25

Geodetic and cartographic problems in archaeological data bases at and within the boundaries of some countries

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25.1 Maps, archaeological databases and their uses

25.1.1 Monument protection

Geographic information systems for archaeology are now quite common. The largest of these are usually found in ancient monument protection organisations or air photo research groups, and thousands of sites over large areas are frequently recorded. Almost all western countries have readily available maps at scales of 1:25000 or larger, and the sites are usually referenced to the grid systems used on these maps. The database systems used for query and data entry are usually common commercial packages, occasionally with some specialised add-ons which provide supplementary functions. However, typical commercial database programs have very limited computational ability.

Queries to the database which concern the coordinates of sites usually take the form of:

1. Find all sites within a certain distance of a given site.
2. Find all sites within a given polygon (perhaps the boundaries of a political division).
3. Find all sites along some kind of linear feature, such as a road or river.

25.1.2 Spatial statistics

Evaluation of the geographic distribution of sites using spatial statistics requires the site coordinates. Typical methods used are Dirichlet tesselations (Voronoi or Theissen Polygons), or computations are carried on Euclidian distances between site coordinates or between sites and random points to determine degrees of randomness, clustering, and surface fitting, (Hodder & Orton 1976; Ripley 1981; Upton & Fingleton 1985). Typical requirements are:

1. All intersite distances
2. All site-random point distances
3. Relative Cartesian coordinates of all sites

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25.1.3 Mapping from oblique aerial photographs

Another use for map coordinates is for the geometric correction and mapping of sites visible in aerial photographs, (Scollar *et al.* 1989). For this purpose, three or more control points must be visible in an image whose coordinates on the ground are known. It is not usually very practical to measure these in the field, and the coordinates are taken from large scale maps (1:5000 or better) which are scanned and processed like images, (Scollar & Weidner 1979).

Required are either:

1. Relative coordinates on a single scanned map (pixel indices)
2. Relative coordinates on up to four adjacent scanned maps if the airphoto has control points lying outside a single map sheet.
3. Adjusted relative coordinates on up to two pairs of two maps each, when the adjacent map pairs are skewed relative to each other at the boundaries of meridian or latitude strips (see below).

25.2 Mapping methods

All would be quite harmless if it were not for the fact that geographic coordinates (latitude and longitude) are not the same as map coordinate systems in most countries. Furthermore, grid references are often supplemented with map names or letter combinations.

Maps are transformations of the surface of the Earth to a plane. Most are not projections although the word is commonly used for historical reasons. Rather, they are functions of two complex variables which describe the transformation from the surface of the ellipsoid to a plane. Mapping transformations were one of the most active research areas in late 18th and early 19th century mathematics, and many well known mathematicians contributed to the subject. The general introduction of computers in the 1960's revived interest in what was thought of as a long-solved problem, and many of the older methods have been revised or abandoned in recent years.

25.2.1 Ellipsoids

The shape of the Earth is approximated by an ellipsoid. It's real shape is obtained through accurate satellite measurements today, but the ellipsoid approximation is used in cartography. One of the early models was due to Bessel, director of the Prussian state mapping service in the early 19th century. The Bessel ellipsoid despite its errors survives to this day in the German, Austrian, Swiss, Dutch, Norwegian and Swedish mapping systems.

There have been many ellipsoid models of the Earth made since the early 19th century, and different countries use different ellipsoids for their mapping systems (see for example Heitz 1985; Sigl & Torge 1981). The ellipsoid is defined as shown in Fig. 25.1. Constants for the most commonly used ones are given in Table 25.1. Although differences are small, they are significant enough so that latitudes and longitudes for the same point will differ somewhat when the ellipsoid is changed. National maps usually use an ellipsoid model which provides a best fit for the local system. Maps which extend over national boundaries usually use one of the international models.

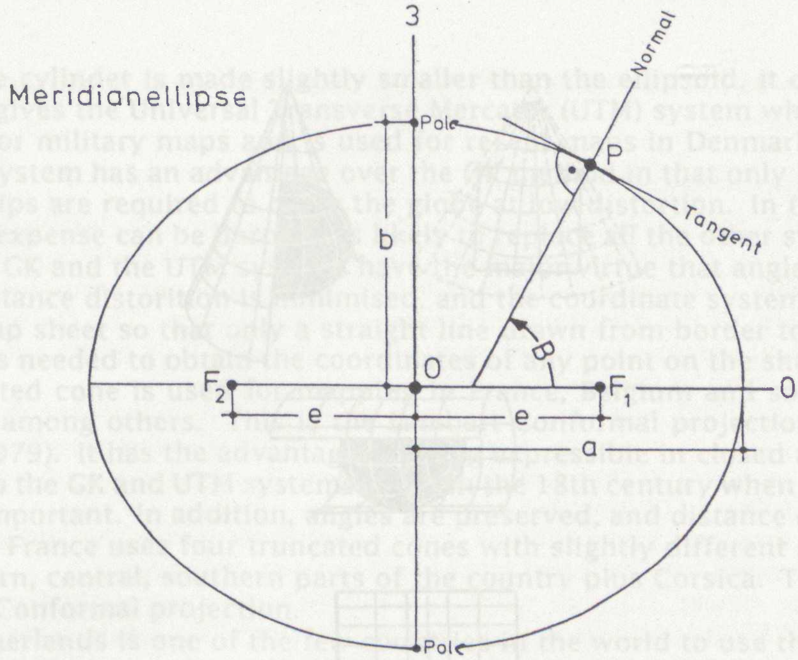

Figure 25.1: Notation for the ellipsoid

Various ellipsoids and constants, the notation following Fig. 1.2:

Name	*Year*	*a*	*b*	*Used in*
Bessel	1842	6377397.2	6356079.0	Germany, Austria, Switzerland, Norway, Sweden
Airy	1849	6377563.4	6356256.9	Great Britain, Ireland
Clarke	1866	6378206.4	6356583.8	USA, Canada
Clarke	1880	6378249.1	6356514.9	France
IUGG 1924	1924	6378388.0	6356911.9	Belgium, Denmark, Italy, Luxemburg
Krassovsky	1944	6378245.0	6356863.0	Warsaw Pact countries
IUGG 1980	1980	6378137±2	6356752.3±1	recommended for new maps

Values of second eccentricity and pole curvature for calculations of GK and UTM coefficients

Name	e'^2	C_o	g_0	g_2	g_4	g_6
Bessel	0.0067192188	6398786.848	111120.6196	15988.6384	16.72997	0.02178
Airy	0.0067153391	6398941.320	111123.6231	15979.8694	16.71117	0.02174
IUGG 1924	0.0067681848	6399936.655	111136.5363	16107.0692	16.97630	0.02226
Krassovsky 1944	0.0067385314	6399698.921	111134.8609	16036.4943	16.82811	0.02197
IUGG 1980	0.0067395013	6399593.640	111132.9524	16038.5193	16.83265	0.02198

Table 25.1: Various ellipsoids and constants

The IUGG 1980 ellipsoid is probably accurate to within the tolerances shown, so that later measurements will not modify these constants significantly. That is why its use is recommended for the future. The values of the coefficients have been computed here using 10 byte extended precision floating point for the most accurate published values of a and b which I could find.

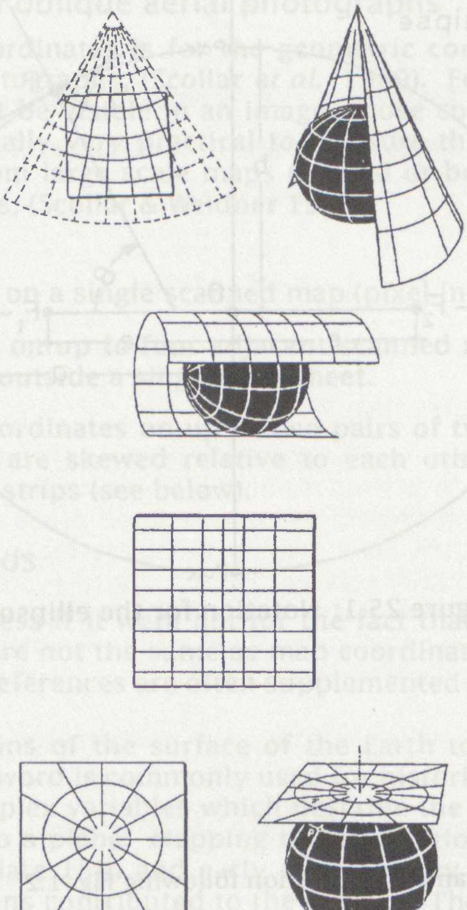

Figure 25.2: Major projections used in large scale maps

25.2.2 Common transformations for large scale maps

Three major types of transformation or projection are used commonly in large scale mapping (Hake 1982; (Maling 1973; Snyder 1982). These are shown in Fig. 25.2.

They are:

1. Mapping to a cone with a transformation, the cone being then unrolled.

2. Mapping to a cylinder which is then unrolled by means of a transformation formula which is not a geometric projection.

3. True projection or transformation mapping to a tangent plane from the opposite side of the ellipsoid.

The most commonly used mapping surface in most European countries is a transverse cylinder tangent to a meridian with axis east-west. This is the transverse Mercator mapping in English-speaking countries, and a Gauss-Krüger (GK) mapping elsewhere. Gauss earned his living as director of the mapping service of the Hanoverian Kingdom. Krüger (1912) systematised and revised the original Gauss (1822) design. It is used in the German-speaking and the Warsaw Pact countries, though with different ellipsoids. Meridian strips of either 3° or 6° are used as discussed below.

When the cylinder is made slightly smaller than the ellipsoid, it cuts two meridians. This gives the Universal Transverse Mercator (UTM) system which is the NATO standard for military maps and is used for recent maps in Denmark and the U.S.A. The UTM system has an advantage over the GK method in that only half the number of map strips are required to cover the globe at low distortion. In the future, when and if the expense can be borne, it is likely to replace all the other systems.

Both the GK and the UTM systems have the major virtue that angles are preserved locally, distance distorition is minimised, and the coordinate system is Cartesian on a given map sheet so that only a straight line drawn from border to border in each direction is needed to obtain the coordinates of any point on the sheet.

A truncated cone is used for mapping in France, Belgium and some of the older U.S. maps among others. This is the Lambert Conformal projection (Snyder 1982; Boucher 1979). It has the advantage of being expressible in closed analytic form as opposed to the GK and UTM systems. Back in the 18th century when it was invented, this was important. In addition, angles are preserved, and distance distortion is not excessive. France uses four truncated cones with slightly different scale factors for the northern, central, southern parts of the country plus Corsica. This is a Lambert Polyconic Conformal projection.

The Netherlands is one of the few countries in the world to use the Stereographic projection, a true projection to a plane from a point opposite The Hague on the other side of the ellipsoid. Stereographic projections are also standard for polar latitudes above 80° north and south.

All of the common transformations preserve angles well enough and are of sufficiently low distortion so that linear distance measurements can be made on a map sheet of scales 1:25000 and larger. Preservation of angles was very important to surveyors since most measurements were made with theodolites before the invention of electronic distance measuring devices. Consistent mathematical treatment of these transformations can be found in (Großmann 1976;, Snyder 1982; and Kuntz 1983) and a complete explanation from a theoretical geodetic point of view can be found in (Heitz 1985). Series expansions suitable for rapid computation are given for the GK and UTM systems by (Schödlbauer 1982). Transformations of small scale maps from one system to another are described by (Brandenberger 1985) and details of programs used by the US Geological Survey are given in (Snyder 1985). The mainframe program package GCTP for the cross-conversion of 17 different small scale projections based on an earlier CIA package was published by the US Geological Survey in 1981. The large scale maps which are of interest in archaeology have other problems.

25.2.3 Grid reference in various countries

Grid references are two numbers which are in the Cartesian coordinates of the map transformation used. In Britain, they are preceded by a two letter code which designates a 100 km side length square in a modified transverse Mercator system, (Harley 1975; Ordnance Survey 1967). In West Germany, Austria, Luxemburg, Italy, the Scandinavian countries and the Warsaw Pact, the coordinates may be given as x and y values (sometimes designated R and H) within a given 1:5000 or 1:25000 map sheet. For historical reasons, cartographers outside France use x for north-south and y for east-west. In France, the kilometer digits of the Lambert coordinate system appear along the border of the map, but either the name or number or conic segment of the map must also be supplied to prevent ambiguity. The borders of the French 1:25000 maps give geographic coordinates in 'gon', 1/400th of a circle, but the conversion by multiplying by 0.9 to get degrees is trivial. The map border markings are based on the meridian of Paris rather than Greenwich on the older maps, so that a constant has to be subtracted as well. Newer maps have both Greenwich and Paris meridian markings.

25.2.4 Map naming conventions

Maps at scales of 1:100000 or larger have widely different nomenclatures, depending on the country. The 1:25000 maps in Germany are bounded by 6' and 10' latitude and longitude. The 1:5000 maps use the GK grid as a border and are numbered using the first four digits of the x and y coordinates in that system. The German 1:25000 maps for historical reasons are numbered separately with four digits starting in the north and terminating in the south of the country.

In France, the 1:50000 maps are numbered with arabic numerals for the rows and Roman numerals for the columns, and these are further divided into four sheets of two half-sheets each at 1:25000, numbered from 1 to 8, each bearing the name of the principal town in the 1:50000 sheet.

In Britain, the maps are given the two letter combination from the 100 km national grid, plus the numbers of the lower left coordinates on that grid.

In Belgium the newer 1:50000 maps are numbered sequentially from north to south on with 1:25000 subdivisions with subnumbering. The Lambert projection is a national one and is not compatible with the French projection. The 1924 international ellipsoid is used instead of the Clarke 1880 ellipsoid used in France.

In Switzerland, maps at 1:100000 have two, 1:50000 three, and 1:25000 four digit numbers. The mapping transformation is a unique variant of GK (see below). Luxemburg numbers its maps from 1 to 30 and uses a GK system passing through the meridian of the city of Luxemburg.

Austria bases the zero meridian of its GK mapping system according to the convention of 1634 in the Habsburg monarchy in the westernmost Canary island Fero and numbers the 1:25000 maps in 1:50000 squares from 1 to 4. Since 1976, the 1:25000 maps are merely enlargements of the 1:50000 and have the same numbering system.

Denmark uses arabic numbers for the 1:100000, Roman numerals for the numbering of the 1:50000, and for the 1:25000 subnumbers these as northwest, northeast, southwest and southeast, using a UTM model although the standarised UTM notation is not followed for the map names.

Further details can be found in United Nations (1976), Ewald (1988), and in the excellent lists of large scale maps which are available commercially in the Geo-Katalog Volume 2 from GeoMap Center in Stuttgart. These catalogues also give overviews of the mapping systems of the other Scandinavian and various Mediterranean countries which will not be discussed here. The examples cited here give some idea of the variety of nomenclature and the resulting problems which may arise when this information is used in an archaeological database.

25.2.5 Meridian and latitude strips

Given this map maker's Tower of Babel, it is not surprising that archaeological databases on or across international boundaries are not easily implemented. Even within a given country, searches for sites on adjacent but non-numerically named maps is a major chore. The situation is made even more complicated by the fact that a given transformation may introduce too much distortion in a large country. Therefore both the transverse Mercator and the Lambert Conformal methods break the ellipsoidal surface up into a series of meridian or latitude strips. This is shown for the GK case in Fig. 25.3 for the whole globe. Seen in detail for a given country like West Germany, the strip boundaries at 3° intervals are shown in Fig. 25.4. The strips are numbered beginning at the Greenwich meridian according to the central meridian of the strip, so that West Germany has three strips numbered 2 to 4 corresponding

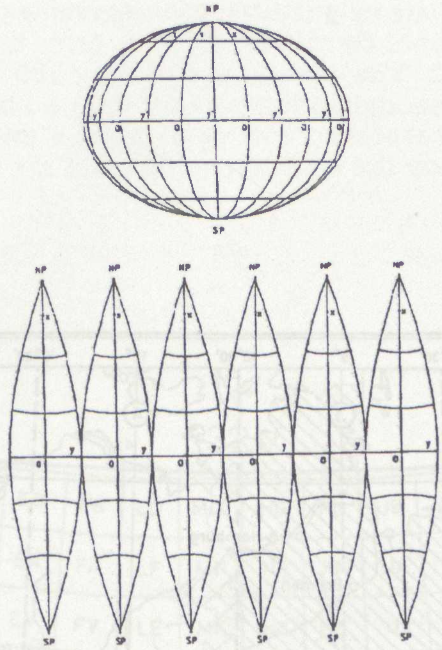

Figure 25.3: Gauss-Krüger Meridian Strips

to the 6°, 9° and 12° strips. At the strip boundaries, the map coordinate numbers jump abruptly and no database query system can find adjacent map coordinates by simple arithmetic. In fact, none of the requirements stated above for distance and coordinate measurements can be met near such strip boundaries. Similarly, the UTM map has an overlapping wedge shaped boundary zone every 6° as shown in Fig. 25.5. Fortunately for the Danes, their country is contained entirely within one UTM strip. Britain is also fortunate in being elongated from north to south, so that only one strip is used tangentially to 2° west of Greenwich. Although computed using the UTM method, the map lettering is incompatible with standard UTM nomenclature. Austria has three strips, with the zero meridian at Fero as mentioned above and is compatible only with Spain (the Habsburg connection!). The Swiss system with a inclined transverse cylinder centered on the old Observatory in Bern is independent of all its European neighbours, (Bolliger 1967). Like that of the Netherlands, it is unique in Europe. Conversion formulae have been worked out to obtain conventional GK and UTM coordinates from the Swiss system (Benzing & Kimmig 1987b). These will not be discussed here.

25.3 Transformations from geographic to grid reference and inversely

25.3.1 Choice of an ellipsoid

Transformation from a national grid system to geographic coordinates and back again requires the use of one of the ellipsoids in Table 25.1. For national use, it is sensible to use the ellipsoid chosen by the national mapping agency, but for international use with data crossing national boundaries, it is more sensible to use one of the international ellipsoids. That of 1924 is the basis for the UTM system and it is also used for a number of national maps. However, that of 1980 is not likely to change very much in the future, being the result of careful satellite measurement of the figure of the Earth (Sigl & Torge 1981). If databases are to be portable so that data from various

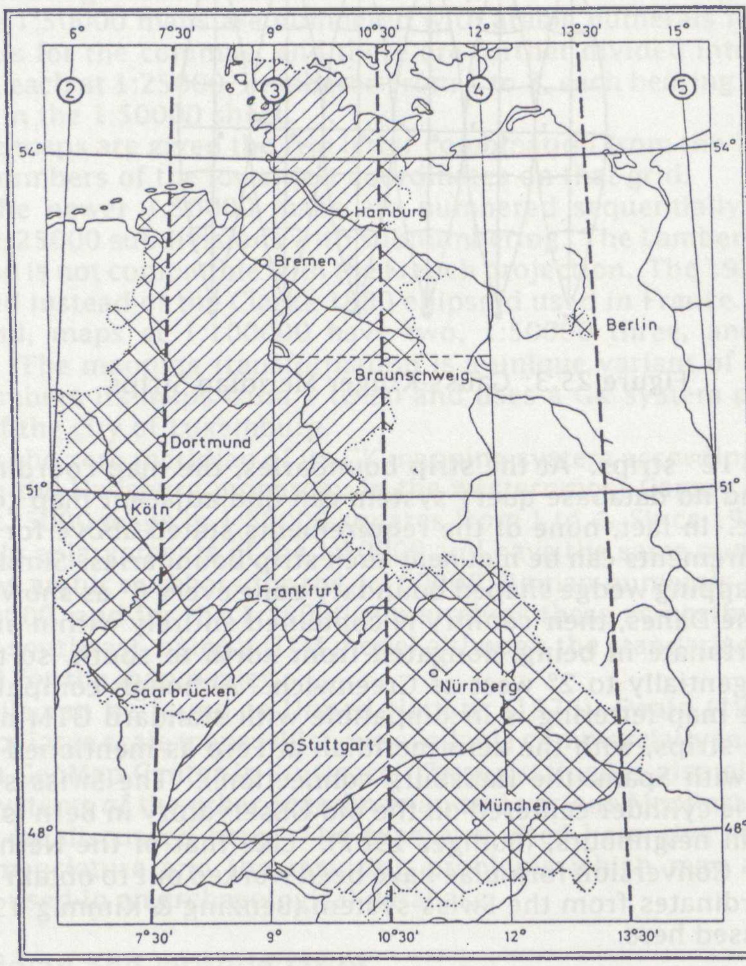

Figure 25.4: The Gauss-Krüger Meridian Strips in Germany

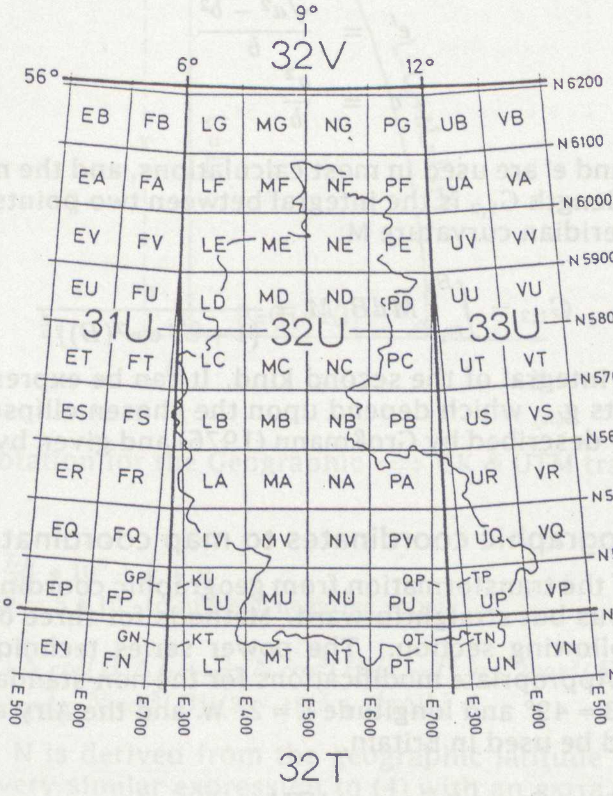

Figure 25.5: The UTM grid in West Germany

countries can be used by their neighbours, then agreement on a common ellipsoid and conversion of the national coordinate system prior to exchange saves a lot of calculation for the recipient. The general use of the 1980 ellipsoid parameters and geographic coordinates is recommended here for data exchange.

The flattening f, first and second eccentricities e and e' and radius of curvature c at the pole of the ellipsoid using the notation of Fig. 25.1 are given by:

$$\begin{aligned} f &= \frac{a-b}{a} \\ e &= \frac{\sqrt{a^2-b^2}}{a} \\ e' &= \frac{\sqrt{a^2-b^2}}{b} \\ c &= \frac{a^2}{b} \end{aligned} \tag{25.1}$$

The squares of e and e' are used in most calculations, and the notation is e^2 and e'^2.

The meridian arc length $G_{1,2}$ is the integral between two points of latitude B_1, B_2 as a function of the meridian curvature M:

$$G_{1,2} = \int_{B_1}^{B_2} M dB, M = \frac{c}{\{1+e'^2\cos^2(B)\}^{\frac{3}{2}}} \tag{25.2}$$

This is an elliptic integral of the second kind. It can be expressed as the sum of a series with constants $g_{(n)}$ which depend upon the chosen ellipsoid, Table 25.1. The derivation is clearly described by Großmann (1976) and given by (Schödlbauer 1982, Vol.2, part 1, p.14).

25.3.2 From geographic coordinates to map coordinates

The computation of the transformation from geographic coordinates to national map coordinates is tedious but straightforward. Methods for three of the major systems are given in the following section. The power series techniques are taken from Schödlbauer 1982. Appropriate modifications for the non-standard UTM system with its zero at latitude B = 49° and longitude L = 2° W. and the Airy ellipsoid coefficients in Table 25.1 should be used in Britain.

25.3.2.1 Geographic to Gauss-Krüger or UTM

The Gauss-Krüger coordinate H (north) is derived from the geographic latitude B and the meridian arclength G(B) following the notation of Fig. 25.6 with H=x:

$$\begin{aligned} H = \;& G(B) + \tfrac{1}{2\rho^2}\bar{N}\cos^4(B)t\Delta L^2 \\ & + \tfrac{1}{24\rho^4}\bar{N}\cos^2(B)t(5-t^2+9e'^2\cos^2(B))\Delta L^4 \end{aligned}$$

with

$\Delta L =$ $L - L_H$ as shown in Fig. 25.6, $t = \tan(B)$

$\bar{N} =$ $\frac{c}{\sqrt{1-e'^2\cos^2(B)}}$ the radius of the curvature normal to the meridian at B

$c =$ the radius of curvature at the pole, Table 25.1 and (Benzing & Kimmig 1987a)

e'^2 as given in Table 25.1 for the chosen ellipsoid

(25.3)

The GK R value is computed as:

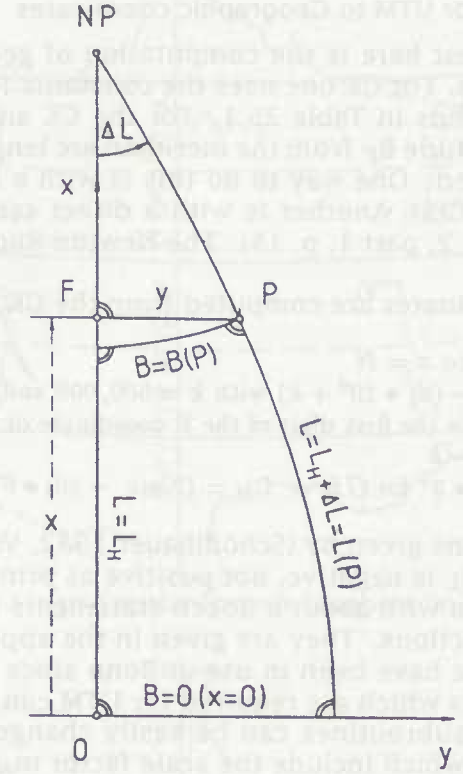

Figure 25.6: Notation for the Geographic $\Longleftrightarrow$ GK & UTM transformations

$$
\begin{aligned}
R = &\ \tfrac{L_H}{3^\circ} \bullet 10^6 + k + y \\
\text{with} &\quad k = 5 \bullet 10^5 \text{ to prevent negative values} \\
\text{and:} & \\
y = &\ \tfrac{1}{\rho} N \cos(B)\Delta L + \tfrac{1}{6\rho^3} N \cos^3(B)(1 - t^2 + e'2cos^2(B))\Delta L^3 \\
& + \tfrac{1}{120\rho^5} N \cos^5(B))(5 - 18t^2 + t^4)\Delta L^5
\end{aligned}
\tag{25.4}
$$

The UTM northing N is derived from the geographic latitude B and the meridian arclength G(B) by a very similar expression to (4) with an extra 6th order term and a scale factor m_H=.9996 which causes the cylinder to cut the ellipsoid along two meridians:

$$
\begin{aligned}
H = &\ m_H G(B) + \tfrac{m_H}{2\rho^2} \bar{N} \cos^2(B) t \Delta L^2 \\
& + \tfrac{m_H}{24\rho^4} \bar{N} \cos^4(B) t (5 - t^2 + 9\, e'^2 \cos^2(B))\Delta L^4 \\
& + \tfrac{m_H}{720\rho^6} \bar{N} \cos^6(B) t (61 - 58\, t^2 + t^4)\Delta L^6 \\
c &\quad \text{and } e'2 \text{ as given in Table 25.1 for the International 1924 ellipsoid}
\end{aligned}
\tag{25.5}
$$

The UTM easting E and Zone number are computed as:

$$
\begin{aligned}
E = &\ y + k \\
\text{Zone} = &\ \tfrac{L_H + 3^\circ}{6^\circ} + 30 \\
\text{and :} & \\
y = &\ \tfrac{m_H}{\rho} N \cos(B)\Delta L + \tfrac{m_H}{6\rho^3} \bar{N} \cos^3(B)(1 - t^2 + e'2\cos^2(B))\Delta L^3 \\
& + \tfrac{m_H}{120\rho^5} N \cos^5(B)\{5 - 18t^2 + t^4 + e'^2 \cos^2(B)(14 - 58t^2)\}\Delta L^5
\end{aligned}
\tag{25.6}
$$

25.3.2.2 Gauss-Krüger or UTM to Geographic coordinates

The main item of interest here is the computation of geographic coordinates from various grid coordinates. For GK one uses the constants for the Bessel, and for UTM, the international ellipsoids in Table 25.1. For the GK and UTM mappings, the first step is to find a base latitude B_F from the meridian arc length G from which the actual latitude can be computed. One way to do this is with a Newton-Raphson approach to obtain the function G(B). Another is with a direct series expansion as given by (Schödlbauer 1982, Vol. 2, part 1, p. 15). The Newton-Raphson method is somewhat simpler (see appendix).

The geographic coordinates are computed from the GK and UTM definitions:

$$\begin{aligned}
x =\ & H \text{ or } x = N \\
y =\ & R - (d_1 \bullet 10^6 + k) \text{ with } k = 500,000 \text{ and} \\
\text{where}\ & d_1 \text{ is the first digit of the R coordinate or} \\
y =\ & E - k \\
L_H =\ & d_1 \bullet 3° \text{ for } GK \text{ or } L_H = (\text{Zone} \ - 30) \bullet 6° - 3° \text{ for UTM}
\end{aligned} \tag{25.7}$$

and the series expansions given by (Schödlbauer 1982, Vol. 2, part 2, p. 80). In that reference, coeffcient $[3]_L$ is negative, not positive as printed.

Two small subroutines with about a dozen statements each accomplish the transformations in both directions. They are given in the appendix for the Gauss-Krüger case in FORTRAN. These have been in use at Bonn since 1981. For GK some of the higher order coefficients which are required for UTM can be left out without serious loss of accuracy. The subroutines can be easily changed for the UTM and British cases by modifications which include the scale factor m_H as shown above. It would be easy to reprogram the routines in C or Pascal and use the code called from within a database. If done on an industry standard PC, a hardware coprocessor is highly recommended not so much for speed, but for the precision of the computation of the trigonometric functions and the availability of an extended floating point data type.

25.3.2.3 Geographic to Lambert Polyconic Conformal

In this section, the French system is used as an example. The Lambert Conformal mapping is applied in other countries with different parameters, but the principles are much the same. The ellipsoid is mapped conformally to a truncated cone which is then mathematically unwrapped. Local angles are preserved well enough, and to minimise linear distortion, strips are used for the same reasons as with GK and UTM. The cone has its apex at some point above the north or south pole, and in all but the smallest countries cuts the ellipsoid along two lines of latitude. The Clarke 1880 ellipsoid used in France is divided into four equal zones defined by a central latitude L_o in gon with slightly different scale factors m_f. The Lambert calculation here is taken from Benzing & Kimmig 1987a.

Zone	*Name*	*$L_o$gon*	*m_f*
I	North	55.0	0.9998773
II	Central	52.0	0.9998774
III	South	49.0	0.9998775
IV	Corsica	46.85	0.9999447

The longitude of Paris (2° 20′ 14″ E. of Greenwich) is used as the central meridian of the map system, Fig. 25.7. In the polar coordinate system the parameters are r and γ, as shown in Fig. 25.8. r is the radius, and γ the angle relative to the central meridan. In contrast with the other mapping systems, X is used for the east-west and Y for the north-south directions. The central meridian is assigned the value 600 (Km) and

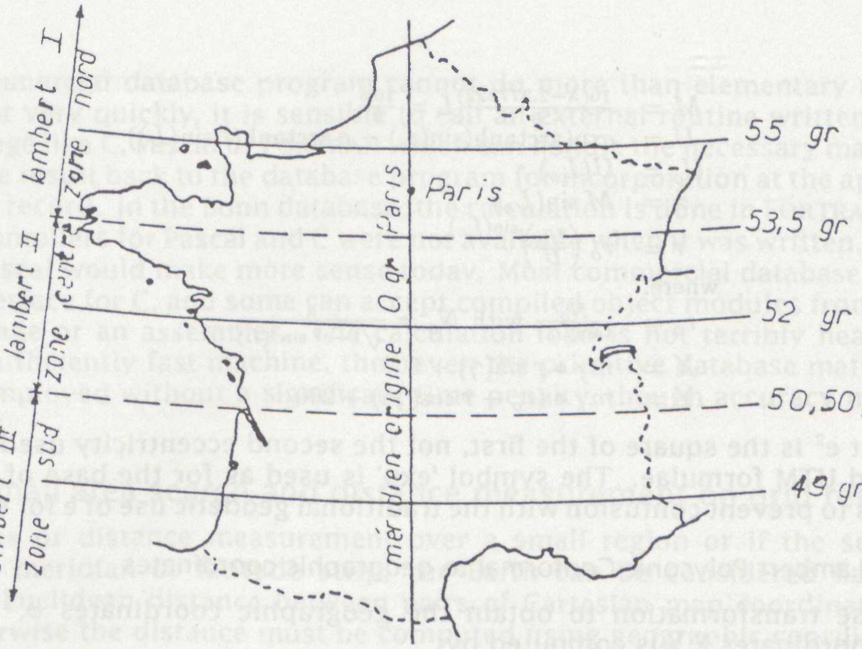

Figure 25.7: Latitude strips for the French Lambert Polyconic system

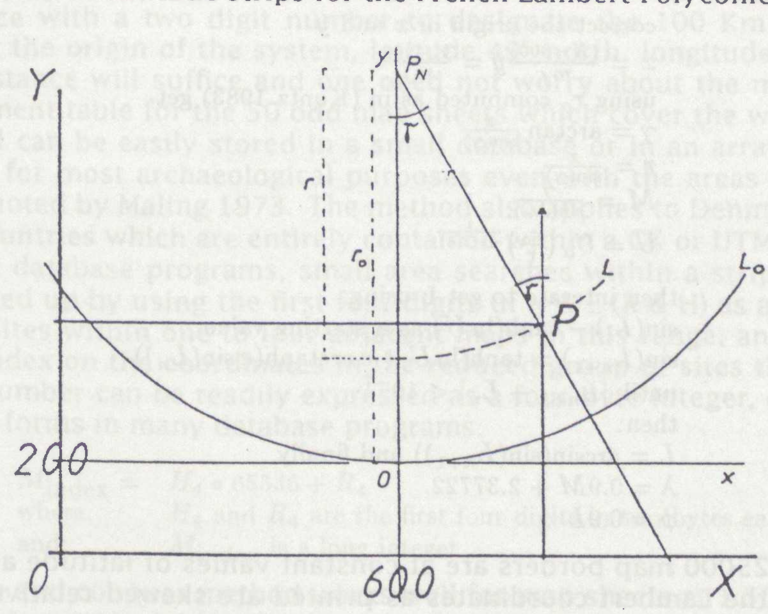

Figure 25.8: Notation for the Geographic $\Longleftrightarrow$ Lambert transformation in France

that of each L_o 200 (Km) to avoid negative values. The French Lambert coordinates X (east), Y (north) of a point with Greenwich longitude λ, and latitude ϕ in decimal degrees are computed as:

$$
\begin{aligned}
M &= \tfrac{10(\lambda-2.377222)}{9} L = \tfrac{10\phi}{9}\\
U &= \exp(\operatorname{arctanh}(\sin(L) - e \operatorname{arctanh}(e\sin(L))\\
U_o &= U(L_o)\\
\gamma &= M\sin(L_o)\\
r &= r_o\left(\tfrac{U_o}{U}\right)^{\sin(L_o)}\\
&\text{where:}\\
r_o &= \tfrac{N_o}{\tan(L_o)} \text{ with } N_o = \tfrac{a}{\sqrt{1-e^2\sin^2(L)}}\\
X &= m_f \bullet r\sin(\gamma) + 600\\
Y &= m_f \bullet (r_o - r\cos(\gamma)) + 200
\end{aligned}
\tag{25.8}
$$

Note that e^2 is the square of the first, not the second eccentricity used so often in the GK and UTM formulae. The symbol 'exp' is used as for the base of the natural logarithms to prevent confusion with the traditional geodetic use of e for eccentricity.

25.3.2.4 Lambert Polyconic Conformal to geographic coordinates

The inverse transformation to obtain the geographic coordinates ϕ, λ from the Lambert coordinates X,Y is computed by:

$$
\begin{aligned}
&\text{correct the origin in } x \text{ and } y\\
&x = \tfrac{(X-600)}{m_f} y = \tfrac{Y-200}{m_f}\\
&\text{using } r_o \text{ computed as in (Kuntz 1983) get}\\
&\gamma = \arctan \tfrac{x}{\sin(\gamma)}\\
&r = \tfrac{x}{\sin(\gamma)}\\
&M = \tfrac{\gamma}{\sin(L_o)}\\
&U = U_o\left(\tfrac{r_o}{r}\right)^{\frac{1}{\sin(L_o)}}
\end{aligned}
\tag{25.9}
$$

$$
\begin{aligned}
&\text{then interate to get L using:}\\
&\sin(L_i) - \tanh(\ln U) \text{ as a starting value}\\
&\sin(L_{n+1}) = \tanh(\ln U + e \operatorname{arctanh}(e\sin(L_n))\\
&\text{until } |L_{n+1} - L_n| < 10^{-7}\\
&\text{then:}\\
&L = \arcsin(\sin(L_{n+1})) \text{ and finally}\\
&\lambda = 0.9M + 2.37722\\
&\phi = 0.9L
\end{aligned}
\tag{25.10}
$$

The French 1:25000 map borders are at constant values of latitude and longitude in gon, so that the Lambert coordinates as printed are skewed relative to the map borders. This makes it somewhat inconvenient to determine site coordinates in X and Y at the same time using a T-Square or a drafting machine, since the map must be rotated. The same is true near the meridian boundaries in the GK and UTM systems.

25.4 Searching databases containing grid references

25.4.1 Incorporating grid reference—geographic coordinate transformation at data entry

The obvious solution to all of these problems is to use ellipsoidal geographic coordinates internally in all data base files, making the conversion from the local map coordinates automatically on data entry. This is done in the Bonn database which the author developed some years ago, (Scollar *et al.* 1986; Scollar 1988). This allows all distance measurements and area or polygon searching to be done without difficulty.

Since a commercial database program cannot do more than elementary arithmetic and that not very quickly, it is sensible to call an external routine written in a high level language like C, Pascal or FORTRAN which can handle the necessary mathematics and pass the result back to the database program for incorporation at the appropriate place in the record. In the Bonn database, the calculation is done in FORTRAN because adequate compilers for Pascal and C were not available when it was written, but doing it in C or Pascal would make more sense today. Most commercial database programs have an interface for C, and some can accept compiled object modules from any high level language or an assembler. The calculation load is not terribly heavy, and if one has a sufficiently fast machine, then even the primitive database math routines might be employed without a significant time penalty, though accuracy may not be satisfactory.

25.4.2 Small area search and distance measurement on grid references

For searches or distance measurement over a small region or if the search area is within a meridian or latitude strip, the Earth can be considered flat. Hence the simple Euclidean distance between pairs of Cartesian map coordinates can be used. Otherwise the distance must be computed using geographic coordinates. For Britain, the simplest method is to replace the two letter map code in the national grid reference with a two digit number to designate the 100 Km maps, starting from zero at the origin of the system, latitude 49° north, longitude 2° west. Then Euclidian distance will suffice and one need not worry about the map boundaries. The replacement table for the 50 odd map sheets which cover the whole of England and Scotland can be easily stored in a small database or in an array. This method can be used for most archaeological purposes even with the areas of considerable distortion quoted by Maling 1973. The method also applies to Denmark, Luxemburg and other countries which are entirely contained within a GK or UTM strip.

With many database programs, small area searches within a strip can be significantly speeded up by using the first four digits of N & E (R & H) as a primary index, find all the sites within one to four adjacent maps in this range, and then search a secondary index on the coordinates in the reduced group of sites thus found. The eight digit number can be readily expressed as a four byte integer, one of the most efficient key forms in many database programs.

$$M_{\text{index}} = H_4 \bullet 65536 + R_4$$

where: H_4 and R_4 are the first four digits in two bytes each (25.11)

and: M_{index} is a long integer

This divide and conquer method works well for map sheets at 1:5000 in Germany. The Km search area is adequate for monuments protection and air photo needs where one wants to find neighbouring sites to a given coordinate pair or zone. The method could also be used with low precision decimal geographic coordinates for quick search over strip boundaries. For 1:25000 maps, use the map number or name of up to four adjacent sheets in a similar way.

25.4.3 Large area search or distance measurements on geographic coordinates

If a strip is crossed, then a simple spherical model of the Earth can be used with modest accuracy. From elementary spherical trigonometry, the distance D depends on the subtended arc δ in Fig. 25.9 and is:

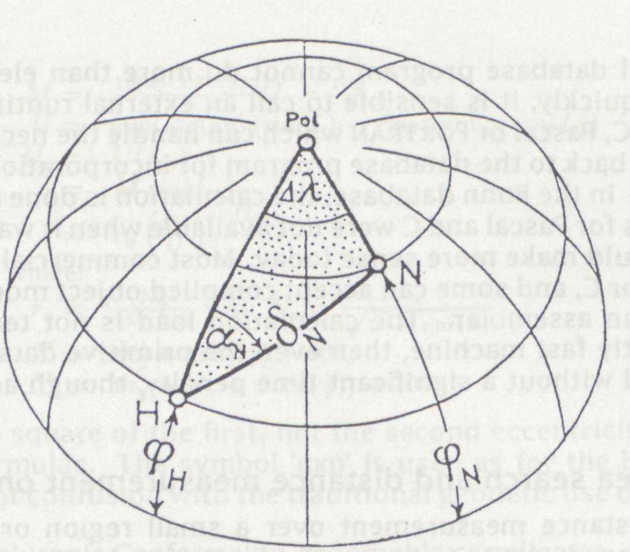

Figure 25.9: Distance calculation on a sphere

$$
\begin{aligned}
D &= \operatorname{arc}(\delta) r \\
\cos(\delta) &= \sin(\phi_H)\sin(\phi_N) + \cos(\phi_H)\cos(\phi_N)\cos(\Delta\lambda) \\
&\text{where } \phi_1, \phi_1 = \text{ latitudes at points H and N} \\
&\text{and } \Delta\lambda = \text{ difference in longitude at points H and N}
\end{aligned}
\tag{25.12}
$$

For points with spacings likely to be of archaeological interest, this expression is numerically unstable. A better method due to Ehlert 1978 is to use:

$$
\begin{aligned}
z_1 &= \cos\left(\tfrac{\Delta L}{2}\right)\cos\tfrac{\Delta B}{2} \quad z_2 = -\cos\left(\tfrac{\Delta L}{2}\right)\sin\left(\tfrac{\Delta B}{2}\right) \\
n_1 &= \sin\left(\tfrac{\Delta L}{2}\right)\sin(B) \quad n_2 = \sin\left(\tfrac{\Delta L}{2}\right)\cos(B) \\
\delta &= 2N \arctan\sqrt{\tfrac{z_2{}^2 + n_2{}^2}{z_1{}^2 + n_1{}^2}}
\end{aligned}
\tag{25.13}
$$

where:

$\Delta B = B_2 - B_1 \quad \Delta L = L_2 - L_1$

N is the radius of the curvature at the average latitude $(B_1 + B_2)/2$:

$$N = \frac{a}{\sqrt{1 + e'^2 \cos^2(B)}}$$

This method will yield results accurate to roughly 0.2% at middle latitudes which suffices for most statistical, plotting and database search purposes. Obtaining near-geodetic accuracy at moderate distances requires only a bit more computation. The ellipsoidal distance can be computed following the original Gauss-Helmert method (Helmert 1880, reformulated by Schödlbauer 1982, Vol. 2, part 1, p. 73) which is more than accurate enough for any archaeological application, ca. 13 cm at 535 km when compared with the high precision method described by Schödlbauer 1980.

For rough distance estimates which may be adequate for some spatial statistical purposes, a spherical model of the Earth can be used for computing the geographic from the GK, UTM or Lambert coordinates. Given the relative complexity of the Ehlert formulae (Schödlbauer 1982) needed for spherical distances there is little gain in speed, and accuracy is lost compared with the Gauss-Helmert ellipsoidal distance. Even on a small PC the computation is nearly instantaneous, so that there is no reason to use spherical calculations as in the past when things were done by hand.

25.4.4 Database for map names or computation of map sheet numbers

Site coordinates are usually incorporated in the database by grid reference. This may require the map sheet name or alphabetical identification. A separate small database with the coordinate reference of the lower left and upper right corners of the map indexed by the map name or identification is useful. If map names are to be found after locating sites, then the same information is also needed. This is quite a lot of work for France, where both the Roman numeral and map name identifiers are needed for the large number of map sheets. It is not worthwhile programming arithmetic for Roman numerals, although some may find this an amusing exercise. A not quite so complex database is required for the alphanumeric scheme used in the Netherlands. In Germany, there is a simple formula which will relate the numbering of the 1:25000 map system to the geographic coordinates in latitude and longitude as well as its inverse. It is given in the appendix.

Similar equations might be set up for the Swiss, Austrian, Belgian, Luxemburg, Spanish and Italian schemes. The Danish combination of numbering, Roman numerals and orientation requires a database which is comparable to that of the French or Dutch systems. The British system requires a simple table or small database giving the letter combinations used. These range from HL to HY, NA to NZ, SC to SZ and TA to TV with a few gaps.

25.5 A map independent storage convention for site and find recording for data exchange

Since rounding errors in distance calculations which involve differences between very large nearly equal values are important, it seems best to store the ellipsoidal coordinates as two fixed point signed integers of four bytes each after computation in extended precision floating point. This permits resolution to 3 cm. anywhere on the Earth's surface, and it is even adequate for small scale find recording. Searching for eight bytes is very rapid with any database programme. An additional four byte integer is used at Bonn as a unique identifier for each item at a given coordinate. In the first record, a single four byte integer records the last unique number assigned. This is referenced before and after adding a record, and if the addition is successful, it is incremented by one. This technique allows erasure of records without requiring renumbering. Record locking in a network or on a shared system is essential to prevent duplicates. If data is entered independently at different physical locations, then blocks of reference numbers can be allocated to the different groups in advance. A further four byte integer is uses as a pointer to join up related sites or finds either dynamically as a result of a common search operation or statically as the result of a spatial clustering algorithm. Using this storage technique with values derived from the International Ellipsoid of 1980 would enable data interchange between all countries, and the notation uniquely identifies all archaeological finds and sites with only 20 bytes.

Acknowledgements

B. Weidner programmed the FORTRAN subroutines given in the appendix for conversions between geographic and GK coordinates and for operations on German map numbers.

The author is indebted to J. Schoppmeyer of the Bonn University Cartographic Institute for calling his attention to an unpublished master's thesis by D. Ewald. It summarises the geodetic properties and nomenclature of all of the readily accessible European mapping systems and contains a copious bibliography which we have freely drawn upon here. Dr. Schoppmeyer also provided me with a number of other references which I have found extremely useful in preparing this paper, and kindly read and corrected the first draft.

For details of the Swiss system together with photocopies of several inaccessible references, I express herewith my gratitude to Jörg Leckebusch, now at the University of Lausanne. The unpublished course notes from the ETH Zürich included a murky program in BASIC for conversions between geographic and Swiss coordinate systems which was easier to follow than some of the references.

Appendix

DEC (VAX, PDP11) Fortran subroutines for the calculation of German Map Sheet numbers, and converting between geographic and Gauss-Krüger coordinates

```
C+------------------------------------------------------------------------------
C
C  T K T G E O  -  CONVERT TK NUMBER TO LONGITUDE/LATITUDE
C
C  CALLING SEQUENCE:
C
C  CALL TKTGEO(ITK,B,L,K)
C
C    ITK - NUMBER OF THE "TOPOGRAPHIC MAP 1:25000"
C    B   REAL*8 LATITUDE OF LOWER LEFT CORNER
C    L   REAL*8 LONGITUDE OF LOWER LEFT CORNER
C    K   - NUMBER OF MERIDIAN STRIP FOR GK COORDINATES
C
C  SIDE LENGTH IS 6' HEIGHT, 10' WIDTH
C
C-------------------------------------------------------------------------------
      SUBROUTINE TKTGEO(ITK,B,L,K)
      REAL*8 B,L
      I=ITK/100
      J=MOD(ITK,100)
      B=-1D-1*I+55.9D0
      L=(J-2)/6D0+6
      K=(J+7)/18+2
      RETURN
      END
C+------------------------------------------------------------------------------
C
C  G E O T T K  -  FIND TK SHEET CONTAINING GIVEN COORDINATE
C
C  CALLING SEQUENCE:
C
C  CALL GEOTTK(ITK,B,L)
C    ITK - TK NUMBER RETURNED
C    B   - REAL*8 LATITUDE OF POINT
C    L   - REAL*8 LONGITUDE OF POINT
C
C-------------------------------------------------------------------------------
      SUBROUTINE GEOTTK(ITK,B,L)
      REAL*8 B,L
      J=6.*( (L-DMOD(L,1./6D0)) - 6. ) + 2
      I=-10.*( (B-DMOD(B,1D-1))   55.9 )
      ITK=100*I + J
      RETURN
      END
C+------------------------------------------------------------------------------
C
C  G E O T G K  -  CONVERT GEOGRAPHICAL TO GAUSS-KRUEGER COORDINATES
C
C  CALLING SEQUENCE:
C
C  CALL GEOTGK(B,L,K)
C    B - REAL*8 LATITUDE OF POINT ON ENTRY, R ON EXIT
C
C    L - REAL*8 LONGITUDE OF POINT ON ENTRY, H ON EXIT
C
C    K - MERIDIAN STRIP NUMBER
C
C-------------------------------------------------------------------------------
      SUBROUTINE GEOTGK(B,L,K)
C
```

```
C     B AND L ARE IN DECIMAL DEGREES
C     K = MERIDIAN NUMBER
C
      IMPLICIT REAL*8 (A-Z)
      INTEGER K
      PARAMETER E2=.0067192188D0     !Second excentricity **2
      PARAMETER RHO=57.2957795131D0 !Degrees -> Radians
      PARAMETER C=6398786.849D0      !Radius of curvature at the pole
      PARAMETER G0=111120.61962D0    !Coefficients for the meridian arc length
      PARAMETER G2=15988.63853D0
      PARAMETER G4=16.72995D0
      PARAMETER G6=0.02178D0
      PARAMETER G8=0.00003D0
C
C     Arithmetic statement function for the meridian arc length
C
      G(B)=G0*RHO*B-G2*DSIN(2*B)+G4*DSIN(4*B)-G6*DSIN(6*B)+G8*DSIN(8*B)
C
C
      B=B/RHO
      CO=DCOS(B)
      ETA2=E2*CO**2
      N=C/DSQRT(1+ETA2)
      T=DTAN(B)
      T2=T**2
C
C     Computation of the longitude difference
C
      Z=CO*(L-3*K)/RHO
      Z2=Z**2
      X=G(B) + N*T*Z2*(0.5 + (5.-T2+9.*ETA2)*Z2/24. )
C
C     Computation of latitude
C
      Y=N*Z*( 1. + Z2*((1.-T2+ETA2)/6. + (5.-18.*T2+T2**2)*Z2/120. ))
C
C     Return results in the input variables
C
      B=K*1D6+5D5+Y
      L=X
      RETURN
      END

C+-----------------------------------------------------------------------------
C
C   G K T G E O  -  CONVERT GAUSS-KRUEGER TO GEOGRAPHICAL COORDINATES
C
C   CALLING SEQUENCE:
C
C   CALL GKTGEO(R,H)
C     R - "R" VALUE, LATITUDE UPON RETURN
C     H - "H" VALUE, LONGITUDE UPON RETURN
C
C------------------------------------------------------------------------------
      SUBROUTINE GKTGEO(R,H)
      IMPLICIT REAL*8 (A-Z)
      INTEGER K
      PARAMETER E2=.0067192188D0
      PARAMETER RHO=57.2957795131D0
      PARAMETER C=6398786.849D0
      PARAMETER G0=111120.61962D0
      PARAMETER G2=15988.63853D0
      PARAMETER G4=16.72995D0
      PARAMETER G6=0.02178D0
      PARAMETER G8=0.00003D0
C
C     Computation of the meridian arc length and its derivative with
```

```
C     arithmetic statment functions
C
      G(B)=G0*RHO*B-G2*DSIN(2*B)+G4*DSIN(4*B)-G6*DSIN(6*B)+G8*DSIN(8*B)
      DG(B)=G0*RHO-2.*G2*DCOS(2*B)+4.*G4*DCOS(4*B)-6.*G6*DCOS(6*B)+8.*G8*DCOS(8*B)
C
C     Newton - Raphson method for meridian arc -> BF
C
      BF=H/C                          !Starting value for the iterations
10    BS=BF
      BF=BS-(G(BS)-H)/DG(BS)
      IF(DABS(BF-BS).GE.1D-9) GO TO 10
C
C     Decompose R
C
      L=IFIX(SNGL(R*1D-6))
      Y=R-L*1D6-5D5
      L=3*L
C
      CO=DCOS(BF)
      ETA2F=E2*CO**2
      Z=Y/(C/DSQRT(1+ETA2F))
      Z2=Z**2
      TF=DTAN(BF)
      T2F=TF**2
C
      BS=BF-TF*Z2*( (1.+ETA2F)/2. + (5.+3.*T2F+6.*ETA2F-6.*ETA2F*T2F)*Z2/24. )
      DL=RHO/CO*Z*( 1. - Z2*( (1.+2.*T2F+ETA2F)/6. +
     X                (5.+28.*T2F+24.*T2F**2)*Z2/120. ) )
C
C     An additional term may be included here for higher accuracy if desired.
C     It must be included when modifying for UTM.
C
C     Return with results in the input variables
C
      H=L+DL
      R=BS*RHO
      RETURN
      END
```

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