visibility determination (as described above) is then performed using each of these simulated DEMs, resulting in different binary viewshed maps. As such, the visibility values (0 or 1) of a raster cell in each simulated viewshed map can be seen as Bernoulli trials out of a binomial distributed population. The probability that a cell lies within or without the real viewshed (i.e. fuzzy visibility) can then be estimated by dividing the sum of all these values by the total number of simulations for each cell. This results in a fuzzy viewshed map that takes the inaccuracy of the DEM into account.

For the simulation process, information about the distribution of the DEM error (interpolation and data-structure related elevation error) and an estimation of the number of simulations is necessary. Too many simulations will cause excessive computation times while too few simulations lead to inaccurate results (see below). The first is achieved by sampling the DEM error. This was done by comparing the interpolated elevation value with the manually measured elevation in the same sample point. No spatial autocorrelation in the DEM errors and an equal distribution of the DEM error over the study area are assumed. However, these elements are not extensively researched and a discussion of them would lead too far from the subject of this study (Fisher 1991, 1322).

3 Number of simulations

3.1 Exploratory approach

The estimation of the number of simulations was accomplished by an examination of the evolution of the fuzzy visibility values in 100 uniform randomly spread sampling points. This evolution can be visualised by plotting the fuzzy visibility against the total number of simulations. A visual interpretation of the behaviour of the fuzzy visibility values showed a stabilisation of these values after approximately 50 simulations for the study area.

3.2 Quantitative approach

The simulation process can also be compared with Bernoulli trials (Sachs 1982):

1. The result of each simulation is either a success (a cell is visible) or a failure (a cell is invisible).
2. A result is independent of the result of a previous simulation.
3. The probability of success is a constant as can be derived from the asymptotic behaviour of the fuzzy visibilities as described above.

The cumulative visibility of a given cell (Y) is called a binomial random variable. This variable follows a binomial distribution. This characteristic enables the calculation of confidence intervals using the statistics specifically developed for binomial distributions.

If n is greater than 20, the normal approximation may be used (Conover 1971):

$$L = \frac{Y}{n} - x_{(1-\alpha/2)}\sqrt{Y(n-Y)/n^3} \qquad [1]$$

$$U = \frac{Y}{n} + x_{(1-\alpha/2)}\sqrt{Y(n-Y)/n^3} \qquad [2]$$

where $x_{1-\alpha/2}$ is the quantile of a normally distributed random variable.

4 Results

Idrisi for Windows (Eastman 1995) was used for this study. The simulation process was automated by the Idrisi Macro Language (IML).

4.1 Number of simulations

Using the exploratory method, performing more than 50 simulations seemed not to result in a decrease of the variance of the fuzzy visibility values. These results were compared for observation points located in different positions in the landscape of the study area. Since the results of this comparison were positive, it was decided to use at least 50 simulations for the determination of the different fuzzy viewsheds.

Confidence intervals can be calculated using functions [1] and [2] for a given level of significance, number of simulations and a given fuzzy visibility. A steep increase of accuracy is visible during the first simulations. After 100 simulations, the confidence interval for a fuzzy visibility of 0.5 (this corresponds to the widest confidence interval) is approximately equal to 0.5 +/- 0.1 (α=0.05). If, for example, 25 simulations result in a fuzzy visibility of 0.5, the real value lies between 0.3 (rather invisible) and 0.7 (rather visible) with a significance level of 5%. Out of this information we decided to use 100 simulations, delivering an acceptable accuracy against acceptable calculation times.

4.2 Fuzzy viewsheds

A comparison of a Boolean and a fuzzy viewshed map from the same observation point clearly demonstrates the deceptive accuracy of a binary viewshed map. In our test case, important areas are Boolean classified as visible although the fuzzy viewshed map indicates fuzzy visibilities of less than 0.5. The probability that a cell lies within the viewshed is smaller in these areas than the probability that the cell lies outside the viewshed. The mean fuzzy visibility of the Boolean viewshed of our testcase equalled 0.5. This means that a cell indicated as visible in the Boolean viewshed has an equal probability of being visible as being invisible, given the inaccuracies in the DEM. Binary viewsheds should therefore be used with the necessary care.

The fuzzy viewsheds make shaded decisions possible and give important information about the accuracy of possible decisions. These fuzzy documents are used in the study of the function of some fortifications around the ancient city of Sagalassos (SW-Turkey) as described by L. Loots et al. (this volume).

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Contact details

K. Nackaerts
Sagalassos Archaeological Research Project
Laboratory for Experimental Geomorphology
Catholic University of Leuven
Redingenstraat 16
B-3000 Leuven
BELGIUM
email: kris.nackaerts@agr.kuleuven.ac.be

G. Govers
Fund for Scientific Research
Laboratory for Experimental Geomorphology
Catholic University of Leuven
Redingenstraat 16
B-3000 Leuven
BELGIUM

L. Loots
Sagalassos Archaeological Research Project
Catholic University of Leuven
Faculteit Letteren
Blijde Inkomstraat 21
B-3000 Leuven
BELGIUM
email: lieven.loots@arts.kuleuven.ac.be