EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

University of Tübingen
Working Papers in
Economics and Finance

No. 33

The Home Market Effect, Regional Inequality, and Intra-Industry Reallocations

by

Gabriel Felbermayr & Benjamin Jung

Faculty of Economics and Social Sciences
www.wiwi.uni-tuebingen.de

The Home Market Effect, Regional Inequality, and Intra-Industry Reallocations*

Gabriel Felbermayr† and Benjamin Jung‡

April 4, 2012

Abstract

In New Trade Theory models, the larger region hosts an overproportionate share of producers. This Home Market Effect (HME) exacerbates regional income discrepancies caused by trade frictions or technology differences. With homogeneous firms, it requires *inter*-industry reallocations to emerge. We present a heterogeneous firms single-sector model with fixed market access costs, in which the HME arises exclusively from empirically more relevant *intra*-industry reallocations. It is magnified by lower trade costs or higher heterogeneity. In contrast to multi-industry models, a more pronounced HME leads to regional income convergence as adjustment of the firm size distribution counteracts the effects of firm entry.

JEL-Classification: F12, R12.

Keywords: Home Market Effect; Regional Inequality; Monopolistic Competition; Heterogeneous Firms; Economic Geography.

---

*We are grateful to Costas Arkolakis, Daniel Bernhofen, Johannes Bröcker, Jonathan Eaton, Carsten Eckel, Peter Egger, Udo Kreickemeier, Mario Larch, Thierry Mayer, Horst Raff, and Tobias Seidel for discussion and suggestions, as well as to seminar participants at the Munich-Tübingen International Economics Workshop 2011, the GEP-ifo conference 2011, and at the Universities of Bielefeld, Kiel, and Mainz for comments.

†Ifo Institute–Leibniz Institute for Economic Research, Poschingerstraße 5, 81679 Munich, Germany; LMU Munich; CESifo; GEP; felbermayr@ifo.de.

‡University of Tübingen, Economics Department, Nauklerstraße 47, 72074 Tübingen, Germany, benjamin.jung@uni-tuebingen.de.

1 Motivation

Multi-industry *New Trade Theory* models predict that lower trade frictions invite firms to more strongly cluster in the larger market, exacerbating the natural advantages conferred by market size (Helpman and Krugman, 1985).[1] This result builds on reallocation of resources *between* industries. The larger region increasingly specializes on the increasing-returns sector while the smaller region specializes more strongly on the homogeneous outside good. Central to this argument is the so-called *Home Market Effect* (HME), by which the bigger economy hosts a more than proportionate share of producers, and the *Home Market Magnification Effect* (HMME), by which the HME is magnified when trade costs fall.[2] Both HME and HMME are important building blocks of economic geography models.

However, a large body of empirical literature shows that lower trade costs mostly induce resource reallocation *within* industries, not between them.[3] This evidence begs the question as to whether a HME can arise from intra-industry reallocations alone, and how, if it exists, the HME interacts with trade costs in shaping regional per capita income discrepancies. To provide an answer, we work with an asymmetric Melitz (2003) *single*-sector model where monopolistically competitive producers differ with respect to productivity and there are fixed costs of market access. We show that intra-industry reallocation generates a HME in complete absence of cross-sector effects. The HME arises because market size differences affect the firm size distribution *within* a single sector. When reallocations are exclusively intra-industry, lower trade costs do not lead to divergence of real incomes across regions but to *convergence*. More pronounced productivity dispersion enforces this process.

[1] E.g., a larger fraction of consumed goods is subject to trade costs, and, so, the price index tends to be lower.

[2] The presence of a HME is a distinctive characteristic of models, where monopolistic firms produce variants of a differentiated goods under increasing returns to scale and where the number of firms adjusts due to free entry; when the number of goods is fixed, the larger region suffers a disadvantage due its lower terms of trade.

[3] Empirical evidence on developing countries includes Haltiwanger, Kugler, Kugler, Micco, and Pages (2004), who "*observe that, for the whole sample and for each country, most of sector reallocation is within sector*" (p. 200) and Wacziarg and Wallack (2004), who find that "*trade liberalization has far smaller effects on intersectoral labor shifts than is often presumed*" (p. 413). In a linked employer-employee data set on Brazilian firms, Menezes-Filho and Muendler (2011) report that "*worker flows within sectors are consistent with the idea that reallocations between employers in an industry are dominant*" (p. 12).

Similar results hold for developed countries. Using U.S. data, Bernard, Jensen, and Schott (2006) show "*that these aggregate [productivity] gains [from trade liberalization] are driven by a reallocation of activity toward more productive plants within industries*" (p. 934).

We develop a new graphical device based on a *market crowding* curve and a *market potential* curve. The tool allows for easy comparative statics despite country-level heterogeneity in endowments and technology. In particular, the much debated but often introduced *linear outside sector*, which leads to factor price insensitivity, can be easily dispensed with. We find that both the HME and regional income inequality can be characterized by a single endogenous variable, the relative probability of successful innovation´.

As in the Helpman and Krugman (1985) model, a region commanding a larger share of world population is a more attractive business location. However, in our model, there are two margins on which higher demand for labor is accommodated: first, the relative wage of the region goes up; second, the average size of firms goes down. The presence of a linear outside sector would mute the first channel, thereby exaggerating the size of the HME. If firms are homogeneous, firm size cannot adjust, and the wage rises until the number of active firms is exactly proportional to the labor force.[4]

Importantly, the upward adjustment in wages entailed by the *crowding* of firms in the larger region, does not fully offset the advantages of increased *market potential.* Home market size is particularly relevant for domestic firms whose competitive cost disadvantage relative to foreign firms is attenuated by trade costs. Importantly, while the mass of firms *attempting entry* is strictly proportional to the endowment size, the likelihood of a given firm to *successfully* cover its fixed costs is greater in the larger market.

Firm-level heterogeneity, selection into markets, and regional asymmetries are crucial for the emergence of a single-sector HME. Without asymmetries in regional size, intra-industry reallocations are the same in all regions, leaving no room for a HME. If there is no selection into the domestic market, the mass of firms active in a region is not affected by trade, which again does not induce a HME. In the absence of fixed exporting costs, trade does not trigger reallocation across firms. Fixed export costs alone, however, do not suffice (Venables, 1994; Medin, 2003). In contrast, the single-sector HME and income convergence rely on the fact that some firms do not export. Their share is endogenous and positively related to the relative size of regions. When sorting becomes more pronounced due to a larger degree of

[4]So, there is no HME when firms are homogeneous, as in the single-sector Krugman (1980) model.

productivity dispersion, or lower trade costs, the HME becomes stronger.

In a single-sector model with inelastic labor supply, variety cannot be increased without lowering aggregate productivity. This trade off does not arise in models with elastic labor labor. For instance, in the Helpman-Krugman (1985) framework, labor is drawn from a linear outside sector, which allows for increasing variety without reducing labor demand per firm. As a consequence, magnification of the HME through lower trade costs or higher productivity dispersion is coupled with convergence of real per capita income.

Our work is related to at least four important strands of literature. First, following Krugman (1980), much work has examined the generality of the HME prediction. The HME appears robust to assuming a non-CES demand structure (Ottaviano and van Ypersele, 2005), oligopolistic rather than monopolistic market structure (Feenstra, Markusen, and Rose, 2001), many differentiated industries (Hanson and Xiang, 2004), more than two regions (Behrens, Lamorgese, Ottaviano, and Tabuchi, 2009), or firm-level heterogeneity (Demidova, 2008; Okubo et al., 2010).[5] In contrast, the assumption of a linear outside sector and the implied factor price insensitivity have been much debated. If the wage does adjust, the HME can disappear (Head and Mayer, 2004). Similarly, trade costs in the outside sector can make the HME go away, too (Davis, 1998). Crozet and Trionfetti (2008) have qualified this prediction. They introduce Armington differentiation and trade costs into the outside sector, nesting Helpman and Krugman (1985) and Davis (1998). Their numerical results suggest that the HME survives but becomes non-linear. All those papers rely on some form of inter-industry reallocation. We seem to be the first to study in full analytical detail how a HME can arise from intra-industry reallocation and how it relates to regional per capita income inequality.

Second, due to its prevalence in models of *increasing returns to scale*, the HME has been used as a discriminating criterion to test for the validity of New Trade Theory in empirical work (Davis and Weinstein, 1999, 2003). A number of prominent empirical papers are Feenstra et al., (2001), Head and Ries (2001), Davis and Weinstein (2003), and Hanson and Xiang (2004). In their survey, Head and Mayer (2004) conclude that "*The evidence on HMEs accumu-*

[5]Also, as shown by Helpman and Krugman (1985), one can have more than one factor of production if the production technology is homothetic.

lated in those papers is highly mixed". More recent research finds stronger results in favor of the HME; see Crozet and Trionfetti (2008) or Brülhart and Trionfetti (2009). The existence of a single-sector HME is important for empirical work. Against the background of the Helpman and Krugman (1985) model, rejection of the HME prediction could be interpreted as failure of the linear outside sector assumption or, more broadly, of the relevance of inter-industry resource reallocation. Moreover, we show that empirical work based on industry-data should control for technology levels and for the degree of productivity heterogeneity.[6]

Third, the HME is core to *New Economic Geography* (NEG) models, where regional per capita inequality results from from market crowding and market potential effects and their interaction with regional factor mobility, see, e.g., Fujita, Krugman, and Venables (1999) or Head and Mayer (2004) for surveys. Inter-industry reallocation is crucial in the typical NEG models. We abstract from factor mobility but allow for interregional technology differences. Similar to NEG, our analysis is motivated by the strong evidence on the role of market size for regional per capita incomes, see Redding and Venables (2004) for cross-country evidence and Hanson (2005) for evidence on U.S. regions.

Finally, we relate to the growing literature on the asymmetric Melitz model. So far, most papers have used the outside sector simplification.[7] Recently, Demidova and Rodríguez-Clare (2011) use a small economy Melitz (2003) model to show that eliminating the assumption of an outside sector reverses the result in Melitz and Ottaviano (2008) or Demidova (2008), where a region that unilaterally lowers trade costs experiences a decline in welfare.[8] Their framework bears resemblance to ours; however, we analyze the case of two *large* economies. Arkolakis, Costinot, and Rodríguez-Clare (2012) discuss a general class of models encompassing the Melitz framework for asymmetric countries or regions. Their ambition is not to fully characterize endogenous variables in terms of exogenous ones, but to demonstrate the validity of a simple welfare function that is isomorphic across different models with and without firm-level heterogeneity. They argue that heterogeneity and selection matter less for aggre-

[6]To make this claim, we alternatively define a HME in demand shares, as is customary in the empirical literature.

[7]Prominent examples are Helpman, Melitz and Yeaple (2004), Grossman, Helpman and Szeidl (2006), Baldwin and Forslid (2010), and Ossa (2011).

[8]Demidova and Rodríguez-Clare (2009) use this framework to analyze trade policy.

gate welfare than what has been hitherto believed. In contrast, for our result, without both heterogeneity and selection the HME disappears. Bernard, Redding, and Schott (2007) embed intra-industry reallocation in a Heckscher-Ohlin model of comparative advantage. Their focus is on comparisons across sectors and factors, while we are interested in regional per capite income inequality. They do not discuss the effect of changes in productivity dispersion; we do. Their analytical results refer to the transition from autarky to free trade; we present analytical results for gradual trade liberalization.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 proves the existence of the HME and shows that lower trade costs or higher productivity dispersion magnifies the HME and leads to per capita income convergence. Section 4 provides a quantitative illustration. Section 5 discusses several extensions. Section 6 concludes. Details to the derivations are relegated to the Appendix.

2 The Model

2.1 Basic environment

The model extends Melitz (2003) to the case of two large asymmetric regions, indexed by $i \in \{H, F\}$. Each region is populated by L_i identical households. Without loss of generality, we assume that the distribution of world endowments is such that $\lambda \equiv L_H/(L_H + L_F) \geq 1/2$. Labor is the only factor of production. Each household *inelastically* supplies one unit of labor.[9] We will denote wages by w_i. The representative consumer has standard Dixit-Stiglitz preferences over a continuum of differentiated varieties

$$U_i = \left[\int_{z \in \Omega_i} q_i [z]^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

[9]In the standard Krugman (1980) framework, Fujita et al. (1999) consider a two-sector model with flexible elasticity of labor supply to the differentiated good sector. With perfectly elastic labor supply, the HME always appears, but if we approach the perfectly inelastic labor supply case, the HME will be reversed for some level of trade costs; see Head and Mayer (2004), p. 29f.

where the measure of the set Ω_i is the mass of available varieties, $q_i[z]$ is the quantity of variety z consumed, and $\sigma > 1$ is the elasticity of substitution.

Firms compete monopolistically in a *single* sector. After paying fixed innovation costs $w_i f^e$, they obtain information about their productivity level φ which is sampled from a Pareto distribution whose c.d.f. is given by $G_i[\varphi] = 1 - (b_i/\varphi)^\beta$. The shape parameter β is inversely related to productivity dispersion and fixed in both regions.[10] The minimum admissibly productivity level b_i potentially differs across regions. Without loss of generality, we assume $B \equiv b_H/b_F \geq 1$.[11] Output is linear in φ.[12] A firm in region i pays fixed market access costs $w_i f_{ij}$ to serve consumers in region j. Selection implies that a firm does not necessarily serve both markets. For simplicity, we assume $f_{ij} = f_{ji} = f^x$ and $f_{ii} = f_{jj} = f^d$. As usual, exporting involves symmetric iceberg trade costs $\tau_{ij} = \tau_{ji} = \tau \geq 1$, where $\tau_{ii} = 1$. Then, $\tau_{ij} w_i/\varphi$ is the marginal cost of producing one unit of output in i and selling to j.

2.2 Equilibrium conditions

The first set of equilibrium conditions are *zero cutoff profit conditions.*[13] They pin down the minimum productivity level φ^*_{ij} required for a firm in region i to make at least zero profits by selling in region j. Since we have two regions, there are *four* of those conditions:

$$\frac{R_j}{\sigma}\left(\frac{\rho P_j}{\tau_{ij} w_i}\varphi^*_{ij}\right)^{\sigma-1} = w_i f_{ij}, \quad i \in \{H, F\}, j \in \{H, F\}, \tag{2}$$

where $\rho = (\sigma - 1)/\sigma \in (0, 1)$ is the inverse of the mark-up. Since we assume balanced trade, aggregate expenditure R_j is equal to national income $w_j L_j$. The left hand side of equation (2) denotes profits of a firm with labor productivity φ^*_{ij}. They are proportional to aggregate

[10]The Pareto distribution is a standard assumption in the literature, see, e.g., Helpman et al. (2004), Falvey et al. (2006), Egger and Kreickemeier (2009), and many others. In extensions we show that our core results hold under a general productivity distribution.

[11]B is the relative productivity bound. Given symmetry in the shape parameter, $B > 1$ implies that the *ex ante* productivity distribution in Home stochastically dominates the productivity distribution in Foreign. B is therefore a measure of the technology gap.

[12]Note that each variety z is produced by a single firm with productivity level φ. We henceforth index varieties by φ.

[13]The equilibrium conditions are derived in detail in section A.1 of the Appendix. See also Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008).

profits R_j/σ. Firm-level profits increase in the price level P_j as the firm's competitive position there is improved; they decrease in w_i for the opposite reason. The right hand side denotes the value of fixed market entry costs. The price index is given by

$$P_i^{1-\sigma} = \theta \sum_{j \in \{H,F\}} m_{ji} M_j \left(\frac{\rho \varphi_{ji}^*}{\tau_{ji} w_j} \right)^{\sigma - 1}, \tag{3}$$

where M_i denotes the (endogenous) mass of active producers located in region i, $m_{ij} \equiv \left(1 - G\left[\varphi_{ij}^*\right]\right)/p_i^{in}$ is the fraction of firms which serve market j, and $\theta \equiv \beta/(\beta - (\sigma - 1))$ is a strictly positive constant.[14]

Note that $p_i^{in} \in (0,1)$ denotes the probability of successful innovation.[15] Empirical evidence suggests that only the most productive firms export. The model reproduces this stylized fact if parameters are such that $\varphi_{ij}^* > \varphi_{ii}^*$. Then,

$$p_i^{in} = 1 - G\left[\varphi_{ii}^*\right].$$

We refer to this situation as to the case of *conventional sorting.* Unconventional sorting obtains if Home becomes very large relative to Foreign. For given fixed costs, the sorting condition can reverse: then, only the more productive foreign firms serve the small foreign market, so $p_F^{in} = 1 - G\left[\varphi_{FH}^*\right]$. In line with the evidence, for the main part of the paper we assume that conventional sorting holds.[16] This happens in equilibrium, if Home's share in the world labor endowment is not too big, i.e., if $\lambda < \bar{\lambda} \equiv \lambda\left[B, \eta, \beta, \rho; f^x/f^d\right]$, where $\eta \equiv \tau^{-\beta}\left(f^x/f^d\right)^{1-\beta/(\sigma-1)} \in (0,1)$ is a measure of the freeness of trade.[17]

The second set of equilibrium conditions are *free entry conditions.* In each region, firms invest fixed setup costs until expected profits from entering $(\theta - 1) w_i \sum_j m_{ij} f_{ij}$ are equal to innovation costs discounted by the probability of successful innovation p_i^{in}. The two free

[14]This is to ensure that the variance of the size distribution is finite.

[15]Innovation is successful if a firm in country i draws a productivity φ which allows to make non-negative profits on at least one market.

[16]We relax this assumption in Section 5.1.

[17]The function $\lambda\left[B, \eta, \beta, \rho; f^x/f^d\right]$ is characterized in Appendix A.2.4. A sufficient condition for $\eta < 1$ is $f^x > f^d$.

entry conditions therefore are

$$(\theta-1)p_i^{in}\sum_{j\in\{H,F\}} m_{ij}f_{ij}=f^e. \tag{4}$$

Note that wages have dropped out from this condition.

Finally, there are two *labor market clearing conditions*. With the above equilibrium conditions and using the Pareto distribution, they simplify to

$$M_i=p_i^{in}L_i\frac{\rho}{\beta f^e}, i\in\{H,F\}, \tag{5}$$

where the mass of active producers in region i, M_i, is linked to labor supply L_i and the likelihood of successful entry p_i^{in}.[18]

Summarizing, we have four zero cutoff profit conditions (2), two free entry conditions (4), and two labor market clearing conditions (5) to pin down eight unknown endogenous variables of the model $\{\varphi^*_{HH},\varphi^*_{FF},\varphi^*_{HF},\varphi^*_{FH};M_H,M_F;w_H,w_F\}$. Knowledge of these equilibrium objects allows determining p_i^{in} and m_{ij}. In the following, we use labor in Foreign as the numeraire and denote $w_H/w_F\equiv\omega$ as the relative wage.

2.3 Intra-industry reallocation

Notice that the labor market clearing condition (5) implies a positive link between the mass of firms and endowments. In the closed economy, the probability of successful entry does not depend on size, and there is a one-to-one relationship between the mass of firms and endowments. It is well-known that opening up to trade induces reallocation of resources to more productive firms. When the average firm is more productive, it charges a lower price and employs more resources. As labor supply is inelastic, the mass of active firms must decline. We show that these reallocation effects are asymmetric if regions differ in size or technology, which has interesting implications for the spatial selection of firms.

Notice that export fixed costs are crucial for this result. In their absence, the probability

[18]We are interested in the mass of active producers, not in the mass of firms that attempt entry $M_i^e=M_i/p_i^{in}$.

of successful innovation is independent of region size, which results in a one-to-one relationship.[19] Interestingly, intra-industry reallocation is inactive in the presence of a linear outside sector. In that case, the labor market clearing condition reads

$$M_i = p_i^{in} \xi_i L_i \frac{\rho}{\beta f^e}, i \in \{H, F\},$$

where ξ_i is the endogenous share of labor devoted to the differentiated good sector and p_i^{in} is *independent* of region size.[20] So, with the outside sector, the overproportional relationship between M_i and L_i results exclusively through *inter*-industry reallocation: the larger region devotes a larger share of labor to the differentiated good sector, and the HME is completely independent from heterogeneity and export selection.

3 The single-sector Home Market Effect and regional inequality

In this section we develop a simple graphical tool to discuss the emergence of a HME in our asymmetric two-region framework, and to conduct comparative statics with respect to trade costs or the firm-level productivity dispersion parameter.

3.1 Endowment and technology differences

We characterize the equilibrium of the asymmetric Melitz model with the help of two separate equilibrium conditions in a diagram with the relative wage ω on the x-axis and the relative probability of successful innovation $\chi \equiv p_H^{in}/p_F^{in} = \left(\frac{\varphi_{FF}^*/b_F}{\varphi_{HH}^*/b_H}\right)^\beta$ on the y-axis. Notice that this probability is inversely related to the relative competitiveness of Home's domestically active firms. χ will turn out to be the key variable driving regional inequality.

Lemma 1 *In a two-region single-sector Melitz (2003) model with Pareto distributed productivities and $\lambda \geq 1/2$ and $B \geq 1$, the equilibrium exhibits conventional sorting if $\lambda < \bar{\lambda} \equiv$*

[19] Appendix E shows that in the absence of export selection, the probability of successful innovation is independent of relative country size. This result generalizes Melitz' (2003) remark on the role of export fixed costs for intra-industry reallocations to asymmetric countries; see his footnote 24.

[20] See Appendix D for details.

$\lambda\left[B,\eta,\beta,\rho;f^x/f^d\right]$. *There always exists a unique equilibrium at the intersection between a strictly downward-sloping convex* ***market crowding*** *curve (MCC):*

$$\chi=\frac{\lambda}{1-\lambda}B^{2\beta}\omega^{-\frac{2\beta-\rho}{\rho}}, \tag{6}$$

and a strictly increasing convex (the latter under mild parameter restrictions) ***market potential*** *curve (MPC):*

$$\chi=\frac{1-\eta B^{\beta}\omega^{-\frac{\beta}{\rho}}}{1-\eta B^{-\beta}\omega^{\frac{\beta}{\rho}}},\eta B^{-\beta}\omega^{\frac{\beta}{\rho}}\in(0,1). \tag{7}$$

Proof. In the Appendix. ■

Lemma 2 *The region with the larger labor force or the leading technology pays the higher wage and has the higher probability of successful innovation.*

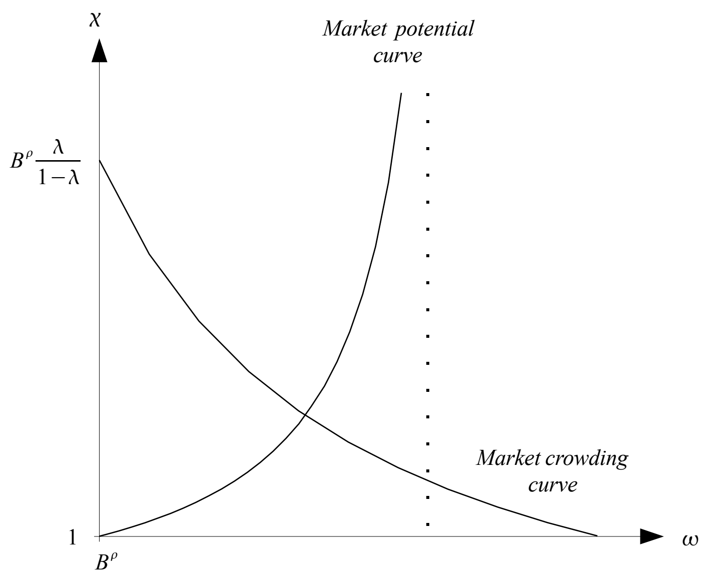
Proof. Follows from Lemma 1. ■

Market crowding curve. Under the assumption of conventional sorting, the market crowding condition curve (6) is obtained by combining all four zero cutoff profit conditions (2) and the balanced trade condition. While balanced trade is implicitly given by representative agents in both regions being on their respective budget constraints, we now make it explicit. Balanced trade can be written as $M_H\bar{r}_{HF}=M_F\bar{r}_{FH}$, where $M_j\bar{r}_{ji}$ denotes aggregate sales of firms located in region j in market i. One can show that $\bar{r}_{ij}=\sigma\theta w_i m_{ij} f_{ij}$. Using the definition of m_{ij} along with equation (5), the relative wage ω appears as a function of region size and technology differences and the relative export productivity cutoffs:

$$\omega=\frac{1-\lambda}{\lambda}B^{-\beta}\left(\frac{\varphi^*_{FH}}{\varphi^*_{HF}}\right)^{-\beta}. \tag{8}$$

The MCC is downward-sloping and convex. At $\omega=B^{\rho}$, we have $\chi=\lambda\left(1-\lambda\right)^{-1}B^{\rho}$. Figure 1 illustrates the locus. We refer to it as the market crowding curve (MCC): if ω increases, Home's relative labor costs go up and it becomes a less attractive location for production. In turn, the domestic entry cutoff φ^*_{HH} relative to φ^*_{FF} has to go up. So, the likelihood of successful innovation falls. The MCC curve illustrates a dispersion force, i.e., a negative equilibrium correlation between relative costs and locational advantage.[21]

Figure 1: Equilibrium of the two-region Melitz (2003) model



Market potential curve. Equation (7) constitutes a second relationship between the relative probability of successful innovation χ and the relative wage ω. The derivation starts from the free entry conditions (4). It employs equations (2) to eliminate productivity cutoffs and makes use of balanced trade (8). The curve is strictly upward-sloping. It features an asymptote at $\omega = B^{\rho}\eta^{-\frac{\rho}{\beta}} > 1$ and goes through the point $(B^{\rho}, 1)$. The MPC is convex if $\eta > \rho/(2\beta + \rho)$; a sufficient condition for this is $\eta > 1/3$.[22] Figure 1 illustrates the locus. We refer to this schedule as to the market potential curve (MPC): if ω increases, Home's relative income goes up so that Home's market potential increases. This makes entry of firms more attractive, the domestic entry cutoff φ^{*}_{HH} relative to φ^{*}_{FF} has to fall. So, the likelihood of successful innovation goes up. The MPC illustrates an agglomeration force, i.e., a positive equilibrium correlation between

[21] The MCC takes an *ex post* perspective in that it summarizes firm behavior after the resolution of uncertainty about productivity.

[22] The analysis of Section 4 implies that this requirement is likely to be met in all reasonable circumstances.

relative market potential and locational advantage.[23]

Wage inequality. Figure 1 shows that when regions are asymmetric in size, we have $\omega > 1$. The intuition for this result is simple: at given factor costs, firms find it more profitable to produce in the larger market as this minimizes payments of variable trade and market access costs. To keep labor employed in both regions, this advantage must be offset by a wage differential.[24] A similar logic applies when regions differ in terms of technology. At given factor costs, firms would prefer to produce in the market with the better technology as this reduces marginal costs. Again, full employment requires that this advantage is offset by a wage differential.

Reallocation effects. While the wage result is well-known from the single-sector Krugman (1980) framework, the model with heterogeneous firms and export selection features a second channel through which the model adjusts to regional size or technology differences: the richer region exhibits a higher probability of successful innovation, $\chi > 1$. Put differently, domestic firms in the large region are less competitive compared to their counterparts in the small region. The intuition is that the reallocation effect is stronger in the small or technologically backward region, which is more open to trade. Presence in the large region is particularly valuable for firms with intermediate productivity levels. Since they do not export, the higher wage in Home puts them at a competitive disadvantage in Home but not in Foreign. The adjustment of entry margins makes it possible for the large economy to host firms with lower productivities.

Home Market Effect. The asymmetric reallocation effect has important implications for the mass of domestically producing firms. From equation (5) we know that the mass of active firms in each region is proportional to the labor force times the probability of successful in-

[23] In contrast to the MCC, the MPC takes an *ex ante* perspective in that it relates to potential firms' decisions to sink setup costs and learn about their productivities.

[24] This prediction is also present in the single-sector Krugman (1980) model, where the wage rate follows from equating (6) and (7) and setting $\beta \to \sigma - 1$; see Burstein and Vogel (2011) on how Melitz (2003) nests the single-sector Krugman (1980) model. In the single-sector Krugman (1980) model, however, a HME cannot arise.

novation. In relative terms, we have $M_H/M_F = \chi L_H/L_F$. Hence, the larger region hosts relative more firms than would be consistent with the relative size of the region. We can also write Home's share of firms as a function of the relative probability of successful innovation χ and of Home's labor share λ:

$$\phi \equiv \frac{M_H}{M_H + M_F} = \gamma\lambda, \text{ with } \gamma \equiv \frac{\chi}{1 + \lambda(\chi - 1)}. \tag{9}$$

Clearly, γ increases in χ and falls in λ. Before we proceed, we need a precise definition:

Definition 1 *A **home market effect** exists, if the share of firms located in Home is larger than Home's share in the world labor endowment, i.e., if $\phi > \lambda$.*

Our definition of the HME conforms with the literature.[25] With $\lambda > 1/2$, equation (9) and Definition 1 imply that

$$\gamma > 1 \Leftrightarrow \chi > 1. \tag{10}$$

So, a *HME* exists if and only if the probability of successful innovation is greater in the larger home region than in Foreign. Figure 1 establishes condition (10) indeed holds. Notice the crucial role of *selection*: when selection is inactive, all firms would find it worthwhile to produce. Then, in both regions we would have $p_i^{in} = 1$, and hence $\chi = 1$. It follows that $\gamma = 1$ and the relationship between ϕ and λ would be one-to-one: there would not be a HME.[26]

We know from equation (9), that a change in the relative probability of successful innovation χ translates into a change in the share of firms ϕ. So, a shock on λ has both a "price effect" and a "quantity effect" (Head and Mayer, 2004). The higher relative wage of Home shifts the price distribution since it affects unit labor costs. It also affects the share of firms that do not find it worthwhile to operate (besides the obvious effect of increasing the number of firms that attempt entry.) In the one-sector Krugman (1980) model, only the price effect

[25]For prominent examples, see Helpman and Krugman (1985), Hanson and Xiang (2004), or Behrens et al. (2009). In the standard case with a linear outside sector (without firm-level heterogeneity or with heterogeneity, but without technology differences), γ is equal to a constant $\bar{\gamma}$ and so $\phi = \bar{\gamma}\lambda$.The HME materializes if and only if $\bar{\gamma} > 1$.

[26]Since γ is not a constant in our setup, one can strengthen the definition of the HME. Therefore, in Section 5.4 we define a strong (or dynamic) HME, as a more than proportionate increase of ϕ due to an increase in λ, i.e., $\phi'(\lambda) > 1$.

exists. Note that the equilibrium relative probability of successful innovation χ is concave in relative size λ. The reason is that (i) the market potential curve is concave in the relative wage, (ii) it is not shifted itself by a region size shock, and (iii) the relative wage is strictly increasing in the share of consumers. We summarize:

Proposition 1 *(**Home market effect**) Consider a two-region single-sector Melitz (2003) model where productivity is Pareto distributed,* $1/2 \leq \lambda < \bar{\lambda}$ *and* $B \geq 1$. *This model exhibits a HME.*

Proof. In the text. ■

Proposition 1 applies also when $\lambda < 1/2$, where Foreign would exhibit a HME over the interval $\left(1 - \bar{\lambda}, 1/2\right)$. Also note that Proposition 1 can be strengthened insofar as the HME can be shown to extend into the region of unconventional sorting; see the extension in Section 5.1. Finally, it can be shown that the existence of a linear outside sector in the Melitz (2003) model exaggerates the HME relative to the case where the outside sector is absent.[27]

Real per capita income. The relative probability of successful innovation is the key endogenous determinant of regional real per capita income inequality. Using the domestic cutoff profit conditions, we can express relative real per capita income as a function of technology differences, relative region size, and the relative probability of successful innovation χ :

$$\frac{W_H}{W_F} = \left(\frac{\lambda}{1-\lambda}\right)^{\frac{1}{\sigma-1}} B\chi^{-\frac{1}{\beta}}. \tag{11}$$

Increasing region size asymmetries and widening the technology gap directly raises relative real per capita income, but also results in a higher relative probability of successful innovation, which dampens the direct effect. $W_H > W_F$ requires $\chi < B^{\beta}\left(\frac{\lambda}{1-\lambda}\right)^{\frac{\beta}{\sigma-1}}$. We know from Figure 1, that $\chi < \frac{\lambda}{1-\lambda}B^{\rho}$. Because $\beta > \sigma - 1$ and $\beta > \rho$, we always have $W_H > W_F$. So, the larger or technologically leading region has the higher real per capita income.[28] For the

[27]To see this, we compare the slopes of $\phi(\lambda)$ at $\lambda = 1/2$ across the two modeling frameworks; see Appendix D for details. With the outside sector, the slope of the locus $\phi(\lambda)$ is equal to $1 + 2\eta$. In the single-sector case, it is equal to $1 + \eta/\left(2 - \rho(1-\eta)/\beta\right)$. The linear outside sector exaggerates the HME as $1 - \eta < \beta/\rho$.

[28]This result extends to the case of unconventional sorting; see A.5.3 in the Appendix.

same reason, it is easy to see that W_H/W_F increases with λ and B so that a more unequal distribution of endowments or technology lead to higher inequality.

Proposition 2 ***(Per capita income)*** *Consider a two-region single-sector Melitz (2003) model where productivity is Pareto distributed,* $1/2 \leq \lambda < \bar{\lambda}$ *and* $B \geq 1$.
(a) The larger and technologically leading region exhibits the higher real per capita income.
(b) Larger endowment or technology differences across regions lead to higher regional income inequality.

Proof. In the text. ■

3.2 Trade liberalization

In the Helpman-Krugman model with an outside sector and regardless whether firms are homogeneous or heterogeneous, lower variable trade costs magnify the HME and exacerbate regional income inequality. These effects are entirely driven by inter-industry reallocations. In this subsection, we show that in our single-sector model with heterogeneity, higher freeness of trade magnifies the HME, but leads to real per capita income convergence.

When variable trade costs τ and/or relative entry costs f^x/f^d fall, freeness of trade $\eta \equiv \tau^{-\beta}\left(f^x/f^d\right)^{1-\beta/(\sigma-1)}$ rises. Conveniently, η only appears in the market potential curve. So, in Figure 1, the increase of η rotates the MPC upwards, and the new equilibrium features a higher χ and a lower ω. Intuitively, for a given wage (market potential), higher freeness of trade favors the larger region as serving the smaller region through exports is now cheaper. The market crowding locus is not affected since freeness of trade rises symmetrically.[29] Hence, the equilibrium relative probability of successful innovation goes up, which translates into a larger HME since γ rises in χ for given λ. Moreover, the equilibrium relative wage declines, so that higher freeness of trade leads to convergence of nominal wages.[30] Under conventional sorting, one can show that γ increases so that the HME becomes stronger.[31]

[29] See Section 5.2 on asymmetric trade costs.

[30] Higher freeness of trade makes it more likely that the conventional sorting conditions fail to hold, so $\bar{\lambda}$ falls.

[31] We provide a generalization of this result to the case of unconventional sorting in Section 5.1.

While trade liberalization magnifies the HME, it leads to *convergence* of real income per capita across regions. Intuitively, higher freeness of trade favors the more open, i.e., the small, region. Importantly, trade liberalization does not lead to convergence when the model features a linear outside sector.[32] The reason is that in the presence of an outside sector, the relative probability of successful innovation depends on the freeness of trade and technology differences B, but is independent of the relative size of the regions λ. With $B > 1$, trade liberalization reduces the relative probability of successful innovation and therefore leads to *divergence* of real per capita income.

Proposition 3 *(**Trade liberalization**) Consider a two-region single-sector Melitz (2003) model where productivity is Pareto distributed,* $1/2 \leq \lambda < \bar{\lambda}$ *and* $B \geq 1$.
*(a) (**Home Market Magnification Effect**) Trade liberalization magnifies the home market effect.*
*(b) (**Per capita income converge**) Trade liberalization leads to real per capita income convergence across regions. In the presence of a linear outside sector, there is no convergence if regions only differ in relative region size and divergence if regions differ in relative productivity.*

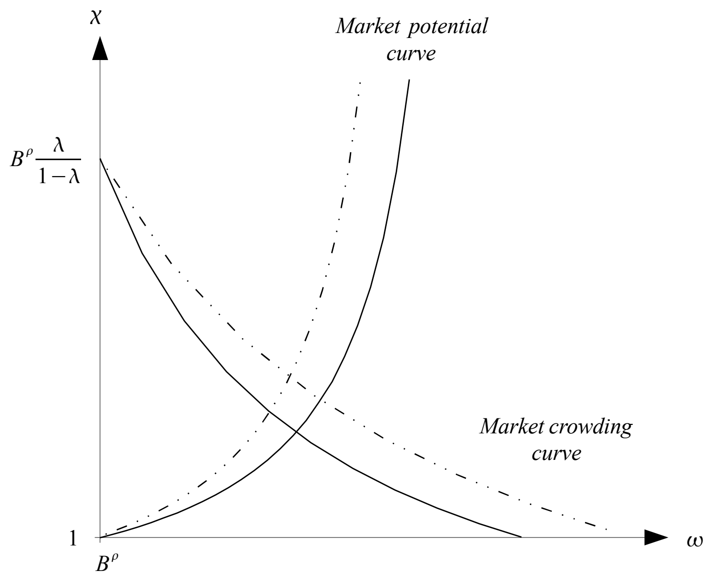
Proof. See the Appendix. ■

3.3 Productivity dispersion

Next, we consider the role of productivity dispersion. Empirical evidence suggests that the degree of productivity dispersion increases with the level of economic development; see Poschke (2011). First, for given variable and fixed trade costs, higher dispersion (lower β) raises the freeness of trade, which only affects the MPC. Second, higher dispersion dampens the effect of technology differences B. For a given wage rate and freeness of trade, this shifts the MCC and the MPC upwards. Finally, the effect of a given wage on MCC and MPC is conditioned on the productivity dispersion. Considering all these effects, higher productivity dispersion rotates both curves up; see Figure 2. This leads to an increase in the relative probability of successful innovation χ. A larger χ translates into larger γ so that the HME is magnified when

[32] See Appendix D for a detailed description of this model.

Figure 2: The effects of higher productivity dispersion



the degree of productivity rises.[33] The intuition for this result is that higher productivity dispersion magnifies differences in domestic entry cutoffs due to size differentials. Dispersion turns out to favor location in the larger region *ex ante* and *ex post.*

The effect of productivity dispersion on the distribution of per capita income is slightly more involved. Higher productivity dispersion (lower β) raises the relative probability of successful innovation and therefore magnifies the HME. At the same time, the elasticity of relative welfare in the relative probability of successful innovation $-1/\beta$ increases in absolute terms, which magnifies the direct effect of χ on relative welfare. Hence, higher productivity dispersion also leads to convergence.

Complementarity between trade liberalization and productivity dispersion. There is an interesting interaction between productivity dispersion and trade liberalization. An increase in productivity dispersion results in an inward rotation of the MPC and outward rotation of the MCC. Variable trade cost liberalization (lower τ) magnifies the rotation of the MPC, but has no additional bearing on the MCC. Hence, the effect of trade liberalization is magnified

[33] This finding has important implications for empirical studies on the HME, such as Hanson and Xiang (2004). The present model suggests that one important industry characteristic that shapes the size of the HME is the degree of productivity dispersion as captured by the shape parameter of the Pareto distribution.

by a high degree of productivity dispersion. By Young's Theorem, the effect of higher productivity dispersion is magnified by lower variable trade costs, too.

Selection. The comparative statics result with respect to productivity dispersion can be used to analyze the role of *selection* for income disparities. Burstein and Vogel (2011) show that the Melitz (2003) model nests the Krugman (1980) model when β reaches its lower bound. Then, productivity dispersion is maximized. In equilibrium, the mass of firms is concentrated at very few firms, which are highly productive and are all exporters. So, the selection channel is effectively shut off. Given that real per capita income converges when β is lowered, income disparities are larger when selection is active than when selection is inactive.

Proposition 4 *(**Productivity dispersion**) Consider a two-region single-sector Melitz (2003) model where productivity is Pareto distributed,* $1/2 \leq \lambda < \bar{\lambda}$ *and* $B \geq 1$.
*(a) (**Home Market Magnification Effect**) Higher productivity dispersion magnifies the home market effect.*
*(b) (**Per capita income convergence**) Higher productivity dispersion lead to real per capita income convergence across regions. In the presence of a linear outside sector, there is no convergence if regions only differ in relative region size and divergence if regions differ in relative productivity.*
*(c) (**Complementarity**) Higher productivity dispersion magnifies the effect of trade liberalization on relative real per capita income and vice versa.*
*(d) (**Selection**) Income disparities are larger when selection is active than when selection is inactive.*

Proof. In the Appendix. ■

3.4 Discussion

The opposite predictions on interregional wage inequality between the single-sector model and the Krugman-Helpman (1985) model arise from opposite reallocation effects. In the presence of an outside sector, trade liberalization and higher productivity dispersion lead to a lower probability of successful entry. These effects, however, do not interact with region size, which implies that magnification then only works through *inter*-industry reallocation. They do, however, interact with technology differences. Then, lower trade costs and higher dispersion lead to reversed intra-industry reallocation effects, which, inter alia, dampens the HME.

Intra-industry reallocations are an important driver of real per capita income. In the single-sector model, trade liberalization and higher dispersion lead to stronger reallocation in the smaller, more open region, and therefore induce *convergence*. In the model with a linear outside sector with technology differences, trade liberalization and higher dispersion lead to stronger reallocation in the technologically leading country, which therefore induces *divergence*.

4 Quantitative illustration

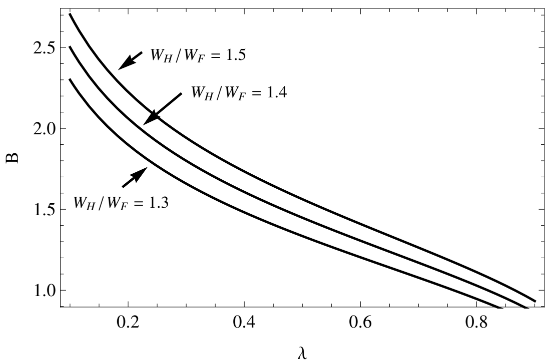
In this section, we quantify the importance of the distribution of endowments for the HME and for interregional inequality. We study the roles of trade costs and productivity dispersion. Moreover, we illustrate the quantitative relevance of wage adjustment.

The parametrization of the model for our simple numerical analysis is very standard and follows the literature; see Bernard, Redding, and Schott (2007) for a leading example. Our model is too stylized to realistically capture any two real-world regions; moreover, it is hard to obtain comparable statistics to match. For this reason, we chose parameters such that the symmetric version of the model replicates (i) the observed standard deviation of domestic US plant sales of 0.84, (ii) the export participation rate of US firms of 21% (Bernard et al., 2004), and (iii) an openness measure (exports over GDP) of 27%.[34] Amongst other things, this implies setting $\tau = 1.3$ and $\beta = 3.3$. One important issue relates to the weight of the outside good in the two-sector model. In line with the working paper version of Demidova (2008), we set the share of income spent on differentiated goods $\mu = 0.5$. Note that this choice has no bearing on the strength of the HME, but potentially affects welfare considerations.

[34]This calibration strategy implies that in the baseline equilibrium $\lambda = 1/2$ and $B = 1$. Details of the calibration are explained in Appendix B.

First, we examine the relative roles of interregional differences in endowments and technology in generating per capita income inequality. For that purpose we allow for $B \neq 1$. For the benchmark specification, Figure 3 shows combinations of relative region size λ and relative productivity bound B which yield the same relative per capita income. A higher relative per capita income implies that the locus is farther away from the origin. We present curves for relative per capita income of $1.3, 1.4$, and 1.5. For a symmetric endowment distribution $(\lambda = 1/2)$, those relative welfare outcomes are achieved by technology levels of $1.32, 1.43$, and 1.54, respectively. The link between relative technology and relative welfare is almost but not exactly one-to-one as B affects W_H/W_F non-linearly. The same relative positions are supported for symmetric technology $(B = 1)$ if endowment shares vary between 0.76 and 0.87. Hence, relative minor endowment share variation can have substantial implications for regional inequality. In the following, we focus on the role of endowment differences $(B = 1)$.

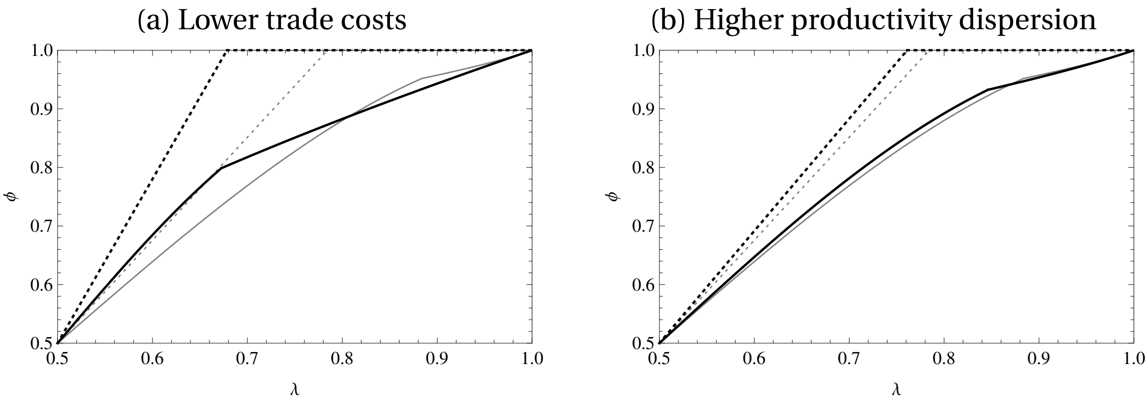
Figure 3: Iso relative per capita income curves



Notes: Regions differ in endowments and technology, but are otherwise identical with $\tau = 1.3, \beta = 3.3$, $\sigma = 3.8$, and $f^x/f^d = 1.8$ (Bernard, Redding and Schott, 2007). The curves represent combinations of λ and B which yield the same relative per capita income.

Figure 4 quantitatively illustrates the HME. Gray lines represent the benchmark scenario $(\tau = 1.3, \beta = 3.3)$. Dashed curves relate to the model with the linear outside sector; the solid ones to the single-sector case. It is obvious that the presence of the outside sector exaggerates the HME. At $\lambda = 0.66$, Home has 72% of all firms when wages are allowed to adjust, but 78% when they cannot. Clearly, the gap increases further as λ rises. Note that the sorting pattern reverses at $\bar{\lambda} = 0.88$. Trade liberalization is modeled by setting $\tau = 1$. Unconventional sorting obtains for $\lambda > \bar{\lambda} = 0.68$. Within the range of conventional sorting (and diversification in the two-sector case), Figure 4 illustrates the home market magnification effect. It is stronger when wages are not allowed to adjust. In panel (b), we increase productivity dispersion by setting $\beta = 2.9$. Again, the magnification effect is visible, and it turns out stronger in the absence of wage adjustment.

Figure 4: Home Market Magnification Effects



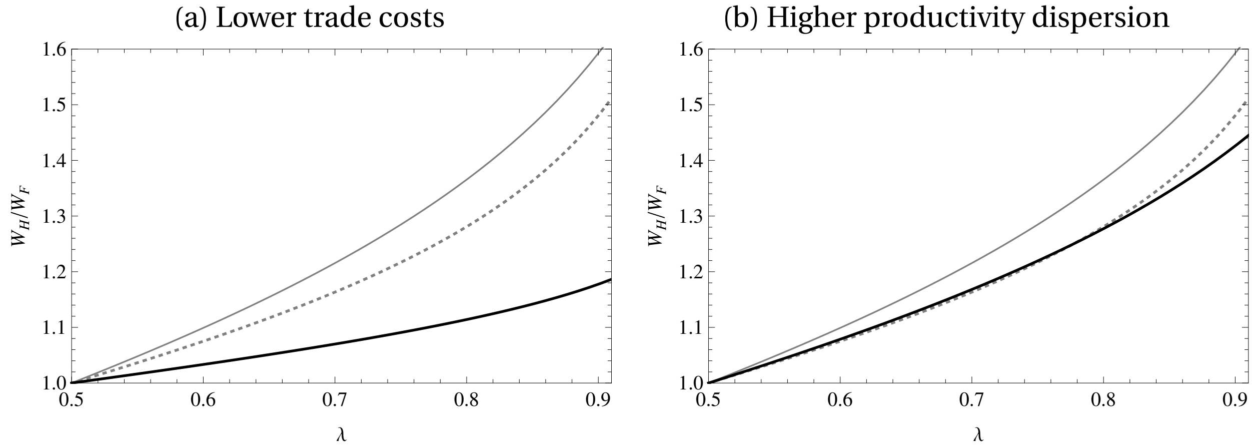
Notes: Gray lines represent the benchmark scenario with $\tau = 1.3, \beta = 3.3$, $\sigma = 3.8$, and $f^x/f^d = 1.8$. Dashed lines relate to model with a linear outside sector. Lower trade costs (bold line) are modeled as $\tau = 1$. Higher productivity dispersion (bold line) is modeled as $\beta = 2.9$. Note that, with a linear outside sector (and $B = 1$), β and τ have no effect on regional inequality.

Figure 5 turns to regional inequality as a function of the distribution of the labor endowment and studies how trade costs and firm level heterogeneity shift this function.[35] Again, the dotted line represents the case with a linear outside sector. Also note that we keep $B = 1$. So, according to Propositions 3 and 4, the presence of a linear outside sector impedes all convergence (or divergence). This prediction does not depend on the (arbitrary) choice of $\mu = 0.5$. Figure 5 illustrates that complete abolishment of variable trade cost ($\tau = 1$, panel (a)) and higher productivity dispersion ($\beta = 2.9$, panel (b)), lead to convergence of real per capita incomes as the W_H/W_F schedule shifts down from the (gray) baseline locus to the bold function. The quantitative importance of convergence is substantial: at $\lambda = 0.66$, the gap between H and F falls from 17% to 5%. Importantly, in the Melitz (2003) model, a more pronounced HME does not imply wider interregional inequality. Although Home hosts more firms, its relative wage falls and so does relative welfare. The reason is a deterioration in average productivity. Without firm-level heterogeneity or with wage insensitivity, such a thing could not

[35]We vary λ in the interval $0.5 \leq \lambda \leq 0.9$. While relative welfare is well defined for all $\lambda < 1$, for $\lambda > 0.9$ it becomes extremely large.

occur.

Figure 5: Regional inequality



Notes: Gray lines represent the benchmark scenario with $\tau = 1.3$, $\beta = 3.3$, $\sigma = 3.8$, and $f^x/f^d = 1.8$. Dashed lines relate to model with a linear outside sector. Lower trade costs are modeled as $\tau = 1$. Higher productivity dispersion is modeled as $\beta = 2.9$.

Table 1 illustrates the complementarity between trade liberalization and productivity dispersion. For a given endowment distribution ($\lambda = 2/3$), we report the effect of a reduction in variable trade costs on relative per capita income for different degrees of productivity dispersion. With the benchmark dispersion, inequality falls by 13.7% if variable trade costs drop from 1.6 to 1.0. With a higher productivity dispersion, this reduction amounts to 15.4%.

Table 1: Relative per capita income

Productivity	Variable trade costs						
dispersion	0%	10%	20%	30%	40%	50%	60%
Benchmark	1.06	1.11	1.15	1.17	1.20	1.21	1.22
Higher	1.01	1.06	1.10	1.14	1.16	1.18	1.20

Notes: Variable trade costs τ are denoted in their ad valorem equivalent. Relative region size is $\lambda = 2/3$. Productivity dispersion is inversely related to β. In the benchmark specification, $\beta = 3.3$; higher dispersion refers to $\beta = 2.9$. Moreover, $\sigma = 3.8$ and $f^x/f^d = 1.8$.

5 Extensions and Additional Results

In the following, we show four further results. First, we prove that a HME generally exists for *all* distributions of labor endowment across regions, i.e., also, when the conventional sorting assumption is violated. Second, we analyze the effects of *unilateral* trade cost reductions on regional inequality and the HME. Third, to better relate our work to empirical studies, we show that the economy with the larger share of world *demand* has an overproportionate share of firms if the underlying demand difference is due to different endowments, but not if it is due to technological differences. Finally, we work with an alternative, stronger definition of the HME. In separate Appendices, we show that the Pareto assumption is not crucial for our results (Appendix C) and that fixed market access costs are necessary for the single-sector HME to arise (Appendix E).

5.1 Unconventional Sorting

If $\lambda < \bar{\lambda}$, i.e., the distribution of labor endowments is not too asymmetric, "conventional sorting" obtains and only the most productive firms engage in exporting.[36] However, when regions are asymmetric in size, the sorting pattern can reverse in the smaller region. Less productive firms would make negative profits on their small domestic market, for which fixed costs are high relative to revenue, while they can make profits on the large export market, where fixed costs are lower relative to revenue. When the sorting pattern reverses, Figure 1 continues to characterize the equilibrium. So, our results on the HME and income convergence continue to hold. The meaning of χ changes, though, since p_F^{in} is now given by $1 - G[\varphi_{FH}^*]$.

We show in Appendix A.5 that the *market crowding curve* (MCC) now is given by

$$\chi = \frac{f^x/f^d}{\eta} B^\beta \omega^{-\frac{\beta}{\rho}}. \tag{12}$$

Clearly, the MCC is strictly decreasing in ω. In contrast to the case of conventional sorting, it is now independent of λ but depends on trade costs. The *market potential curve* (MPC) now reads

$$\chi = \frac{f^x/f^d}{1 - \frac{1-\lambda}{\lambda}\omega^{\frac{\beta-\rho}{\rho}}\left(\omega^{\beta/\rho} - \eta\right)}. \tag{13}$$

[36] See Bernard, Eaton, Jensen, and Schott (2007) for empirical support for this pattern.

The MPC is strictly increasing in ω, and now depends on λ and on trade costs. Similar to the case of conventional sorting, equilibrium can be determined by the intersection of the two curves. Evaluated at $\omega = 1$, the MPC curve yields $\chi > 1$. Hence, when $\lambda > 1/2$ and/or $B > 1$, the HME also occurs under unconventional sorting. We summarize these results in the following proposition:

Proposition 5 *(**Unconventional sorting**). Consider a two-region single-sector Melitz (2003) model with $\lambda > \bar{\lambda}$.*
*(a) (**Home Market Effect**) The larger and technologically leading region has a higher relative wage (ω) and higher relative probability of successful innovation (χ), so that a home market effect exists.*
*(b) (**"Magnification"**) Higher freeness of trade (η) or higher asymmetries in region size (λ) reduce χ, so that the home market effect is diminished.*
*(c) (**Per capita income convergence**) Trade liberalization leads to real per capita income convergence across regions.*

Proof. In the Appendix. ■

As a corollary, the model features a HME over *all* possible interregional allocations of the labor endowment. As in Crozet and Trionfetti (2008) the HME is non-linear. Their numerical exercise suggests that the HME is concave in λ for values of λ around the symmetric equilibrium and convex thereafter. In our setup, the HME is concave for $\lambda < \bar{\lambda}$ and convex for $\lambda > \bar{\lambda}$.

5.2 Asymmetric trade liberalization

So far, we have studied the effect of symmetric iceberg trade costs on the size of the home market effect and on regional real income disparities. Now, we focus on *asymmetries* in variable iceberg trade costs; $\tau_{FH} \geq 1$ measures Home's import barrier which is equivalent to Foreign's export barrier.

As usual, we derive the *market crowding curve* (MCC) by combining the relative zero cutoff profit conditions; for simplicity and without loss of generality, we set $B = 1$. The only difference to the analysis in earlier sections is that trade costs no longer drop out from the resulting expression, which is now given by:

$$\chi = \frac{\lambda}{1-\lambda}\left(\frac{\tau_{FH}}{\tau_{HF}}\right)^{\beta}\omega^{-\frac{2\beta-\rho}{\rho}}; \tag{14}$$

a detailed derivation is in Appendix A.6. The MCC is still downward-sloping in the relative wage ω, but it is shifted by asymmetric changes in trade costs. The *market potential curve* (MPC) is also very similar to the one derived earlier with the difference that the freeness of trade η now is directional, i.e., $\eta_{ij} \equiv \tau_{ij}^{-\beta}\left(f^x/f^d\right)^{1-\beta/(\sigma-1)}$. So, the MPC becomes

$$\chi = \frac{1-\eta_{HF}\omega^{-\frac{\beta}{\rho}}}{1-\eta_{FH}\omega^{\frac{\beta}{\rho}}}. \tag{15}$$

The slope of this locus is still positive in ω and it possesses an asymptote. For similar reasons than those discussed above, a HME arises even if $\tau_{FH} \neq \tau_{HF} \geq 1$.

Unilateral liberalization of the large country's import barriers, τ_{FH}, holding fixed the wage, raises the export profits of the average firm in the small region. In order to restore zero expected profits, the probability of successful innovation has to go down, which implies a shift of the domestic entry cutoff to the right. Hence, similar to Melitz (2003), we have reallocation from less to more productive firms in the small region. This reallocation effect is reflected by an upward shift of the MPC. There is, however, a countervailing effect. By balanced trade, the lower export cutoff of the small region translates into a lower export cutoff in the large region.[37] In order to restore balance, the competitiveness of the small region has to go down. This adjustment is represented by an upward shift of the MCC. It follows from the diagram that, with lower τ_{FH}, the relative wage falls. However, the effect on the relative probability of successful innovation χ (and, hence, relative per capita income) is unclear *a priori* and requires more algebra. In the Appendix, we show that the reallocation effect always dominates the anti-competitiveness effect, so that a higher χ materializes.[38]

[37] Notice that we hold the wage rate fixed.

[38] In a companion paper (Felbermayr and Jung, 2012), we generalize the insight of Demidova and Rodríguez-Clare (2011) who have demonstrated welfare gains from unilateral liberalization in a small economy Melitz model. In a setting with two large regions, we show that unilateral trade liberalization in the form of lower iceberg trade

Unilateral liberalization of the small region's import barriers induces reallocation and anti-competitiveness effects in the large region. Again, the reallocation effect dominates, which leads to a lower χ and real per capita income divergence.[39] The results are summarized in the following proposition:

Proposition 6 ***(Unilateral Trade Liberalization)****. Consider a two-region single-sector Melitz (2003) model with $\lambda > \bar{\lambda}$. A unilateral reduction of import barriers by the large region (Home)*
*(a) (**Home Market Magnification Effect**) magnifies the HME and*
*(b) (**Per capita income convergence**) leads to convergence of real per capita incomes.*
Opposite results obtain if the small region liberalizes.

Proof. In the Appendix. ■

5.3 The HME in demand shares

Empirical work typically analyzes how shocks on *demand shares* affect production patterns on the industry level.[40] Home's share in world demand (GDP) is given by

$$\delta \equiv (1 + (1 - \lambda) / (\omega\lambda))^{-1},$$

which is of course endogenous to the model. In the standard setup, with an identically parameterized linear outside sector in both regions, we would have $\omega = 1$ and therefore $\delta = \lambda$. The HME in demand shares is then identical to the HME in endowment shares. In our case, this is different since $\omega > 1$. The relative wage can be affected through exogenous changes in relative size of regions and in relative productivity.

We define the home market effect in demand shares as an overproportional relationship between the share of firms and the demand share, so that $\delta > 1/2 \Rightarrow \phi > \delta$. From equation

costs increases welfare in both regions. In Felbermayr and Jung (2012) we do not analyze *relative* per capita income.

[39]With symmetric trade liberalization, the competitiveness effects exactly cancel out, while trade-induced reallocation is stronger in the small region, which is more open.

[40]We do not extend our model to a multi-industry setup as Hanson and Xiang (2004). Under the assumption of industry-specific labor, this would be straight-forward. Allowing for inter-industry mobility of labor is an interesting avenue for further research.

(9), it is easy to see that the HME in demand shares requires $\chi > \omega$. Note that the condition for a HME in demand shares is stronger than the one for a HME in labor shares, which is $\chi > 1$.

When regions are symmetric in technologies but differ in size, one can show that a HME arises if and only if $\eta > \rho/(2\beta+\rho)$, which is the same condition as required for convexity of the MPC. When regions are identical in size but differ in technologies, however, the necessary condition for a HME can never hold. Hence, there exists a reverse HME: the richer region hosts an underproportional share of firms. This is due to the fact that higher average productivity translates into larger average firm size so that the number of firms has to adjust downwards. Widening the technology gap (as modeled by an increase in b_H) therefore leads to lower prices, but also reduces the number of domestically produced varieties that are available without trade costs.

These results are summarized in the following proposition:

Proposition 7 ***(Home market effect in demand shares)****. Let δ be Home's share in GDP. Assume that conventional sorting holds $(\lambda < \bar{\lambda})$ and that the freeness of trade is not too low $(\eta > 1/3)$.*
(a) Let $\lambda > 1/2$ and $B = 1$. Then, the larger region exhibits a HME in demand shares $(\phi > \delta)$.
(b) Let $\lambda = 1/2$ and $B > 1$. Then, the richer region exhibits a reverse HME in demand shares $(\phi < \delta)$.

Proof. In the Appendix. ■

If regions simultaneously differ in size and technologies, the situation is more complicated. A HME obtains if the relative size of regions is large enough relative to B. The finding that the underlying cause for interregional variation in demand shares matters for whether or not a HME exists, is important for empirical work. If a researcher runs a regression of ϕ on δ, it is important to control for some measure of b, for example average TFP. Failing to do so could explain why the empirical literature has found mixed support for the HME so far.

5.4 The strong (or dynamic) HME

Since with intra-firm reallocation effects, γ in equation (9) is not a constant, we can define the HME in a stronger than standard fashion as follows:

Definition 2 *A **strong (dynamic) home market effect** exists, if an increase in Home's labor share yields a more than proportionate increase in Home's share of firms, i.e., if* $\phi'(\lambda) > 1$.

The *strong HME* is slightly more involved to analyze than the conventional one ($\phi > \lambda$). It obtains when an increase in the labor share of a region leads to a more-than-proportionate increase in its share of firms. Denote by ϵ_x the elasticity of some variable x with respect to λ. Then,

$$\epsilon_\phi = 1 + \epsilon_\gamma > 1 \Leftrightarrow \epsilon_\gamma > 0. \tag{16}$$

To verify the validity of the above condition, one needs to understand how γ, and hence χ, depend on λ. This can be easily seen with the help of Figure 1, where the effect of an increase in Home's share of labor affects only the market crowding curve. It shifts upwards if λ increases; the shift is larger, the smaller λ is initially. Clearly, an increase in λ leads to a higher relative wage ω and to a higher relative probability of successful innovation.

So, as long as the conventional sorting condition holds, $d\chi/d\lambda > 0$ and $d^2\chi/d\lambda^2 < 0$. Equation (9) implies that around the symmetric equilibrium $(\lambda = 1/2, \chi = 1)$, the derivative of γ with respect to λ is given by $(d\chi/d\lambda)/2$.[41] It follows that, around the symmetric equilibrium, ϵ_γ is positive and a strong HME exists. As λ grows away from symmetry, the positive increments to χ become smaller; moreover, a higher λ also has a direct negative effect on γ. It follows that $\epsilon_\gamma > 0$ cannot hold for all λ. Let $\bar{\lambda}$ denote the endowment share at which conventional sorting does no longer hold. It can be proved that the strong HME exists over an interval $(1/2, \lambda^*)$ with the critical value λ^* bounded by $\lambda^* < \bar{\lambda}$.

Proposition 8 *(**Strong HME**). Consider a two-region single-sector Melitz (2003) model where productivity is Pareto distributed,* $1/2 \leq \lambda < \bar{\lambda}$ *and* $B \geq 1$. *This model exhibits a **strong** HME if* $\lambda < \lambda^*$, *where* $\lambda^* \leq \bar{\lambda}$.

Proof. In the Appendix. ■

[41] For a general characterization, see Appendix A.8.

6 Conclusion

This paper provides a tractable way to characterize a two-region single-sector asymmetric Melitz (2003) model for the purpose of conducting comparative statics. It does so without imposing a linear, perfectly competitive and frictionless outside sector, as the literature has usually chosen to do. The outside sector assumption has been criticized to be unrealistic and possibly important for aggregate results, such as welfare (Demidova and Rodríguez-Clare, 2011), or for the ability of the model to predict a Home Market Effect (HME), by which a large region attracts a more than proportionate share of producing firms (Davis, 1998).

We show that firm-level productivity heterogeneity and selection effects interact to generate a HME even in the absence of inter-sectoral reallocation. Hence, the existence of a HME is a more robust characteristic of increasing-returns-to-scale Krugman (1980) type trade models than previously thought. The HME is non-linear, as the empirical analysis of Crozet and Trionfetti (2008) suggests. It is magnified by falling trade costs and by a higher degree of firm-level productivity dispersion. The HME translates into interregional per capita welfare differences. In contrast to the model with a linear outside sector, trade liberalization attenuates these interregional differences and leads to real wage convergence. Firm-level heterogeneity is absolutely crucial for these results: in the Krugman (1980) single-sector model, no HME can arise. Moreover, convergence is stronger when the degree of firm-level heterogeneity is more pronounced. Lower trade costs and more pronounced firm-size dispersion make interregional disparities based on market size and technology differences less pronounced.

While our analysis sheds new light on the role of trade costs and firm-level productivity dispersion for interregional economic disparities, it also has a number of empirical implications. While the assumption of a linear outside sector does not stand up against empirical evidence, firm level heterogeneity is well-documented. Since the outside-sector is not crucial for the existence of a HME, an empirical rejection of an overproportionate relation between a region's share of firms and its share of endowments is indeed evidence against increasing returns theories of international (or interregional) trade. Moreover, empirical tests that fail to control for the level of technology may wrongly reject the existence of an endowment-driven HME. Finally, tests should also control for industry productivity dispersion.

References

[1] Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012), 'New Trade Models, Same Old Gains?', *American Economic Review* **102**(1): 94–130.

[2] Arkolakis, C., S. Demidova, P. Klenow, and A. Rodríguez-Clare (2008), 'Endogenous Variety and the Gains from Trade', *American Economic Review* **98**(2): 444-450.

[3] Baldwin, R. E. and R. Forslid (2010), 'Trade Liberalization with Heterogeneous Firms', *Review of Development Economics* **14**(2): 161-176.

[4] Behrens, K., A.R. Lamorgese, G.I.P. Ottaviano, and T. Tabuchi (2009), 'Beyond the Home Market Effect: Market Size and Specialization in a Multi-Country World', *Journal of International Economics* **79**(2): 259-265.

[5] Bernard, A., J. Eaton, B. Jensen, and S. Kortum (2003), 'Plants and Productivity in International Trade', *American Economic Review* **93**(4):1268-1290.

[6] Bernard, Andrew, B. Jensen, and P. Schott (2006), 'Trade Costs, Firms, and Productivity', *Journal of Monetary Economics* **53**(5): 917–37.

[7] Bernard, A., B. Jensen, S. Redding, and P. Schott (2007), 'Firms in International Trade', *Journal of Economic Perspectives*, Summer 2007.

[8] Bernard, A., S. Redding, and P. Schott (2007), 'Comparative Advantage and Heterogeneous Firms', *Review of Economic Studies **74**: 31-66.*

[9] Brülhart, M. and F. Trionfetti (2009), 'A Test of Trade Theories when Expenditure is Home Biased', *European Economic Review* **53**(7): 830–45.

[10] Burstein, A.T. and J. Vogel (2011), 'Factor Prices and International Trade: A Unifying Perspective'. *NBER Working Paper* No. 16904.

[11] Crozet, M. and F. Trionfetti (2008), 'Trade Costs and the Home Market Effect', *Journal of International Economics* **76**: 309–321.

[12] Davis, D.R. (1998), 'The Home Market, Trade, and Industrial Structure', *American Economic Review* **88**(5): 1264–76.

[13] Davis, D.R. and D.E. Weinstein (1999), 'Economic Geography and Regional Production Structure; An Empirical Investigation', *European Economic Review* **43**: 379–407.

[14] Davis, D.R. and D.E. Weinstein (2003), 'Market Access, Economic Geography and Comparative Advantage: An Empirical Test', *Journal of International Economics* **59**(1): 1–23.

[15] Demidova, S. (2008), 'Productivity Improvements and Falling Trade Costs: Boon or Bane?', *International Economic Review* **49**(4): 1437-1462.

[16] Demidova, S. and A. Rodríguez-Clare (2009), 'Trade Policy under Firm-Level Heterogeneity in a Small Economy', *Journal of International Economics* **78**(1): 100-112.

[17] Demidova, S. and A. Rodríguez-Clare (2011), 'The Simple Analytics of the Melitz Model in a Small Open Economy', NBER Working Paper 17521.

[18] Egger, H. and U. Kreickemeier (2009), 'Firm Heterogeneity and the Labor Market Effects of Trade Liberalization', *International Economic Review* **50**: 187-216.

[19] Falvey, R., D. Greenaway and Z. Yu (2006), 'Extending the Melitz Model to Asymmetric Countries', GEP Research Paper 2006/07, University of Nottingham.

[20] Feenstra, R.C., J.R. Markusen, and A.K. Rose (2001), 'Using the Gravity Equation to Differentiate among Alternative Theories of Trade', *Canadian Journal of Economics* **34**(2): 430–47.

[21] Felbermayr, G. and B. Jung (2012), 'Unilateral Trade Liberalization in the Melitz Model: A Note', University of Tübingen Working Papers in Economics and Finance 30.

[22] Fujita, M., P.R. Krugman, and A.J. Venables (1999), 'The Spatial Economy. Cities, Regions and International Trade (Cambridge, MA: MIT Press).

[23] Grossman, G.M., E. Helpman, and A. Szeidl (2006), 'Optimal Integration Strategies for the Multinational Firm', *Journal of International Economics* **70**(1): 216-238.

[24] Haltiwanger, J., A. Kugler, M. Kugler, A. Micco, and C. Pages (2004), 'Effects of tariffs and real exchange rates on job reallocation: evidence from Latin America', *Journal of Policy Reform* **7**(4): 191–208.

[25] Hanson, G.H. (2005), 'Market Potential, Increasing Returns and Geographic Concentration', *Journal of International Economics* **67**(1): 1-24.

[26] Hanson, G.H. and C. Xiang (2004), 'The Home-market Effect and Bilateral Trade Patterns', *American Economic Review* **94**(4): 1108–29.

[27] Head, K. and T. Mayer (2004), 'The Empirics of Agglomeration and Trade', in J.V. Henderson and J.-F. Thisse (eds.), Handbook of Regional and Urban Economics, Vol. 4 (Amsterdam: New Holland): 2609-2669.

[28] Head, K. and J. Ries (2001), 'Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of US-Canada Trade', *American Economic Review* **91**(4): 858-876.

[29] Helpman, E. and P. Krugman (1985), Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy (Cambridge, MA: MIT Press).

[30] Helpman, E., M. Melitz, and S. Yeaple (2004), 'Export versus FDI with Heterogeneous Firms', *American Economic Review* **94**(1): 300-316.

[31] Krugman, P. (1980), 'Scale Economies, Product Differentiation, and the Pattern of Trade', *American Economic Review* **70**(5): 950-959.

[32] Krugman, P. and A. Venables (1999), 'How Robust Is the Home Market Effect?', Mimeo, Massachusetts Institute of Technology and London School of Economics.

[33] Mayer, T. and G.I.P. Ottaviano (2007), 'The Happy Few: The Internationalisation of European Firms. New Facts Based on Firm-level Evidence', *Bruegel Blueprint Series* 3.

[34] Medin, H. (2003), 'Firms' Export Decisions–Fixed Trade Costs and the Size of the Export Market', *Journal of International Economics* **61**: 225-241.

[35] Melitz, M.J. (2003), 'The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity', *Econometrica* **71**: 1695-1725.

[36] Melitz, M.J. and G.I.P. Ottaviano (2008), 'Market Size, Trade, and Productivity', *Review of Economic Studies* **75**(1): 295-316.

[37] Menezes-Filho, A.N. and M.-A.Muendler (2011), 'Labor Reallocation in Response to Trade Reform', *NBER Working Papers* No. 17372.

[38] Obstfeld, M. and K.S. Rogoff (2001), 'The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?' In: B.S. Bernanke and K.S. Rogoff (eds.), NBER Macroeconomics Annual 2000 (Cambridge, MA: MIT Press: 339-412).

[39] Okubo, T., P. Picard, and J.-F. Thisse (2010), 'The spatial selection of heterogeneous firms', *Journal of International Economics* **82**(2): 230–237.

[40] Ossa, R. (2011), 'A "New Trade" Theory of GATT/WTO Negotiations', *Journal of Political Economy* **119**(1): 122-152.

[41] Ottaviano, G.-M. and Tabuchi (2009), 'Market Size and Tax Competition', *Journal of International Economics* **67**: 25-46.

[42] Ottaviano, G.-M. and T. van Ypersele (2005), 'Market Size and Tax Competition', *Journal of International Economics* **67**: 25-46.

[43] Poschke, M. (2011), 'The Firm Size Distribution across Countries and Skill-Biased Change in Entrepreneurial Technology', *CIREQ Working Paper* 08-2011.

[44] Redding, S. and A.J. Venables (2004), 'Economic geography and international inequality', *Journal of International Economics* **62**(1): 53-82.

[45] Venables, A.J. (1994), 'Integration and the Export Behavior of Firms–Trade Costs, Trade Volume and Welfare', *Review of World Economics* **130**: 118-132.

[46] Wacziarg, R. and J. Wallack (2004), 'Trade Liberalization and Intersectoral Labor Movements', *Journal of International Economics* **64**(2): 411–439.

A Proofs of Lemmata and Propositions, Details to Derivations

A.1 Derivation of equilibrium conditions

Zero cutoff profit conditions. Demand for any variety is given by

$$q_{ij}[z] = R_j P_j^{\sigma-1} p_{ij}[z]^{-\sigma},$$

where the price index to (1) is given by $P_i^{1-\sigma} = \int_{z\in\Omega_i} p[z]^{1-\sigma} dz$ and R_i denotes aggregate expenditure.[42] Given the demand function, the price charged at the factory gate is $w_i/(\rho\varphi)$. Then, operating profits of a firm from region i on market j are

$$\pi_{ij}[\varphi] = R_j P_j^{\sigma-1} \left(\frac{\rho\varphi}{\tau_{ij} w_i}\right)^{\sigma-1} /\sigma - w_i f_{ij}.$$

The zero cutoff profit conditions follow from noting that $\pi_{ij}\left[\varphi_{ij}^*\right] = 0$.

Price index. Using the zero cutoff profit condition, we can write the price level P_i as

$$\begin{aligned} P_i^{1-\sigma} &= \sum_{j\in\{H,F\}} \int_{\varphi_{ji}^*}^{\infty} \left(\frac{\tau_{ji} w_j}{\rho\varphi}\right)^{1-\sigma} M_j m_{ji} \frac{dG[\varphi]}{1-G\left[\varphi_{ji}^*\right]} \\ &= \sum_{j\in\{H,F\}} \left(\frac{\tau_{ji} w_j}{\rho}\right)^{1-\sigma} M_j m_{ji} \theta \left(\varphi_{ji}^*\right)^{\sigma-1} \\ &= \theta \sum_{j\in\{H,F\}} m_{ji} M_j \left(\frac{\rho\varphi_{ji}^*}{\tau_{ji} w_j}\right)^{\sigma-1}, \end{aligned}$$

where $\theta \equiv \beta/(\beta-\sigma+1)$ is a positive constant.

Free entry condition. Using optimal demand and the zero cutoff profit condition, we obtain the following expression for expected profits of a firm in region i from entering:

$$\begin{aligned} \bar{\pi}_i &= \sum_{j\in\{H,F\}} \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}[\varphi] \frac{dG[\varphi]}{1-G[\varphi_{ii}^*]} \\ &= \sum_{j\in\{H,F\}} m_{ij} \left(\theta \frac{R_j P_j^{\sigma-1}}{\sigma} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{1-\sigma} \left(\varphi_{ij}^*\right)^{\sigma-1} - w_i f_{ij}\right) \\ &= \sum_{j\in\{H,F\}} m_{ij} \left(\theta R_j P_j^{\sigma-1} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{1-\sigma} R_j^{-1} P_j^{1-\sigma} \left(\frac{\tau_{ij} w_i}{\rho}\right)^{\sigma-1} w_i f_{ij} - w_i f_{ij}\right), \end{aligned}$$

which reduces to the expression in the text.

---

[42] Note that each variety z is produced by a single firm with productivity level φ. We henceforth index varieties by φ.

Labor market clearing condition. Labor market clearing is given by

$$L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^*} \frac{\tau_{ij} q_{ij} [\varphi]}{\varphi} \frac{dG[\varphi]}{1 - G\left[\varphi_{ij}^*\right]} = M_i \theta \sigma \sum_j m_{ij} f_{ij},$$

where the second equality follows from inserting $M_i^e = M_i / p_i^{in}$, using the free entry condition to substitute out f^e, and using the zero cutoff profit conditions to substitute out the cutoff productivity levels. The formula in the text follows from using the free entry condition to substitute out $\sum_j m_{ij} f_{ij}$ and noting that $\theta\sigma / (\theta - 1) = \beta/\rho$.

Trade balance condition. In analogy to expected profits, we can write expected revenues of a firm in region i from selling to region j as

$$\bar{r}_{ij} = \int_{\varphi_{ij}^*}^{\infty} r_{ij}[\varphi] \frac{dG[\varphi]}{1 - G[\varphi_{ii}^*]} = \sigma \theta w_i m_{ij} f_{ij}.$$

Using this expression, the labor market clearing condition, the definition of m_{ij} and exploiting symmetry of fixed cost, we obtain the balanced trade condition (8).

A.2 Proof of Lemma 1

A.2.1 Derivation of market crowding curve

In order to derive the market crowding curve (MCC), we use the **zero cutoff profit conditions** in relative terms and the balanced trade condition. Taking F as the target market and using the two associated zero cutoff profit conditions:

$$\left(\frac{\varphi_{HF}^*}{\varphi_{FF}^*}\right)^{\sigma-1} = \tau^{\sigma-1} \omega^{\sigma} \frac{f^x}{f^d}. \tag{17}$$

Taking H as the target market and dividing the two associated zero cutoff profit conditions:

$$\left(\frac{\varphi_{FH}^*}{\varphi_{HH}^*}\right)^{\sigma-1} = \tau^{\sigma-1} \omega^{-\sigma} \frac{f^x}{f^d}. \tag{18}$$

Using equations (17) and (18) together with the trade balance condition (8), we obtain

$$\chi = \frac{\lambda}{1-\lambda} B^{2\beta} \omega^{-\frac{2\beta - \rho}{\rho}},$$

where $\chi \equiv \left(\frac{\varphi_{FF}^*/b_F}{\varphi_{HH}^*/b_H}\right)^{\beta}$ denotes Home's relative probability of successful innovation.

A.2.2 Derivation of market potential curve

In order to derive the market potential curve (MPC), we use the **free entry conditions** in relative form along with the zero cutoff profit conditions and the balanced trade condition.

In relative form, the free entry conditions read:

$$\chi = \frac{f^d + m_{FH} f^x}{f^d + m_{HF} f^x}. \tag{19}$$

Expanding m_{ij} by φ^*_{jj}, b_H, and b_F and employing the definition of χ, we obtain

$$m_{FH} = \chi \left(\frac{\varphi^*_{HH}/b_H}{\varphi^*_{FH}/b_F} \right)^{\beta} \text{ and } m_{HF} = \chi^{-1} \left(\frac{\varphi^*_{FF}/b_F}{\varphi^*_{HF}/b_H} \right)^{\beta},$$

where the zero profit conditions can be used to substitute out the terms $\frac{\varphi^*_{ii}/b_i}{\varphi^*_{ji}/b_j}$.

Using m_{ij} in equation (19) along with the zero profit conditions and $B = b_H/b_F$, and collecting terms, we obtain

$$\chi = \frac{1 - \eta B^{\beta} \omega^{-\frac{\beta}{\rho}}}{1 - \eta B^{-\beta} \omega^{\frac{\beta}{\rho}}}.$$

A.2.3 Characteristics of market potential and market crowding curves

Market crowding curve. The market crowding curve implies a downward-sloping and convex relationship between χ and ω as

$$\frac{\partial \chi}{\partial \omega} < 0 \text{ and } \frac{\partial^2 \chi}{\partial \omega^2} > 0,$$

where the inequalities follow from $2\beta > \rho$. Evaluated at $\omega = B^{\rho}$, the market crowding curve takes the value $\chi = \frac{\lambda}{1-\lambda} B^{\rho} \geq 1$. χ is bounded from below by 0.

Market potential curve. The MPC features an asymptote at $\omega = \eta^{-\frac{\rho}{\beta}} B^{\rho}$. Evaluated at $\omega = B^{\rho}$, both the numerator and denominator are $1-\eta > 0$, which results in $\chi = 1$. The nominator is increasing in ω. Hence, for $\omega \geq B^{\rho}$, the nominator is always positive. The denominator is decreasing in ω. It becomes zero at $\omega = \left(B \eta^{-\frac{1}{\beta}} \right)^{\rho}$. Combining these observations, we can conclude that the MPC takes positive values on the interval $[B^{\rho}, \eta^{-\frac{\rho}{\beta}} B^{\rho})$.

The market potential curve implies an upward-sloping relationship between χ and ω

$$\frac{\partial \chi}{\partial \omega} = \frac{\beta \eta}{\rho \omega} \frac{B^{\beta} \omega^{-\frac{\beta}{\rho}} + B^{-\beta} \omega^{\frac{\beta}{\rho}} - 2\eta}{\left(1 - \eta B^{-\beta} \omega^{\frac{\beta}{\rho}}\right)^2} = \frac{\beta \eta}{\rho \omega} \frac{\left(B^{\beta} \omega^{-\frac{\beta}{\rho}} - \eta\right) + \left(B^{-\beta} \omega^{\frac{\beta}{\rho}} - \eta\right)}{\left(1 - \eta B^{-\beta} \omega^{\frac{\beta}{\rho}}\right)^2} > 0.$$

The inequality follows from $B^{\beta} \omega^{-\frac{\beta}{\rho}} - \eta > 0 \Leftrightarrow \omega < \left(B \eta^{-\frac{1}{\beta}} \right)^{\rho}$ and $B^{-\beta} \omega^{\frac{\beta}{\rho}} - \eta > 0 \Leftrightarrow \omega > B^{\rho} \eta^{\frac{\rho}{\beta}} > B^{\rho}$, where the last inequality follows from $\eta < 1$.

Convexity of the market potential curve requires that the freeness of trade is not too small.

In order to see this, we compute:

$$\frac{\partial^2 \chi}{\partial \omega^2} = \frac{1}{\omega}\frac{\partial \chi}{\partial \omega}\left[-1 + \frac{\beta}{\rho}\left(\frac{B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}}{B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta} + \frac{2\eta B^{-\beta}\omega^{\frac{\beta}{\rho}}}{1 - \eta B^{-\beta}\omega^{\frac{\beta}{\rho}}}\right)\right].$$

At $\omega = B^{\rho}$, the sign is given by

$$-1 + \frac{\beta}{\rho}\frac{2\eta}{1-\eta} > 0 \Leftrightarrow \eta > \frac{\rho}{2\beta + \rho}.$$

Moreover, one can show that the term in round brackets is increasing in ω. The derivative of the first term is given by

$$\frac{2\beta}{\rho\omega}\frac{2 - \eta\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)}{\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta\right)^2} > 0 \Leftrightarrow 2 > \eta\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}}\right).$$

Given the boundaries of ω, a sufficient condition for $2 > \eta\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)$ is $\eta < 1$. Moreover, it is easy to check that the second term is also increasing in ω, which completes the proof of convexity of the MPC curve.

A.2.4 Derivation of conventional sorting cutoff $\bar{\lambda}$

Cutoff level $\bar{\lambda}$ up to which Foreign first serves domestic and then export market is implicitly defined by $\varphi^*_{FF} = \varphi^*_{FH}$:

$$\varphi^*_{FF} = \varphi^*_{FH} \Leftrightarrow 1 = \tau^{-1}\bar{\omega}^{-\frac{\beta-\rho}{\rho\beta}}\left(\frac{\bar{\lambda}}{1-\bar{\lambda}}\right)^{\frac{1}{\beta}} B\left(\frac{f^x}{f^d}\right)^{-\frac{1}{\sigma-1}} \tag{20}$$

Market potential and market crowding curves imply

$$\frac{\bar{\lambda}}{1-\bar{\lambda}} = \frac{B^{-\beta}\bar{\omega}^{\frac{\beta}{\rho}} - \eta}{1 - B^{-\beta}\bar{\omega}^{\frac{\beta}{\rho}}\eta}\bar{\omega}^{\frac{\beta-\rho}{\rho}}B^{-\beta}. \tag{21}$$

Using this expression to substitute out $\frac{\bar{\lambda}}{1-\bar{\lambda}}$ from equation (20) and solving for $\bar{\omega}$, we obtain:

$$\bar{\omega}^{\frac{\beta}{\rho}} = \frac{\eta\tau^{-\beta}\left(\frac{f^x}{f^d}\right)^{-\frac{\beta}{\sigma-1}} + 1}{\tau^{-\beta}\left(\frac{f^x}{f^d}\right)^{-\frac{\beta}{\sigma-1}}B^{-\beta} + B^{-\beta}\eta}.$$

Using the definition of η, we can write $\bar{\omega}^{\frac{\beta}{\rho}}$ as

$$\bar{\omega}^{\frac{\beta}{\rho}} = B^{\beta}\frac{\eta + \frac{1}{\eta}\frac{f^x}{f^d}}{1 + \frac{f^x}{f^d}}.$$

This expression can be used to back out $\bar{\lambda}$ from equation (21).

A.3 Proof of Proposition 3

A.3.1 Home Market Magnification Effect

Evaluated at $\omega = B^{\rho}$, the market potential curve takes the value $\chi = 1$, which does not depend on η. The market potential curve rotates upwards in response to a freeness of trade shock since

$$\frac{\partial \chi}{\partial \eta} = \frac{B^{-\beta}\omega^{\frac{\beta}{\rho}}\left(1 - B^{2\beta}\omega^{-\frac{2\beta}{\rho}}\right)}{\left(1 - \eta B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)^2} > 0,$$

where the inequality follows from $\omega > B^{\rho}$. The locus of the market crowding curve is unaffected. Then, χ has to increase, which raises γ and therefore magnifies the HME. The equilibrium value of ω declines in response to a freeness of trade shock.

A.3.2 Per capita income convergence

Real per capita income directly follows from the region's domestic zero cutoff profit condition:

$$W_i \equiv \frac{w_i}{P_i} = \rho\left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \varphi_{ii}^{*}. \tag{22}$$

Equation (11) directly follows from rewriting equation (22) in relative terms. It constitutes a negative relationship between relative real per capita income and freeness of trade. Then, higher freeness of trade leads to real per capita income convergence.

A.4 Proof of Proposition 4

A.4.1 Home Market Magnification Effect

Higher productivity dispersion (lower β) leads to an upward rotation of the MCC curve as

$$\frac{\partial \chi}{\partial \beta} = 2\chi\left(\ln B - \frac{1}{\rho}\ln \omega\right) < 0$$

for $\omega > B^{\rho}$.

The effect of productivity dispersion on the MPC curve is more involved. We have:

$$\frac{\partial \chi}{\partial \beta} = \frac{\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta\right)\eta\left(\frac{\ln\omega}{\rho} - \ln B\right) + B^{-\beta}\omega^{\frac{\beta}{\rho}}\left(1 - B^{2\beta}\omega^{-\frac{2\beta}{\rho}}\right)\frac{\partial\eta}{\partial\beta}}{\left(1 - B^{-\beta}\omega^{\frac{\beta}{\rho}}\eta\right)^2},$$

where the sign of $\partial\chi/\partial\beta$ is given by the sign of the numerator and

$$\frac{\partial \eta}{\partial \beta} = -\eta\left(\ln\tau + \frac{1}{\sigma - 1}\ln\frac{f^x}{f^d}\right) < 0.$$

We conjecture that $\partial\chi/\partial\beta < 0$, which requires

$$\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta\right)(\sigma\ln\omega - (\sigma-1)\ln B) < B^{-\beta}\omega^{\frac{\beta}{\rho}}\left(1 - B^{2\beta}\omega^{-\frac{2\beta}{\rho}}\right)\left((\sigma-1)\ln\tau + \ln\frac{f^x}{f^d}\right),$$

where

$$\ln\omega \leq \rho\ln B + \rho\ln\tau + \frac{\beta-(\sigma-1)}{\beta\sigma}\ln\frac{f^x}{f^d}.$$

This inequality follows from taking logs on both sides of $\omega < \eta^{-\frac{\rho}{\beta}}B^{\rho}$ and using the definition of η. Hence, a sufficient condition for $\partial\chi/\partial\beta < 0$ reads:

$$\begin{aligned}&\sigma\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta\right)\left(\rho\ln B + \rho\ln\tau + \frac{\beta-(\sigma-1)}{\beta\sigma}\ln\frac{f^x}{f^d} - \rho\ln B\right)\\ < \;& B^{-\beta}\omega^{\frac{\beta}{\rho}}\left(1 - B^{2\beta}\omega^{-\frac{2\beta}{\rho}}\right)\left((\sigma-1)\ln\tau + \ln\frac{f^x}{f^d}\right),\end{aligned}$$

which is equivalent to

$$-\frac{\sigma-1}{\beta}\ln\frac{f^x}{f^d}\left[1 + B^{\beta}\omega^{-\frac{\beta}{\rho}}\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} - 2\eta\right)\right] + 2B^{\beta}\omega^{-\frac{\beta}{\rho}}\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} - \eta\right)\left((\sigma-1)\ln\tau + \ln\frac{f^x}{f^d}\right) < 0.$$

The inequality follows from the following observations. First, the second term is negative since $B^{\beta}\omega^{-\frac{\beta}{\rho}} - \eta < 0 \Leftrightarrow \omega < \left(B/\eta^{\frac{1}{\beta}}\right)^{\rho}$. The latter inequality follows from the characteristics of the MPC curve. Second, the term in squared brackets is positive. To see this, notice the characteristics of the MPC curve imply that:

$$B^{\beta}\omega^{-\frac{\beta}{\rho}} - 2\eta > B^{\beta}\left(B/\eta^{\frac{1}{\beta}}\right)^{-\frac{\beta\rho}{\rho}} - 2\eta = -\eta.$$

Moreover, the characteristics of the MPC curve imply

$$1 - \eta B^{\beta}\omega^{-\frac{\beta}{\rho}} > 1 - \eta B^{\beta}\left(B^{\rho}\right)^{-\frac{\beta}{\rho}} = 1 - \eta > 0,$$

where the last inequality follows from $\eta < 1$. Hence, higher dispersion leads to a higher χ, which raises γ and magnifies the HME.

A.4.2 Per capita income convergence

The effect of productivity dispersion (lower β) on relative real per capita income is given by

$$\left(\widehat{\frac{W_H}{W_F}}\right)/\hat{\beta} = -\frac{1}{\beta}\frac{\hat{\chi}}{\hat{\beta}} + \frac{1}{\beta}\ln\chi > 0,$$

where the inequality follows from $\chi > 1$ and $\frac{\hat{\chi}}{\hat{\beta}} < 0$. Hence, higher dispersion leads to convergence.

A.4.3 Complementarity

The complementarity between variable trade cost liberalization and higher productivity dispersion follows from

$$\frac{\partial\left(\widehat{\frac{W_H}{W_F}}\right)/\hat{\beta}}{\partial\tau}=\frac{1}{\beta}\left(-\frac{\partial\frac{\hat{\chi}}{\hat{\beta}}}{\partial\tau}+\frac{\frac{\partial\chi}{\partial\tau}}{\chi}\right)<0,$$

where $\partial\chi/\partial\tau<0$ and $\partial\frac{\hat{\chi}}{\hat{\beta}}/\partial\tau>0$; see Appendix A.3.1.

A.4.4 Selection

The claim follows from noting that selection is effectively shut off if β approaches its lower bound. The claim follows from the fact that higher dispersion (lower β) leads to convergence.

A.5 Proof of Proposition 5

Under unconventional sorting, equations (6) and (7) continue to hold. This claim is obvious for equation (6), which follows from the zero cutoff profit conditions. It is also true for equation (7) because the free entry condition that prevails in the small region under unconventional sorting is isomorphic to the free entry condition under conventional sorting:

$$(\theta-1)\left(\frac{\varphi_{ij}^*}{b_i}\right)^{-\beta}\left(\left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*}\right)^{\beta}f^d+f^x\right)=f^e\Leftrightarrow(\theta-1)\left(\frac{\varphi_{ii}^*}{b_i}\right)^{-\beta}\left(f^d+\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^{\beta}f^x\right)=f^e.$$

As a consequence, we can still use the graphical tool displayed in Figure 1 to characterize equilibrium. Notice, however, that we can no longer read the y-axis as the relative probability of successful innovation. It follows that also under unconventional sorting, the larger or technologically leading region pays the higher wage. Moreover, higher freeness of trade unambiguously reduces the relative wage.

A.5.1 Home market effect

The relative probability of successful innovation χ is given by

$$\chi=\frac{1-G\left[\varphi_{HH}^*\right]}{1-G\left[\varphi_{FH}^*\right]}=\left(\frac{\varphi_{FH}^*/b_F}{\varphi_{HH}^*/b_H}\right)^{\beta}.$$

The two zero cutoff profit conditions from targeting Home imply:

$$\chi=\tau^{\beta}\left(\frac{f^x}{f^d}\right)^{\frac{\beta}{\sigma-1}}B^{\beta}\omega^{-\frac{\beta}{\rho}}=\frac{f^x}{f^d}\eta^{-1}B^{\beta}\omega^{-\frac{\beta}{\rho}},\tag{23}$$

which constitutes our market crowding curve.

Using equation (23) to substitute out $\omega^{\frac{\beta}{\rho}}$ in equation (21) and solving for χ, we obtain the market potential curve:

$$\chi = \frac{f^x}{f^d} \frac{1}{1 - \frac{1-\lambda}{\lambda} B^{-\beta} \omega^{\frac{\beta-\rho}{\rho}} \left(B^{-\beta} \omega^{\frac{\beta}{\rho}} - \eta \right)},$$

which constitutes an upward-sloping relationship between χ and ω. Notice that the MPC curve implies $\chi = \frac{f^x}{f^d}$ if

$$B^{-\beta} \omega^{\frac{\beta}{\rho}} - \eta = 0 \Leftrightarrow \omega = B^{\rho} \eta^{\frac{\rho}{\beta}}.$$

Moreover, at $\omega = B^{\rho} \eta^{\frac{\rho}{\beta}}$, the MCC curve implies:

$$\chi = \frac{f^x}{f^d} \eta^{-2} > \frac{f^x}{f^d},$$

where the inequality follows from $\eta < 1$. Hence, the equilibrium relative probability of successful innovation is larger than $f^x/f^d > 1$. Together with equation (9), this proves the existence of a HME under unconventional sorting.

One can show that $\omega > 1$. Evaluating the MCC curve and the MPC curve at $\omega = 1$, we obtain $\chi = \frac{f^x}{f^d} B^{\beta} \eta^{-1}$ and $\chi = \frac{f^x}{f^d} \frac{1}{1 - \frac{1-\lambda}{\lambda} B^{-\beta} \left(B^{-\beta} - \eta \right)}$, respectively. $\omega > 1$ requires that:

$$\frac{f^x}{f^d} B^{\beta} \eta^{-1} > \frac{f^x}{f^d} \frac{1}{1 - \frac{1-\lambda}{\lambda} B^{-\beta} \left(B^{-\beta} - \eta \right)} \Leftrightarrow 1 - \frac{1-\lambda}{\lambda} B^{-\beta} \left(B^{-\beta} - \eta \right) > \eta B^{-\beta}.$$

Rearranging terms, we can rewrite this inequality as

$$\eta < \frac{\lambda B^{\beta} - (1-\lambda) B^{-\beta}}{2\lambda - 1}.$$

Notice that the right hand side is equal to unity when $B = 1$ and larger than one when $B > 1$, while by definition $\eta < 1$.

A.5.2 "Magnification"

Lower variable trade costs shift down both the market crowding and the market potential curves. Lower export fixed costs additionally shift both curves proportionally. Hence, a higher freeness of trade lowers the relative probability of successful innovation under unconventional sorting, which implies that the home market effect is *diminished* under unconventional sorting.

A.5.3 Per capita income convergence

Using the MCC (12), the trade balance condition (8), and equation (17), we can rewrite relative welfare as

$$\frac{W_H}{W_F} = \left(\frac{\lambda}{1-\lambda} \right)^{\frac{\beta-(\sigma-1)}{\beta(\sigma-1)}} B^{-1} \omega^{\frac{2\beta-\rho}{\beta\rho}} \geq 1.$$

This is a general expression which holds under unconventional and conventional sorting.[43] Under conventional sorting, the inequality follows from $\lambda \geq 1/2$, $B \geq 1$, $\omega > B^{\rho}$, $\beta > (\sigma - 1)$, and $\beta > \rho$. Given that ω is falling in the freeness of trade, higher freeness of trade leads to convergence in real per capita income.

A.6 Proof of Proposition 6

A.6.1 Preliminaries

Market crowding curve. Taking F as the target market and using the two associated zero cutoff profit conditions, we obtain:

$$\left(\frac{\varphi^*_{HF}}{\varphi^*_{FF}}\right)^{\sigma-1} = \tau_{HF}^{\sigma-1}\omega^{\sigma}\frac{f^x}{f^d} \Leftrightarrow \left(\frac{\varphi^*_{HF}}{\varphi^*_{FF}}\right)^{\beta} = \tau_{HF}^{\beta}\omega^{\frac{\beta}{\rho}}\left(\frac{f^x}{f^d}\right)^{\frac{\beta}{\sigma-1}}.$$

Taking H as the target market, we obtain:

$$\left(\frac{\varphi^*_{FH}}{\varphi^*_{HH}}\right)^{\sigma-1} = \tau_{FH}^{\sigma-1}\omega^{-\sigma}\frac{f^x}{f^d} \Leftrightarrow \left(\frac{\varphi^*_{FH}}{\varphi^*_{HH}}\right)^{\beta} = \tau_{FH}^{\beta}\omega^{-\frac{\beta}{\rho}}\left(\frac{f^x}{f^d}\right)^{\frac{\beta}{\sigma-1}}.$$

Using equations (17') and (18'), we obtain:

$$\chi = \frac{\lambda}{1-\lambda}\left(\frac{\tau_{FH}}{\tau_{HF}}\right)^{\beta}\omega^{-\frac{2\beta-\rho}{\rho}}.$$

Market potential curve. With asymmetric variable trade costs, the relative free entry conditions read:

$$\chi = \frac{f^d + \chi\left(\frac{\varphi^*_{HH}}{\varphi^*_{FH}}\right)^{\beta} f^x}{f^d + \chi^{-1}\left(\frac{\varphi^*_{FF}}{\varphi^*_{HF}}\right)^{\beta} f^x} \Leftrightarrow \chi = \frac{1 - \omega^{-\frac{\beta}{\rho}}\tau_{HF}^{-\beta}\left(\frac{f^x}{f^d}\right)^{1-\frac{\beta}{\sigma-1}}}{1 - \omega^{\frac{\beta}{\rho}}\tau_{FH}^{-\beta}\left(\frac{f^x}{f^d}\right)^{1-\frac{\beta}{\sigma-1}}}.$$

Endogenous wage adjustment. We borrow the following observations from Felbermayr and Jung (2012). First, the relative wage and the export cutoff of the larger region are positively correlated

$$\hat{\omega} = \xi\hat{\varphi}^*_{HF}, \text{ where } \beta > \xi \equiv \frac{\beta\rho}{\rho + \alpha_F(\beta - \rho)} > \rho,$$

where $dx/x = \hat{x}$; see their equation (12). α_i denotes the share of income spent on domestic goods. Second, the export cutoffs of both regions are positively correlated

$$\hat{\varphi}^*_{HF} = \beta\hat{\varphi}^*_{FH}/(\beta - \xi);$$

[43] In the case of conventional sorting, it follows from substituting out χ by means of the MCC (6).

see their equation (13). Finally, the Foreign's export cutoff falls if Home lowers its import barriers τ_{FH}

$$\hat{\varphi}^*_{FH} = \kappa\rho\hat{\tau}_H, \text{ where } \kappa \equiv \left(\rho + \frac{\beta\rho}{\beta-\xi}\left(\frac{\xi}{\rho} + \frac{1-\alpha_H}{\alpha_H}\right)\right)^{-1} > 0;$$

see text below their equation (13).[44]

Combining these observations, we can quantify the percentage drop in the relative wage in response to lower import barriers

$$\hat{\omega} = \xi\frac{\beta}{\beta-\xi}\kappa\rho\hat{\tau}_{FH}.$$

A.6.2 Home Market Magnification Effect

The change in the relative entry probability is given by

$$\hat{\chi} = \beta\hat{\tau}_{FH} - \frac{2\beta-\rho}{\rho}\hat{\omega} = \beta\left(1 - \frac{2\beta-\rho}{\beta-\xi}\xi\kappa\right)\hat{\tau}_{FH}.$$

Magnification of the HME requires $1 - \frac{2\beta-\rho}{\beta-\xi}\xi\kappa < 0$. Employing the definition of κ, we can rewrite this condition as

$$(2\beta-\rho)\,\xi > \rho\,(\beta-\xi) + \beta\rho\left(\frac{\xi}{\rho} + \frac{1-\alpha_H}{\alpha_H}\right) \Leftrightarrow \xi > \frac{\rho}{\alpha_H},$$

where the last inequality holds since $\xi > \rho$ and $\alpha_H < 1$. Hence, lowering the larger region's import barriers magnifies the HME.

A.6.3 Per capita income convergence

Equation (11) continues to hold under asymmetric trade liberalization and implies that a magnification of the HME translates into convergence of real per capita income.

A.7 Proof of Proposition 7

A.7.1 Preliminaries

Home's share of demand is

$$\delta \equiv \frac{1}{1+\omega^{-1}\frac{1-\lambda}{\lambda}}.$$

Comparing δ to the Home's share of firms ϕ in equation (9), we find that a HME in demand shares requires $\chi > \omega$.

[44] Note that Home's import barriers τ_{FH} are denoted τ_H in Felbermayr and Jung (2012).

A necessary condition for the equilibrium χ to exceed the equilibrium ω is that the MPC curve lies above the MCC curve at the intersection of the latter with $\omega = \chi$. The wage $\tilde{\omega}$ for which the MCC curve yields $\omega = \chi$ is given by:

$$\tilde{\omega} = \frac{\lambda}{1-\lambda} B^{2\beta} \tilde{\omega}^{-\frac{2\beta-\rho}{\rho}} \Leftrightarrow \tilde{\omega}^{\frac{2\beta}{\rho}} = \frac{\lambda}{1-\lambda} B^{2\beta} \Leftrightarrow \tilde{\omega} = \left(\frac{\lambda}{1-\lambda}\right)^{\frac{\rho}{2\beta}} B^{\rho}.$$

Then, the necessary condition reads:

$$\frac{1 - B^{\beta}\tilde{\omega}^{-\frac{\beta}{\rho}}\eta}{1 - B^{-\beta}\tilde{\omega}^{\frac{\beta}{\rho}}\eta} > \tilde{\omega}.$$

Solving for η and substituting out $\tilde{\omega}$, we obtain:

$$\eta > \ell^{\frac{1}{2}} \frac{B^{\rho}\ell^{\frac{\rho}{2\beta}} - 1}{B^{\rho}\ell^{\frac{2\beta+\rho}{2\beta}} - 1},$$

where $\ell \equiv \lambda / (1-\lambda)$ with $\ell \geq 1$.

A.7.2 Region size differences

First, consider $B = 1$ and $\ell > 1$.Then, the necessary condition reads:

$$\eta > \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

If ℓ increases, the numerator increases, but the denominator increases faster, such that the term on the right hand side declines. Then, a sufficient condition for a HME in demand shares is

$$\eta > \lim_{\ell \to 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1}.$$

Employing l'Hospital's rule, we find:

$$\lim_{\ell \to 1} \ell^{\frac{1}{2}} \frac{\ell^{\frac{\rho}{2\beta}} - 1}{\ell^{\frac{2\beta+\rho}{2\beta}} - 1} = \lim_{\ell \to 1} \ell^{\frac{1}{2}} \lim_{\ell \to 1} \frac{\frac{\rho}{2\beta}\ell^{-\frac{2\beta-\rho}{2\beta}}}{\frac{2\beta+\rho}{2\beta}\ell^{\frac{\rho}{2\beta}}} = \frac{\rho}{2\beta+\rho}.$$

A.7.3 Technology differences

Second, consider $B > 1$ and $\ell = 1$. Then, the necessary condition for a HME in demand shares reduces to

$$\eta > 1,$$

which can never hold. Quite the opposite is true. Hence, if regions are symmetric in size but differ in their average productivity, the richer (technologically leading) region hosts an underproportional share of firms.

A.8 Proof of Proposition 8

Remember that

$$\phi = \gamma\lambda, \gamma = \frac{\chi}{1+\lambda\chi-\lambda}$$

with

$$\frac{\partial\gamma}{\partial\lambda} = \frac{\frac{\partial\chi}{\partial\lambda}\left[1+\lambda\left(\chi-1\right)\right]-\chi\left(\chi+\lambda\frac{\partial\chi}{\partial\lambda}-1\right)}{\left[1+\lambda\left(\chi-1\right)\right]^2}$$

Then

$$\begin{aligned}\epsilon_\gamma &\equiv \frac{\lambda\frac{\partial\gamma}{\partial\lambda}}{\gamma} = \frac{\frac{\partial\chi}{\partial\lambda}\lambda\left[1+\lambda\left(\chi-1\right)\right]-\lambda\chi\left(\chi+\lambda\frac{\partial\chi}{\partial\lambda}-1\right)}{1+\lambda\left(\chi-1\right)}\frac{1}{\chi}\\ &= \epsilon_\chi\left(1-\phi\right)-\phi\left(\frac{\chi-1}{\chi}\right),\end{aligned}$$

where $\epsilon_\chi \equiv \frac{\partial\chi}{\partial\lambda}\frac{\lambda}{\chi}$. We have $\epsilon_\gamma > 0$ if

$$\epsilon_\chi > \frac{\phi}{1-\phi}\left(\frac{\chi-1}{\chi}\right).$$

An alternative way to write this is

$$\epsilon_\chi > \frac{\frac{\chi\lambda}{1+\lambda\chi-\lambda}}{\frac{1-\lambda}{1+\lambda\chi-\lambda}}\left(\frac{\chi-1}{\chi}\right) = \frac{\lambda}{1-\lambda}\left(\chi-1\right) \Leftrightarrow \frac{\epsilon_\chi}{\chi-1} > \frac{\lambda}{1-\lambda}.$$

The conjecture is that we can identify a downward- and an upward-sloping curve such that the left hand side and the right hand side are equal for some unique $\lambda^* \in (1/2, 1)$.

Using $\epsilon_\chi = \frac{\partial\chi}{\partial\lambda}\frac{\lambda}{\chi} = \frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}$, we can rewrite the inequality as

$$\frac{\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}}{\chi-1} > \frac{\lambda}{1-\lambda}\frac{1}{\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}}.$$

We prove that the left hand side is downward-sloping in ω and therefore downward-sloping in λ, whereas the right hand side is increasing in λ.

The $\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)$ locus. Recall that

$$\frac{\partial\chi}{\partial\omega} = \frac{\eta\beta}{\rho}\frac{\left(B^\beta\omega^{-\frac{\beta}{\rho}}-\eta\right)+\left(B^{-\beta}\omega^{\frac{\beta}{\rho}}-\eta\right)}{\omega\left(1-\eta B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)^2} > 0.$$

The elasticity of χ in ω is then

$$\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi} = \frac{\eta\beta}{\rho}\frac{\left(B^\beta\omega^{-\frac{\beta}{\rho}}-\eta\right)+\left(B^{-\beta}\omega^{\frac{\beta}{\rho}}-\eta\right)}{\left(1-\eta B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)\left(1-\eta B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)},$$

where

$$\Leftrightarrow \frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi} = \epsilon_{\chi} / \left(\frac{\partial \omega}{\partial \lambda}\frac{\lambda}{\omega} \right).$$

Moreover,

$$\chi - 1 = \eta \frac{B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}}{1 - \eta B^{-\beta}\omega^{\frac{\beta}{\rho}}} > 0,$$

and therefore

$$\frac{\frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi}}{\chi - 1} = \frac{\beta}{\rho}\frac{\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} - \eta\right) + \left(B^{-\beta}\omega^{\frac{\beta}{\rho}} - \eta\right)}{\left(1 - \eta B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)}.$$

Slope of $\frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi} / (\chi - 1)$ in ω. We conjecture that $\partial \left[\frac{\partial \chi}{\partial \omega}\frac{\omega}{\chi} / (\chi - 1)\right] / \partial \omega < 0$. To check this, define

$$\begin{aligned} f[\omega] &= B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta > 0 \\ g[\omega] &= \left(1 - \eta B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}\right) \\ &= B^{-\beta}\omega^{\frac{\beta}{\rho}} - \eta - B^{\beta}\omega^{-\frac{\beta}{\rho}} + \eta B^{2\beta}\omega^{-\frac{2\beta}{\rho}} > 0. \end{aligned}$$

We have to show that

$$f'[\omega]\, g[\omega] - f[\omega]\, g'[\omega] < 0,$$

where

$$\begin{aligned} f'[\omega] &= -\frac{\beta}{\rho}B^{\beta}\omega^{-\frac{\beta}{\rho}-1} + \frac{\beta}{\rho}B^{-\beta}\omega^{\frac{\beta}{\rho}-1} = \frac{\beta}{\rho}\left(\frac{B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}}{\omega}\right) > 0 \\ g'[\omega] &= \frac{\beta}{\rho}B^{-\beta}\omega^{\frac{\beta}{\rho}-1} + \frac{\beta}{\rho}B^{\beta}\omega^{-\frac{\beta}{\rho}-1} - \frac{2\beta}{\rho}\eta B^{2\beta}\omega^{-\frac{2\beta}{\rho}-1} = \frac{\beta}{\rho}\left(\frac{B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}} - 2\eta B^{2\beta}\omega^{-\frac{2\beta}{\rho}}}{\omega}\right) > 0. \end{aligned}$$

The last inequality follows from $B^{2\beta}\omega^{-\frac{2\beta}{\rho}} < 1 \Leftrightarrow \omega > B^{\rho}$ and $f[\omega] > 0$.

We can rewrite the necessary condition as

$$\begin{aligned} & \frac{\beta}{\rho}\frac{1}{\omega}\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)^{2}\left(1 - \eta B^{\beta}\omega^{-\frac{\beta}{\rho}}\right) \\ < \;& \frac{\beta}{\rho}\frac{1}{\omega}\left(B^{\beta}\omega^{-\frac{\beta}{\rho}} + B^{-\beta}\omega^{\frac{\beta}{\rho}} - 2\eta\right)\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}} - 2\eta B^{2\beta}\omega^{-\frac{2\beta}{\rho}}\right). \end{aligned}$$

Since $1 - \eta B^{\beta}\omega^{-\frac{\beta}{\rho}} < 1$ and $B^{2\beta}\omega^{-\frac{2\beta}{\rho}} < 1$, a sufficient condition is

$$\left(B^{-\beta}\omega^{\frac{\beta}{\rho}} - B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)^{2} < \left(B^{-\beta}\omega^{\frac{\beta}{\rho}} + B^{\beta}\omega^{-\frac{\beta}{\rho}} - 2\eta\right)^{2} \Longleftrightarrow 0 < B^{\beta}\omega^{-\frac{\beta}{\rho}} - \eta\omega < B^{\rho}\eta^{-\frac{\rho}{\beta}}\omega < B^{\rho}\eta^{-\frac{\rho}{\beta}}.$$

The $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus. Using equation (21), we can define

$$H\left[\lambda,\omega\right]\equiv\omega^{-\frac{\beta-\rho}{\rho}}\frac{\eta\omega^{\frac{\beta}{\rho}}-B^{\beta}}{\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}}-\frac{1-\lambda}{\lambda}=0.$$

By the implicit function theorem, we have:

$$\frac{\partial\omega}{\partial\lambda}=-\frac{\partial H}{\partial\lambda}/\frac{\partial H}{\partial\omega},$$

where

$$\frac{\partial H}{\partial\lambda}=\frac{1}{\lambda^{2}}$$

and

$$\frac{\partial H}{\partial\omega}=-\left(\frac{\beta-\rho}{\rho}\frac{\omega^{-\frac{\beta}{\rho}}\left(\eta\omega^{\frac{\beta}{\rho}}-B^{\beta}\right)}{\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}}+\frac{\beta}{\rho}\frac{1-\eta^{2}}{\left(\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)^{2}}\right)<0.$$

Then,

$$\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}=\frac{\lambda}{1-\lambda}\lambda\omega\left(\frac{\beta-\rho}{\rho}\frac{\omega^{-\frac{\beta}{\rho}}\left(\eta\omega^{\frac{\beta}{\rho}}-B^{\beta}\right)}{\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}}+\frac{\beta}{\rho}\frac{1-\eta^{2}}{\left(\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)^{2}}\right)$$

Using the trade balance condition to substitute out $\lambda/\left(1-\lambda\right)$ on the right hand side, we obtain:

$$\begin{aligned}\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1} &= \omega^{\frac{\beta-\rho}{\rho}}\frac{\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}}{\eta\omega^{\frac{\beta}{\rho}}-B^{\beta}}\lambda\omega\left(\frac{\beta-\rho}{\rho}\frac{\omega^{-\frac{\beta}{\rho}}\left(\eta\omega^{\frac{\beta}{\rho}}-B^{\beta}\right)}{\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}}+\frac{\beta}{\rho}\frac{1-\eta^{2}}{\left(\eta-B^{-\beta}\omega^{\frac{\beta}{\rho}}\right)^{2}}\right)\\ &= \lambda\left(\frac{\beta-\rho}{\rho}+\frac{\beta}{\rho}\frac{1-\eta^{2}}{\eta^{2}+1-\eta\left(B^{-\beta}\omega^{\frac{\beta}{\rho}}+B^{\beta}\omega^{-\frac{\beta}{\rho}}\right)}\right).\end{aligned}$$

Slope of the $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus. It is easy to check that $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ rises in λ, given that $B^{-\beta}\omega^{\frac{\beta}{\rho}}+B^{\beta}\omega^{-\frac{\beta}{\rho}}$ increases in ω and therefore in λ.

Hence, the downwards-sloping $\frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}/\left(\chi-1\right)$ locus and the upward-sloping $\frac{\lambda}{1-\lambda}\left(\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}\right)^{-1}$ locus determine a unique λ^{*} such that for $\lambda<\lambda^{*}$ a strong HME occurs.

B Calibration

We use standard results from the literature to calibrate the model at symmetric equilibrium.[45] An important source of information is Bernard, Eaton, Jensen, and Kortum (2003). They argue that $\sigma = 3.8$ fits their data well and report that the standard deviation of domestic US plant sales is 0.84. In terms of the Melitz (2003) model, this value has to equal $(\sigma - 1)/\beta$. With the estimate for σ at hand, we obtain $\beta = 3.3$, which meets the restriction $\beta > \sigma - 1$. Moreover, it is close to estimates from other sources.[46] According to Bernard et al. (2003), the export participation rate of US firms is about 21%.[47] Using $\tau = 1.3$, which we take from Obstfeld and Rogoff (2001), and the corresponding values for σ and β, the implied relative export fixed costs amount to $f^x/f^d = 1.8$.

Table 2: Parametrization

Parameter	Benchmark
σ	3.8
β	3.3
f^x/f^d	1.8
τ	1.3

Under this parameter constellation, the freeness of trade is given by $\eta \approx 0.38$. The implied conventional sorting amounts to $\bar{\lambda} \approx 0.88$. Since it is only implicitly defined, we have to back out λ^* from our simulations. The market potential curve is convex, and a HME in demand shares arises.

[45] We follow Demidova (2008).

[46] Mayer and Ottaviano (2007) report that the shape parameters for total manufacturing are 3.03 and 2.55 for Italy and France, respectively.

[47] In Europe, the UK features a similar export participation rate (28%), but German and French export participation rates are higher; see Mayer and Ottaviano (2007).

Complementary Appendices C-E

to

Felbermayr & Jung

Home Market Effect, Regional Inequality, and Intra-Industry Reallocations

April 4, 2012

C The HME with a General Sampling Distribution

In this subsection, we generalize the argument to a situation where firms' productivity levels are sampled from a general productivity distribution. Our MCC and MPC curves hinge on the assumption of the Pareto distribution and are therefore of no help in the general case. While it is difficult to derive results on the strong HME or on magnification effects, it is possible to establish the HME for the case of conventional sorting.

To this end, it appears useful to reduce the equilibrium conditions (2) - (5) to two equations in ω and Home's export cutoff φ^*_{HF}, which have opposite slopes. The two loci are substantially more complicated than our MCC and MPC schedules, but they are still useful for our purposes. Labor endowment shares affect only one of these curves, which allows inference on the effect of a region size shock on the relative wage and the various cutoffs. Moreover, drawing on a generalized labor market clearing condition, we can derive our result on the home market effect.

Lemma 2' *Assume that productivity levels are sampled from a general productivity distribution. Then, the larger region pays the higher wage.*

Proof. As proposed by Demidova and Rodríguez-Clare (2011), we reduce the model's equilibrium conditions (2), (4), and (5) to a system of two equations in ω and φ^*_{HF}. The first curve draws on zero cutoff profit and free entry conditions and is independent of the productivity distribution. The relative cutoff profit conditions for targeting Foreign is given by

$$\varphi^*_{HF} = \tau \left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma-1}} \omega^{\frac{1}{\rho}} \varphi^*_{FF}. \tag{24}$$

Foreign's domestic entry cutoff φ^*_{FF} is a function of its export cutoff φ^*_{FH} by free entry. Moreover, using the relative cutoff profit conditions for entry into Home, φ^*_{FH} can be expressed as a function of the relative ω and φ^*_{HH}. The latter is a function of φ^*_{HF} by free entry. Hence, our first equilibrium condition constitutes an upward sloping and concave relationship between the relative wage and Home's export cutoff. It is important to note that the locus of this curve is not affected by region size.

Under a general productivity distribution, the balanced trade condition can be rewritten as

$$\sigma \left(\varphi^*_{HH}\right)^{-\rho} \left(\psi_H + 1\right) = \frac{\lambda}{1-\lambda} \tau^{\rho} \left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma}} \left(\varphi^*_{FH}\right)^{-\rho} \left(\psi_F + 1\right), \tag{25}$$

where

$$\psi_i = \frac{f^d}{f^x} \left(\frac{\varphi^*_{ij}}{\varphi^*_{ii}}\right)^{\sigma-1} \frac{\int_{\varphi^*_{ii}} \varphi^{\sigma-1} dG_i\left[\varphi\right]}{\int_{\varphi^*_{ij}} \varphi^{\sigma-1} dG_i\left[\varphi\right]}.$$

The left hand side of equation (25) is independent of ω. Moreover, it rises in φ^*_{HF}. The right hand side can be expressed as a function of φ^*_{FH}. φ^*_{FH} has to rise in φ^*_{HF}. By free entry, φ^*_{HH} falls. Relative entry in Home reads

$$\varphi^*_{FH} = \tau \left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma-1}} \omega^{-\frac{1}{\rho}} \varphi^*_{HH}. \tag{26}$$

Since φ^*_{FH} rises and φ^*_{HH} falls, ω must fall to restore equilibrium. Hence, trade balance estab-

lishes a negative relationship between Home's export cutoff and the relative wage.
Consider a region size shock. For a given relative wage, the right hand side of the trade balance curve must be larger, which can only come about by an increase in φ^*_{HF}. Hence, a region size shock shifts the trade balance locus upwards. We also conclude that the larger region pays a higher wage, which extends Lemma 1 to the general case. ■

Proposition 1' *Assume that productivity levels are sampled from a general productivity distribution. Moreover, assume that conventional sorting holds* $(1/2 < \lambda < \bar{\lambda})$. *Then, the model exhibits a HME.*

Proof. Using the free entry conditions, we can write the ratio of active firms as

$$\frac{M_H}{M_F} = \frac{\lambda}{1-\lambda} \frac{f^e/p_F^{in} + f^d + f^x m_{FH}}{f^e/p_H^{in} + f^d + f^x m_{HF}}. \tag{27}$$

We have argued in the proof of Lemma 1' that a region size shock raises Home's export cutoff φ^*_{HF}. By free entry, Home's domestic entry cutoff falls. Hence, the denominator of the above expression falls. It follows from equation (26) that Foreign's export cutoff φ^*_{FH} falls since φ^*_{HH} falls and ω rises. By free entry, φ^*_{FF} rises. Then, the numerator of the above equation rises, which implies that the effect of the region size shock on the relative mass of firms is magnified.
It is easy to check that $M_H/M_F = \phi/\left(1-\phi\right) > \lambda/\left(1-\lambda\right)$ directly translates into $\phi > \lambda$, which constitutes a home market effect. ■

D Melitz (2003) with outside sector

In this appendix, we derive equilibrium in the presence of an outside sector. We show that the economy exhibits a home market effect. As in Helpman and Krugman (1995), the home market effect is linear in λ. Moreover, we discuss welfare implications.

Basic environment. The model is augmented by a homogeneous good produced under constant returns to scale and perfect competition. Utility takes the Cobb-Douglas form, where μ denotes the share of expenditure spent on differentiated varieties. The outside good is freely tradable. Hence, wages are equalized and henceforth normalized to unity. Welfare per worker is given by the inverse of the aggregate price index. Using P_i to denote the price index of the differentiated goods sector and defining $\tilde{\mu} \equiv (1-\mu)^{1-\mu}\,\mu^{\mu}$, we can write welfare per worker as

$$W_i = \tilde{\mu}/P_i^{\mu}.$$

Equilibrium. The free entry conditions are unaffected. In the zero cutoff profit conditions, however, wages drop. In relative form, the zero cutoff profit condition becomes

$$\varphi_{ji}^{*} = \tau \left(\frac{f^x}{f^d}\right)^{\frac{1}{\sigma-1}} \varphi_{ii}^{*}.$$

Hence, we can substitute out export cutoffs from the free entry conditions. We obtain two equations in two unknowns, which can be used to solve for the domestic entry cutoffs as

$$\left(\varphi_{ii}^{*}\right)^{\beta} = (\theta-1)\,\frac{f^d}{f^e}\,\frac{1-\eta^2}{1-\eta\left(\frac{b_i}{b_j}\right)^{\beta}}\,b_i^{\beta}.$$

It is important to note that the cutoff productivity levels do not depend on region size. With symmetric technology $b_H = b_F = b$, they reduce to

$$\left(\varphi_{ii}^{*}\right)^{\beta} = (\theta-1)\,\frac{f^d}{f^e}\,(1+\eta)\,b^{\beta}.$$

The free entry conditions along with the relative zero cutoff profit conditions allow to solve for the relative probability of successful innovation in closed form as

$$\chi = \frac{1-\eta B^{\beta}}{1-\eta B^{-\beta}} \leq 1,$$

where the equality appears for $B=1$. Notice that χ does not depend on relative region size.

Labor market clearing implies:

$$\xi_i L_i = M_i^e f^e + M_i \sum_j m_{ij} f_{ij} + M_i \sum_j \int_{\varphi_{ij}^{*}} \frac{\tau_{ij} q_{ij}\left[\varphi\right]}{\varphi}\,\frac{dG\left[\varphi\right]}{1-G\left[\varphi_{ij}^{*}\right]} = M_i \theta\sigma \sum_j m_{ij} f_{ij},$$

where ξ_i denotes the fraction of workers employed in the differentiated good sector.

Using the free entry condition, we obtain:

$$M_i = \frac{\xi_i L_i}{\bar{r}_i};$$

where $\bar{r}_i \equiv \sum_j \bar{r}_{ij} = \frac{\beta f^e}{\rho} \left(\frac{\varphi^*_{ii}}{b_i} \right)^\beta$. Expected revenues $\bar{r}_{ij}$ are given by

$$\bar{r}_{ij} = \int_{\varphi^*_{ij}} r_{ij} [\varphi] \frac{dG [\varphi]}{1 - G \left[\varphi^*_{ij} \right]} = \theta \sigma m_{ij} f_{ij}.$$

Assuming $b_H = b_F$, cutoffs are symmetric. We henceforth suppress the subscripts of the revenue terms. Note that $\bar{r}^x / \bar{r}^d$ reduces to η.

Balanced trade is given by

$$M_i \bar{r}_{ij} = M_j \bar{r}_{ji} + (1 - \mu) L_i - (1 - \xi_i) L_i,$$

where the term on the left hand side represents region i's exports of the differentiated good. The first term on the right hand side represents i's imports of the differentiated good. The remaining terms reflect i's imports of the homogeneous good (spending on the homogeneous good minus value of domestic homogeneous good production).

Substituting out M_i and rearranging terms, we obtain:

$$\xi_i = \mu \frac{\bar{r}_i}{\bar{r}_{ij}} - \frac{L_j}{L_i} \frac{\bar{r}_{ji}}{\bar{r}_{ii}} \frac{\bar{r}_i}{\bar{r}_j} \xi_j, i \in \{H, F\}.$$

The solution to this system of two equations is given by

$$\xi_i = \mu \frac{\bar{r}_i}{\bar{r}_{ii}} \frac{1 - \frac{\bar{r}_{ji}}{\bar{r}_{jj}} \frac{L_j}{L_i}}{1 - \eta^2}, i \in \{H, F\}.$$

Using the above expressions, we find that

$$\xi_H > \xi_F \Leftrightarrow \frac{1 + \eta \frac{L_H}{L_F} B^\beta}{1 + \eta \frac{L_F}{L_H} B^{-\beta}} > 1 > \chi,$$

where the first inequality follows from $\lambda \geq 1/2$ and $B \geq 1$. Hence, the smaller region is the net exporter of the homogeneous good.

Home market effect. The labor market clearing conditions implies that Home's share of firms active in the differentiated good sector can be written as

$$\phi = \frac{1}{\lambda + (1 - \lambda) \frac{\xi_F}{\xi_H} \chi^{-1}} \lambda.$$

Using $\bar{r}_{HH} = \bar{r}_{FF} = \bar{r}^d$, we obtain:

$$\phi = \frac{1}{1 + \frac{\frac{1-\lambda}{\lambda}\bar{r}_F - \bar{r}_{HF}}{\bar{r}_H - \bar{r}_{FH}\frac{1-\lambda}{\lambda}}\chi^{-1}} = \frac{\lambda\left(\bar{r}^d + \bar{r}_{HF} + \bar{r}_{FH}\right) - \bar{r}_{FH}}{\chi^{-1}\bar{r}^d + \left(\chi^{-1} - 1\right)\left[\lambda\left(\bar{r}^d + \bar{r}_{HF} + \bar{r}_{FH}\right) - \bar{r}_{FH}\right]}$$

with

$$\frac{\partial\phi}{\partial\lambda} = \frac{\chi^{-1}\bar{r}^d\left(\bar{r}^d + \bar{r}_{HF} + \bar{r}_{FH}\right)}{\left(\chi^{-1}\bar{r}^d + \left(\chi^{-1} - 1\right)\left[\lambda\left(\bar{r}^d + \bar{r}_{HF} + \bar{r}_{FH}\right) - \bar{r}_{FH}\right]\right)^2}.$$

In the absence of technology differences ($B = 1$), we have $\chi = 1$, and the derivative simplifies to

$$\frac{\partial\phi}{\partial\lambda} = 1 + 2\frac{\bar{r}^x}{\bar{r}^d} = 1 + 2\eta.$$

Comparison to single-sector model. It is easy to check that $\partial\phi/\partial\lambda = \gamma + \lambda\partial\gamma/\partial\lambda$, where the first term is the direct effect of a region size shock on λ. The second term represents the indirect effect due to adjustments in the relative probability of successful innovation. We have already shown that

$$\frac{\partial\gamma}{\partial\lambda} = \frac{\gamma}{\lambda}\left[\epsilon_\chi\left(1 - \phi\right) - \phi\left(\frac{\chi - 1}{\chi}\right)\right],$$

where ϵ_χ denotes the elasticity of χ in λ. Applying the chain rule, we can rewrite ϵ_χ as $\epsilon_\chi = \frac{\partial\chi}{\partial\omega}\frac{\omega}{\chi}\frac{\partial\omega}{\partial\lambda}\frac{\lambda}{\omega}$. At the symmetric equilibrium, we have $\phi = \lambda = 1/2$ and $\omega = \chi = \gamma = B = 1$. Hence, we have

$$\frac{\partial\chi}{\partial\omega} = \frac{2\eta\beta}{\rho\left(1 - \eta\right)} \text{ and } \frac{\partial\omega}{\partial\lambda} = \frac{1}{\lambda^2}\frac{\rho}{2\frac{\beta}{1-\eta} - \rho}.$$

Moreover, we have

$$\epsilon_\chi = \frac{\partial\gamma}{\partial\lambda} = 2\eta\left(1 - \frac{\rho\left(1 - \eta\right)}{2\beta}\right)^{-1}.$$

At the symmetric equilibrium, the slope of the ϕ-curve is given by

$$\frac{\partial\phi}{\partial\lambda} = 1 + \eta\left(1 - \frac{\rho\left(1 - \eta\right)}{2\beta}\right)^{-1},$$

which is strictly smaller than the slope of ϕ in λ for the case with the outside sector since $\rho\left(1 - \eta\right)/\beta < 1$.

Per capita income. Using the domestic zero cutoff profit condition, we can rewrite welfare per worker as

$$W_i = \tilde{\mu}\left(\frac{\mu L_i}{\sigma f^d}\right)^{\frac{\mu}{\sigma-1}}\left(\rho\varphi_{ii}^*\right)^\mu,$$

where $\tilde{\mu} \equiv \left(1 - \mu\right)^{1-\mu}\mu^\mu$ is a constant. Relative real per capita income is given by

$$\frac{W_H}{W_F} = \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{\mu}{\sigma-1}} B^\mu \chi^{-\frac{\mu}{\beta}}.$$

With $B = 1$, the relative probability of successful innovation is fixed at unity and neither trade liberalization nor productivity dispersion affect relative per capita income. For $B > 1$, we have $\chi < 1$ and

$$\frac{\partial \chi}{\partial \eta} = -\frac{B^{\beta} - B^{-\beta}}{\left(1 - \eta B^{-\beta}\right)^2} < 0.$$

Hence, for $B > 1$, trade liberalization leads to divergence of real per capita income.

E Melitz (2003) without export selection

In the absence of fixed exporting costs ($f^x = 0$), all firms export as in Krugman (1980). In a setting with symmetric countries, Melitz (2003) argues that fixed export costs are crucial for trade to induce "distributional changes among firms" (footnote 24, p. 170). We generalize this remark to the case of asymmetric countries, which stresses the importance of intra-industry reallocation for our results.

Zero cutoff profit conditions. We only have to characterize the domestic entry conditions. As the elasticity of demand is constant across countries, firms charge the same ex-factory price in all markets, which is a constant mark-up over marginal costs. Given optimal demand in both regions, profits of a firm with productivity φ in country i are given by

$$\pi_i [\varphi] = \left[\frac{R_i}{\sigma} P_i^{\sigma-1} + \frac{R_j}{\sigma} \left(\frac{P_j}{\tau_{ij}} \right)^{\sigma-1} \right] \left(\frac{\rho\varphi}{w_i} \right)^{\sigma-1} - w_i f^d.$$

The entry cutoff φ_i^* is determined by the zero cutoff profit condition

$$\pi_i [\varphi_i^*] = 0.$$

Free entry conditions. As in Melitz (2003), free entry requires that the expected profits must be equal to the innovation fixed costs $w_i f^e$. Expected profits of a firm in region i from entering are given by

$$\begin{aligned}
\bar{\pi}_i &= \int_{\varphi_i^*} \pi_i [\varphi] \frac{dG [\varphi]}{1 - G [\varphi_i^*]} \\
&= \left[\frac{R_i}{\sigma} P_i^{\sigma-1} + \frac{R_j}{\sigma} \left(\frac{P_j}{\tau_{ij}} \right)^{\sigma-1} \right] \theta \left(\frac{\rho\varphi_i^*}{w_i} \right)^{\sigma-1} - w_i f^d.
\end{aligned}$$

Using the zero cutoff profit condition, we obtain:

$$\bar{\pi}_i = (\theta - 1) w_i f^d.$$

Then, the free entry condition reads:

$$(\theta - 1) p_i^{in} f^d = f^e.$$

Equilibrium. Recall that $p_i^{in} = (\varphi_i^*/b_i)^{-\beta}$. Hence, the free entry condition implies that the equilibrium entry cutoff φ_i^* is independent of region size and variable trade costs. This, in turn, means that the mass of firms is fixed, and there is no HME. Trade liberalization has no effect on real per-capita income.