

Arguments against the existence of the 'Megalithic Yard'.

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It has long been conjectured that a standard measure was used in the construction of Megalithic Monuments. Barclay (1926) suggested that an 'Ancient Greek Foot' (12.1608 inches) was used in the building of Stonehenge. C.A. Newham (1964) reasoned that a 'Roman Measure' may have also been used. Hammersley and Morton (1954) claimed that a statistical examination of the misnamed Druid Circle problem was beyond resolution. However using the Quantum Hypothesis of S.R. Broadbent (1955, 1956), A. Thom (1955, 1962, 1967) concluded that many stone circles and rings were constructed using the Megalithic Yard, which he states, remained constant to within 0.003 inches of 2.72 feet, from Scotland to Brittany, during the whole Megalithic Era. Professor Thom then used this measure to produce sophisticated geometries for the many non-elliptical rings. In the previous two conferences I gave two alternative geometries for these sites (Angell 1977, 1978), and in these papers I referred to my misgivings about the Megalithic Yard. So in this paper I will examine Thom's derivation of the 'Yard' and also question the validity of applying the Quantum Hypothesis to this problem.

2. An outline of the Statistical Ideas involved.

We consider a set of n observations y_1, \dots, y_n (the diameters of the megalithic rings). The hypothesis is that these measurements may be represented in the form:-

$$y_i = 2\delta m_i + \beta + \epsilon_i \quad (i=1, \dots, n) \quad \dots (i)$$

β and 2δ are constants (2δ is called the quantum), the m_i are positive integers, and the ϵ_i the inevitable error in the i 'th measurement. This means that the data is assumed to be grouped around equally spaced nodes.

2δ is the linear spacing between the nodes.

β is the error in placing the origin of measurement.

There are two distinct classes of problem.

I. There is an 'a priori' knowledge that a quantum exists, and we have a value for its magnitude.

II. The quantum must come from the data.

There are two subdivisions in each case:-

(1) β is known to be zero.

(2) β is non-zero and is obtained from the data.

For each of these cases there is a method of estimating the value of 2δ and β . $2d_1$ and $2d_2$ are the estimates for 2δ in the respective subdivisions and b is the estimate for β .

(In what follows the symbol \sum will stand for $\sum_{i=1}^n$.)

$$\text{Case I(1)} \quad 2d_1 = \sum m_i y_i / \sum m_i^2 \quad \dots (ii)$$

with variance $\text{var}(2d_1) = \frac{\left(\sum y_i^2 - \left(\sum m_i y_i \right)^2 / \sum m_i^2 \right)}{(n-1) \sum m_i^2}$.. (iii)

Case I(2) $2d_2 = \left(n \sum m_i y_i - \sum m_i \sum y_i \right) / \Delta$.. (iv)

$b = \left(\sum m_i^2 \sum y_i - \sum m_i \sum m_i y_i \right) / \Delta$.. (v)

where $\Delta = n \sum m_i^2 - \left(\sum m_i \right)^2$.. (vi)

and $\text{var}(2d_2) = s_2^2 / \Delta$.. (vii)

$\text{var}(b) = s_2^2 \left(1 + \left(\sum m_i \right)^2 / n\Delta \right) / n$.. (viii)

where $s_2^2 = \left(n \sum y_i^2 - \left(\sum y_i \right)^2 - (2d)^2 \Delta \right) / n(n-2)$.. (ix)

Then using the statistic $s^2/d^2 = \sum \epsilon_i^2 / nd^2$ ($d=d_1$ or d_2) .. (x)

together with Fig.2.1 (Thom [1], page 10), a 'probability level' for any pair n and s^2/d^2 is found. Thom states 'probability level ... refers to the probability that a quantity is real and is not a spurious result obtained by accident'. The probability level is in fact the probability (usually expressed as a percentage) of the result occurring by accident. These two sentences are self-contradictory: the latter is used by Thom. This statement is not strictly correct when we consider Broadbent's test and the quantum hypothesis, since this test is of the rectangular hypothesis (the converse to the quantum hypothesis). A low 'level' only gives an indication of the truth of the quantum hypothesis.

Case II. Using the previous estimators $2d_2, b$ and the statistic s^2/d_2^2 , Broadbent gives the criterion for accepting the hypothesis to be

$$c = \sqrt{n} \left(\frac{1}{3} - \frac{s^2}{d_2^2} \right) > 1 \quad \text{.. (xi)}$$

3. The suitability of Thom's Data to this Statistical Model.

The hypothesis initially assumes that the errors involved are not proportional to the diameter size; a doubtful assertion.

In both cases I and II an initial choice of 2δ is made, we call this value the unit. From the unit and the y_i we calculate the unique values of the m_i and ϵ_i ($i=1, \dots, n$) using the equations

$$m_i = \left[\frac{y_i}{2\delta} \right] \quad \text{if } y_i - 2m_i\delta \leq \delta$$

$$= \left[\frac{y_i}{2\delta} \right] + 1 \quad \text{otherwise.}$$

(Here $\left[\frac{y_i}{2\delta} \right]$ is the integral part of $y_i/2\delta$.)

and $\epsilon_i = y_i - 2m_i\delta$.

The choice of m_i limits the values of ϵ_i to the range $]-\delta, \delta]$. Thom states that his calculations show that these residuals do not increase

seriously with increased circle diameter: implying that he was correct to make the assumption that the errors are not proportional to the circle diameter. In fact, the limiting of the error range, together with the existence of 'large' errors in the circles of small diameter will naturally ensure this result. Thus his calculations exhibit a biproduct of that assumption, and is not a verification of its validity.

We now consider whether it is possible to even test a claim such as 'the Megalithic Yard is 2.72 ± 0.003 ft.' from the data supplied in table 5.1 (Thom [1], pages 36-39). The first objection is that the data "is known to ± 1 foot or better" but all values are given correct to 0.1 feet, and it is this questionable accuracy which is used in the calculations.

Suppose we have a unit 2δ ft., which varies $\pm \epsilon$ ft., so that the largest possible value of the yard is $2\delta + \epsilon$ ft., and the smallest is $2\delta - \epsilon$ ft.

Consider a circle of diameter y ft. say. What is the correct value for m ? It is possible that

$$|y - m_1(2\delta - \epsilon)| \leq \delta - \frac{\epsilon}{2}$$

$$|y - m_2(2\delta + \epsilon)| \leq \delta + \frac{\epsilon}{2}$$

where $m_1 - m_2 \geq 1$ even for small y .

As y gets larger, there is a point where the yard $2\delta - \epsilon$ will always give a different value of m to the one given by $2\delta + \epsilon$.

This is so when

$$\left| \frac{y}{2\delta - \epsilon} - \frac{y}{2\delta + \epsilon} \right| > 1$$

$$\text{i.e. } y > \frac{(2\delta)^2 - \epsilon^2}{2\epsilon}$$

Thus when y is greater than $((2\delta)^2 - \epsilon^2)/2\epsilon$, the extreme values of the yard give different values of m . This indicates the difficulty in testing whether the yard used was some human feature (a man's stride for example). The variation of such a feature would imply inevitable errors in the value of m_i for larger circles, and once the m_i have been defined, the existence of such errors are ignored and this tends to bias the estimators for the yard toward the unit 2δ . Similarly the variance of the yard about these estimators will also be undervalued, giving the impression of a very small variation about the mean.

For example, when testing a yard, which for the sake of argument was known to be $2.72\text{ft} \pm 1$ " every circle of diameter greater than 45 feet will have different values of m for the extreme range of the yard. In fact more than 50% of circles with diameter greater than 23 feet would encounter the same difficulty.

Thus by using $2\delta = 2.72$ ft., and hence defining the m_i , the variance given by the computation would be far less than its actual value. Assuming the m_i to be correct then the accuracy implied in the measurement of the y_i would be assumed to be better than

$$\pm \frac{\delta}{2m_i \delta} \times 100\% = \pm \frac{50}{m_i} \%$$

and this implied accuracy affects both the year and variance estimates.

The extent of bias may be examined by considering firstly the two estimators $2d_1$, $2d_2$ for the yard; and secondly the variance estimate used by Thom $\text{var}(2d_1)$.

We assume the initial choice of unit is 2δ .

$$2d_1 = \frac{\sum_i m_i y_i}{\sum_i m_i^2} = \frac{\sum_i (2\delta m_i + \epsilon_i)}{\sum_i m_i^2}$$

$$= 2\delta + \frac{\sum_i \epsilon_i}{\sum_i m_i^2}$$

$$2d_2 = \frac{n \sum_i m_i y_i - \sum_i m_i y_i}{\Delta}$$

$$= 2\delta + \frac{n \sum_i \epsilon_i - \sum_i \epsilon_i}{\Delta}$$

and we consider the 'variation factors'

$$V_1 = \left| \frac{\sum_i \epsilon_i}{\sum_i m_i^2} \right| \cdot \frac{100}{2\delta} \%$$

$$V_2 = \left| \frac{n \sum_i \epsilon_i - \sum_i \epsilon_i}{\Delta} \right| \cdot \frac{100}{2\delta} \%$$

Using the 145 sites given in Thom's table 5.1, V_1 , V_2 were calculated for 2δ varying from 1.0 ft. to 6.00 ft. in steps of 0.005 ft. The vast majority of units examined gave V_1 less than 0.3%, and V_2 is not much better. Over 900 of the 1000 units considered returned this negligible variation factor; questioning both the suitability of the quantum hypothesis test for the data supplied, and the validity of the estimators (especially $2d_1$).

Since $\frac{\sum_i \epsilon_i}{\sum_i m_i^2}$ is linearly dependent on the ϵ_i , the numerator is subject to

cancelling factors among the positive and negative values of the $m_i \epsilon_i$. The test would be more valid if the data produced a small range of units which gave V_1 much smaller than the rest of the units. This is not the case, $2d_1$ is simply a re-statement of the original assumed unit. So the test revolves around the probability levels associated with s^2/d^2 and c .

$$\text{var}(2d_1) = \frac{\sum_i y_i^2 - (\sum_i m_i y_i)^2 / \sum_i m_i^2}{(n-1) \sum_i m_i^2}$$

$$= \frac{\sum_i (2m_i \delta + \epsilon_i)^2 - (\sum_i (2m_i \delta + m_i \epsilon_i))^2 / \sum_i m_i^2}{(n-1) \sum_i m_i^2}$$

$$= \frac{\sum_i m_i^2 \epsilon_i^2 - (\sum_i m_i \epsilon_i)^2}{(n-1) (\sum_i m_i^2)^2}$$

$$\frac{\Sigma c^2}{(n-1)\Sigma m_i^2} < \frac{n\delta^2}{(n-1)\Sigma m_i^2}$$

As is readily seen, the upper bound of the variance value is highly dependent on the m_i , so that if any large circles are considered, no matter what the accuracy, then the variance estimate is going to be small. The accuracy claimed for the yard (± 0.003 ft.) was obtained from the variance by

$$\text{standard error} = 0.67 \times \sqrt{\text{var}(2d_1)}$$

This value was calculated for the 1000 units examined, and all produced an error of this small order of magnitude, once more stressing the failure of this test to produce meaningful results.

We have seen that a small variation about the unit causes a substantial change in the m_i , which in turn invalidates the c_i , and these inaccuracies bias the statistic s^2/d^2 toward a low value, implying a far better probability level than should be taken.

The whole test hinges on this statistic (and on c , which is dependent on s^2/d^2), again demonstrating the ineffectiveness of the quantum hypothesis for the analysis of Megalithic Stone Rings.

Throughout, the basis of the problem is seen to be the choice of m_i : if there was an independent method of producing their correct values then the test would have some significance. Instead we have the circular procedure

1. Choose a unit and produce the m_i .
 2. From these produce an estimator for the yard, which is of course biased to the original choice of unit, and we end up back where we started.
4. A discussion of Thom's calculations.

We consider subsets of the 145 sites given by the Professor in his table 5.1. He first examines a unit 5.44 ft., (which he calls the megalithic fathom) and obtains from (x) and (xi)

$$s^2/d^2 = 0.22 \quad c = 1.32$$

giving an impressive probability level of 0.001%, with c greater than 1, implying acceptance of the unit. However with $n=145$ any unit having $s^2/d^2 < 0.24$ has a probability level better than 0.01%. All units in the range [5.39, 5.48] are within this level, raising doubts about the choice of m_i , which in turn implies that the calculated probability level is better than can ordinarily be taken.

$$\text{We have } b = 0.27 \quad 2d_1 = 5.43 \quad 2d_2 = 5.42$$

Thom assumes $\beta=0$ (there are no grounds for this assumption) and then uses estimator $2d_1$, when $2d_2$ should have been taken. (Note 5.42 is outside the range 5.44 ± 0.006 proposed.) This is the first of many occasions, when the assumption ' $\beta=0$ ' is made, when the estimator b is significantly different from zero. Furthermore, if we ignore those sites which are classified as ellipses, flattened circles and egg-shaped rings - because the proposed megalithic yard was used to define their geometry, and hence may have subtly affected the measured dimensions of the sites, we are left with 112 rings whose

statistics are

$$s^2/d^2 = 0.26$$

$$c = 0.80$$

c is less than 1, which, by the Broadbent criterion, does not give any information about accepting the quantum hypothesis.

Thom now concentrates on the half fathom or Megalithic Yard, since he reasons that his calculations were done on diameters but the original builders would probably have used radii.

In order to 'verify' the universal use of the Megalithic Yard, five subsets of the data were also considered by Thom.

1. England and Wales (all sites).
2. England and Wales (circles only).
3. Scotland (all sites).
4. Scotland (circles only).
5. Britain (circles only).

However since the value of the Yard was derived from the data as a whole, the consideration of subsets of this data cannot be taken as an independent test. We overlook this detail, and consider the results for $2\delta = 2.72$ feet as does Thom.

<u>Area</u>	<u>Type</u>	<u>b</u>	<u>2d₁</u>	<u>2d₂</u>	<u>c</u>
Britain	All	-0.069	2.720	2.723	0.814
England & Wales	All	0.217	2.721	2.714	0.390
Scotland	All	-0.258	2.719	2.729	0.737
Britain	Circles only	-0.006	2.719	2.719	0.639
England & Wales	Circles only	0.432	2.719	2.704	0.356
Scotland	Circles only	-0.293	2.719	2.730	0.535

Apart from the results for Britain as a whole, the values of b are shown to be significantly non-zero; the former become zero due to the averaging effect of the positive Scottish b and the negative English b . Thom then uses the argument that because the estimator b was zero for Britain as a whole, then he could assume β is zero for each part of Britain taken separately. He then uses this doubtful if not invalid assumption to justify the use of the inadequate estimator $2d_1$ for all five subsets. Because $2d_1$ is almost equal to 2δ in all cases (a fact which we have demonstrated to be a property of the algorithm, and hence is not a pointer towards acceptance) he claims that the Yard was universally used. If instead, the $2d_2$ estimator had been used (as is more realistic) then we see a definite difference between the respective English and Scottish results (2.704 to 2.730) which is far outside the ± 0.003 ft. range claimed (a range which we have also shown is always small because of the algorithm). If anything these results would indicate that different units were used in the two countries, and we can say with confidence that 2.72 ft. unit was not used in England and Wales. Furthermore the values of c in the table are all well below unity, implying that no conclusion can be drawn from these results anyway.

With such a mass of contradictory evidence it must be concluded that the case for the 2.72 ± 0.003 ft. Megalithic Yard is far from being proved.

References

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Site
Profile designation
Reference (standard or optional) points:
 "latitudes" and
 elevations

Fig. 2 - List of entries for the cut boundary values

Site
Item category
Cut
Item inventory number
Square
Horizontal co-ordinates
Elevation
Item width
Item height
Morphoscopy
Roundness
Lithology
Strike
Dip
Profile designation
Profile "latitude"

Fig. 3 - General list of entries