6

Matrix processing of stratigraphic graphs: a new method

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6.1 Introduction

A Harris matrix is a way of representing a complex set of archaeological layers by a graph which is a conventional synthesis of the stratigraphy of the archaeological site (Harris 1979a, Harris 1979b).

There is a formal analogy between the Harris matrix approach and graphs used in Operational Research for sequencing problems, like PERT and related methods (Fig. 6.1) in particular with the MPM method (Degos 1976).

In this last method, tasks (i.e. stratigraphic units) are figured as nodes of a graph while anteriority or posteriority constraints are figured as the links of the graph (Fig. 6.1b).

Associated with the graph, a presence-absence matrix is defined, where the value 1 signifies the presence of a direct chronological constraint between two units. The aim of this paper is to present a simple, interactive method of matrix processing to help to build a stratigraphic graph. Computerisation of some parts of the method is discussed.

6.2 Method

6.2.1 A new representation of a Harris 'matrix'

The classical representation of the Harris 'matrix' has problems with non-logical node locations and crossing lines which involve a mix of graphical logic and archaeological interpretation (Fig. 6.2a).

A new representation is proposed in the following manner:

- A unit is figured on the same horizontal line each time the unit has a direct anterior or posterior connection. Therefore, each diachronic connection is represented by a (I) vertical line while each synchronic connection is represented by a (=) horizontal line : then no connections are figured by an angular line (Fig. 6.2b).

6.2.2 Step one: data entry

Direct stratigraphic connections, observed during excavation (Fig. 6.3a) are recorded in a unit by unit matrix where conventionnally a value 1 signifies the column unit is stratigraphically posterior to the row unit (Fig. 6.3b).

6.2.3 Step two: inferred and non-redundant connections

The chronological connections between stratigraphic units are *transitive*.

If a unit (A) is posterior to a unit (B) and if the unit (B) is posterior to a unit (C), then the unit (A) is posterior to the unit (C). By applying the rule of transitivity, the matrix is completed by these new connections called inferred connections.

In the same step, non-redundant connections may be determined in the matrix. If a unit (A) is posterior to a unit (B), if the unit (B) is posterior to a unit (C) and if the unit (A) is posterior to the unit (C), then the connection (A)-(C) is redundant. Therefore, by definition, the non-redundant connections are the minimal set of connections from which all the other connections may be inferred.

The determination of inferred and non-redundant connections may be achieved by differents techniques. With matricial processing, an easy algorithm may be defined with the following manner, figured by the Figs. 6.4a to 6.4g.

```
for each column j
  for each row k>j
    if Ukj = 1 then
      for each column k and for each row l
        if Ulk = 1 then
          if Ulj = 1 then
            the link is redundant
            set Ulj = 2
          else if Ulj = 0 then
            a link is inferred by transitivity
            set Ulj = 3
          endif
        endif
      endfor
    endif
  endfor
endfor
```

Conventionally a redundant connection is valued 2 and an inferred connection is valued 3.

At the end of the step two, the matrix is completed with observed non-redundant connections (value 1), observed redundant connections (value 2), inferred connections (value 3).

6.2.4 Step three: error detection

Observed and recorded data can contain two types of errors :

Forgotten but existing connections. Unfortunately, in the case of forgotten non-redundant connections, due to errors during the excavation recording, there is no issue to detect and correct such errors.

Added non-existing connections. If the connections are in contradiction with the rule of antisymmetry, after the application of the rule of transitivity, unit (1) is posterior to unit (1), such errors can be detected. In

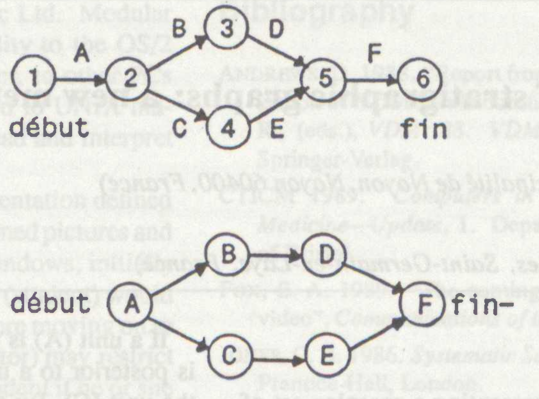


Figure 6.1: Sequencing graphs: Pert (a) and MPM (b) methods

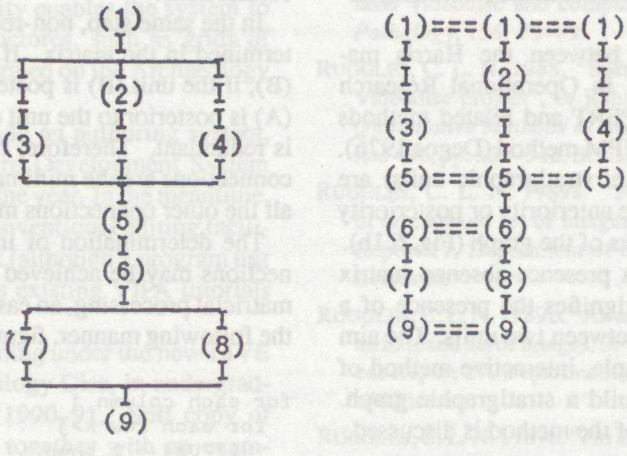


Figure 6.2: Statigraphic graphs: Harris (a) and Desachy-Djindjian (b) representations

that case, connections appear in the diagonal of the matrix, revealing the units concerned.

6.2.5 Step four: Reorganisation of the matrix

The reorganization of the matrix is achieved by counting non-empty cases for rows and columns of the matrix (Fig. 6.5a) and then by reorganizing the matrix following the decreasing numbers in column or in row (Fig. 6.5b). The result is a matrix giving the stratigraphic sequence of the units:

- for a given column j, the number is the number of units posterior to the unit j;
- for a given raw i, the number is the number of units anterior to the unit i.

The reorganized matrix shows two main features:

- the matrix is triangular,
- the non-redundant connections are located near the diagonal.

The reorganized matrix shows characteristic structures of the stratigraphy:

- Regular stratigraphies are represented by a full triangular matrix, where all the non-redundant connections are located just under the diagonal (Fig. 6.6a).
- Parallel stratigraphies are represented by vertical or horizontal breaks (Figs. 6.6b and 6.6c).

The reorganization of the matrix can have problems with equal scores, involving multiple choices in the reorganization of rows and columns. In such cases, in order to locate the non-redundant connections nearer to the diagonal, a reciprocal averaging algorithm is applied only to non-redundant connections of each submatrix of equal scores.

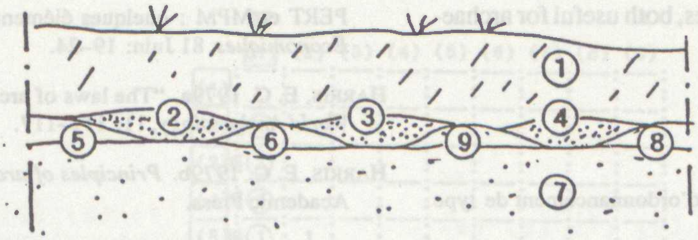
At this step of the process, it is important to focalise several points:

- the reorganized triangular matrix can be use as a stratigraphic tool, even without the help of any graph,
- the reorganization of the triangular matrix can be entirely computerised.

6.2.6 Step five: constructing a graph

The stratigraphic graph is the result of the former mathematical processing and of the interpretations of the archaeologist constructed from other data in the field.

The graph is plotted following the convention of a stratigraphic graph described in section 6.2.1 (Fig. 6.2b). In this graph, a horizontal line corresponds to a single unit. The sequence of the horizontal lines is given by the sequence of the units in the reorganized matrix. An orthogonal frame is built allowing the plotting of all the non-redundant connections for a given unit. Each non-redundant connection involves a vertical link between the two given units, for which the location and the length is already determined by the sequence of the units.



	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)									
(2)	1								
(3)	1								
(4)	1								
(5)	1	1							
(6)	1	1	1						
(7)		1	1	1	1	1		1	1
(8)	1			1					
(9)	1		1	1					

Figure 6.3: Step one

An easy algorithm to plot the graph may be defined, in the following manner:

- find successively non-redundant connections, starting with the first column as showed in the example of the Fig. 6.7a to 6.7f, and balancing from columns to rows until all the non-redundant connections have been used.

6.2.7 Step six: minimisation of crossing lines

A stratigraphic graph is a two dimensional map of a three dimensional reality. From a theoretical point of view, not only are crossing lines to be expected, but the elimination of all th crossing lines is likely to be impossible. It is only the minimisation of the number of crossing lines which is discussed in this step.

The algorithm of minimisation of crossing lines is based on the choice of particular paths during the plotting of the graph. A way to obtain such a path is the following:

- after having plotted the first vertical line (Fig. 6.8a), the algorithm is searching the first convergent node unit from the bottom to the top, or, if not, the first divergent node unit from the top to the bottom (Fig. 6.8b), until all the matrix is processed.

Fig. 6.8 shows an application of the algorithm from Fig. 6.8a to Fig. 6.8i. Figures 6.8h and 6.8i are two equivalent graphs without crossing lines.

6.3 Archaeological interpretation of a stratigraphic graph: from stratigraphy to chronology

The construction of a stratigraphic graph has showed that several possible graphs can be plotted from the reorganized matrix.

The final optimisation of the graph can only be made manually by the archaeologist, using other information.

One of the most important pieces of information used in this step is the contemporeneity of units, as given by archaeological finds or sedimentological structures.

Fig. 6.9 shows an example of such an interpretation based on the contemporaneous layers 5 and 6.

From the graph in Fig. 6.9a, Fig. 6.9b shows a long chronology while Fig. 6.9d shows a short chronology.

6.4 Conclusion

A new approach to the problem of stratigraphic graphs has produced the following results:

- a clear distinction between computerised matrix processing and archaeological interpretation,
- a new stratigraphic graph, with a better formalism than the Harris graph,
- a six step algorithm, easy to implement on a micro-computer, giving a reorganized matrix, and a graph

with minimized crossing lines, both useful for archaeological interpretations.

Bibliography

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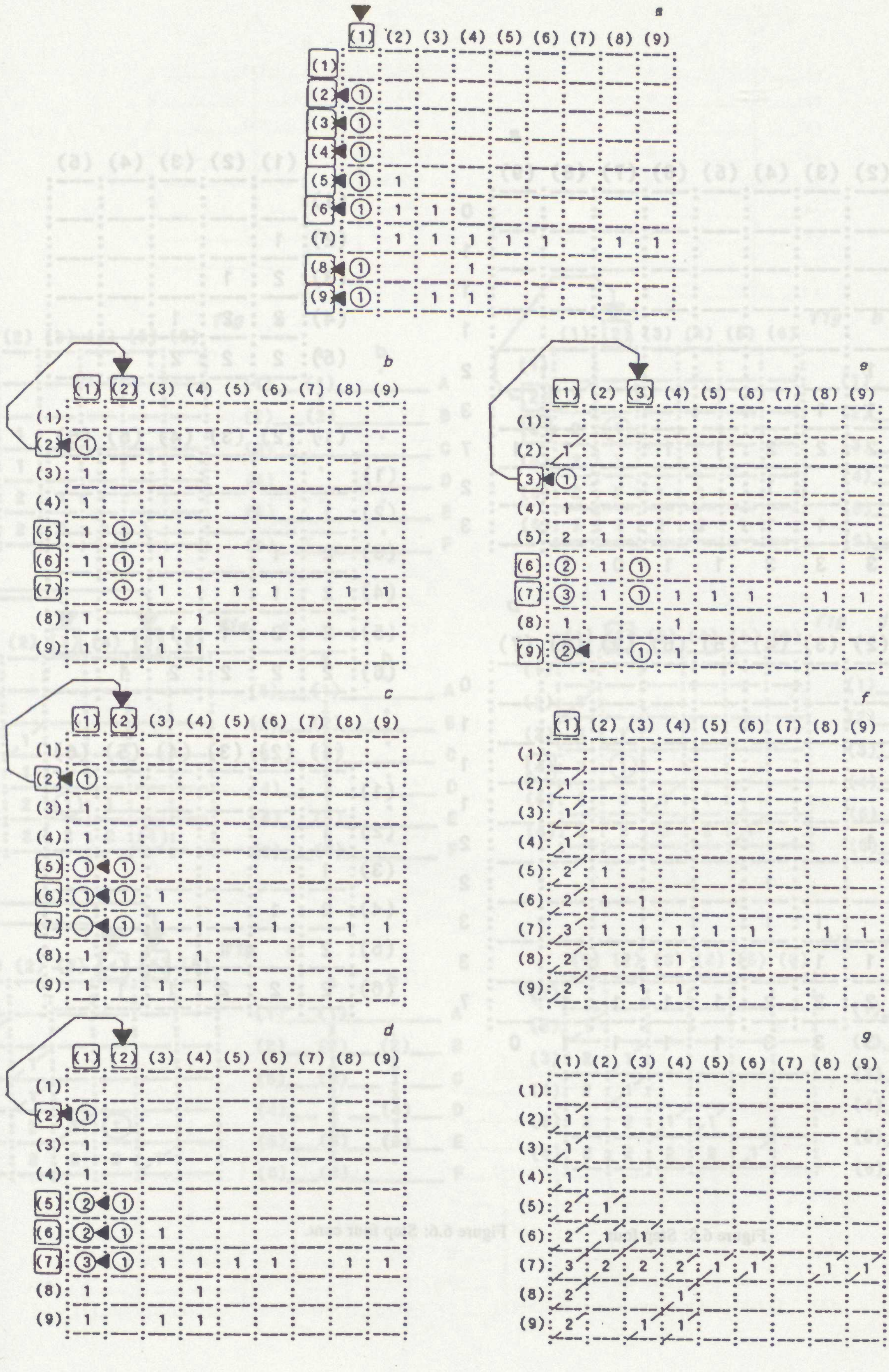

Figure 6.4: Step two

a

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
(1):										0
(2):	1									1
(3):	1									1
(4):	1									1
(5):	2	1								2
(6):	2	1	1							3
(7):	3	2	2	2	1	1		1	1	7
(8):	2			1						2
(9):	2		1	1						3
	8	3	3	3	1	1	0	1	1	

b

	(1)	(2)	(3)	(4)	(5)	(6)	(9)	(8)	(7)	
(1):										0
(2):	1									1
(3):	1									1
(4):	1									1
(5):	2	1								2
(8):	2			1						2
(9):	2		1	1						3
(6):	2	1	1							3
(7):	3	2	2	2	1	1	1	1		7
	8	3	3	3	1	1	1	1	0	

Figure 6.5: Step four

a

	(1)	(2)	(3)	(4)	(5)
(1):					
(2):	1				
(3):	2	1			
(4):	2	2	1		
(5):	2	2	2	1	

```
(1)
 I
(2)
 I
(3)
 I
(4)
 I
(5)
```

b

	(1)	(2)	(3)	(4)	(5)	(6)
(1):						
(2):	1					
(3):	2	1				
(4):	2	1				
(5):	2	2	1	1		
(6):	2	2	2	2	1	

```
(1)
 I
(2)===(2)
 I     I
(3)   (4)
 I     I
(5)===(5)
 I
(6)
```

c

	(1)	(2)	(3)	(4)	(5)	(6)
(1):						
(2):	1					
(3):	1					
(4):	2	1				
(5):	2		1			
(6):	2	2	2	1	1	

```
(1)===(1)
 I     I
(2)   (3)
 I     I
(4)   (5)
 I     I
(6)===(6)
```

Figure 6.6: Step four cont.

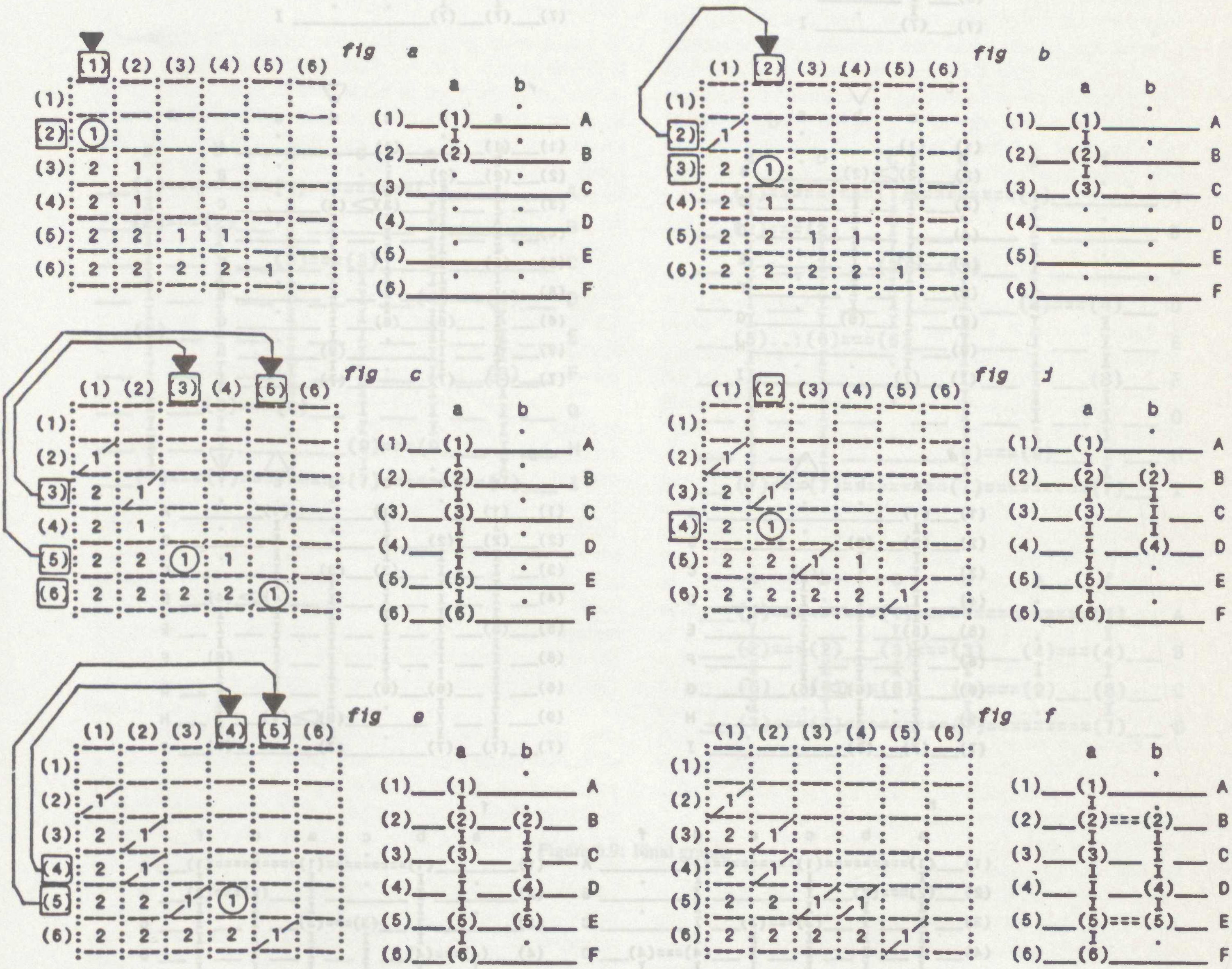

Figure 6.7: Step five

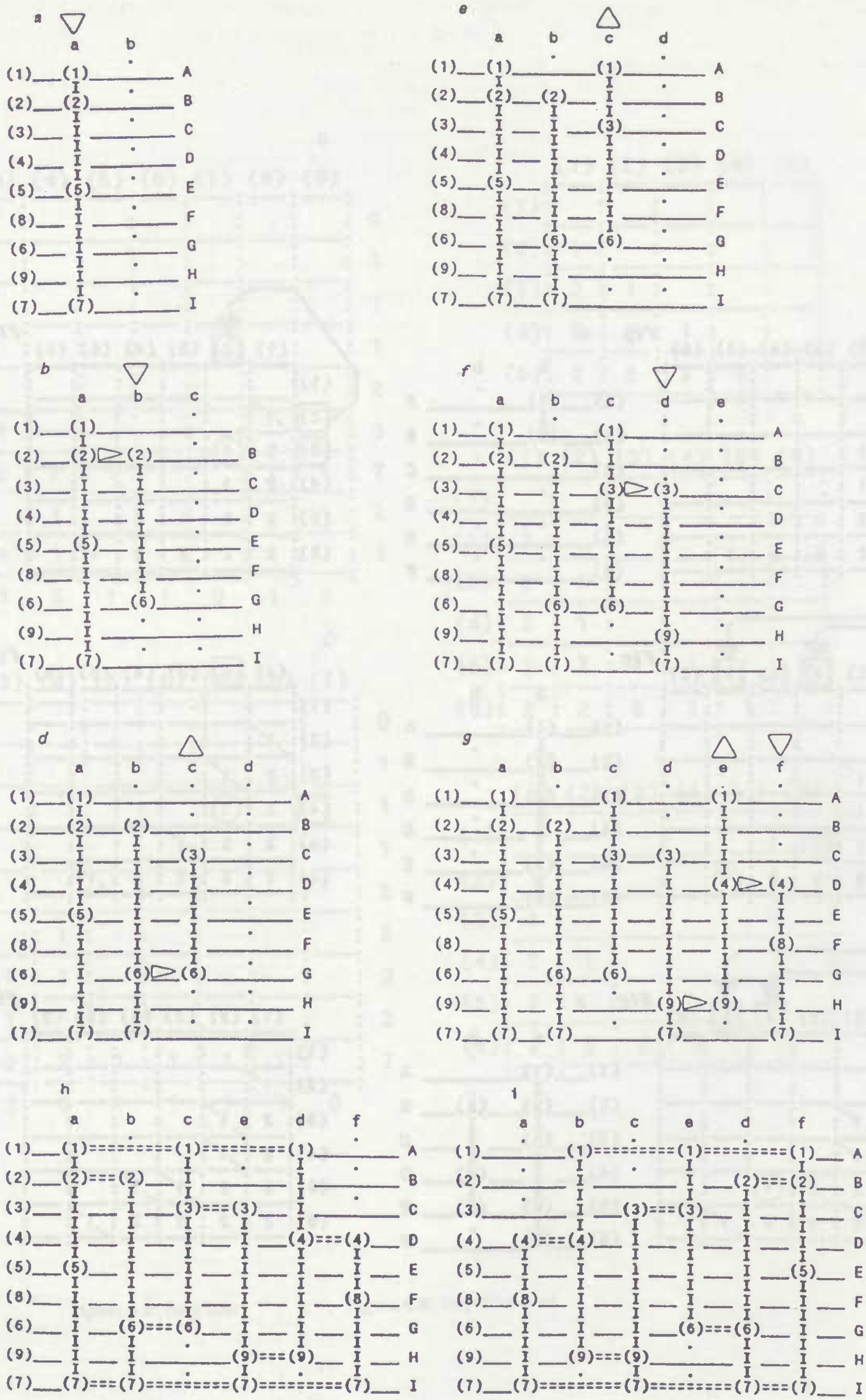

Figure 6.8: Step six

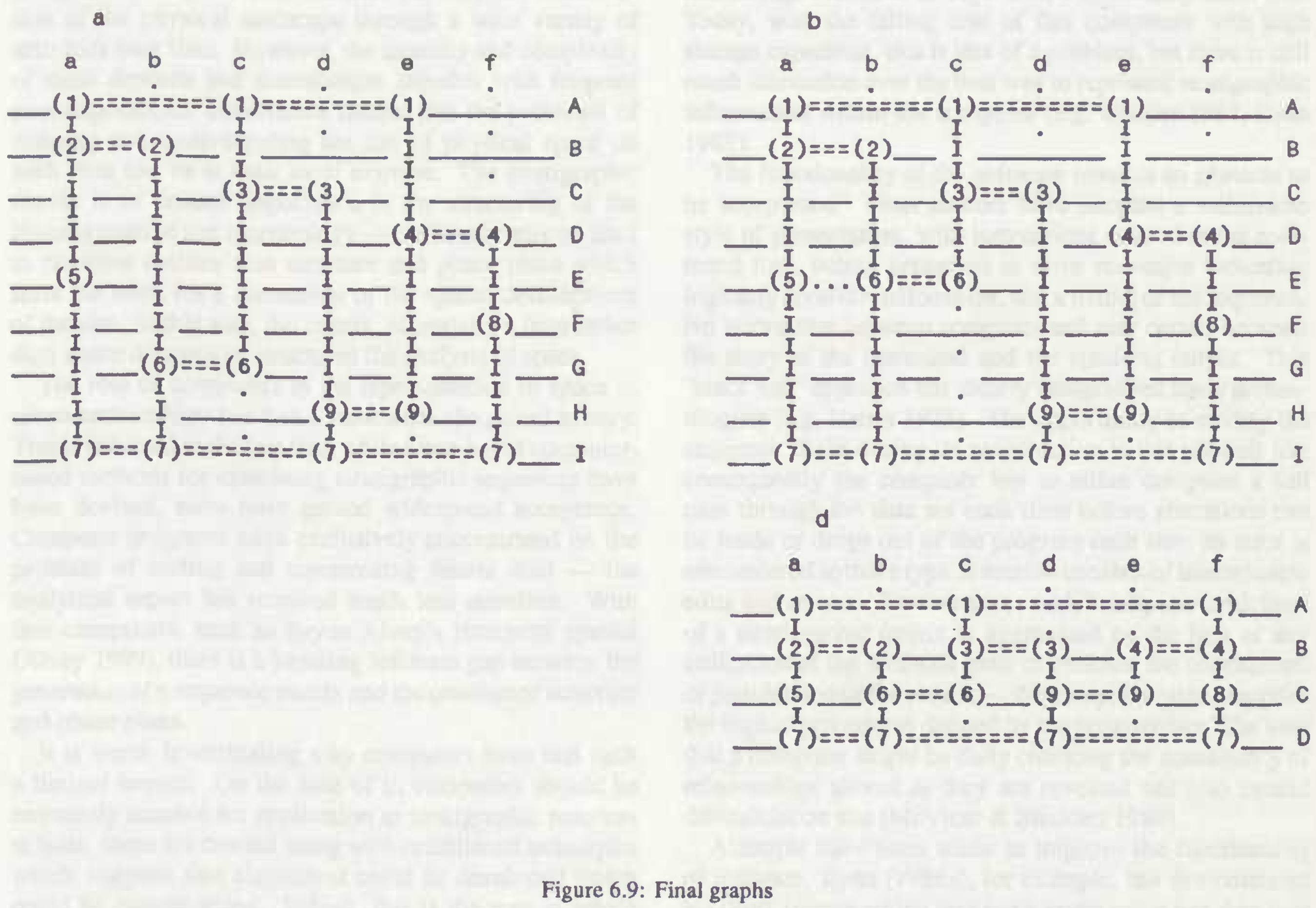

Figure 6.9: Final graphs