When Do Firms Exchange Information?*

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Abstract

This paper further develops the standard modelling of information exchange between firms in the presence of demand uncertainty which applies to firms in new industries and insecure regions or markets. We replace the normal distribution of the random variables, commonly used because of its convenient mathematical properties, by an alternative one, namely a random variable with a binomial positive outcome. For the symmetric case, we confirm the results of the existing literature. However, for the non-symmetric case, we derive the new result that in the resulting Bayesian Nash equilibrium, the firms will disclose their information more often than they would under the standard modelling.

<u>JEL classification:</u> L13, D43, D82, C72, C73

Zusammenfassung

Wann tauschen Unternehmen Informationen aus?

Dieser Beitrag erweitert die Standardmodellierung der Literatur über Informationsaustausch zwischen Unternehmen bei Nachfrageunsicherheit und läßt sich vor allem auf Unternehmen in neuen Branchen oder in unsicheren Märkten anwenden. Die bisher insbesondere aus technischen Gründen unterstellte Normalverteilung der Zufallsvariablen wird durch eine alternative, nur im positiven Bereich definierte Zweipunkt-Verteilung ersetzt. Für den symmetrischen Fall lassen sich die Ergebnisse der bestehenden Literatur bestätigen. Zusätzlich zeigt sich jedoch für den asymmetrischen Fall, daß die Unternehmen im resultierenden Bayes-Nash-Gleichgewicht noch häufiger ihre Informationen austauschen als bei einer symmetrischen Verteilung.

<u>JEL-Klassifikation:</u> L13, D43, D82, C72, C73

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1 Introduction

Why should a firm with private information about its demand disclose this information to its rivals? A first quick answer could be that it intends to weaken the intensity of competition via collusive behaviour in order to increase its profits.[1] However, as will be shown below, this is not the only possible explanation.

This paper deals with the question whether firms in a duopolistic market will disclose their private information to their rivals even when there is no deliberate intention to limit competition. Thus, this paper belongs to the class of oligopoly models investigating the exchange of private information in a world with incomplete information. Pioneers in this field are *Basar*, *Ho* (1974), *Ponssard* (1979) and *Novshek*, *Sonnenschein* (1982). Two main directions of analysis have evolved: Models analysing cost uncertainty (cf. for example *Fried* 1984, *Li* 1985, *Gal-Or* 1986, *Shapiro* 1986) and models analysing demand uncertainty (cf. for example *Clarke* 1983, *Vives* 1984, *Gal-Or* 1985, *Li* 1985, *Sakai* 1986, *Kirby* 1988, *Sakai*, *Yamato* 1989, *Hviid* 1989, *Hornig* 1999).[2] *Sakai* (1990, 1991), *Jin* (1992) and *Raith* (1996) form general models that contain most of the results obtained in the papers cited above as special cases. Very recently, *Vives* (2000, Ch. 8) provides a non-formal and very comprehensive overview. Also, *Stadler*, *Hornig* (2000) show the effects of information sharing in a simple general model.

All the models treating demand uncertainty have in common that uncertainty about the common ordinate intercept of the market demand functions exists. This uncertainty is modelled by assuming that the random variable is normally distributed. This is done mainly for technical reasons concerning the mathematics of the models. Although the normal distribution is defined over the range $-\infty$ to $+\infty$, the authors implicitly assume the realisations of the random variable to be "very close" to the expected intercept in order to guarantee that the non-negativity constraint of the ordinate intercept of the demand function is fulfilled. However, this obviously contradicts the assumption made about the distribution.[3]

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[1] The controversy whether information exchange between firms weakens the intensity of competition prevailing in a market begins with *Stigler's* (1964) classical article about oligopoly theory and continues with *Shapiro* (1986), *Jin* (1992) and the contributions in *Albach et al.* (1996).

[2] By investigating the case of capacity uncertainty, *Farmer* (1994) begins an additional direction of analysis.

[3] *Li* (1985) shows that the non-negativity constraint could be imposed by assuming gamma-Poisson or beta-Binomial distributions. Another argument in order to heal the non-negativity problem might be that random variables that result of the additive clustering of many singular

The model presented here solves this drawback: The distribution used guarantees the condition of non-negativity. Further, the random variables here do not necessarily have to be symmetrically distributed (as is implicitly assumed by using the normal distribution). The most simple way of meeting these conditions (non-negativity and non-symmetry) is to choose a random variable with two possible (positive) realisations that do not have to be equally probable.

Beyond that, this distribution is a good approximation of the way firms in new industries and insecure regions analyse their markets. Due to the lack of existing data or experiences about the respective market, the firms have no choice but to form a strictly limited number of the most probable scenarios about the demand for their products. As examples for firms in new industries with missing data and experiences one may think of firms in the software, pharmaceuticals, biotechnical or genetic engineering industries. However, the same arguments also apply to situations when firms set off as pioneers for so far cut-off markets or countries. Similarly, imagine markets in which established firms are forced to fundamentally reorientate because of unforeseen political or military imponderabilities. Then, the firms are confronted with a relatively poor information base: The well known structures and data from the past do not count anymore. The best the firms can do is to form scenarios about their market and demand situation. Therefore, there is neither a theoretical nor an empirical justification to limit the analysis by the assumption of a symmetric probability distribution. Additionally, besides theoretical interest, applying a discrete and not a continuous distribution for the above reasons seems empirically more plausible.

The above cited papers concerning demand uncertainty and using the normal distribution show that Cournot firms producing very close substitutes do not exchange their information, whereas for not very close substitutes and for the whole range of complementary goods, disclosure is always favourable. In contrast, under Bertrand competition, firms producing substitutes and not very close complements always disclose their private information, otherwise they do not. For the case of a symmetric distribution we will confirm these results. However, for a non-symmetric distribution of the random variable, we derive the new result that in the Bayesian Nash equilibrium, the firms will disclose their information more often than for a symmetric one.

effects applying the central limit theorem generally can be approximated by the normal distribution. However, then it would be convenient to model all these singular effects explicitly or at least to mention them.

The next section of the paper presents the assumptions and the information structure of the model. Then, section 3 analyses the market equilibria for the two basic competition forms of Industrial Economics, Cournot and Bertrand competition. Section 4 concludes, summarising the results and comparing them with the ones obtained in models assuming the normal distribution.

2 The Model

The market structure is compromised of two risk neutral and profit maximising firms $i = 1, 2$. Both duopolists produce one differentiated good. The (inverse) demand functions for the two products are given by

$$p_i (q_i, q_j) = \alpha - \beta (q_i + g q_j) , \qquad i, j = 1, 2, \ i \neq j \tag{1}$$

with $\alpha, \beta > 0$, $|g| \leq 1$ as parameters, p_i the price of firm i and q_i, q_j the respective outputs of both duopolists.[4] For $g > 0$ the goods are characterised as substitutes, whereas for negative values of the substituibility parameter g, the firms produce complements and for $g = 0$ independent goods. The unit production costs of both firms are assumed to be identical ($c_i = c_j = c$). Consequently, without loss of generality and in order to simplify the notation of the following analysis, from now on α denotes the inverse demand net of unit costs, i.e. $\alpha = \tilde{\alpha} - c$ with $\tilde{\alpha}$ as the "true" parameter of the demand function (cf. *Sakai* 1986 for similar arguments).

In order to model demand uncertainty, the demand parameter α is assumed to be stochastic. In contrast to the existing literature which uses the normal distribution (as for example *Vives* 1984, *Gal-Or* 1985, *Sakai* 1986, 1990, 1991, *Sakai, Yamato* 1989, *Jin* 1992, *Raith* 1996), in this paper we will analyse the case of a non-symmetric distribution. Specifically, there are two states market demand may take on: It can be high (H) or low (L) represented by the index $k = L, H$. Thus, high demand is denoted by a high parameter value (α_H) and low demand by a low one ($\alpha_L = h\alpha_H$), with $0 \leq \alpha_L \leq \alpha_H$, $0 \leq h \leq 1$. As the parameter h indicates the ratio between the two possible demand levels, it may be labelled as the demand

[4] This demand function results from an appropriate quadratic utility function of the type

$$U (q_0, q_1, q_2) = q_0 + \alpha (q_1 + q_2) - \frac{\beta}{2} \left(q_1^2 + 2g q_1 q_2 + q_2^2\right)$$

of a representative consumer with q_0 indicating the consumed quantity of the numéraire good.

variability parameter, too. Both firms know that a low or a high demand occurs with the probabilities $\mathrm{P}(\alpha_L) = \kappa$ and $\mathrm{P}(\alpha_H) = 1-\kappa$, respectively. This means that with a higher value of κ, the probability of a low demand increases.

Before the duopolists start competing, they independently observe a private signal s_i about the stochastic demand parameter α. The signal may indicate high (s_{iH}) or low demand (s_{iL}).[5] The relation between the private signal s_i and the real demand level (represented by α) is assumed to be determined by the following conditional probabilities $\mathrm{P}(s_{ik}|\,\alpha_k)$ which are common knowledge to both firms:

$\mathrm{P}(s_{ik}\|\,\alpha_k)$		s_i	
		s_{iL}	s_{iH}
α	α_L	ξ	$1-\xi$
	α_H	$1-\xi$	ξ

Table 1: Conditional probabilities $\mathrm{P}(s_{ik}|\,\alpha_k)$.

Consequently, the quality of the signal improves with an increasing probability ξ. The private signal s_i can also be viewed as the firms' a priori beliefs about the demand level which are different because the information source or interpretation method may differ. To ensure, as is realistic, that the firms will actually consider α_L (α_H) most probable after having received the signal s_{iL} (s_{iH}), we assume $0.5 \leq \xi \leq 1$.[6] From the conditional probabilities in table 1 it follows that, as long as $\xi < 1$, the problem of incomplete information becomes less severe due to the additional signal s_i, but does not disappear.

In this situation of incomplete information the firms are able to mutually exchange their private information. They can do that before they start engaging in competition on their market. To exclude the possibility of "strategic" information exchange as modelled for example by *Crawford*, *Sobel* (1982) or *Okuno-Fujiwara et al.* (1990), the firms are assumed to choose their exchange strategies before receiving their private signals. For this purpose, they enter into a binding agreement of either disclosing or keeping the private information to themselves. As usual in this literature, a trustee or a trade association will guarantee this information exchange agreement.

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[5] In this context, "independently" means that in spite of an identical value of the realised demand parameter α for both duopolists, one firm may observe a private signal indicating high and the other a signal for low demand.

[6] This assumption implies no loss of generality because the probability ξ is exogenous and common knowledge to both firms. For a value $\xi < 0.5$ they just would presume the opposite demand level more probable, i.e. for the signal s_{iL} (s_{iH}) they would expect a high (low) demand.

If the two competitors commit themselves to complete disclosure, the amount of information concerning the expected demand level both firms possess increases from only containing their own private information ($z_i = \{s_i\},\ z_j = \{s_j\}$) before exchange, to containing both signals ($z_i = z_j = \{s_i, s_j\}$) afterwards. When subsequently competing on the commodity market they make use of this better information base. Of course, in case of no disclosure the information remains unchanged: $z_i = \{s_i\}$, $z_j = \{s_j\}$. Basically, the firms will always exchange their private information if they expect competition intensity to reduce as a result, thereby ensuring higher profits.

To summarise, in this two stage game of incomplete information, the time and information structure of the firms results as:

1. information exchange:
   (a) common knowledge of the two states of demand α_H and α_L with the corresponding probabilities $1 - \kappa$ and κ;
   (b) decision about the information exchange behaviour;
   (c) signal s_i about the demand level, given the probability ξ;
   (d) information exchange or not depending on the result of stage 1(b);
   (e) information set z_i.
2. commodity market competition.

3 Derivation of the Equilibria

In this section we will analyse the basic market equilibria of Industrial Economics: the Cournot equilibria with quantity competition and the Bertrand equilibria with price competition. In the course of this analysis, these equilibria will be derived depending on the existing information set available to the firms.

In this two-stage model, the firms have two decision parameters by which maximise their expected profits. In the following, we will derive the respective equilibria comparing the expected profits that result with "no information exchange" and "complete information exchange". In order to obtain the resulting Bayesian Nash equilibria, we will solve this optimisation problem by the standard backward induction method.

3.1 Cournot Equilibria

Formally, the Cournot equilibria of this model consist of the output levels and each competitor's decision whether to exchange information or not. Therefore, the firms choose their respective output levels dependent on their information sets z_i, $i = 1, 2$, in order to maximise expected profits. With the demand function (1) and given the information set z_i, the expected profit of firm i is

$$\mathrm{E}\left(\pi^{Ci}\middle| z_i\right) = \mathrm{E}\left\{\left\{\left[\alpha - \beta\left(q_i + gq_j\right)\right] q_i\right\} |z_i\right\} \tag{2}$$

with the index C indicating Cournot competition. Maximising this expected profit (2) leads to the reaction function of firm i:[7]

$$q_i^*\left(z_i\right) = \frac{1}{2\beta}\left[\mathrm{E}\left(\alpha\,|z_i\right) - g\beta\mathrm{E}\left(q_j^*\,|z_i\right)\right] \tag{3}$$

The firms are symmetric in all aspects with the exception of their information set. Consequently, if the information set is identical for both, they also behave symmetrically in the equilibrium. This means that they choose an identical output $q_{1k}^* = q_{2k}^* = q_k^*$ for the signal s_{ik}, indicating the state of demand k ($k = L, H$).

Inserting the reaction function (3) into the profit function (2) leads to the following expected profit of firm i in reduced form which depends on the available information set determined by the exchange behaviour of the first step:

$$\mathrm{E}\left(\pi^{Ci}\right) = \beta\mathrm{E}\left\{\left[q_i^*\left(z_i\right)\right]^2\right\} \tag{4}$$

Using (4) it is now possible to determine the expected equilibrium profits. As is standard in the existing information exchange literature, we will derive the incentives to share the private information in a comparative static manner, analysing only the two extreme cases of "no information exchange" and "complete information exchange".

[7] The sufficient condition for profit maximising is globally met: $\frac{\partial^2 \mathrm{E}\left(\pi^i\right)}{\partial q_i^2} = -2\beta < 0$. An asterisk always symbolises equilibrium values.

3.1.1 No Information Exchange

If the competitors do not exchange their private information, the information set of firm i only consists of the own private signal about the demand level ($z_i = \{s_i\}$). Because of the assumption $\xi \geq 0.5$ and no additional information from the competitor, firm i will infer α_k ($k = L, H$) from s_{ik} and will choose the equilibrium output q_k.

Using the respective probabilities, from the reaction function (3) of firm i, we obtain for the private signal s_{iL} which indicates low demand

$$\begin{aligned} \mathrm{P}(s_L)\, q_L &= \frac{1}{2\beta}\{\mathrm{P}(\alpha_L, s_L)\,\alpha_L + \mathrm{P}(\alpha_H, s_L)\,\alpha_H \\ &\quad -g\beta\,[\mathrm{P}(s_L, s_L)\, q_L + \mathrm{P}(s_L, s_H)\, q_H]\} \end{aligned} \tag{5}$$

and for s_{iH} indicating high demand:

$$\begin{aligned} \mathrm{P}(s_H)\, q_H &= \frac{1}{2\beta}\{\mathrm{P}(\alpha_L, s_H)\,\alpha_L + \mathrm{P}(\alpha_H, s_H)\,\alpha_H \\ &\quad -g\beta\,[\mathrm{P}(s_H, s_L)\, q_L + \mathrm{P}(s_H, s_H)\, q_H]\} \end{aligned} \tag{6}$$

These equations (5) and (6) can be combined to the following equation system:

$$\begin{aligned} &\frac{\alpha_H}{\beta}\begin{pmatrix} h\mathrm{P}(\alpha_L, s_L) + \mathrm{P}(\alpha_H, s_L) \\ h\mathrm{P}(\alpha_L, s_H) + \mathrm{P}(\alpha_H, s_H) \end{pmatrix} \\ &= \begin{pmatrix} 2\mathrm{P}(s_L) + g\mathrm{P}(s_L, s_L) & g\mathrm{P}(s_L, s_H) \\ g\mathrm{P}(s_H, s_L) & 2\mathrm{P}(s_H) + g\mathrm{P}(s_H, s_H) \end{pmatrix}\begin{pmatrix} q_L \\ q_H \end{pmatrix} \end{aligned} \tag{7}$$

Corresponding to table 1, the various probabilities of the equation system (7) are:

$$\mathrm{P}(\alpha_L, s_L) = \kappa\xi \tag{8}$$

$$\mathrm{P}(\alpha_H, s_L) = (1-\kappa)(1-\xi) \tag{9}$$

$$\mathrm{P}(\alpha_L, s_H) = \kappa(1-\xi) \tag{10}$$

$$\mathrm{P}(\alpha_H, s_H) = (1-\kappa)\,\xi \tag{11}$$

$$\mathrm{P}(s_L) = \kappa\xi + (1-\kappa)(1-\xi) \tag{12}$$

$$\mathrm{P}(s_H) = \kappa(1-\xi) + (1-\kappa)\,\xi \tag{13}$$

$$\begin{aligned}
\mathrm{P}(s_L, s_L) &= \kappa\xi^2 + (1-\kappa)(1-\xi)^2 & (14)\\
\mathrm{P}(s_L, s_H) &= \xi(1-\xi) & (15)\\
\mathrm{P}(s_H, s_L) &= \xi(1-\xi) & (16)\\
\mathrm{P}(s_H, s_H) &= \kappa(1-\xi)^2 + (1-\kappa)\xi^2 & (17)
\end{aligned}$$

Using these probabilities (8) - (17), the equation system (7) solves for the equilibrium outputs for the respective signals indicating low or high demand as follows:

$$\begin{aligned}
q_L = {} & \frac{\alpha_H}{(2+g)\beta\left[2\xi(\xi-1)-(2+g)\kappa(1-\kappa)(1-2\xi)^2\right]}\\
& \cdot\{h\kappa\xi\left[2(\kappa+\xi-2\kappa\xi)+g(\kappa+2\xi-2\kappa\xi-1)\right]\\
& +(1-\kappa)(1-\xi)\left[2\xi+(2+g)\kappa(1-2\xi)\right]\} \qquad (18)
\end{aligned}$$

$$\begin{aligned}
q_H = {} & \frac{\alpha_H}{(2+g)\beta\left[2\xi(\xi-1)-(2+g)\kappa(1-\kappa)(1-2\xi)^2\right]}\\
& \cdot\{h\kappa(1-\xi)\{2\left[1-\xi-\kappa(1-2\xi)\right]+g(1-\kappa)(1-2\xi)\}\\
& +(1-\kappa)\xi\left[2(1-\xi)-(2+g)\kappa(1-2\xi)\right]\} \qquad (19)
\end{aligned}$$

With the equations (4), (12), (13), (18) and (19) in the no information sharing Cournot equilibrium (NN), the expected profit of firm i is:[8]

$$\begin{aligned}
\pi_{NN}^{Ci} &= \beta \mathrm{E}\left\{\left[q_i(s_i)\right]^2\right\}\\
&= \beta\left[\mathrm{P}(s_L)q_L^2+\mathrm{P}(s_H)q_H^2\right]\\
&= \frac{\alpha_H^2}{(2+g)^2\beta\left[(2+g)\kappa(1-\kappa)(1-2\xi)^2+2\xi(1-\xi)\right]^2}\\
&\quad\cdot\left\{h^2\kappa^2\left\{\xi^2\left[2(\kappa+\xi-2\kappa\xi)+g(\kappa+2\xi-2\kappa\xi-1)\right]^2\right.\right.\\
&\quad\cdot\left[\kappa\xi+(1-\kappa)(1-\xi)\right]\\
&\quad+(1-\xi)^2\{2\left[1-\xi-\kappa(1-2\xi)\right]+g(1-\kappa)(1-2\xi)\}^2\\
&\quad\cdot\left[\kappa(1-\xi)+(1-\kappa)\xi\right]\}\\
&\quad+2h\kappa(1-\kappa)\xi(1-\xi)\\
&\quad\cdot\{\left[2(\kappa+\xi-2\kappa\xi)+g(\kappa+2\xi-2\kappa\xi-1)\right]\left[2\xi+(2+g)\kappa(1-2\xi)\right]
\end{aligned}$$

[8] NN characterises the situation when neither of the two firms discloses any information.

$$
\begin{aligned}
&\cdot \left[\kappa\xi + (1-\kappa)(1-\xi)\right] + \{2\left[1-\xi-\kappa(1-2\xi)\right] + g(1-\kappa)(1-2\xi)\} \\
&\cdot \left[2(1-\xi) - (2+g)\kappa(1-2\xi)\right]\left[\kappa(1-\xi) + (1-\kappa)\xi\right]\} \\
&+ (1-\kappa)^2 \left\{(1-\xi)^2 \left[2\xi + (2+g)\kappa(1-2\xi)\right]^2 \left[\kappa\xi + (1-\kappa)(1-\xi)\right]\right. \\
&\left. +\xi^2 \left[2(1-\xi) - (2+g)\kappa(1-2\xi)\right]^2 \left[\kappa(1-\xi) + (1-\kappa)\xi\right]\right\}\} \qquad (20)
\end{aligned}
$$

As can be seen from equation (20), the expected profit in the no information sharing Cournot equilibrium $\pi^{Ci}_{NN} = \pi^{Ci}_{NN}(\alpha_H, \beta, g, h, \kappa, \xi)$ depends on the demand parameters α_H, β, g and h, as well as on the probabilities κ and ξ.

3.1.2 Complete Information Exchange

If the firms disclose their information completely, the information set of both is identical and consists of the two private demand signals: $z_i = \{s_i, s_j\}$, $i = 1, 2$, $i \neq j$. The optimality condition (3) of firm i can be expressed as:

$$
q_i(s_i, s_j) = \frac{1}{2\beta}\{\mathrm{E}\left[\alpha \left|(s_i, s_j)\right.\right] - g\beta q_j(s_i, s_j)\} \qquad (21)
$$

As the firms are symmetric, they consequently produce the identical equilibrium output $q_i(s_i, s_j) = q_j(s_i, s_j) =: q(s_i, s_j)$. This implies for equation (21):

$$
q(s_i, s_j) = \frac{\mathrm{E}\left[\alpha \left|(s_i, s_j)\right.\right]}{(2+g)\beta} \qquad (22)
$$

Using the probabilities in table 1 and Bayes' theorem, the three possible signal combinations (s_L, s_L), (s_L, s_H) and (s_H, s_H) lead to the three corresponding output levels q_{LL}, q_{LH} and q_{HH}:

$$
\begin{aligned}
q_{LL} &= \frac{1}{(2+g)\beta}\{\mathrm{P}\left[\alpha_L \left|(s_L, s_L)\right.\right]\alpha_L + \mathrm{P}\left[\alpha_H \left|(s_L, s_L)\right.\right]\alpha_H\} \\
&= \frac{\alpha_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]}{(2+g)\beta\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]} \qquad (23)
\end{aligned}
$$

$$
q_{LH} = \frac{1}{(2+g)\beta}\{\mathrm{P}\left[\alpha_L \left|(s_L, s_H)\right.\right]\alpha_L + \mathrm{P}\left[\alpha_H \left|(s_L, s_H)\right.\right]\alpha_H\}
$$

$$= \frac{\alpha_H \left[h\kappa + (1-\kappa)\right]}{(2+g)\,\beta} \tag{24}$$

$$\begin{aligned} q_{HH} &= \frac{1}{(2+g)\,\beta}\left\{\mathrm{P}\left[\alpha_L \left|(s_H, s_H)\right.\right]\alpha_L + \mathrm{P}\left[\alpha_H \left|(s_H, s_H)\right.\right]\alpha_H\right\} \\ &= \frac{\alpha_H \left[h\kappa (1-\xi)^2 + (1-\kappa)\,\xi^2\right]}{(2+g)\,\beta \left[\kappa (1-\xi)^2 + (1-\kappa)\,\xi^2\right]} \end{aligned} \tag{25}$$

Using the equations (4), (14), (15), (17) and (23) - (25), the expected profit of firm i in the Cournot equilibrium with complete information exchange (RR) is:[9]

$$\begin{aligned} \pi_{RR}^{Ci} &= \beta \mathrm{E}\left\{\left[q_i\left(s_i, s_j\right)\right]^2\right\} \\ &= \beta\left[\mathrm{P}\left(s_L, s_L\right) q_{LL}^2 + 2\mathrm{P}\left(s_L, s_H\right) q_{LH}^2 + \mathrm{P}\left(s_H, s_H\right) q_{HH}^2\right] \\ &= \frac{\alpha_H^2}{(2+g)^2\,\beta\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]} \\ &\quad \cdot\left\{h^2\kappa^2\left\{\xi^4\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]\right.\right. \\ &\quad +2\xi(1-\xi)\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] \\ &\quad \left. +(1-\xi)^4\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\right\} \\ &\quad +2h\kappa(1-\kappa)\,\xi(1-\xi)\left\{\xi(1-\xi)\left[1-2\xi(1-\xi)\right]\right. \\ &\quad \left. +2\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]\right\} \\ &\quad +(1-\kappa)^2\left\{(1-\xi)^4\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]\right. \\ &\quad +2\xi(1-\xi)\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right] \\ &\quad \left.\left. +\xi^4\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]\right\}\right\} \end{aligned} \tag{26}$$

As can be seen from equation (26), the expected profit in the complete information sharing Cournot equilibrium $\pi_{RR}^{Ci} = \pi_{RR}^{Ci}\left(\alpha_H, \beta, g, h, \kappa, \xi\right)$ depends on the demand parameters α_H, β, g and h, as well as on the probabilities κ and ξ.

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[9] RR characterises the situation when both firms completely reveal their private information.

3.1.3 Which Information Exchange Strategy Do Firms Choose?

The decision criterion for the profit maximising firms is the difference in the respective expected profits $\Delta\pi^{Ci}_{RR/NN} := \pi^{Ci}_{RR} - \pi^{Ci}_{NN}$. This reflects the firm's rationale that it wants to choose the strategy that bears the highest possible expected profit. For a positive profit difference, they will exchange their private information, for a negative they will not. Using the complete exchange profit (26) and the no exchange profit (20), for the difference we obtain a function:[10]

$$\begin{aligned}\Delta\pi^{Ci}_{RR/NN} &= \Delta\pi^{Ci}_{RR/NN}\left(\alpha_H, \beta, g, h, \kappa, \xi\right) \\ &\sim \Delta\pi^{Ci}_{RR/NN}\left(g, \kappa, \xi\right) \end{aligned} \tag{27}$$

The sign of this proportionality cannot be analytically determined unambiguously. Both expected profits (20) and (26) depend on the exogenous demand parameters α_H, β, g and h and on the (equally exogenous) probabilities κ and ξ. This also holds for the difference in expected profits (27). However, the sign of the profit difference only depends on the substitution parameter g and on the probabilities κ and ξ.

By contrast, the other demand parameters, i.e. the absolute demand level α_H, the demand variability h and the parameter β, have no influence on the decision as they only function as shift parameters. In addition to these general statements about the relevance of the various parameters of the model, numerical analysis leads to the following results, shown graphically in the figures 1 - 3. These figures represent the expected profit difference $\Delta\pi^{Ci}_{RR/NN}$ depending on the probabilities κ and ξ, each for a different substituability level g of the goods produced. In figures 1 and 2 we cut the plot at zero expected profit difference $\left(\Delta\pi^{Ci}_{RR/NN} = 0\right)$ and only explicitly show the negative regions. Consequently, the flat areas represent parameter combinations with positive, the deepenings combinations with negative profit differences. Figure 3 shows the whole plot which is positive for the entire range.

As can clearly be seen, depending on the parameter g which indicates how strong substitutes or complements the goods are, it is possible to determine when the firms disclose their private information and when they do not exchange it: In general, firms producing complementary goods ($g < 0$) always reveal their private information. In most cases, competition with substitutive goods leads to complete revelation, too.

[10] For interested readers, an extensive mathematical appendix with the derivations of all the results stated in the text is available from the author upon request.

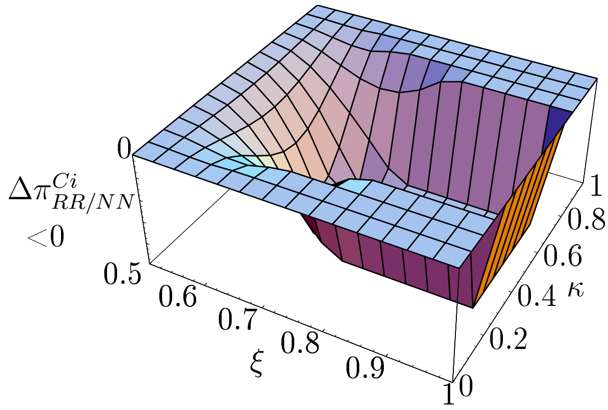

Figure 1: Perfect Substitutes ($g = 1$)

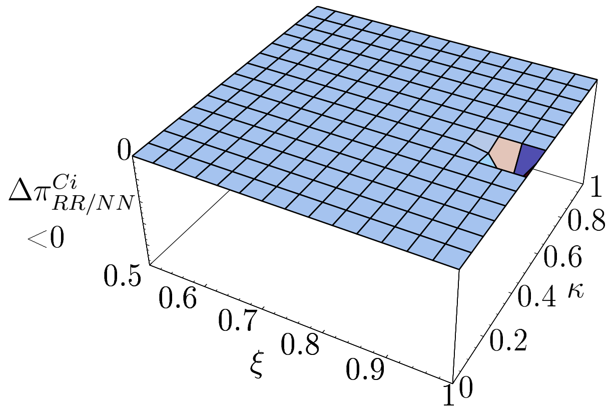

Figure 2: Close Substitutes ($g = 0.7$)

Only for close substitutes ($g > 0.6$; cf. figures 1 and 2) do there exist parameter combinations where the firms do not exchange information. These combinations result with medium values of the probability κ and a high ξ. The reason is that a high probability ξ represents a high signal quality. In this case of very good own information, additional information by the rival is less attractive. On the other hand, because of mutual identical information exchange behaviour, a firm producing substitutive goods would lose a comparative advantage by disclosing additional information to the rival.

In models using the normal distribution (cf. for example *Raith* 1996 and the other authors cited above) Cournot firms producing very close substitutes (g near to 1)

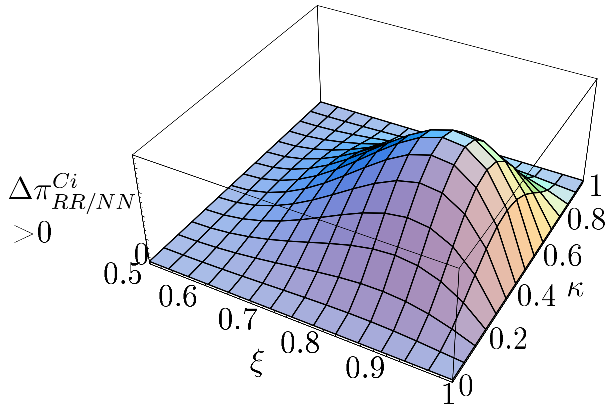

Figure 3: Perfect Complements ($g = -1$)

do not exchange their information, whereas for not very close substitutes and for the whole range of complementary goods, disclosure is always favourable. Thus, for the symmetric case ($\kappa = 0.5$), our distribution reproduces these results. However, with a non-symmetric distribution of the high and low demand realisations (κ close to 1 or close to 0), firms producing close substitutes will also comletely reveal their private information. Further, it can be seen that with decreasing substitutability of the goods, i.e. with increasing g, the information exchange range amplifies under Cournot competition.

3.2 Bertrand Equilibria

The case of Bertrand competition between the two firms will be analysed in a similar way. In this model, the strategies of the two firms in the Bayesian Nash equilibrium consist of the price and the decision on whether to exchange information or not. The firms choose their respective prices dependent on their information sets z_i, $i = 1, 2$, again with the aim of maximising expected profits. With the (inverse) demand functions (1), the demand functions for price-setting firms are

$$q_i(p_i, p_j) = a - b(p_i - gp_j), \qquad i, j = 1, 2 \tag{28}$$

with $a := \frac{\alpha}{(1+g)\beta}$ and $b := \frac{1}{(1-g^2)\beta}$ as positive parameters. Obviously, the parameter a in the Bertrand case can be interpreted in exactly the same way as the parameter α

in the Cournot case. The stochastic properties of α are also identical with those of a. With the demand function (28), the expected profit of firm i, given its information set z_i, is

$$\mathrm{E}\left(\pi^{Bi}\middle| z_i\right) = \mathrm{E}\left\{\left\{\left[a - b\left(p_i - gp_j\right)\right] p_i\right\} |z_i\right\} \tag{29}$$

with B indicating Bertrand competition. Maximising the expected profit (29) leads to the reaction function of firm i:[11]

$$p_i^*\left(z_i\right) = \frac{1}{2b}\left[\mathrm{E}\left(a\left|z_i\right.\right) + gb\mathrm{E}\left(p_j^*\left|z_i\right.\right)\right] \tag{30}$$

The firms are symmetric with the exception of their respective information sets. Consequently, if the information set is identical for both, they also behave symmetrically in the equilibrium. This means that in the Bertrand case they choose an identical price $p_{1k}^* = p_{2k}^* = p_k^*$ for the state of demand k $(k = L, H)$.

Inserting the reaction function (30) into the profit function (29) leads to the following expected profit of firm i in reduced form which depends on the available information set z_i determined by the exchange behaviour decision in the first stage of the game:

$$\mathrm{E}\left(\pi^{Bi}\right) = b\mathrm{E}\left\{\left[p_i^*\left(z_i\right)\right]^2\right\} \tag{31}$$

Using this profit function in reduced form, in the following the expected equilibrium profits which also depend on the exchange behaviour can be determined. For the same reasons as in the Cournot case above, we will limit the analysis to the two extreme cases of "no information exchange" and "complete information exchange".

3.2.1 No Information Exchange

Again, if the firms do not exchange their private information, each information set only consists of the own private signal about the level of demand $(z_i = \{s_i\}$,

---

[11] The sufficient condition for profit maximising is globally met: $\frac{\partial^2 \mathrm{E}\left(\pi^i\right)}{\partial p_i^2} = -2b < 0$.

$i = 1, 2$). With $\xi \geq 0.5$ and no additional information from the competitor, firm i will infer a_k ($k = L, H$) from s_{ik} and will choose the equilibrium price p_k.

Using the respective probabilities, from the reaction function (30) of firm i, we obtain for the private signal s_{iL} which indicates low demand

$$\begin{aligned} \mathrm{P}(s_L)\, p_L &= \frac{1}{2b}\{\mathrm{P}(a_L, s_L)\, a_L + \mathrm{P}(a_H, s_L)\, a_H \\ &\quad + gb\,[\mathrm{P}(s_L, s_L)\, p_L + \mathrm{P}(s_L, s_H)\, p_H]\} \end{aligned} \tag{32}$$

and for s_{iH} indicating high demand:

$$\begin{aligned} \mathrm{P}(s_H)\, p_H &= \frac{1}{2b}\{\mathrm{P}(a_L, s_H)\, a_L + \mathrm{P}(a_H, s_H)\, a_H \\ &\quad + gb\,[\mathrm{P}(s_H, s_L)\, p_L + \mathrm{P}(s_H, s_H)\, p_H]\} \end{aligned} \tag{33}$$

These two equations (32) and (33) can be combined to the following equation system:

$$\begin{aligned} &\frac{a_H}{b}\begin{pmatrix} h\mathrm{P}(a_L, s_L) + \mathrm{P}(a_H, s_L) \\ h\mathrm{P}(a_L, s_H) + \mathrm{P}(a_H, s_H) \end{pmatrix} \\ &= \begin{pmatrix} 2\mathrm{P}(s_L) - g\mathrm{P}(s_L, s_L) & -g\mathrm{P}(s_L, s_H) \\ -g\mathrm{P}(s_H, s_L) & 2\mathrm{P}(s_H) - g\mathrm{P}(s_H, s_H) \end{pmatrix}\begin{pmatrix} p_L \\ p_H \end{pmatrix} \end{aligned} \tag{34}$$

As $\mathrm{P}(\alpha_L, s_L) = \mathrm{P}(a_L, s_L)$, $\mathrm{P}(\alpha_H, s_L) = \mathrm{P}(a_H, s_L)$, $\mathrm{P}(\alpha_L, s_H) = \mathrm{P}(a_L, s_H)$ and $\mathrm{P}(\alpha_H, s_H) = \mathrm{P}(a_H, s_H)$, we will use the probabilities (8) - (17) in order to solve the equation system (34). With

$$\begin{aligned} \Omega &:= b\{[(2-g)\,\kappa\,(1-2\xi) + \xi\,(2-g\xi)]\,\{2\,(2\kappa\xi - \kappa - \xi + 1) \\ &\quad + g\left[\kappa\,(1-2\xi) - (1-\xi)^2\right]\} - g^2\xi^2\,(1-\xi)^2\} \end{aligned} \tag{35}$$

the equilibrium prices for the respective signals indicating either low or high demand result in:

$$\begin{aligned} p_L &= \Omega^{-1} a_H\,\{h\kappa\xi\,[g\,(1-\kappa)\,(1-2\xi) + 2\,(\kappa + \xi - 2\kappa\xi)] \\ &\quad + (1-\kappa)\,(1-\xi)\,[2\xi + (2-g)\,\kappa\,(1-2\xi)]\} \end{aligned} \tag{36}$$

$$p_H = \Omega^{-1} a_H \left\{ h\kappa^2 \xi (1-\xi) [2(2\kappa\xi - \kappa - \xi + 1) - g(1-\kappa)(1-2\xi)] + (1-\kappa)\xi [2(1-\xi) - (2-g)\kappa(1-2\xi)] \right\} \quad (37)$$

With the equations (12), (13), (31), (36) and (37) the expected profit of firm i in the no information sharing Bertrand equilibrium (NN) is:

$$\begin{aligned} \pi_{NN}^{Bi} &= b\mathrm{E}\left\{[p_i(s_i)]^2\right\} \\ &= b\left[\mathrm{P}(s_L) p_L^2 + \mathrm{P}(s_H) p_H^2\right] \\ &= b\Omega^{-2} a_H^2 \left\{h^2\kappa^2\xi^2 \left\{[g(1-\kappa)(1-2\xi) + 2(\kappa + \xi - 2\kappa\xi)]^2 \right.\right. \\ &\quad \cdot [\kappa\xi + (1-\kappa)(1-\xi)] \\ &\quad + \kappa^2 (1-\xi)^2 [2(2\kappa\xi - \kappa - \xi + 1) - g(1-\kappa)(1-2\xi)]^2 \\ &\quad \cdot [\kappa(1-\xi) + (1-\kappa)\xi]\} \\ &\quad + 2h\kappa(1-\kappa)\xi(1-\xi)\{[g(1-\kappa)(1-2\xi) + 2(\kappa + \xi - 2\kappa\xi)] \\ &\quad \cdot [2\xi + (2-g)\kappa(1-2\xi)][\kappa\xi + (1-\kappa)(1-\xi)] \\ &\quad + \kappa\xi [2(2\kappa\xi - \kappa - \xi + 1) - g(1-\kappa)(1-2\xi)] \\ &\quad \cdot [2(1-\xi) - (2-g)\kappa(1-2\xi)][\kappa(1-\xi) + (1-\kappa)\xi]\} \\ &\quad + (1-\kappa)^2 \left\{(1-\xi)^2 [2\xi + (2-g)\kappa(1-2\xi)]^2 [\kappa\xi + (1-\kappa)(1-\xi)] \right. \\ &\quad \left.\left. + \xi^2 [2(1-\xi) - (2-g)\kappa(1-2\xi)]^2 [\kappa(1-\xi) + (1-\kappa)\xi]\right\}\right\} \end{aligned} \quad (38)$$

As can be seen from equations (35) and (38), the expected profit in the no information sharing Bertrand equilibrium $\pi_{NN}^{Bi} = \pi_{NN}^{Bi}(a_H, b, g, h, \kappa, \xi)$ depends on the demand parameters a_H, b, g and h, as well as on the probabilities κ and ξ.

3.2.2 Complete Information Exchange

Just as in the Cournot case, if the firms disclose their information completely, the information sets of both are identical and consist in the two private demand signals: $z_i = \{s_i, s_j\}$, $i = 1, 2$, $i \neq j$. The optimality condition (30) of firm i, can be expressed as:

$$p_i(s_i, s_j) = \frac{1}{2b}\{\mathrm{E}[a|(s_i, s_j)] + gbp_j(s_i, s_j)\} \quad (39)$$

With symmetric firms, they choose the equilibrium price $p_i(s_i, s_j) = p_j(s_i, s_j) =: p(s_i, s_j)$, implying for equation (39):

$$p(s_i, s_j) = \frac{\mathrm{E}\left[a\,|(s_i, s_j)\right]}{(2-g)\,b} \tag{40}$$

Using the probabilities in table 1 and Bayes' theorem, the three possible signal combinations (s_L, s_L), (s_L, s_H) and (s_H, s_H) lead to the three corresponding prices p_{LL}, p_{LH} and p_{HH}:

$$\begin{aligned} p_{LL} &= \frac{1}{(2-g)\,b}\left\{\mathrm{P}\left[a_L\,|(s_L, s_L)\right]a_L + \mathrm{P}\left[a_H\,|(s_L, s_L)\right]a_H\right\} \\ &= \frac{a_H\left[h\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]}{(2-g)\,b\left[\kappa\xi^2 + (1-\kappa)(1-\xi)^2\right]} \end{aligned} \tag{41}$$

$$\begin{aligned} p_{LH} &= \frac{1}{(2-g)\,b}\left\{\mathrm{P}\left[a_L\,|(s_L, s_H)\right]a_L + \mathrm{P}\left[a_H\,|(s_L, s_H)\right]a_H\right\} \\ &= \frac{a_H\left[h\kappa + (1-\kappa)\right]}{(2-g)\,b} \end{aligned} \tag{42}$$

$$\begin{aligned} p_{HH} &= \frac{1}{(2-g)\,b}\left\{\mathrm{P}\left[a_L\,|(s_H, s_H)\right]a_L + \mathrm{P}\left[a_H\,|(s_H, s_H)\right]a_H\right\} \\ &= \frac{a_H\left[h\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]}{(2-g)\,b\left[\kappa(1-\xi)^2 + (1-\kappa)\,\xi^2\right]} \end{aligned} \tag{43}$$

Using equations (14), (15), (17), (31), and (41) - (43) the expected profit of firm i in the Bertrand equilibrium with complete information exchange (RR) is:

$$
\begin{aligned}
\pi_{RR}^{Bi} &= b\mathrm{E}\left\{\left[p_i\left(s_i,s_j\right)\right]^2\right\} \\
&= b\left[\mathrm{P}\left(s_L,s_L\right)p_{LL}^2+2\mathrm{P}\left(s_L,s_H\right)p_{LH}^2+\mathrm{P}\left(s_H,s_H\right)p_{HH}^2\right] \\
&= \frac{a_H^2}{\left(2-g\right)^2 b\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right]} \\
&\quad \cdot\left\{h^2\kappa^2\left\{\xi^4\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right]\right.\right. \\
&\quad +2\xi\left(1-\xi\right)\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right] \\
&\quad \left.+\left(1-\xi\right)^4\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\right\} \\
&\quad +2h\kappa\left(1-\kappa\right)\xi\left(1-\xi\right)\left\{\xi\left(1-\xi\right)\left[1-2\xi\left(1-\xi\right)\right]\right. \\
&\quad \left.+2\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right]\right\} \\
&\quad +\left(1-\kappa\right)^2\left\{\left(1-\xi\right)^4\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right]\right. \\
&\quad +2\xi\left(1-\xi\right)\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\left[\kappa\left(1-\xi\right)^2+\left(1-\kappa\right)\xi^2\right] \\
&\quad \left.\left.+\xi^4\left[\kappa\xi^2+\left(1-\kappa\right)\left(1-\xi\right)^2\right]\right\}\right\}
\end{aligned} \tag{44}
$$

Equation (44) shows that the expected profit in the complete information sharing Bertrand equilibrium $\pi_{RR}^{Bi} = \pi_{RR}^{Bi}\left(a_H, b, g, h, \kappa, \xi\right)$ depends on the demand parameters a_H, b, g and h, as well as on the probabilities κ and ξ.

3.2.3 Which Information Exchange Strategy Do Firms Choose?

Again, the decision criterion for the profit maximising firms is the difference of the respective expected profits $\Delta\pi_{RR/NN}^{Bi} := \pi_{RR}^{Bi} - \pi_{NN}^{Bi}$. Using the complete exchange profit (38) and the no exchange profit (44), the profit difference function is determined as:

$$
\begin{aligned}
\Delta\pi_{RR/NN}^{Bi} &= \Delta\pi_{RR/NN}^{Bi}\left(a_H, b, g, h, \kappa, \xi\right) \\
&\sim \Delta\pi_{RR/NN}^{Bi}\left(g, h, \kappa, \xi\right)
\end{aligned} \tag{45}
$$

The sign of this equation cannot be unambiguously determined analytically. Both expected profits (38) and (44) depend on the exogenous demand parameters a_H, b, g and h and on the (also exogenous) probabilities κ and ξ. Generally, the same holds for the difference of expected profits (45). However, we can show that the sign of the profit difference does not depend on the demand parameters a_H and b. In contrast to the Cournot case, here the variability of the demand h is also of importance in determining the sign of the expected profit difference (45).

Again, the results regarding the information exchange decision by the firms are derived via numerical analysis. These decision rules are shown graphically in the figures 4 - 6.[12] It is possible to determine for the whole probability ranges of κ and ξ whether the firms exchange their private information or not depending on

- how strong substitutes or complements the goods are (denoted by g) and
- how different the demand levels are (denoted by h).

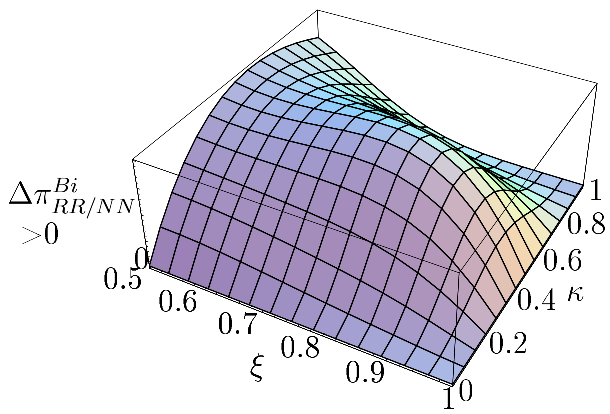

Figure 4: Perfect Substitutes ($g = 1$) with $h = 0.5$

Generally, for firms producing substitutes ($g > 0$), the numerical analysis shows that complete revelation of the private information is always optimal. One can see that competition with complementary goods mostly leads to complete revelation, too. Only for a high signal quality (ξ close to 1), are there parameter combinations which lead to negative expected profit differences for all values of demand variability $h \in [0; 1]$ (cf. figures 5 and 6 plotted for the case of $h = 0.5$). It can be seen that with diminishing variability of the demand (i.e. increasing values of h) information

[12] Figure 4 shows the whole plot of the expected profit difference $\Delta\pi^{Bi}_{RR/NN}$ which is positive for the entire range. In figures 5 and 6, the plot is cut at zero expected profit difference. Therefore, the flat areas again represent parameter combinations with positive, the deepenings combinations with negative profit differences.

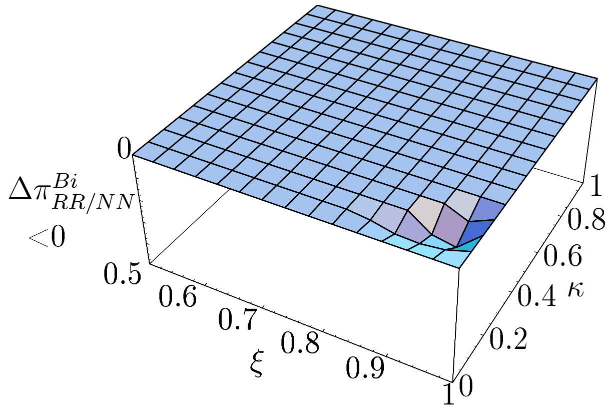

Figure 5: Close Complements ($g = -0.8$) with $h = 0.5$

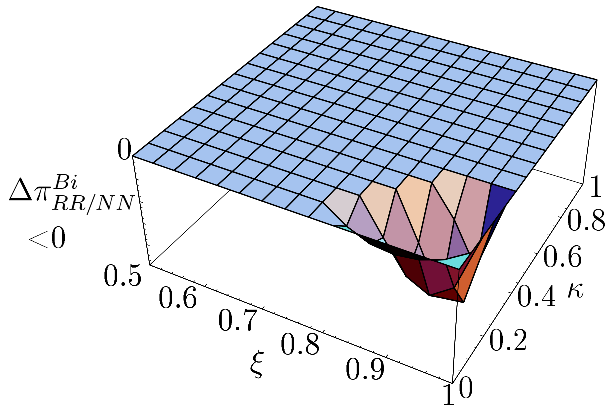

Figure 6: Perfect Complements ($g = -1$) with $h = 0.5$

exchange already becomes unfavourable for very weak complements (i.e. low values of g; increasing until $g = -0.3$).[13]

Comparing these results with the ones in the existing literature we know that in models using the normal distribution Bertrand firms producing substitutes and not very close complements always disclose their private information, otherwise (g near 1; very close complements) they do not. This means that similar to Cournot competition, our distribution reproduces these results for the symmetric case ($\kappa = 0.5$).

[13] In addition, it can be shown that with diminishing variability in demand, the range of negative profit differences moves more and more into the region of high probabilities κ, i.e. into the region of a high probability of low demand.

In the non-symmetric case (κ close to 1 or close to 0) however, firms producing close complements will reveal their private information completely, too. Further, we show that with decreasing complementarity of the goods, i.e. with decreasing g, the information exchange range amplifies under Bertrand competition.

4 Concluding Remarks

This paper further develops the standard modelling in the literature which deals with information exchange between firms in the presence of demand uncertainty. However, it assumes an alternative distribution of the random variables and signals. We have replaced the normal distribution, commonly used because of its convenient mathematical properties, by an alternative one, a random variable with a non-symmetric binomial outcome. Moreover, we argued that this discrete and non-symmetric distribution is a good approximation to the scenario technique firms in new industries like software, pharmaceuticals or genetic engineering or firms in unknown and insecure markets apply.

As usual in the information exchange literature, we analysed under which conditions firms mutually exchange their private demand information and under which conditions they do not. With our new distribution for the symmetric case, we were able to confirm the results of the existing literature using the normal distribution, namely that Cournot firms producing very close substitutes do not exchange their information, whereas for not very close substitutes and for the whole range of complementary goods, disclosure is always favourable. In contrast, under Bertrand competition, firms producing substitutes and not very close complements always disclose their private information, otherwise they do not. However, with the non-symmetric distribution, we were able to additionally prove: In addition to the parameter ranges for information exchange in the symmetric scenario ($\kappa = 0.5$), we showed that for an asymmetric probability distribution between the high and low demand (κ close to 1 or close to 0), for Cournot firms producing close substitutes and Bertrand firms producing close complements, complete revelation of the private information is ruled out as the optimal strategy, too. Further, we showed that with decreasing substitutability (decreasing complementarity) of the goods, i.e. with increasing (decreasing) g, under Cournot (Bertrand) competition, the information exchange range amplifies.

Summarising, the presented model could solve the two drawbacks of the existing literature concerning information exchange by assuming a non-symmetric distribution in which only positive realisations of the random variable are allowed. Although only a very simple assumption was made about the distribution, we provided a broader analysis than one using the normal distribution. Most importantly, the new results for the asymmetric case justify the use of our distribution as an interesting complement to the existing literature.

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