

Fast and High-Resolution MRI with Nonlinear Spatial Encoding Functions

Dissertation

der Mathematisch-Naturwissenschaftlichen Fakultät
der Eberhard Karls Universität Tübingen
zur Erlangung des Grades eines
Doktors der Naturwissenschaften
(Dr. rer. nat.)

vorgelegt von
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Tübingen
2026

Gedruckt mit Genehmigung der Mathematisch-Naturwissenschaftlichen Fakultät der
Eberhard Karls Universität Tübingen.

Tag der mündlichen Qualifikation:

09.06.2026

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To my family, especially my parents and grandparents.

Abstract

This thesis presents recent advances in image acceleration and high-resolution methods for Magnetic Resonance Imaging (MRI), driven by a deepened understanding of how to characterize and utilize nonlinear spatial encoding functions. Conventionally, MRI spatial encoding relies on linear gradients and RF receivers (i.e., in high field) – a process well understood by k-space theory. However, despite decades of progress, MRI remains fundamentally constrained by sequential sampling in the Fourier-domain, which limits both speed and spatial resolution. To further boost performance, this work develops two separate categories of techniques for fast and high-resolution MRI, respectively, which are ultimately found to be unified under a novel mathematical perspective.

The first category accelerates MRI sampling by exploiting a local B_0 coil array, as a novel gradient hardware system that can generate a wide range of linear and nonlinear magnetic field modulations. This disrupts the traditional k-space framework, and pushes acquisition into a largely unexplored regime. Consequently, k-space is sampled by time-varying local neighborhoods (i.e., patches), rather than point-by-point trajectory (due to linear gradients) or time-invariant kernels (due to RF receivers), allowing more information to be captured per unit time. Despite significant theoretical and practical challenges, this thesis establishes a comprehensive pipeline for this approach, encompassing hardware design, signal encoding, magnetic field calibration, data compression, and image reconstruction.

The second category achieves high-resolution MRI through a novel self-navigation approach for multi-shot Echo-Planar-Imaging (EPI). By treating the notorious shot-to-shot phase fluctuations as a specialized form of nonlinear spatial encoding functions, this thesis enables robust self-navigation across a wide range of multi-shot trajectories. Our technique provides strong resilience to inter-shot physiological signal instability in flexibly designed multi-shot trajectories, shortens total scan time, and fundamentally expands the theoretical scope of self-navigation in MRI.

Behind these technical progresses, one of the key insights is that, signal encoding with nonlinear B_0 gradient modulations can follow similar mathematical rules as RF receiver coils in well-established parallel imaging. By formulating linear and nonlinear spatial encoding functions within a reproducing kernel Hilbert space (RKHS) framework – originally developed for parallel imaging – we can treat continuous B_0 field modulations and shot-to-shot phase fluctuations similar to the way people treat RF receivers. This perspective inspired several practical techniques: group-kernel auto-calibration, group-patch joint compression and RKHS reconstruction for MRI accelerated by B_0 gradient modulations, and overlap-kernel EPI for multi-shot self-navigation. While substantially boosting speed and spatial resolution, these methods achieve unprecedented field calibration robustness and reconstruction speed, inheriting the beauty from parallel imaging methods such as GRAPPA and SENSE-ESPIRiT. Ultimately, nonlinear gradient modulation and multi-shot EPI become mathematically bridged to parallel imaging, where different forms of nonlinear spatial encoding functions provide independent, complementary encoding dimensions.

Zusammenfassung

In dieser Dissertation werden aktuelle Fortschritte in der Magnetresonanztomographie (MRT) präsentiert, wobei insbesondere Technologien mit Zielsetzung der beschleunigten Bildgebung und erhöhten Auflösung vorgestellt werden. Diese Fortschritte werden im Wesentlichen durch ein vertieftes Verständnis nichtlinearer räumlicher Kodierungsfunktionen ermöglicht. Die räumliche Kodierung der untersuchten Spins basiert in der konventionellen MRT typischerweise auf linearen Gradienten, sowie auf mehreren verbauten Hochfrequenz (HF)-Empfangspfaden (Rx-Path, im Hochfeldbereich). Diese Kodierung kann sehr gut durch das bekannte mathematische Konzept des „k-Raumes“ beschrieben werden. Trotz vieler Fortschritte in den vergangenen Jahrzehnten bleibt die MRT Bildgebung bei typischem Betrieb grundlegend durch die sequentielle Abtastung im k-Raum langsam und somit in ihrer Auflösung begrenzt. Um die Leistung der MRT weiter zu steigern, werden in dieser Arbeit zwei separate technische Ansätze für die schnelle, bzw. hochauflösende, MRT entwickelt, die sich letztlich unter einer neuen mathematischen Perspektive vereinen lassen.

Der erste Forschungsansatz adressiert die Beschleunigung der MRT-Abtastung durch die Nutzung eines lokalen B_0 -Spulen-Arrays als neuartiges Gradienten-Hardware-System. Somit kann ein breites Spektrum linearer und nichtlinearer Magnetfeldmodulationen erzeugt werden. Dies erweitert das traditionelle Konzept des k-Raumes und verschiebt die Datenakquisition in einen weitgehend unerforschten Bereich: Folglich wird der k-Raum durch zeitlich variierende lokale Blocks abgetastet – anstatt durch Punkt-für-Punkt-Trajektorien oder zeitinvariante Sensitivitäten der HF-Empfangseinheiten (welches auch „Calibration Kernel“ genannt). Damit können mehr Informationen pro Zeiteinheit erfasst werden. Trotz erheblicher theoretischer, sowie praktischer Herausforderungen etabliert dieser Forschungsansatz eine umfassende Pipeline, die Hardware-Design, Signalkodierung, Magnetfeldkalibrierung, Datenkompression und Bildrekonstruktion umfasst.

Der zweite Forschungsansatz adressiert die hochauflösende MRT mithilfe eines neuartigen Ansatzes der Selbstnavigation (Self-Navigation) für die Bildgebungsmethode Multi-Shot Echo-Planar-Imaging (EPI). Indem die bekannten Shot-to-Shot-Phasenfluktuationen als eine spezielle Form nichtlinearer Ortskodierungsfunktionen behandelt werden, ermöglicht diese Methode eine robuste Selbstnavigation über einen weiten Bereich von Multi-Shot-Trajektorien. Die entwickelte Technik bietet eine hohe Resilienz gegenüber physiologischen Signalinstabilitäten zwischen den Aufnahmen bei flexibel gestalteten Multi-Shot-Trajektorien, verkürzt die Gesamtscanzeit und erweitert den theoretischen Umfang der Selbstnavigation in der MRT grundlegend.

Die zentrale Erkenntnis hinter diesen technischen Fortschritten ist, dass die Signalkodierung mit nichtlinearen B_0 -Gradientenmodulationen ähnlichen mathematischen Regeln folgen kann wie HF-Empfangsspulen in der etablierten parallelen Bildgebung (Parallel Imaging). Durch die Formulierung linearer und nichtlinearer Ortskodierungsfunktionen innerhalb eines reproduzierenden Hilbertraums (englisch Reproducing Kernel Hilbert Space, abgekürzt RKHS) – einer Methode die ursprünglich für die parallele Bildgebung entwickelt wurde – können kontinuierliche B_0 -Feldmodulationen und Shot-to-Shot-Phasenfluktuationen mathematisch analog zu HF-Empfänger behandelt werden. Diese Perspektive inspiriert im Folgenden mehrere praktische Techniken, wie die Group-Kernel Auto-Calibration, die Group-Patch Joint Compression und RKHS-Rekonstruktion für durch B_0 -Gradientenmodulationen beschleunigte MRT, sowie die Overlap-Kernel EPI für die Multi-Shot-Selbstnavigation. Während diese Methoden die Geschwindigkeit und räumliche Auflösung steigern, sind sie ebenfalls robust bei Feldkalibrierung und Rekonstruktionsgeschwindigkeit und übernehmen die Eleganz von Methoden der parallelen Bildgebung wie GRAPPA und SENSE-ESPIRiT. Zusammenfassend werden in dieser Arbeit die nichtlineare Gradientenmodulation und die Multi-Shot-EPI mathematisch mit der parallelen Bildgebung verknüpft, wobei verschiedene Formen nichtlinearer Ortskodierungsfunktionen unabhängige, komplementäre Kodierungsdimensionen bieten.

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Acronyms and Definitions

MR: Magnetic Resonance

MRI: Magnetic Resonance Imaging

SNR: Signal to Noise Ratio

SENSE: Sensitivity Encoding

GPRPPA: Generalized Autocalibrating Partially Parallel Acquisitions

ACS: Auto-Calibrating-Signal

CAIPIRINHA/CAIPI: Controlled Aliasing in Parallel Imaging Results in Higher Acceleration

FOV: Field-Of-View

BPE: Bunched-Phase-Encoding

Wave-CAIPI: Wave modulation combined with CAIPI

FRONSAC: Fast Rotary Nonlinear Spatial Acquisition

G-map: Geometry-factor map

FFT: Fast Fourier Transform

DW: Diffusion-Weighting or Diffusion-Weighted

EPI: Echo-Planar-Imaging

FLASH: Fast Low Angle Shot

RKHS: Reproducing Kernel Hilbert Space

VD-PD: Variable Density Poisson Disc

PATLOC: Parallel Imaging Technique Using Localized Gradients

O-space: Parallel Imaging Using Second-Order Nonlinear Fields As Encoding Gradients With No Phase Encoding

SAKE: Simultaneous Autocalibrating And K-space Estimation

LORAKS: Low-rank Modeling Of Local K-space Neighborhoods For Constrained MRI

RESOLVE: Readout Segmentation Of Long Variable Echo Trains

MUSE: Multiplexed Sensitivity Encoding

MUSSEL: Multi-shot Sensitivity-Encoded Diffusion Data Recovery Using Structured Low-rank Matrix Completion

MPRAGE: Magnetization Prepared Rapid Gradient Echo

SMS: Simultaneous Multi-Slice

SWI: Susceptibility-Weighted-Imaging

R: Reduction Factor For K-space Undersampling

1. List of publications/manuscripts of the dissertation

This dissertation is written in cumulative form, based on the following publications:

Publication 1

Tian R, Uecker M, Davids M, Thielscher A, Buckenmaier K, Holder O, Steffen T, Scheffler K. Accelerated 2D Cartesian MRI with an 8-channel local B₀ coil array combined with parallel imaging. *Magn Reson Med*. 2024; 91: 443–465. doi: 10.1002/mrm.29799. **I.I. Rabi YIA finalist, ISMRM, 2024.**

Publication 2

Tian R, Uecker M, Holder O, Povolni P, Steffen T, Scheffler K. Group-kernel auto-calibration and group-patch k-space reconstruction: Fast MRI with time-variant B₀ kernels partitioned into time-invariant subsets. *arXiv*. Preprint posted online January 9, 2026:arXiv:2601.05469. doi:10.48550/arXiv.2601.05469. To submit, *Magn Reson Med*.

Publication 3

Tian R, Scheffler K. Group-Patch Joint Compression: Compressing Dynamic B₀ and Static RF Spatial Modulations Across k-Space Subregion Groups for Highly Accelerated MRI. *Magn Reson Med*. 2026; 0:1–20. doi: 10.1002/mrm.70439.

Publication 4

Tian R, Uecker M, Zaitsev M, and Scheffler K, “Overlap-Kernel EPI: Estimating MRI Shot-to-Shot Phase Variations by Shifted-Kernel Extraction From Overlap Regions at Arbitrary k-Space Locations,” *Magnetic Resonance in Medicine* 95, no. 4 (2026): 2131–2152, <https://doi.org/10.1002/mrm.70196>. **I.I. Rabi YIA finalist, ISMRM, 2026.**

2. Statements of contributions

All authors proofread their corresponding manuscripts.

2.1 Publication 1

Rui Tian: Proposed specific scientific ideas, derived mathematical formulation, integrated hardware system, programmed software codes, carried out experiments, analyzed simulation and experimental data, and drafted the manuscript.

Martin Uecker: Discussed mathematics with Rui Tian.

Mathias Davids: Evaluated electromagnetic safety in collaboration with Rui Tian.

Axel Thielscher: Evaluated electromagnetic safety in collaboration with Rui Tian.

Kai Buckenmaier: Discussed hardware issues with Rui Tian.

Oliver Holder: Manufactured coil hardware, and discussed electrical issues with Rui Tian.

Theodor Steffen: Manufactured coil hardware, and discussed electrical issues with Rui Tian.

Klaus Scheffler: Proposed and supervised this project, designed nonlinear gradient coil hardware, discussed technical progress with Rui Tian, managed team members, and provided research resources and financial supports.

2.2 Publication 2

Rui Tian: Proposed this project and specific scientific ideas, derived mathematical formulation, integrated hardware system, programmed software codes, carried out experiments, analyzed data, and drafted the manuscript.

Martin Uecker: Discussed mathematics with Rui Tian.

Oliver Holder: Manufactured coil hardware, and discussed electrical issues with Rui Tian.

Pavel Povolni: Debugged hardware noise with Rui Tian, and designed and produced third-party current monitors.

Theodor Steffen: Manufactured coil hardware, and discussed electrical issues with Rui Tian.

Klaus Scheffler: Supervised this project, designed nonlinear gradient coil hardware, discussed technical progress with Rui Tian, managed team members, and provided research resources and financial supports.

2.3 Publication 3

Rui Tian: Proposed this project and specific scientific ideas, derived mathematical formulation, integrated hardware system, programmed software codes, carried out experiments, analyzed data, and drafted the manuscript.

Klaus Scheffler: Supervised this project, designed nonlinear gradient coil hardware, discussed technical progress with Rui Tian, managed team members, and provided research resources and financial supports.

2.4 Publication 4

Rui Tian: Proposed this project and specific scientific ideas, derived mathematical formulation, programmed software codes, carried out experiments, analyzed data, and drafted the manuscript.

Martin Uecker: Discussed mathematics with Rui Tian.

Maxim Zaitsev: Discussed software codes with Rui Tian, provided supports in research facility.

Klaus Scheffler: Supervised this project, discussed technical progress with Rui Tian, managed team members, and provided research resources and financial supports.

3 Introduction

In this cumulative thesis, the introduction provides only the most basic scientific background needed for Publication 1-4, rather than an exhaustive review. The focus is placed on the essential physics, mathematics and signal processing underlying how macroscopic spins can be spatially encoded to form magnetic resonance (MR) images, with spatially linear and nonlinear magnetic fields. The effects of these fields can be represented as either image-space spatial-encoding maps, or equivalently, k-space kernels. When the kernels are shift-varying across acquisition time, they can be seen as shift-invariant subsets, as a novel idea of this thesis that plays a central role throughout Publication 1-4. The quantum mechanical description of microscopic spins, other MRI engineering aspects, and application-orientated studies are omitted.

3.1 Conventional MRI with linear gradients and RF receivers

To produce images, Magnetic Resonance Imaging (MRI) spatially encode nuclear spins, most commonly those of hydrogen (^1H), using linear gradients¹ and RF receivers²⁻⁵. Typically, an MR scanner comprises a strong main magnet that produces a static B_0 magnetic field, three gradient coils that produces orthogonal spatially linear varying B_0 fields, and RF transmit (B_1^+) and receive (B_1^-) coils that excites and detects spin signals. Each gradient field can be seen as an encoding function that varies linearly across space (note, this linear function is not conceptually equivalent to the terminology linear system). In contrast, the sensitivity profile of the RF receiver coil represents a form of nonlinear spatial encoding function. Within biological tissue, the magnetic moments of many individual nuclear spins combine to form a macroscopic magnetization vector – the vector sum of all spins within a small voxel. The basic spin manipulation can be described by Bloch equations. This net magnetization can be tipped away (i.e., excited) from equilibrium by external electromagnetic waves in Larmor frequency, which increases linearly with the main B_0 field. Subsequently, the net magnetizations precess at this frequency, inducing detectable MR signals in the receive coils. The MR signals decay with the transverse relaxation time T_2 , and recover with the longitudinal relaxation time T_1 .

To generate spatial contrast, linear B_0 gradients are switched to spatially encode the Larmor frequency of the spin magnetization as a linear function of position, allowing the precession signals from distinct spatial positions to be distinguished through their time-varying frequencies. Note, this linear spatial encoding function provides the argument of the nonlinear complex exponential function that represents detected signals in RF receivers. In addition, at high magnetic fields (3T or above) where signal-to-noise ratio (SNR) is sufficient, multiple RF receiver coils with distinct spatial sensitivity profiles can further modulate the amplitude and phase of the detected signals nonlinearly across space, thereby providing additional localization capabilities^{4,5}. These mechanisms can be elegantly described by the k-space theory⁶⁻⁸. In continuous space, the raw time-domain data acquired by the MR receiver chain can be mathematically expressed as the Fourier transform of spin density distribution weighted by the RF sensitivity:

$$S_n(t) = \mathcal{F}\{c_n(r)\rho(r)\} = \int c_n(r)\rho(r) e^{-ik(t)r} dr, (1)$$

with $k(t) = \gamma \int_0^t g(\tau) d\tau$ as the k-space trajectory term (i.e., in rad/m)

Here, $\mathcal{F}$ is the Fourier transform, γ is the gyromagnetic ratio (i.e., in rad/s/T), r is the spatial location, $\rho(r)$ is the spin density distribution, $c_n(r)$ is the n^{th} RF sensitivity distribution, $g(\tau)$ is the shaped pulse of the linear gradients. Spin relaxation effects are neglected here for simplicity.

By the Fourier shift theorem, the above expression corresponds to sampling the k-space (i.e., the Fourier-domain of the image $\rho(r)$) convolved with the Fourier transform of the RF receivers' sensitivity maps:

$$S_n(t) = \mathcal{F}\{\rho(r)\} * \mathcal{F}\{c_n(r)\}, (2)$$

Here, $*$ denotes convolution operation. When data sampling is accelerated, the k-space data become undersampled relative to the Nyquist criterion. With only linear gradient encoding, the reconstructed image will exhibit aliasing artifacts due to violation of the sampling theorem. However, by employing multiple RF receiver coils, a full image can still be reconstructed, since additional measurements from multiple coils improve the conditioning of the linear system. In the parallel imaging approach known as Sensitivity Encoding (SENSE)⁴, the discrete form of the forward model in Equation (1) can be written as below, to represent mapping from image-space pixels to acquired k-space data:

$$\mathbf{y} = \mathbf{E}\mathbf{x}, \quad (3)$$

where $\mathbf{y} \in \mathbb{C}^{TN \times 1}$ is the vectorized acquired k-space data, $\mathbf{E} \in \mathbb{C}^{TN \times R}$ is the encoding matrix incorporating both B_0 gradient and RF coil sensitivity encoding, $\mathbf{x} \in \mathbb{C}^{R \times 1}$ is the vectorized spin density distribution. The capital letters T, N, R denotes the total number of k-space samples, the total number of RF receiver channels, and the total number of spatial voxels/pixels. In the special case of undersampled Cartesian scans, this system can be transformed into the image-domain by applying inverse Fourier transform to the k-space data. Consequently, the reconstruction problem reduces to disentangling aliased image-space pixels by inverting a linear system, which uses the coil sensitivity information to estimate the true image intensities.

A corresponding linear system can also be formulated to describe mapping from acquired to reconstructed k-space, as suggested by Equation (2). In this view, missing k-space samples can be interpolated from neighboring data across receiver channels using k-space kernels, as the Fourier transform of RF sensitivity maps. These kernels can be estimated from a fully-sampled calibration region, known as auto-calibrating-signal (ACS) region, as in the parallel imaging approach called Generalized Autocalibrating Partial Parallel Acquisitions (GRAPPA)⁵:

$$\mathbf{y}^{\text{AC}} = \mathbf{A}^{\text{AC}} \mathbf{w}, \quad (4)$$

where $\mathbf{y}^{\text{AC}} \in \mathbb{C}^{S \times 1}$ is the stacked vector of selected S k-space data points in a receiver channel, $\mathbf{A}^{\text{AC}} \in \mathbb{C}^{S \times DN}$ is the interpolation matrix constructed by reshaping k-space neighborhoods (each including D samples surrounding a data point in $\mathbf{y}^{\text{AC}}$) in N receiver channels into its rows, and $\mathbf{w} \in \mathbb{C}^{DN \times 1}$ is the vectorized interpolation kernel that maps between the interpolating receiver channels to the synthesized ones. Once a kernel $\tilde{\mathbf{w}}$ is learned from Equation (4), it can be applied to synthesize missed k-space data $\mathbf{y}^{\text{Syn}} \in \mathbb{C}^{S \times 1}$ using the acquired data $\mathbf{A}^{\text{Acq}} \in \mathbb{C}^{S \times DN}$, extracted from the same neighborhood pattern surrounding each undersampled k-space location:

$$\mathbf{y}^{\text{Syn}} = \mathbf{A}^{\text{Acq}} \tilde{\mathbf{w}}, \quad (5)$$

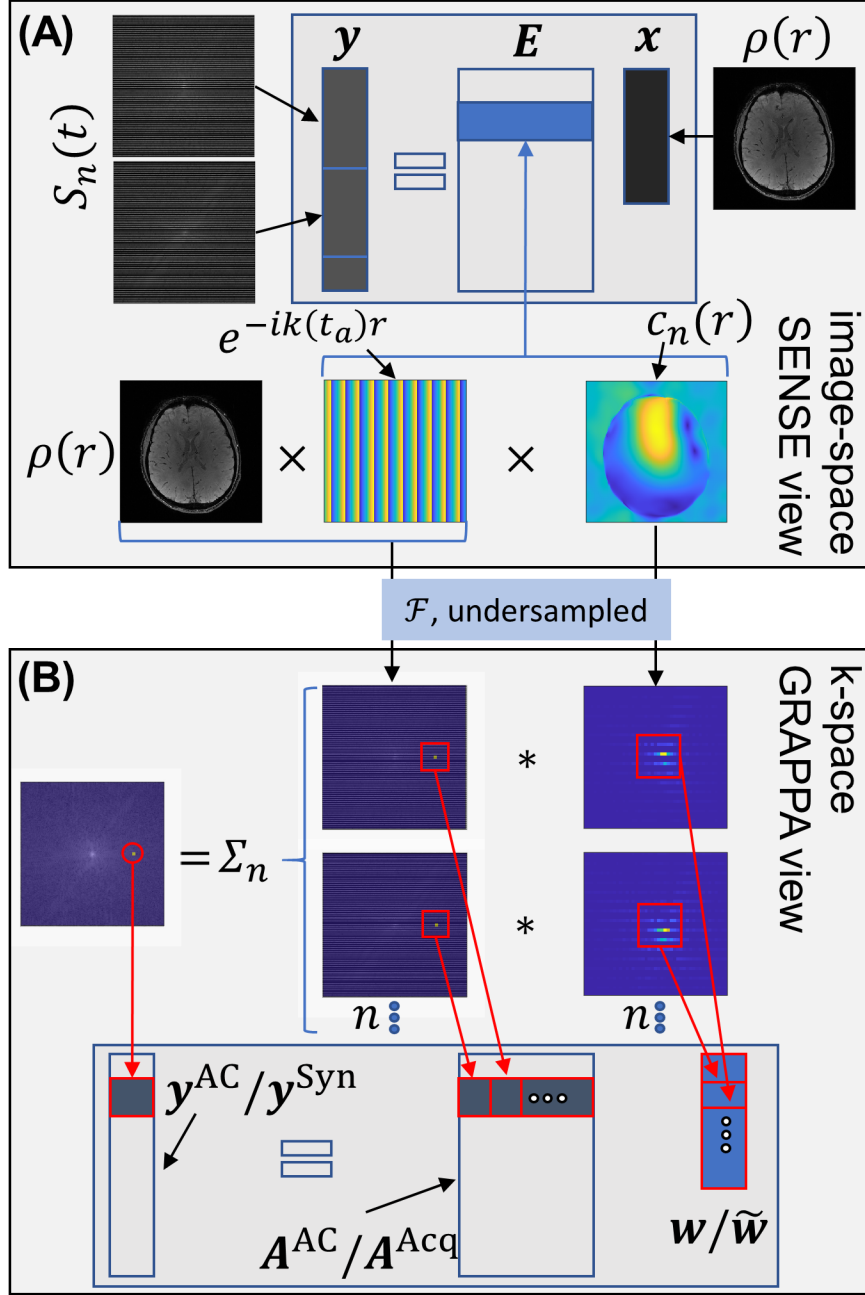


Figure 1. (A) Image-space signal model, used in SENSE. The object is elementwise multiplied with spatial-encoding maps, including phase maps induced by linear gradients at different time instants (e.g., t_a) and RF sensitivity maps, and summed up to a data point within an undersampled k-space region. (B) k-space signal model, used in GRAPPA. Small neighborhoods in the undersampled k-space data from multiple RF receivers are convolved with k-space kernels (with identical undersampling pattern) to interpolate a data point of reconstructed k-space in a RF receiver channel. This operation can be performed to synthesize multiple RF receiver channels that can be combined afterwards. The k-space kernels are estimated by solving the same k-space interpolation relationships from fully-sampled ACS regions acquired by pre-scans. Here, the kernels are the Fourier transform of spatial-encoding maps, zero-filled by the undersampling mask.

3.2 Fast MRI with rapid B_0 gradient modulations

This section introduces the first category of the techniques developed in this thesis – image acceleration with rapid B_0 gradient modulation. As nonlinear spatial encoding function varying across channels, the multiple RF receivers can effectively accelerate k-space acquisition dominated by linear gradient encoding, e.g., typically achieving 2x-3x acceleration along a phase-encoded dimension in Cartesian scans. However, as the undersampling factor increases, parallel imaging artifacts become more pronounced. This limitation is particularly critical for clinical and neuroscientific studies, where subtle pathological or functional features may resemble imaging artifacts, demanding artifact-free, high-fidelity image quality. Although optimized CAIPINRINHA/CAIPI (Controlled Aliasing in Parallel Imaging Results in Higher Acceleration)⁹ sampling patterns and advanced numerical solvers – such as compressed sensing with regularization¹⁰ or neural network-based solvers¹¹ – can mitigate k-space undersampling artifacts to some extent, the achievable acceleration remains fundamentally constrained.

To further boost MRI speed, another form of nonlinear spatial encoding functions that varies across time – rapid modulation of B_0 gradients – can be applied during oversampled signal readout. Normally, this oversampled readout data would be averaged onto a non-oversampled grid that reaches the Nyquist sampling rate and avoid redundant background extension of the field-of-view (FOV). However, with linear (2D: Bunched phase encoding^{12,13}, 3D: Wave-CAIPI¹⁴) or nonlinear (FRONSAC^{15,16}, local B_0 coils^{17,18}) B_0 gradient modulations, these redundant oversampled points can be exploited to encode rapidly varying (typically a few to around 20kHz¹⁹) spin signals, particularly along otherwise-missed phase-encoding steps. As a result, more k-space information is acquired per unit time, as long as the electronics of Analog-to-Digital Converter (ADC) can sample faster than the required readout Nyquist rate (determined by gradient bandwidth). This enables further image acceleration and theoretically, can be combined with parallel imaging.

Compared with the recent image acceleration techniques based on switched RF sensitivity distributions²⁰ during oversampled readout, the use of B_0 gradient modulations can surpass the RF wavelength encoding limit determined by the main field strength^{21,22}, effectively extending the k-space kernel width beyond that of conventional GRAPPA²³. Therefore, to explore the ultimate acceleration limits, we built an 8-channel local B_0 coil array¹⁸ driven by independently controlled current waveforms for in-vivo scans in a 9.4T human MR scanner (Siemens Healthineers, Germany). This system serves as a generalized B_0 field-generation platform capable of producing distinct and spatially-localized field modulation, thereby enabling systematic investigation of optimized spatiotemporal modulation patterns from a large design space^{24–27}, beyond globally fixed linear (Wave-CAIPI) and low-order (2nd-3rd) nonlinear (FRONSAC) gradient fields.

Mathematically, the presence of the rapidly modulated B_0 magnetic fields results in another spatial-temporal phase evolution term, on the top of the conventional Fourier and RF sensitivity encoding operator for MRI acquisition. When local B_0 coils are switched on, the acquired time-domain continuous signals obtained by the n^{th} RF receiver coil at the time instant t can be written as below, neglecting spin relaxation effects for simplicity:

$$S_n(t) = \int c_n(r) \rho(r) \exp \left\{ -i \left\{ k(t)r + \gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^t I_{\lambda}(\tau) d\tau \right] \right\} \right\} dr, \quad (6)$$

with $k(t) = \gamma \int_0^t g(\tau) d\tau$ as the k-space trajectory term (i.e., in rad/m)

where γ is the gyromagnetic ratio (i.e., in rad/s/T), r is the spin spatial location, $\rho(r)$ is the object spin density, $c_n(r)$ is the n^{th} RF sensitivity distribution, $g(\tau)$ is the shaped pulse of the linear gradients, $B_{\lambda}(r)$ is the B_0 field distribution produced by unit current in the λ^{th} local B_0 coil, $I_{\lambda}(\tau)$ is the programmable current waveform in the λ^{th} local coil.

From the perspective of Equation (3), this oversampled readout dimension with additional spatiotemporal spin modulations introduces a longer and information-enriched encoding matrix E , which can inherently improve the invertibility of the underlying linear system²⁸. Due to k-space undersampling, the corresponding image-space noise amplification map can be derived from the composite encoding matrix that incorporate all present spatial encoding maps. This measure, known as the Geometry-Factor map (G-map) originally developed for parallel imaging⁴, can be written as:

$$G_r = \sqrt{[(E^H E)^{-1}]_{r,r} (E^H E)_{r,r}}, \quad (7)$$

Here, G_r is the G-factor value at spatial location r , and $E \in \mathbb{C}^{OTN \times R}$ represents the encoding matrix (modified from the one in Equation (3), but conceptually similar) describing evolution of spin amplitudes and phases under the influence of the oversampling factor O , linear gradients across T timepoints, sensitivities of N RF receivers, and rapidly modulated B_0 fields. The operator $(\cdot)_{r,r}$ denotes the diagonal element corresponding at spatial location index r . The operator $(\cdot)^H$ denotes conjugate transpose.

The spin phase evolution maps caused by local B_0 coils or other gradient systems needs to be measured for forward model reconstruction. This field calibration can be performed by either B_0 mapping of each modulation coil element, or a chemical-shift based sequence to measure the overall phase evolution. To efficiently reconstruct scans additionally accelerated by rapid B_0 fields, the hybrid-space reconstruction technique^{14,17,29} – originally developed for Wave-CAIPI – can be employed. Specifically, when identical B_0 field modulation is maintained across all phase-encoded steps, the phase-encoding dimensions can be efficiently reconstructed using FFT, leaving only the readout dimension as a linear system to be inverted. This framework is applicable to both 2D and 3D acquisitions, with either linear or nonlinear gradient modulations, which remains parallelizable under regular Cartesian undersampling schemes.

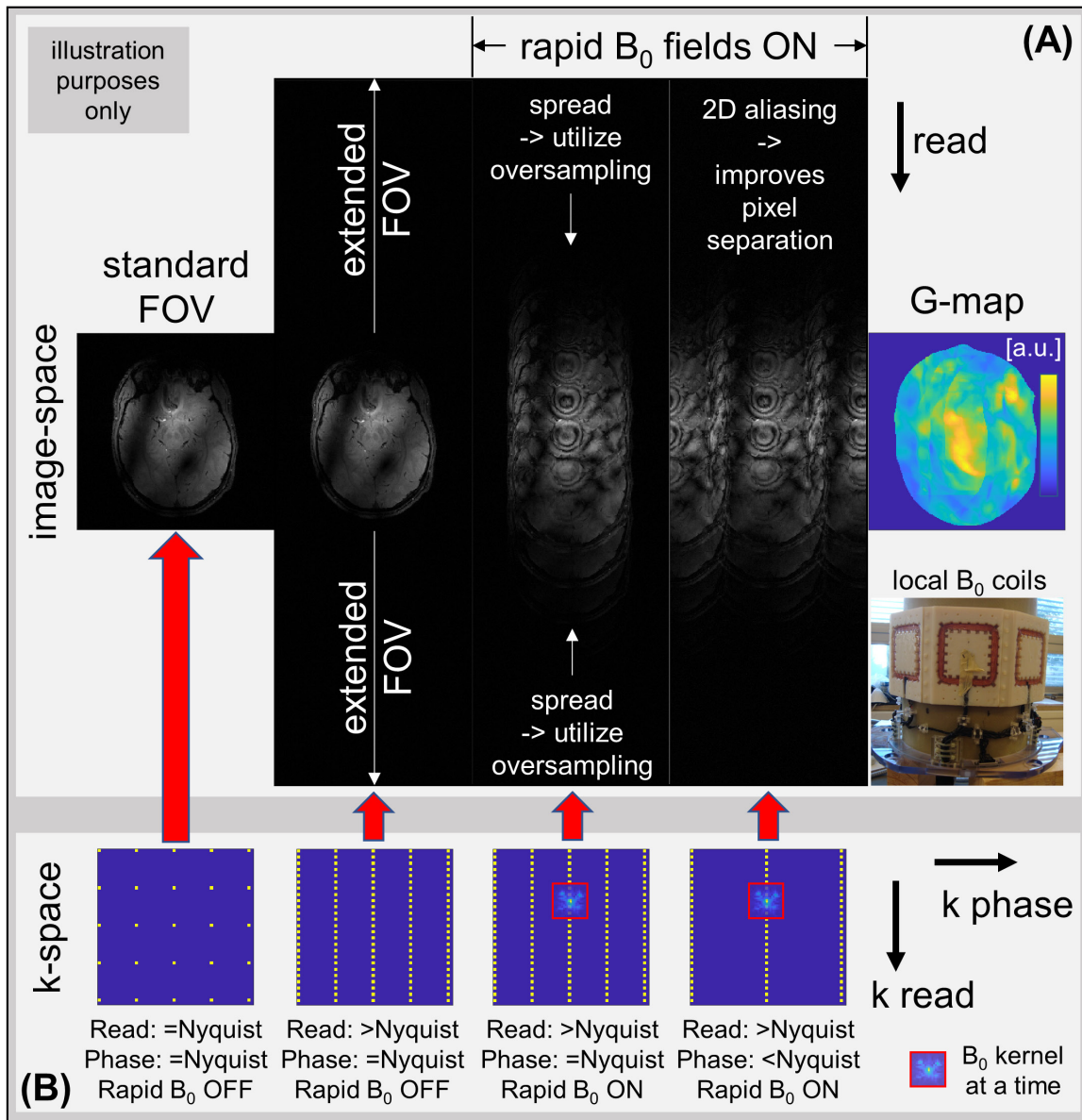


Figure 2. Four line-by-line GRE acquisition strategies are illustrated (left to right): standard readout without oversampling, readout oversampling, readout oversampling with rapid modulations of local B_0 coils, and readout oversampling with rapid B_0 modulations under phase-encoding undersampling. (A) Image-space representation. Readout oversampling induces an extended FOV with noninformative background, which can be exploited for additional spatial encoding via rapid B_0 modulations. These modulations introduce 2D aliasing patterns under phase-encoding undersampling, and can improve pixel separability during reconstruction. Generally, a G-map can be computed from an encoding scheme, representing spatially-varying noise amplification factors with the minimum value of one. (B) k-space representation. The oversampled readout samples local k-space neighborhoods with time-varying kernels. Otherwise, without rapid B_0 modulations, the oversampled k-space will be averaged onto a non-oversampled Fourier grid. The readout/phase directions could be swapped if needed.

3.3 High-resolution MRI with shot-to-shot phase variations

This section introduces the second category of the techniques developed in this thesis – self-navigated multi-shot EPI. Another important form of nonlinear spatial encoding function arises from the notorious shot-to-shot phase fluctuations^{30,31}, most prominently observed in Diffusion-Weighted (DW) multi-shot Echo-Planar Imaging (EPI) scans, within the conventional linear gradient encoding regime. In this case, rather than being actively generated by the hardware of local B_0 coils, such nonlinear spatial distributions of spin phase variations naturally occur in multi-shot diffusion-weighted EPI (DW-EPI) sequences, and hinders their use for high-resolution neuroimaging. In practice, diffusion-weighted scans are usually performed using single-shot spin-echo EPI, in which the entire k-space is sampled within one RF excitation. The achievable spatial resolution is limited by the k-space sampling window with finite width, determined by T_2 decay.

Theoretically, higher resolution can be achieved by acquiring multiple k-space segments over several shots and combining them to form a single image. However, in the presence of diffusion-sensitizing gradients, microscopic motion within the body (e.g., respiration, cardiac pulsation) introduces stochastic, spatially-nonlinear phase variations across shots. As a result, when k-space segments with inconsistent phase patterns are combined, artifacts will emerge in the reconstructed images. Mathematically, the k-space signals in each shot can be written as:

$$S_{n,s}(t) = \int c_n(r) \rho(r) e^{-ik(t)r} e^{-i\varphi_s(r)} dr, \quad (8)$$

Here, $S_{n,s}(t)$ is the k-space signal in the n^{th} RF receiver channel during the s^{th} shot of acquisition, $\varphi_s(r)$ represents the spatially-nonlinear phase distribution induced by diffusion-weighting gradients in the s^{th} shot. Due to the presence of the fluctuating phase term $e^{-i\varphi_s(r)}$, k-space data from different shots (e.g., $S_{n,1}(t)$, $S_{n,2}(t)$, ... with different t ranges) cannot be directly combined. Various strategies have been proposed to mitigate these phase inconsistencies, including navigator-based corrections^{32–35}, self-navigating trajectories^{36–38}, advanced iterative

reconstruction algorithms^{39,40}, post-processing techniques^{41–43}, and magnitude-based super-resolution imaging^{44–47}. Nevertheless, each approach has its own inherent limitation.

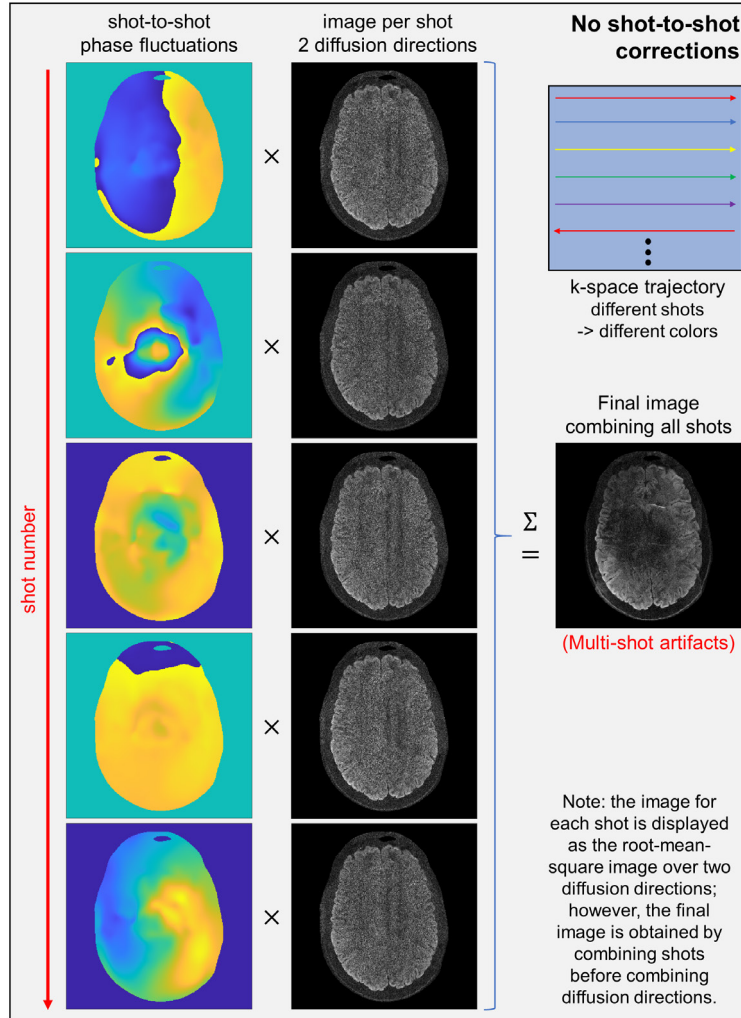


Figure 3. Multi-shot artifacts in a 5-shot phase-interleaved EPI acquisition with shot-to-shot phase fluctuations. Shot-dependent diffusion-induced phase maps are multiplied with the corresponding shot images and then summed to form the final image. Here, each shot image is an intermediate parallel imaging reconstruction (or, stronger ghosts will appear) from a partial k-space acquisition (5x undersampled), indicated by trajectory in one color. Without explicit removal of the phase variations, the combined image exhibits strong ghosting artifacts. The inter-shot phase can be estimated/removed in certain circumstances. When each shot is highly-undersampled (especially around k-space center), reliable estimation of the shot-dependent phase maps becomes more difficult, leading to failure of the multi-shot reconstruction. Data shown here correspond to b-value of 1000 s/mm² with two diffusion directions. Artifact patterns may vary when using other multi-shot trajectories.

3.4 Current limitations of the two categories of MRI techniques

For fast MRI using nonlinear B_0 gradient modulations, although such rapid modulation can accelerate k-space acquisition, the underlying signal encoding mechanism, magnetic field calibration, and image reconstruction remain challenging. First, the presence of nonlinear B_0 gradients disrupts the conventional k-space theory; hence, the cumulative sampling efficiency spanning different temporal durations is difficult to characterize. Consequently, determining the most efficient temporal modulation waveforms or spatial field shapes is not straightforward. The image-space noise amplification map (G-map⁴) or similar concepts⁴⁸ can reflect pixelwise encoding efficiency over the entire acquisition. The local k-space trajectory²⁶ can track k-space for each individual pixel indicating local spatial resolution. But no existing method can quantify the instantaneous sampling efficiency contributed simultaneously from all image-space pixels, under time-varying nonlinear gradient modulations. Second, rapid B_0 modulation fields can induce complex eddy currents that are difficult to calibrate robustly without additional lengthy scans⁴⁹ or auxiliary hardware^{50–53}. Third, the required readout oversampling factor substantially increases data volume, especially with multiple RF receiver channels, hindering the use of compressed-sensing reconstruction as widely-adopted in modern scanners⁵⁴. Because of these challenges, despite its high potential, the investigations of large and unexplored parameter space of nonlinear gradient modulations is confined to only a few MRI groups in the world, and the commercialization has been hindered for a long time.

In multi-shot EPI sequences, shot-to-shot phase variations must be robustly corrected for reconstructing high-resolution images. The navigator-based corrections are time-consuming and can suffer from inconsistencies between navigator and imaging echoes. The iterative reconstruction or low-rank matrix completion (i.e., self-navigated post-processing) approaches are generally limited to phase-interleaved multi-shot trajectories and are incompatible with high parallel imaging acceleration factors. The magnitude-based super-resolution approach suffers from SNR loss. The self-navigated trajectories require

oversampled k-space center to extract phase maps and are limited to a few trajectory patterns.

In summary, nonlinear spatial encoding functions play crucial roles in fast and high-resolution MRI technologies, however, remain difficult to manage for robust imaging, and requires both theoretical advances and practical innovations in signal modeling, calibration and reconstruction.

4 Research objectives

The overall objective of this thesis is to establish theoretical and engineering foundations for time-varying, spatially-nonlinear B_0 modulation in MRI, and address challenges in the two categories of MRI techniques. This aims both to expand nonlinear spatial encoding capabilities beyond conventional linear-gradient systems (Publication 1-3), and to extend these concepts toward calibrating naturally occurring nonlinear modulation effects within the linear gradient encoding regime (Publication 4).

Publication 1 seeks to establish a mathematical framework for nonlinear gradient MRI based on reproducing kernel Hilbert space (RKHS) formalism, to enable quantitative evaluation of k-space sampling efficiency for various nonlinear modulation field shapes within arbitrary sampling windows. It also includes experimental validation using optimized waveforms of local B_0 coils and field calibration techniques based on B_0 mapping, and safety evaluation for in-vivo scans. The custom-built local B_0 coil array is synchronized with a 9.4T human MRI scanner.

Publication 2 aims to develop a much more robust magnetic field calibration strategy that ensures reconstruction accuracy given rapid B_0 field modulations along all spatial orientations, with either nonlinear or linear (e.g., Wave-CAIPI) gradients. Extending the principles of GRAPPA, this should enable auto-calibration of arbitrary spatiotemporal modulations directly from imaging data. Furthermore, within the unified RKHS framework, an efficient k-space subregion-wise reconstruction technique is proposed to speed up image reconstruction for MR scans accelerated by rapid field modulations. This is experimentally tested in a 9.4T human MRI scanner.

Publication 3 further addresses computational challenges associated with B_0 gradient modulations, regardless of linear or nonlinear, by developing a joint-compression approach for k-space data sampled by B_0 and RF nonlinear spatial

encoding functions. This should substantially reduce the k-space data volume than conventional RF compression, and enable highly-accelerated compressed-sensing reconstructions given either linear (Wave-CAIPI) or nonlinear gradient modulations. This is experimentally tested in a 9.4T and a standard 3T human MRI scanner.

Publication 4, inspired by Publication 2, focuses on developing a novel class of multi-shot EPI sequences particularly useful for diffusion-weighted imaging, that allows self-navigation for nonlinear diffusion phase variations. This is achieved through overlapping k-space sampling trajectories between shots, from which shot-to-shot phase maps can be explicitly extracted by applying GRAPPA/ESPIRiT-type operations. The proposed methods are validated on both standard and high-performance-gradient 3T human MRI scanners.

5 Summary of publications

This thesis addresses key limitations mentioned above in fast, high-resolution MRI when spatial encoding departs from linear gradient encoding. Publication 1-4 collectively introduce a novel and complete framework to model, calibrate, compress, and reconstruct signals encoded by nonlinear spatial encoding functions. Notably, they are built upon one novel central principle: nonlinear spatial encoding functions, which can be represented as shift-varying kernels in k-space, can be seen as shift-invariant kernels in subsets based on appropriately chosen k-space subregions. By grouping the k-space encoding functions or data into proper subsets, we derive a series of formulations with experimental validation, which unlock advances in signal encoding, calibration, compression, and reconstruction for fast and high-resolution MRI with nonlinear encoding.

5.1 Publication 1 – Nonlinear gradient encoding in RKHS

First, Publication 1¹⁸ proposes a novel mathematical framework for MRI with nonlinear gradient encoding that quantifies arbitrary gradient modulation fields directly in k-space. The contribution builds upon an MRI sampling theory based on reproducing kernel Hilbert space (RKHS) originally developed for parallel imaging²³, and extends it to explicitly incorporate nonlinear B_0 gradient terms. In its original form, parallel imaging encoding efficiency for different k-space undersampling schemes, such as CAIPINRINHA and Variable-Density-Poisson-Disc (VD-PD)⁵⁵, can be directly compared in k-space. Specifically, cardinal and power function maps are derived from the composite spatial encoding maps – combining linear gradients and RF receivers' sensitivity – to predict noise amplification and approximation error in k-space in arbitrary sampling durations. These maps can guide the design of undersampled trajectory (linear gradient) with GRAPPA kernels (Fourier transform of RF sensitivity distributions) to avoid k-space hot-spots which indicate high noise amplification factors or approximation errors.

This generalization is made based on two observations. The spatiotemporal modulation pattern caused by rapid B_0 fields is spatially nonlinear as RF

sensitivity, and temporally nonlinear (here, sinusoidal) as gradient blips for CAIPINRINHA, sharing the same mathematical property as parallel imaging with different CAIPI patterns. Moreover, direct computation over the full k-space becomes computationally prohibitive: the encoding matrix that enters the RKHS calculations grows with readout oversampling factors (e.g., 8x) and RF receivers channels (e.g., 32x). However, the shift-varying k-space kernels¹⁶ induced by the modulation are periodic. From this perspective, the RKHS formalism can be evaluated on a small, representative k-space subregion – e.g., spanning a few modulation periods along readout and several phase-encoded steps – that can faithfully reflect the whole k-space. Within this subregion, the k-space efficiency maps (cardinal and power function) can quantify the encoding performance of arbitrary B_0 spatiotemporal modulation schemes contributed from all image-space pixels spanning any sampling time duration. This enables comparison of distinct nonlinear and linear gradient strategies within the same k-space. Namely, k-space encoding with shift-varying kernels is evaluated on a subregion sampled by a shift-invariant kernel subset. This theoretical advance is essential for our custom-build local B_0 coil array, which can be seen as a generalization of existed nonlinear gradient techniques, such as PATLOC²⁴/O-space⁵⁶ (static nonlinear fields) and FRONSAC¹⁵ (dynamic low-order B_0 terms). They all lack a quantitative framework to predict temporal encoding efficiency for guiding sequence design, and our extended RKHS framework successfully fills this gap. The k-space efficiency maps across small but representative subregion can also be used as cost function to efficiently optimize channel specific modulation waveforms⁵⁷.

Specifically, we define a reproducing kernel⁵⁸ (note, not GRAPPA or B_0 kernels) as the image-space inner product of two composite spatial-encoding maps ϵ with acquisition indices (t, i) and $(\bar{t}, j)$:

$$K_{ij}(t, \bar{t}) = \langle \epsilon^{t,i}, \epsilon^{\bar{t},j} \rangle_{L^2}, \quad (9)$$

Accordingly, we can build Gram ($\mathbf{M}$) and cross-Gram ($\mathbf{R}$) matrices with encoding matrices $\mathbf{E}$ (i.e., discretized stack of composite maps ϵ , index $(\bar{t}, k)$ for fully-sampled Fourier functions as reference), eliminating image-space dependency and flexibly restricting the signal model to a chosen acquisition time range (i.e., k-space subregions).

$$\begin{cases} \mathbf{M} = K_{ij}(t, \bar{t}) = \langle \epsilon^{t,i}, \epsilon^{\bar{t},j} \rangle_{L^2} = \mathbf{E}_{\bar{t},j} \mathbf{E}_{t,i}^H, \in \mathbb{C}^{TN \times TN}, (10) \\ \mathbf{R} = K_{kj}(\bar{t}, t) = \langle \epsilon^{\bar{t},k}, \epsilon^{t,j} \rangle_{L^2} = \mathbf{E}_{\bar{t},j} \mathbf{E}_{t,k}^H, \in \mathbb{C}^{TN \times T}, (11) \end{cases}$$

The columns of $\mathbf{M}$ or $\mathbf{R}$ correspond to discrete counterparts of representers of evaluation that spans Hilbert-space of k-space signals. T, N refers to the number of acquisition samples and RF receivers. The operator $(\cdot)^H$ denotes conjugate transpose.

Consequently, such RKHS signal model yields subspace interpolation from sampling k-space (source) to reconstructed k-space (target), using the interpolation weights called cardinal functions (i.e., discretized matrix $\mathbf{U}$), with image-space independent interpretation. The k-space sampling efficiency under different gradient modulations can be accessed by Frobenius norm of the cardinal functions (i.e., noise amplification, taking sum-of-squares along the vertical dimension of $\mathbf{U}$), or the power functions $\mathbf{P}^{t,k}$ that quantify difference between source and target functions for interpolation (i.e., approximation errors). The operator $(\cdot)_{\text{diag}}$ denotes taking the diagonal elements of a matrix.

$$\mathbf{M}\mathbf{U} = \mathbf{R}, (12)$$

$$\mathbf{P}^{t,k} = \sqrt{(\mathbf{E}_{t,k} \mathbf{E}_{t,k}^H - \mathbf{U}^H \mathbf{E}_{t,i} \mathbf{E}_{t,k}^H)_{\text{diag}}}, (13)$$

Second, guided by this theory, Publication 1 develops a complete experimental pipeline for image acceleration using nonlinear B_0 modulations. An 8-channel local B_0 coil array, driven by a custom-programmed signal generator (ADwin, Germany) and 8-channel power amplifier (IECO, Finland), are installed onto the scanner during experiments, and operates in synchronization with a modified FLASH sequence (Siemens IDEA). The magnetic field distribution by each local B_0 coil is simulated based on Biot-Savart law in MATLAB, to examine the field strength and gradient given the coil geometry. The gradients need to be made localized, and strong to be capable of synthesizing distinct and effective nonlinear gradient patterns. For image acceleration with modulation scans, the workflow includes magnetic field calibration, ex-vivo validation, safety assessment, and in-vivo imaging scans.

A key requirement for artifact-free reconstruction is accurate calibration of the additional phase modulation maps induced by the local B_0 coils. We perform conventional B_0 field mapping scans²⁶ with 9 repetitions, each with a small blip applied to a local B_0 channel plus a no-blip baseline. To ensure reconstruction robustness, we introduce a novel ESPIRiT-based extrapolation⁵⁹ to transform the low-resolution B_0 field maps onto a high-resolution grid – analogous to its established use for RF sensitivity maps estimation, thereby improving the quality of estimated fields and reconstruction. These calibrated maps, combined with monitored coil currents, are incorporated into forward encoding model for Wave-CAIPI-type hybrid-space reconstruction. Without our proposed ESPIRiT-type B_0 maps extrapolation, residue artifacts are occasionally observed. Note, such calibration cannot capture eddy currents (during B_0 modulations) which are absent during B_0 field mapping. For sagittal and coronal slices with strong z-axis eddy currents – likely related to the RF shield⁶⁰ without capacitors interrupting the z-axis – pronounced artifacts can occur which is addressed by Publication 2.

Based on this calibration approach, experimental reconstructions of ex-vivo scans accelerated by different B_0 modulation schemes validate theoretical predictions via the G-maps and the proposed RKHS-derived k-space efficiency

maps. For 2D Cartesian scans, the most efficient coil modulation scheme we found produces a nearly-linear gradient oscillating along the phase-encoding dimension, whereas alternatives – including different field shapes and randomized sinusoidal frequencies or phase offsets across B_0 coil channels and TRs – do not achieve comparable acceleration. The cardinal and power function maps explain the reason in a visible way: the linear gradient remains the main encoder, and leaves a pattern of missing k-space data that can be naturally fit by simply a zig-zag trajectory with minimal interpolation errors. Furthermore, these k-space efficiency maps also visibly clarify several unintuitive sampling behaviors presented in Publication 1, such as why readout-CAIPI (TR-varying modulation) only helps for 5 kHz modulation frequency with a specific field shape (linear gradient rotating in-plane), and why randomizing modulation frequencies and phase offsets across coils/TRs does not lead to improved sampling efficiency.

A comprehensive safety evaluation is performed with respect to PNS and acoustic noise for our local B_0 coil array. It comprises experimental validation of B-field strengths, numerical calculation of E-fields and their coupling with human neuron models for PNS prediction^{61,62}, in-scanner measurements of acoustic noise generated by the local B_0 coils modulation at 9.4T. When ramping up the amplitude of modulation currents, no PNS or prohibitive acoustic noise was reported by human subjects⁶³. Using the 8-channel B_0 coil array, in-vivo 2D brain scans (transverse plane near the B_0 coil winding) achieve 3x acceleration without SENSE, and 7-8x joint acceleration with SENSE, without noticeable acceleration artifacts.

Publication 1 was nominated as for **finalist of the I.I. Rabi Award in the ISMRM 2024 Young Investigator Awards Competition**, Singapore.

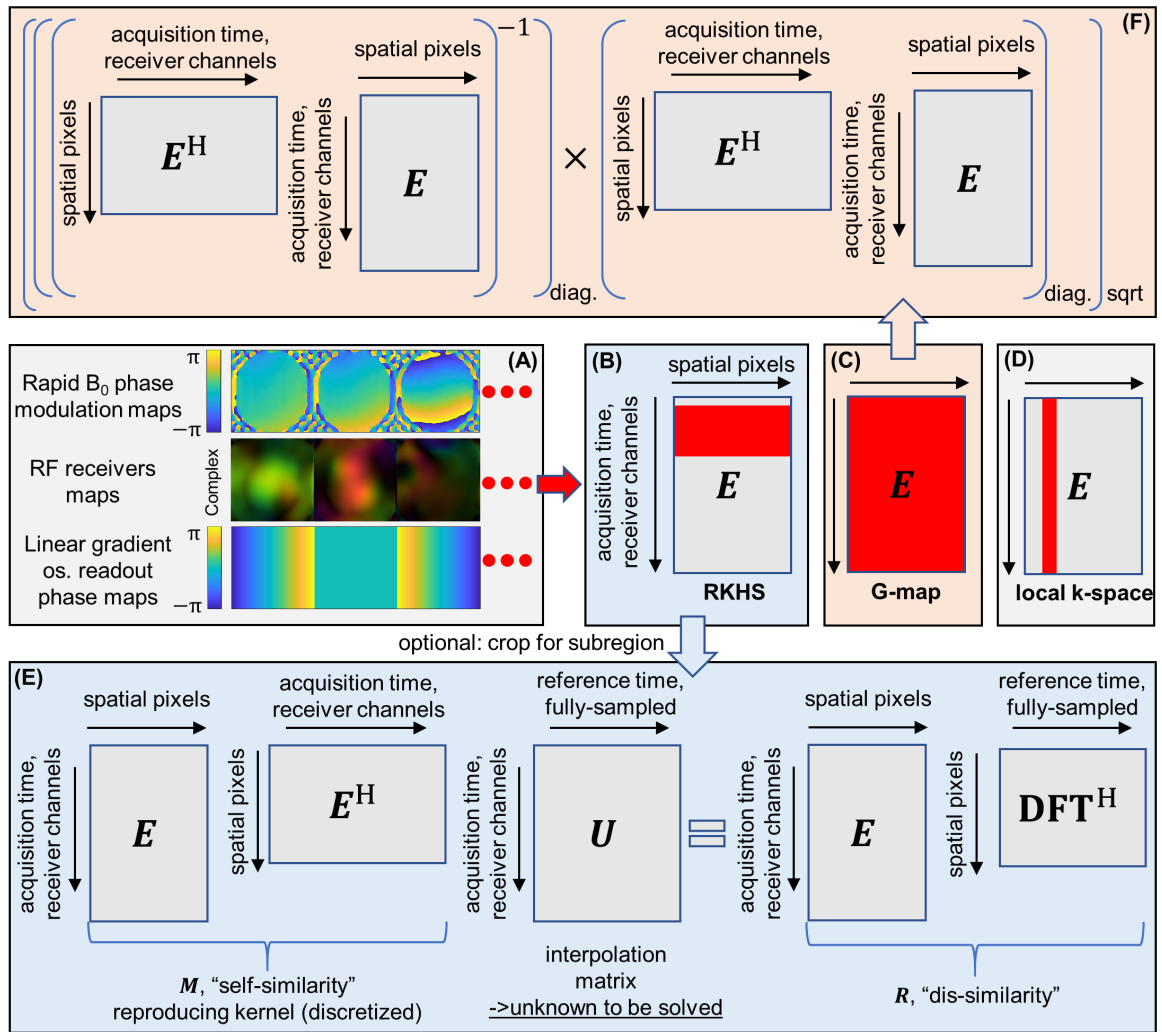


Figure 4. The RKHS formalism incorporating additional rapid B_0 modulations and optional selection of k-space subregions. (A) All spatial-encoding maps are elementwise multiplied to form a composite encoding matrix. (B) The encoding matrix with a selected row subset (red), to compute efficiency maps at k-space subregions. (C) The entire (red) encoding matrix used for G-map calculation. (D) A column of encoding matrix used to compute local k-space at a specific spatial location. (E) RKHS formalism to compute cardinal function as interpolation weights, which can be further used to compute power functions as approximation errors. "DFT" denotes the DFT matrix, representing fully-sampled Fourier sampling as the target k-space for reconstruction. (F) The conventional image-space G-map calculations for comparison.

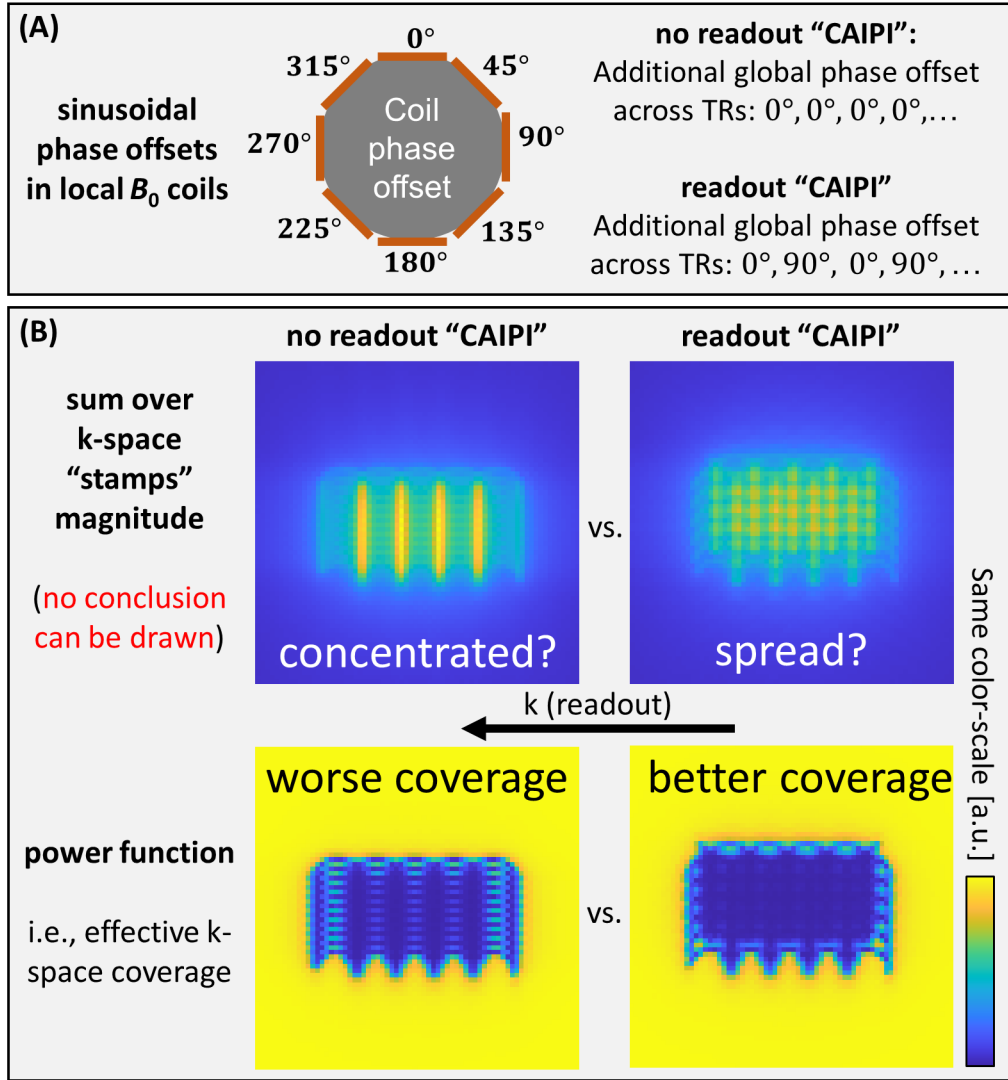


Figure 5. Demonstration of the unique benefits of the RKHS formalism for analyzing nonlinear gradient modulations. (A) Each local B_0 coil is driven with a 5 kHz sinusoidal current, with the phase offsets shown. In the readout “CAIPI” regime, an additional 90-deg phase offset is applied to all B_0 coils, effectively shifting the trajectory along the readout dimension. (B) Conventional inspection of the coverage by k-space kernels across the readout – such as summing kernel magnitudes to estimate k-space coverage – cannot determine which modulation scheme (with or without readout “CAIPI”) is more efficient. For example, this ignores the kernels’ phase, which play a critical role in resolving acquired neighborhoods. In contrast, the RKHS power function incorporates both the amplitude and phase of the kernels, and quantifies the k-space approximation error by solving a forward model that evaluates how well the sampling functions – corresponding to the set of k-space kernels – can be interpolated onto a fully-sampled Fourier basis. It clearly reveals that the readout “CAIPI” strategy provides improved coverage by eliminating the hot spots present in the scheme without readout “CAIPI”.

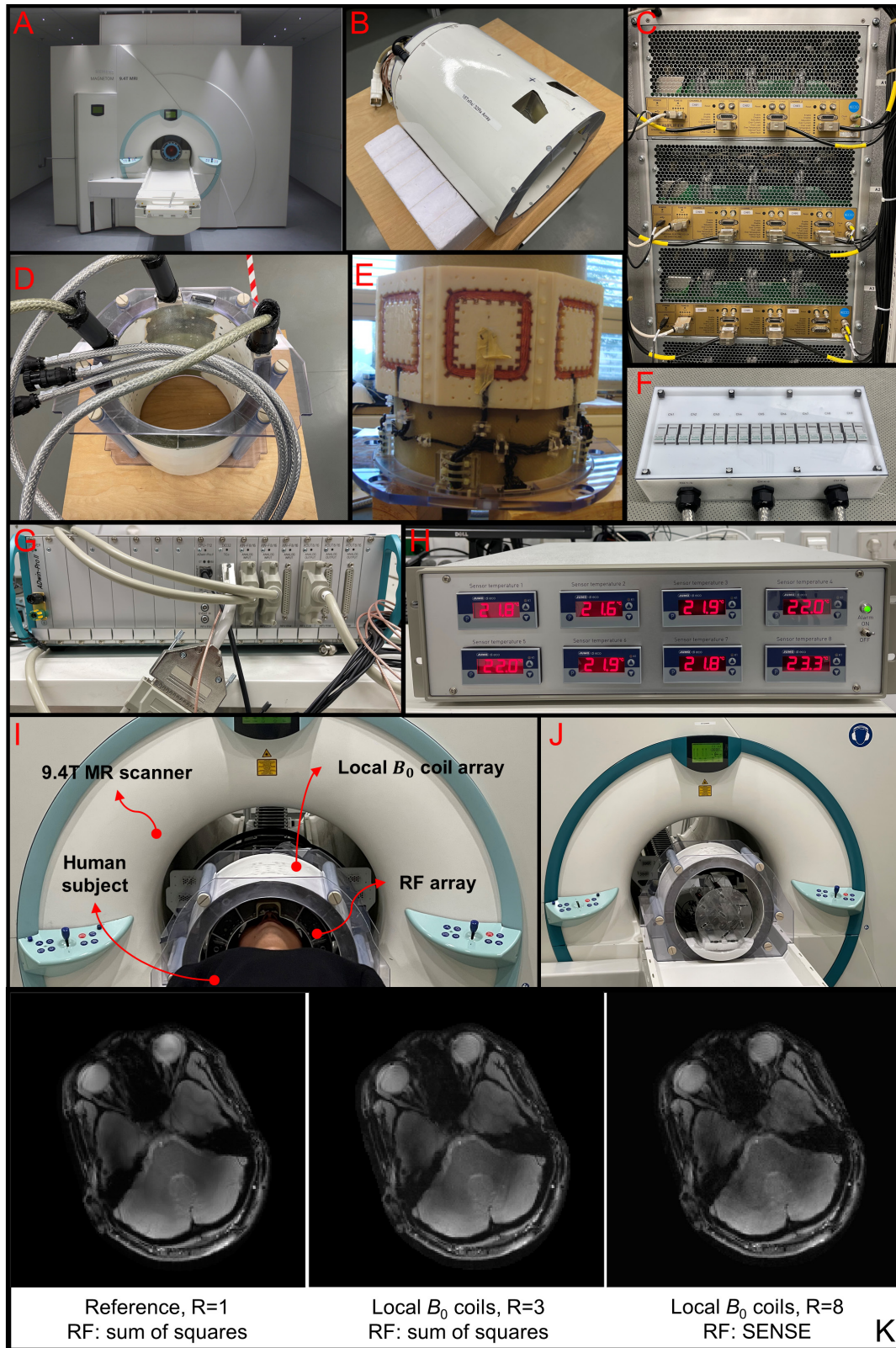


Figure 6. (A) 9.4T human MR scanner. (B) 16-transceive-32-receive RF array. (C) IECO power amplifier (Finland). (D) 8-channel local B_0 coil array. (E) Local B_0 array during construction. (F) Fuse box for local B_0 array. (G) Custom-programmed signal generator for waveform control. (H) Temperature monitors. (I) Setup for in-vivo scans. (J) Field camera monitoring. (K) In-vivo images with acceleration by local B_0 coil array, which can be optionally combined with SENSE. R: acceleration factor.

5.2 Publication 2 – Group-kernel calibration and group-patch RKHS reconstruction

First, Publication 2⁶⁴ develops a more robust magnetic field calibration strategy – called group-kernel calibration – that can auto-calibrate the additional spin phase evolution induced by the local B_0 coil array directly from imaging data, rather than via separate B_0 mapping as Publication 1. The time-varying rapid B_0 modulation kernels are calibrated separately in groups, by partitioning them into time-invariant kernels based on modulation periodicity. Consequently, this approach inherently captures both intended encoding effects and all system imperfections (e.g., eddy currents, waveform distortions, delay), and provides a unified calibration framework that bridges self-calibration of nonlinear gradient and the established parallel imaging concepts. Instead of merely utilizing ESPIRiT to extrapolate B_0 maps which cannot capture modulation-induced eddy currents, the group-kernel calibration exploits the intrinsic kernel-shift data structure during modulation scans to estimate B_0 modulation kernels from two auto-calibration signal (ACS) regions, extending GRAPPA estimations from RF receivers to rapidly modulated B_0 fields. One is a standard ACS region as in parallel imaging, and the other one is a field-modulated ACS acquired with additional B_0 modulations on, while both can be later used as imaging echoes in reconstruction. In our experiments, this approach is essential for 3D and for 2D sagittal/coronal scans, where we observed artifacts likely driven by strong z-axis eddy currents when using the field mapping calibration in Publication 1 – possibly due to lack of capacitors cutting current flow along z-direction on the RF shield. Compared with the conventional field calibration method based on chemical-shift sequences⁴⁹, the group-kernel auto-calibration approach avoids separate lengthy measurements, eliminates temperature stress on the hardware, and requires no debugging of the additional field-calibration sequences.

Similar to RKHS encoding in Publication 1, the core principle behind this auto-calibration technique is to partition the k-space data sampled by shift-varying k-space kernels into data acquired by shift-invariant kernel subsets, but grouped in a more sophisticated way for calibration purpose. Unlike time-static RF sensitivity, the k-space is encoded by dynamic B_0 kernels that change shapes across

oversampled readout time, mixed with Fourier encoding. However, the periodic nature of the B_0 modulation ensures that identical spatial modulation recurs at specific k-space locations as a subset – such as at the same phase of the modulation period along readout, across different phase-encoding steps and all receiver channels. This recurring condition allows the shift-varying kernels to be grouped into shift-invariant subsets, so that each can be auto-calibrated similar to GRAPPA that estimates shift-invariant RF kernels. Specifically, linear systems of k-space interpolation relationships are established, spanning certain readout timepoints, all phase-encoding steps and RF receivers that share the same B_0 modulation states. At each k-space location within a subset, a k-space neighborhood in the standard ACS is multiplied with an unknown B_0 kernel and summed up to interpolate a data point in the field-modulated ACS, similar to Equation (4-5). The linear systems are established and solved to yield B_0 kernels separately, which are Fourier transformed into image-space as spatial modulation maps caused by local B_0 coils, and incorporated into image reconstruction of undersampled k-space data. The group-kernel calibration technique is experimentally tested for calibrating local B_0 coils modulation and Wave-CAIPI in a 9.4T human scanner.

Second, Publication 2 proposes a novel group-patch RKHS¹⁸ reconstruction technique based on k-space subregion⁶⁵ interpolations for B_0 field modulated scans, as an efficient alternative to the conventional Wave-CAIPI-type hybrid-space reconstruction. Namely, the k-space data is reconstructed patch-by-patch in separate groups, based on the RKHS formalism. The cardinal function calculated via the RKHS formalism¹⁸ in Publication 1, which represents k-space interpolation weights, can be used to reconstruct fully-sampled k-space by interpolating undersampled data using matrix multiplication. Thus, for reconstruction purpose, undersampled k-space data encoded by shift-varying but periodic B_0 kernels can be grouped into patches with shift-invariant modulations. For example, a k-space subregion can span an integer number of periods of the sinusoidal B_0 modulation field. Consequently, an interpolation matrix of cardinal function can be computed once for a subregion, and then reused to interpolate all

data patches from subregions spanning the identical modulation subsets, using simple and efficient matrix multiplications.

Relative to the original RKHS framework for parallel imaging²³, we restrict the global interpolation into subregions tailored to the rapid field modulation schemes, similar to visualizing k-space encoding efficiency of nonlinear gradient in single or a few phase-encoded steps as in Publication 1. This turns the originally-demanding RKHS calculation/reconstruction into an advantage: once the cardinal function matrices are available, reconstruction reduces to a sequence of tunable-size, highly-parallelizable matrix multiplications. Compared with hybrid-space Wave-CAIPI-type reconstruction that decouples only the phase-encoding dimensions, our RKHS reconstruction can decouple all k-space dimensions (including readout), flexibly slicing reconstruction data into smaller chunks with low memory requirements, and substantially reducing computational burden further. In a proof-of-concept reconstruction with single receiver data, our RKHS formalism achieves 3x faster reconstruction by reconstructing sliced k-space subregions, compared to the conventional hybrid-space reconstruction. The experiments are performed with local B_0 coils modulation at 9.4T.

A **patent** of the auto-calibration technique for rapid B_0 fields, applicable to both nonlinear and linear (e.g., Wave-CAIPI) gradients, has been filed through Max-Planck-Innovation. The relevant conference abstract was selected as one of **the highly-regarded abstracts in the ISMRM 2024**, Singapore.

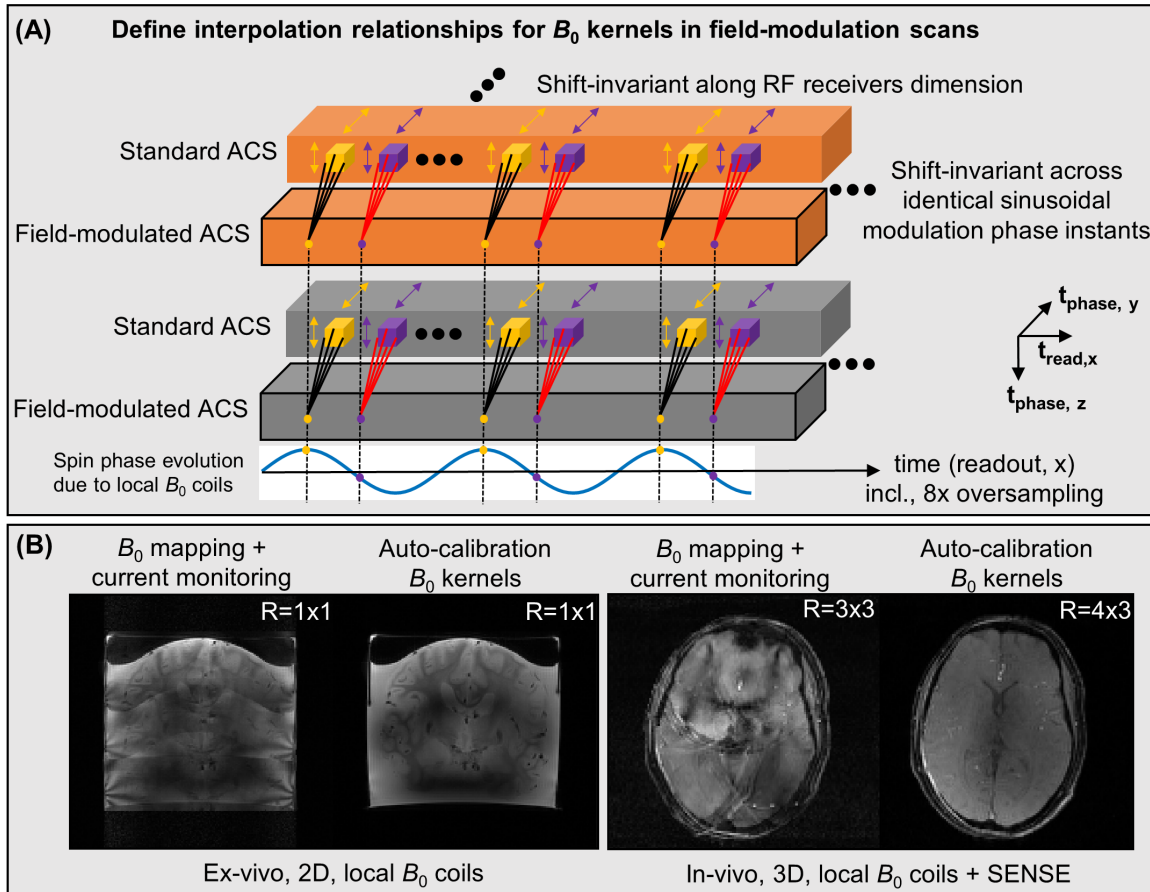


Figure 7. (A) Establish k-space interpolation relationships between standard and field-modulated ACS regions. At k-space locations where object experiences identical additional B_0 field modulation, linear system of kernel interpolation can be established and solved for one B_0 kernel. Here, a data sample in a field-modulated ACS block is the sum value of a time-varying local neighborhood (i.e., a “cube”) of the data in a standard ACS block. Different k-space “cubes” (e.g., yellow, purple) represents different B_0 modulation kernels that vary periodically along readout. The ACS blocks in different colors (e.g., orange, grey) represents ACS datasets from different RF receivers. (B) Comparison between B_0 mapping with current monitoring and our auto-calibration approach, which appears rather robust for 2D coronal and 3D volume scans. In our current MATLAB CPU implementation, the 3D kernel calibration takes about 8 mins for 10-10-11 kernel size, 20h for 18-19-19 kernel size, and might be further accelerated with optimized coding.

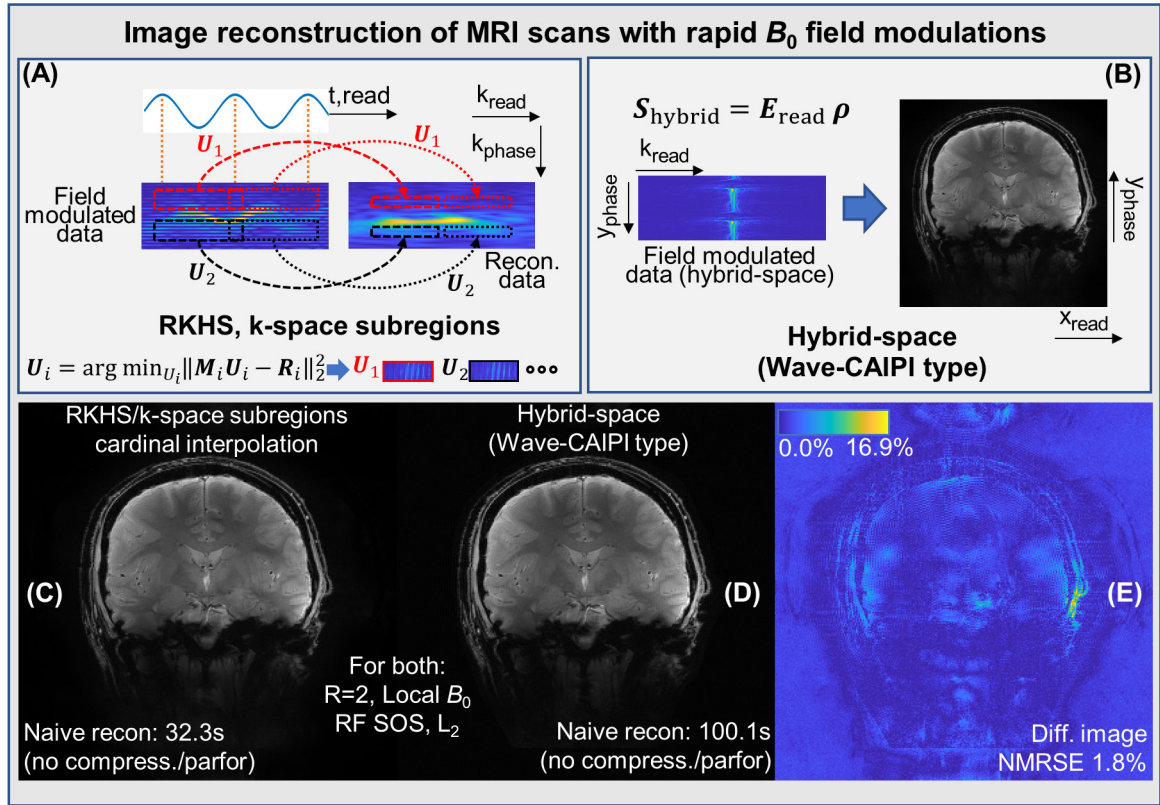


Figure 8. (A) Group-patch RKHS reconstruction by interpolating k-space subregions with identical cardinal function computed via the RKHS formalism. Here, two generalized interpolation matrices U_1, U_2 are computed and reused to interpolate undersampled k-space subregions (encoded by the same set of B_0 - B_1 encoding functions) to reconstructed k-space subregions. (B) Hybrid-space reconstruction (Wave-CAIPI type), which solves the readout forward model that maps image-space to k-space. (C-E) 2D in-vivo scans with different reconstruction algorithms, without compression or parallel computing.

5.3 Publication 3 – Group-patch joint compression

Publication 3⁶⁶ further speeds up image reconstruction of B_0 field modulated scans^{67–73,29,74,75}, by jointly compressing k-space data encoded by both B_0 and RF receiver fields. The k-space data is compressed patch-by-patch in separate groups. This drastically reduces data volume loaded into the reconstruction algorithm, which enables compressed-sensing for the conventional hybrid-space reconstruction¹⁴, and is theoretically compatible with the RKHS reconstruction in Publication 2 for further acceleration of reconstruction speed. The key observation is that rapid B_0 field modulation (e.g., 3-10 kHz) varies slowly across neighboring oversampled readout points (e.g., 3 μ s), which indicates high encoding redundancy and hence high compressibility. Rather than compressing the full readout encoding matrix – which is as inefficient as inverting it – the shift-

varying periodic B_0 kernels are partitioned into groups of shift-invariant subsets, each spanning an interval of the B_0 modulation period. For each subset, one compression matrix is computed to compress data patch-by-patch, reusing it across all identical intervals of the modulation period along the readout, across phase-encoded steps and RF receiver channels. To implement it, all spatial-encoding maps associated for that interval – RF sensitivity maps, phase maps induced by oversampled readout gradients and rapidly modulated B_0 fields – are loaded into a conventional RF array compression algorithm⁷⁶ to obtain the subset's compression basis. This mimics loading RF sensitivity maps to this algorithm to obtain the optimized compression matrix based on RF sensitivity distributions. The same compression matrix then compresses both the spatial-encoding maps and the k-space data, yielding maximal efficiency because B_0 and RF spatial-encoding maps are jointly compressed in the same space. Such k-space subregion-wise compression is performed for all intervals and kernel subsets, which together spans the entire B_0 modulation period. Theoretically, compression matrix can also be calculated from k-space data⁷⁷. As joint compression factor increases from 1, the compressed spatial-encoding maps first become artificially oscillatory (corresponding to broadened k-space kernels) and then become smooth again (narrowed k-space kernels). These compressed maps may not physically exist, but result from the numerical basis induced by singular-value decomposition operation during the compression.

The joint compression technique is tested with local B_0 coils modulations and Wave-CAIPI in a 9.4T human scanner, and Wave-CAIPI in a standard 3T scanner. Thus, MRI data with rapid B_0 field modulation can be compressed by a factor of 10-20x, far beyond the conventional RF array compression of 1.5-3x. The reconstruction speed of the conventional hybrid-space reconstruction is substantially improved. Particularly, the compressed sensing reconstruction becomes feasible within 3-10 mins for 3D datasets (e.g., 180x180x80, 230x230x112, 230x230x144). RKHS reconstruction has not been implemented in Publication 3, but is theoretically compatible for further reduction of runtime and memory.

A **patent** of the joint compression technique for rapid B_0 fields, applicable to both nonlinear and linear (e.g., Wave-CAIPI) gradients, has been filed through Max-Planck-Innovation.

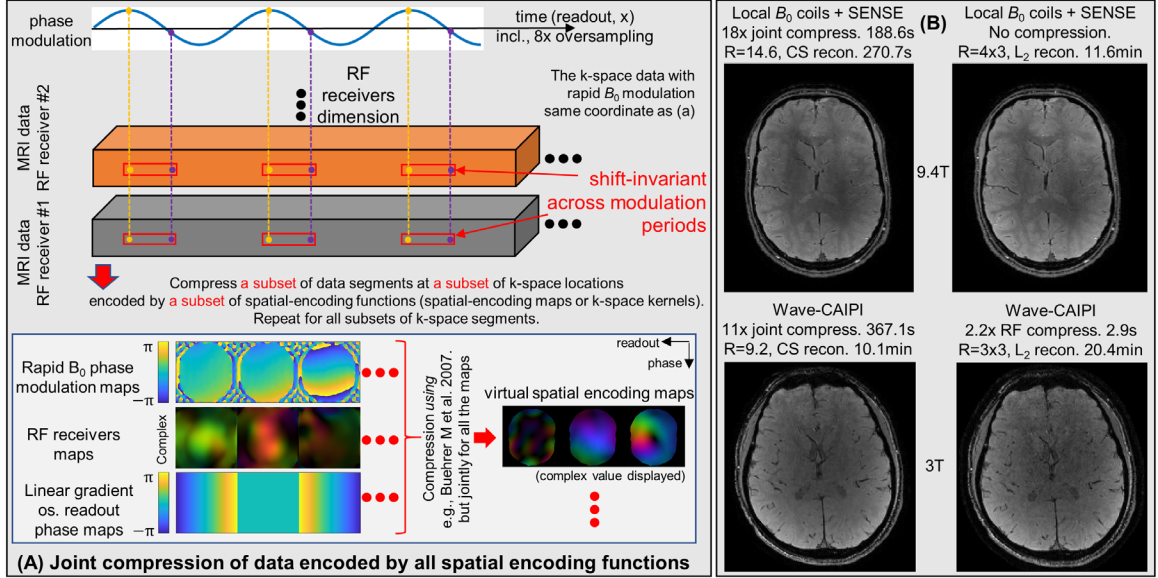


Figure 9. (A) Illustration of joint compression of B_0 and RF encoded k-space data. (B) In-vivo least-squares or compressed-sensing reconstructions with no compression, conventional RF compression, and our joint compression techniques.

5.4 Publication 4 – Overlap-kernel EPI

Publication 4⁷⁸ extends the auto-calibration technique in Publication 2 from nonlinear B_0 modulations to the conventional linear gradient regime, targeting the notorious shot-to-shot nonlinear phase fluctuations that pose threats to high-resolution multi-shot EPI. This technique is termed overlap-kernel EPI, where shot-to-shot diffusion kernels or their corresponding image-space maps are estimated from overlap regions at arbitrary k-space locations. In k-space, the diffusion phase fluctuations in Equation (8) manifest as convolution kernels, shift-varying across different shots but shift-invariant across the entire k-space. Therefore, Publication 4 proposes to slightly overlap neighboring k-space segments acquired in different shots, and explicitly extract the shot-to-shot phase fluctuation kernels from the shot-to-shot overlap regions using a

GRAPPA⁵/ESPIRiT⁵⁹-type operator, called overlap-kernel EPI. These estimated phase kernels are Fourier transformed into maps, and incorporated into multi-shot reconstruction to yield a shot-to-shot corrected high-resolution image⁷⁹. Unlike previous self-navigating trajectories^{36–38,80} that repeatedly oversample k-space center across shots, our approach demonstrates that self-navigation with explicit phase map monitoring can be performed at arbitrary k-space locations given sufficient SNR. Analogous to how SAKE⁸¹/LORAKS⁸² for self-calibrated parallel imaging with iterative low-rank matrix completion evolves into MUSSEL⁴¹ for diffusion scans, Publication 4 represents how GRAPPA/ESPIRiT is extended to achieve multi-shot diffusion EPI with one-step explicit extraction of diffusion phase maps. This technique is broadly-applicable to a wide range of multi-shot trajectory patterns, and four proof-of-concept examples of 2D Cartesian segmentations are tested: 2-shot center-out mosaic EPI, 4-shot center-out mosaic EPI, readout-segmented EPI, and phase-interleaved EPI.

The 2-shot and 4-shot mosaic EPI have advantages of high-SNR for both imaging echoes and calibration regions (compared to separate navigators after imaging acquisitions), due to center-out trajectory as spiral⁸³. Regardless of a point spread function due to T_2 decay, the low-resolution components usually with the largest pixel numbers occupied experience minimal decay, which contributes to visibly higher SNR. The self-navigated readout-segmented EPI with our overlap-kernel calibration achieves identical multi-shot reconstruction quality compared to conventional 2D navigator-based corrections, when diffusion-weighted b-values are equal or below 1500 s/mm² in a standard 3T human scanner. For phase-interleaved EPI, a one-step phase extraction can be performed from the central region of intermediate parallel imaging reconstruction, as called MUSE with kernel extraction. This can be viewed as an optimized form of MUSE⁴⁰ with improved robustness for extracting phase maps, due to kernel-shift rather than total variation. It also optimizes MUSSEL⁴¹, which uses similar kernel-based construction matrix, but can suffer from low-SNR peripheral k-space and long computational time due to iterative low-rank matrix completion. Compared with JULEP⁸⁴, as a MUSSEL-MUSE joint strategy, the overlap-kernel

technique elegantly unifies the two operations into one-step kernel-based phase extraction. In addition, the overlap-kernel calibration can enhance the phase map extrapolation of separate navigator scans or B_0 field mapping, leveraging averaging effect of kernel-shift for greater robustness.

Publication 4 was nominated for **finalist of the I.I. Rabi Award in the ISMRM 2026 Young Investigator Awards Competition**, Cape town, South Africa.

A **patent** of the overlap-kernel EPI technique, applicable to a variety of multi-shot EPI techniques (e.g., RESOLVE in Siemens, MUSE in GE), has been filed through Max-Planck-Innovation.

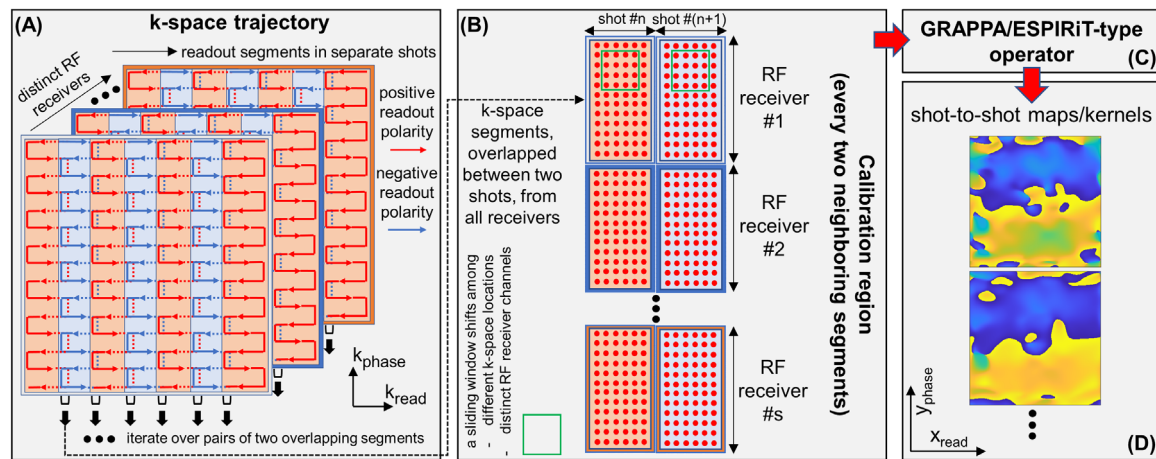


Figure 10. Demonstration of overlap-kernel EPI. (A) Multi-shot k-space trajectories are designed with slight overlaps (typically about 16 pixels along the shorter axis) to create shot-to-shot auto-calibration signal (ACS) regions. Other multi-shot trajectories can be designed in similar ways of inter-shot overlapping. (B) The overlap regions between neighboring shots serve as calibration regions. A sliding mathematical window is used to extract k-space neighborhoods from the calibration region. The shot-to-shot k-space kernels are shift-variant across shot dimension, and shift-invariant across k-space locations and RF receiver channels. Optionally, RF receivers can be combined before estimating diffusion phase kernels. (C) The calibration region is loaded into a GRAPPA/ESPIRiT-type operation, treating shot dimension as the RF receiver dimension in parallel imaging calibrations. (D) The estimated shot-to-shot phase maps, or equivalently, their Fourier domain k-space kernels, are obtained and subsequently used for multi-shot reconstructions.

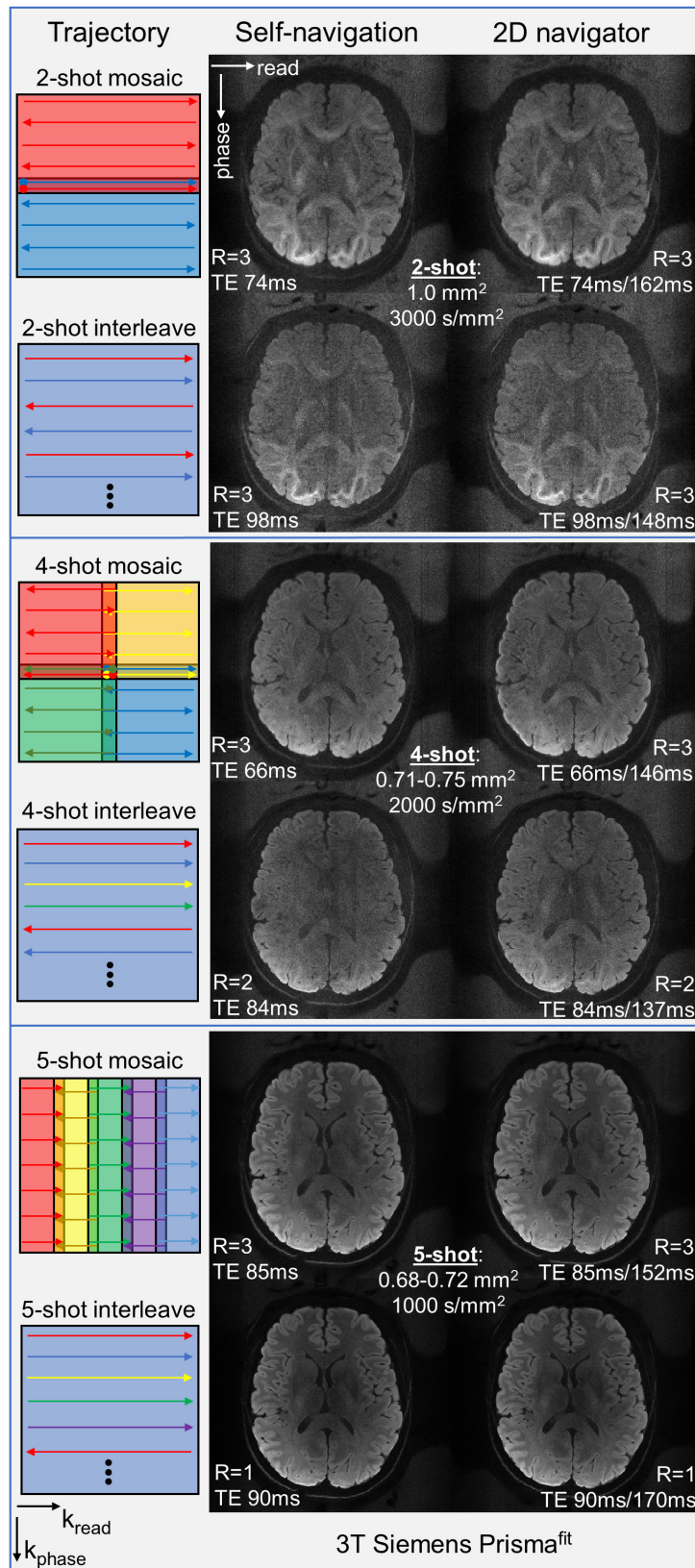


Figure 11. Multi-shot sequences with different EPI segmentations and inter-shot overlaps. Self-navigation based on overlap regions, and 2D navigator corrections are compared. Note, the overlaps in phase-interleaved EPI are the central regions of intermediate parallel imaging reconstruction of all shots as in MUSE.

5.5 Publication 1 to 4 – Mathematically bridging three MRI sub-areas

Beyond individual contributions, Publication 1-4 collectively bridge nonlinear gradient modulation, multi-shot EPI and parallel imaging under the same RKHS framework to manage the associated nonlinear spatial encoding functions, where shift-varying k-space kernels are partitioned into shift-invariant subsets for different purposes⁸⁵. Just as RF receiver sensitivity, these papers show that nonlinear B_0 modulations can be treated the same way, with respect to signal encoding, auto-calibration, compression, and reconstruction. So is the self-navigation for auto-calibrated shot-to-shot diffusion phase fluctuations. Consequently, these three areas are connected far more tightly than before, opening possibilities to new sequence designs and reconstruction strategies for nonlinear gradient encoding or multi-shot MRI, while inheriting the robustness and elegance that have defined parallel imaging over the past two decades.

6 Conclusions, limitations and discussions

As summarized in Publication 1-4, this thesis advances fast and high-resolution MRI by pushing nonlinear spatial encoding functions into an extended RKHS framework, and partitioning shift-varying k-space kernels in shift-invariant subsets depending on purposes. Our custom-built local B_0 coil array, combined with SENSE, accelerates in-vivo Cartesian 3D scans by 14.6x without critical acceleration artifacts or prohibitive compressed-sensing runtimes. Four multi-shot, self-navigated 2D EPI sequences are implemented in Pulseseq and tested in-vivo successfully. The methodology and experimental results can now be clearly explained by the theoretical analysis, and thus, should be simple and robust to reproduce elsewhere.

Despite significant progress, several limitations and unsolved challenges remain, which shall be discussed for future research. From hardware and encoding efficiency perspective (Publication 1), in regimes where rapidly-modulated B_0 fields are superimposed onto the scanner's linear gradients to speed up k-space acquisitions, the linear gradients still provide the dominant spatial encoding that determines the main trajectory. To gain meaningful benefits from additional rapid B_0 phase modulations, the readout gradient bandwidth must be kept modest to low (e.g., 200-300 Hz/Pixel, in this thesis), so that sufficiently dense and broad k-space modulation coverage can be reached along phase-encoding dimensions. In other words, the readout gradient has to be partially slowed down to keep the B_0 modulations (up to 10 kHz) rapid relative to the k-space traversal speed along readout – a long-standing limitation in Wave-CAIPI⁸⁶. This can be less critical when scan time reduction comes primarily from fewer phase-encoded lines rather than slightly faster readout for each line, as in Siemens products of Magnetization-Prepared Rapid Acquisition with Gradient Echoes (MPRAGE)⁸⁷, Susceptibility-Weighted-Imaging (SWI)⁸⁸ and so on. However, under already fast gradient trajectories like spiral⁸⁹, extra B_0 modulation might only slightly perturb the main trajectory, and can be less effective in practice.

To investigate whether this barrier can be removed to make fast trajectory even faster, we have recently designed an integrated, portable B_0 - B_1 array and tested in-vivo. There are 3 rows of 6 B_0 loops mounted perpendicularly onto a 16-channel RF transceiver array, producing stronger and more localized fields that can synthesize more flexible nonlinear spatiotemporal modulations. The extended RKHS formalism in Publication 1 has been utilized to formulate cost function to fast optimize modulation waveforms for 18 channels, which otherwise can become extremely computationally expensive (e.g., using cost function as 3D G-maps). Preliminary results have been submitted for 2026 ISMRM conference abstract⁵⁷. So far, real-time undersampling has not been used, which once implemented, might further improve reconstruction quality due to less subject motions compared to fully-sampled scans with retrospective undersampling.

For the computation side, several aspects can be refined. The group-kernel calibration (Publication 2) with very large B_0 modulation kernels (e.g., width around 18-19-19 k-space pixels) can be time-consuming (e.g., 20 hours) yet necessary for high-order nonlinear fields. Although calibration can begin immediately after reference scans of the two ACS blocks and run efficiently on GPUs, a pre-combination across RF channels for the standard and modulated ACS could potentially reduce the kernel shift number and speed up the solvers, which requires future investigations. The joint compression technique (Publication 3) can also incur computation time in minutes, and improved parallelization or joint B_0 -RF compression completely in k-space could be explored to mitigate it. Finally, deep neural networks could be investigated for improving auto-calibration accuracy of both B_0 gradient⁹⁰ and diffusion kernels, given wide kernel widths or low-SNR calibration regions, similar to RAKI⁹¹ that optimizes GRAPPA reconstruction quality.

For sequence developments, simultaneous-multi-slice (SMS) naturally fits the overlap-kernel EPI technique. However, 3D EPI may require careful design of overlapping regions in three dimensions, which can significantly prolong acquisition times. The b-value limits for different multi-shot trajectories require

more systematic protocol-specific validations. For scanners equipped with high-performance gradients (e.g., Siemens Cima.X, Connectome and Impulse scanners), the overlap-kernel EPI calibrates not only the shot-to-shot phase fluctuations, but also the shot-to-shot eddy currents (as spatially-varying phase offsets). The latter may become prominent with increased gradient performances^{92,93}, and cannot be captured by separate 2D navigators, where self-navigation by overlap-kernel calibration becomes essential.

Beyond aggressive nonlinear gradient encoding and advanced multi-shot EPI, many MRI scenarios involving nonlinear spatial encoding functions remain under-explored. The methodology developed in this thesis does more than solve a set of specific problems. Instead, it aims at providing a general framework with concrete solution examples, to guide how to setup the engineering pipeline for addressing future challenges wherever nonlinear spatial encoding function arises. By visualizing them in RKHS, and partitioning shift-varying kernels into shift-invariant subsets, one can encode, calibrate, compress and reconstruct k-space data in an understandable and robust way. In long term, these could potentially become significant building blocks to understand and pursue the ultimate MRI limits in speed, resolution and encoding efficiency.

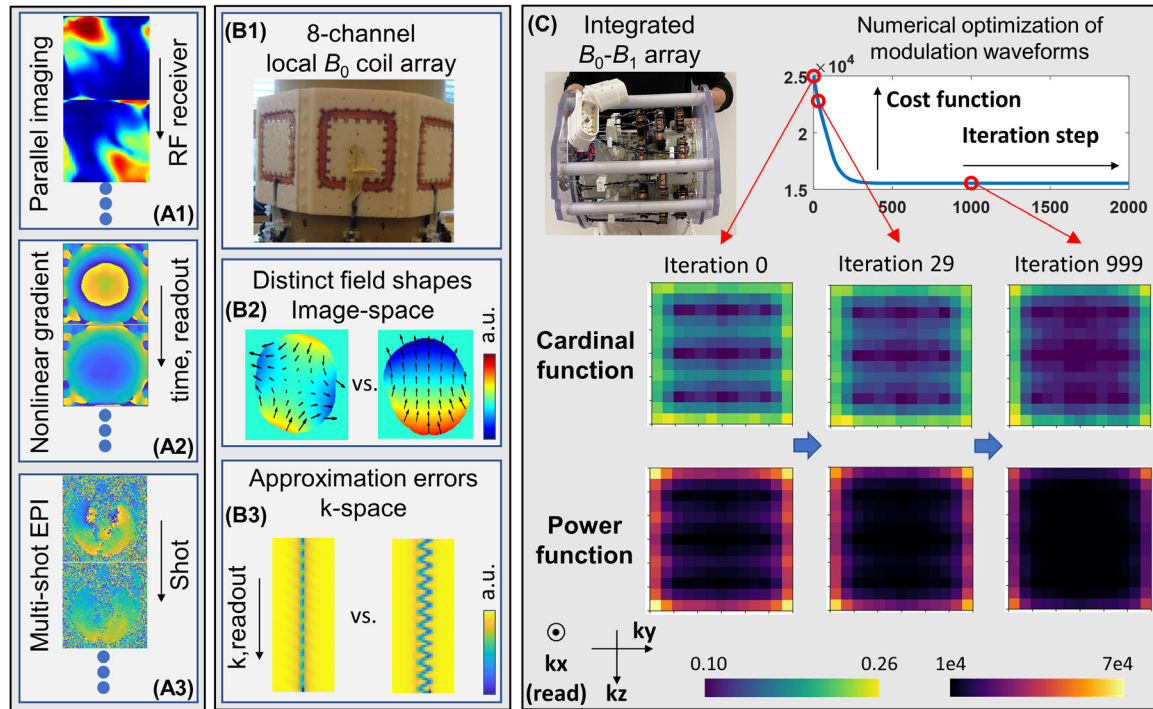


Figure 12. In future work, a unified framework for different nonlinear spatial encoding functions is exploited, which can guide modulation waveform designs for a recently developed B_0 - B_1 array, as submitted for ISMRM 2026, Cape Town, South Africa. (A1-A3) Different nonlinear spatial encoding functions, originating from distinct physical mechanisms, but sharing the same mathematical form. (B1-B3) An 8-channel local B_0 coil array, with efficiency by distinct modulation field shapes visualized in k-space. (C) An integrated B_0 - B_1 array, with 18 B_0 loops mounted perpendicularly onto a 16-channel RF transceiver array. The modulation waveforms are efficiently optimized for minimized values in k-space efficiency maps across a small representative region.

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Acknowledgements

This has been a complicated, challenging, yet deeply enriching Ph.D. journey, much longer and tougher than I initially expected. At the beginning, I simply hoped to find a nonlinear gradient encoding scheme that could outperform the well-established linear gradients, which already felt ambitious enough for a doctorate project. Over time, however, the technical developments evolved far beyond what I could have imagined when I started in March 2020. To truly unlock the full potential of the large but underexplored pool of nonlinear gradient fields – which disrupts the foundational k-space theory for MRI – it gradually became clear to me that blindly searching for a single nonlinear encoding scheme was neither realistic nor the most essential step. Instead, it was crucial to first understand the role of nonlinear spatial encoding functions in a deeper and more systematic manner, and ensure that the benefits they offer do not come at the cost of introducing new critical problems. As a result, the project evolved from aiming for one specific solution, into developing an entire framework for managing nonlinear spatial encoding functions for fast and high-resolution MRI. This broad perspective emerged step by step, built on many incremental insights rather than any one single discovery.

For this, I am deeply grateful to Prof. Klaus Scheffler for the invaluable opportunity to join this ERC Advanced Grant project at its earliest stage, and for trusting me with this technical demanding yet inspiring MRI physics topics to challenge the fundamental limits of this well-established technology. Looking back, his original proposal to accelerate MRI using localized and independent rapid B_0 field modulations insightfully outlined a unique physical platform, capable of generating spatial-encoding fields that conventional acceleration technologies cannot access. This already gave the initial project a level of originality that fundamentally stood out from concurrent ideas, and I feel honored to start my Ph.D. on building such a cutting-edge and forward-looking system. I also appreciate his long-term commitment to investing in basic MRI methodology research, which is increasingly rare in today's application-driven MRI community worldwide. Without his sufficient patience and scientific freedom, my long-term methodology developments could easily have been abandoned midway,

especially since the first journal article took far longer than expected. Additionally, his generous funding allowed me to test my Pulseseq sequences for many times in Magdeburg, Freiburg, and Leipzig, significantly expanding my scientific network and giving me access to state-of-the-art Siemens scanners (Impulse, Cima.X, Connectom). It has been a true pleasure and privilege to work in his department over the last 6 years, as one of the very a few labs worldwide with the resources and ambitions to pursue truly fundamental MRI physics and technology projects. Indeed, this experience has become one of the most important and influential parts of my MRI career.

A key turning point in my thesis was the shift from blindly searching for efficient nonlinear encoding schemes to pushing nonlinear gradients into an RKHS-based mathematical framework. For this, I am sincerely grateful to Prof. Martin Uecker, especially during my first Ph.D. supervisory board meeting. When I asked what the most advanced linear system formulation of MRI was, he sent me his RKHS paper on parallel imaging. Although without discussing its details for a long time afterwards, I gradually realized how naturally this RKHS formulation could incorporate nonlinear gradients, and how essential it was to examine the local B_0 coil array within this framework to make sense of several otherwise confusing encoding behaviors. I would also like to thank him for some follow-up discussions on his RKHS/ESPIRiT theories which inspired many of my later ideas, and guidance over the implementations for compressed-sensing reconstructions. These interactions effectively help me expand my skill sets, from physics-oriented MRI background toward a deeper understanding of the mathematical foundations of MRI.

This project involves building up nonlinear gradient systems for a 9.4T scanner, and developing sequences that were tested in a 3T Prisma, a 7T Impulse, a 3T Cima.X, and a 3T Connectom scanner. It would be impossible without assistance of many colleagues. For this, I would like to thank the electrical (Oliver, Theodor, and many others) and the fine mechanical workshops (the late Mirsat, Markus, and many others) in our institute, for their supports in manufacturing the nonlinear

gradient coils. I would like to thank Stefan from ADwin Germany for his guidance in programming the signal generator for nonlinear gradient waveform control. I would like to thank our RF team – Nikolai, Georgiy, Anton, Dario– for their supports in RF coil maintenance in 9.4T scanner. I would like to thank Matthias at MGH and Prof. Axel Thielscher at DTU for their assistance in evaluating the electromagnetic safety of our custom-built nonlinear gradient systems for in-vivo image acceleration. I would like to thank our MRI sequence and reconstruction colleagues for numerous discussions – Felix, Praveen, Sebastian, Tim, Alexander, Jiazheng, Prof. Gisela Hagberg. I would like to thank Cindy, Astrid, Niklas, Simon, Yi-Hang, Prof. Oliver Speck for supports with the 7T Impulse scanner in Magdeburg, Qingping and Prof. Maxim Zaitsev for supports with the 3T Cima.X scanner in Freiburg, Kerrin and Prof. Nikolaus Weiskopf for supports with the 3T Connectom scanner in Leipzig.

For the evaluation of this thesis, I would like to thank Prof. Sebastian Kozerke for being my 2nd thesis reviewers. Much of the knowledge I relied on during Ph.D. was first learned from his courses during my master studies at ETH Zurich. Moreover, at ETH, I would like to thank Franek for supervising my phaseless encoding MRI project for 3 years, from which numerous discussions significantly inspired the Publication 4 (Overlap-kernel EPI) in this thesis. I would also like to thank Prof. Klaas Pruessmann for his insightful advice regarding choosing scientific projects for my future career, including helpful comments for this nonlinear gradient project before I came to Tuebingen in 2020. I would also like to thank Takuya for helping me equipped with the mindset of experimental physics, which substantially make my Ph.D. work more solid. For my undergraduate studies before coming to ETH, I would like to thank Prof. Joseph Rispoli, who taught me RF coil design step by step via hands-on guidance and made me convinced that MRI is a right major for me. For Ph.D. defense, I would like to thank Prof. Fritz Schick and Prof. Thomas Küstner for attending my examinations. I would also like to thank Prof. Fritz Schick and Prof. Gisela Hagberg for offering me tutor positions to teach courses with them at the University of Tuebingen. I would like to thank Pavel, Felix, and Dr. Kai Buckenmaier for revising the German translation of the abstract in this thesis.

My Ph.D. life would have become rather boring without all my colleagues and friends who accompanied me. For this, I would also like to thank other colleagues in the department of high-field magnetic resonance – Kiley, Ali, Jonas, Egor, Rebekka, Kai Hertz, Michael, Florian, Kai Buckenmaier, Friedemann, Milena, Edyta, Jörn, Nicolas, Nikolas, Vinod, Gabriele, Lucas, Angela, Rolf, Pavel, Dana, Tina, Julius, Aaron, Ju Young, Qi, Hang, Yi. Beyond the MR community, I would like to thank Yuxuan for unconditional support in my career, even far away in China. I would like to thank Chuyu, Kun, Kaidi, Xuming, Yan, Sanqian, Mingyu, Jiatong, Ms. Qingyi Hou, Jan, Xiaoyan, Jiaqian, Lin, Xiaying, Shu, Weigutian, Ruocheng, Jiawei, Weiye, Tianqi, Jingwen, Haoyun, Zihao, Zhongnan, Haijia, Yu, Ran, Ziyuan, and many others for enjoying nice food and drinks at different restaurants and bars. I would like to thank Mr. Ping Liong Tjoa for teaching me the essence of Idogo as internal skill, Daniel for hosting me in the Idogo Haus. I would like to thank Mr. Ming Chen and Mr. Yongjie Sun for hosting me in Yantai and teaching me several essential internal martial arts techniques. I would like to thank Yiyu and Mr. Guoding Wang for helping me pick up the remaining techniques in Xin-Yi-Liu-He, after Mr. Qizhong Wang unfortunately passed away. I would like to thank Mr. Zhonghua Chen and Ms. Ye Jin for their training advice during several Taichi workshops. I would like to thank Mr. Qinsun Ding for guiding me into the form of Yeung family Taichi. I would like to thank Han for interesting discussions about martial arts that quite often pushed me for making progress.

Lastly, I would like to express my deepest gratitude to all my family members, who are the most important part of my life. A huge thank you to my parents for financially supporting my 14-year study abroad journey (2012-2026), and to my grandparents for filling my childhood with love.

Appendix

List of all journal articles, conference abstracts and patents

This section lists all journal articles and conference abstracts published during the time period of this thesis (March 2020 – July 2026).

Journal articles

1. **Tian R**, Uecker M, Holder O, Povolni P, Steffen T, Scheffler K. Group-kernel auto-calibration and group-patch k-space reconstruction: Fast MRI with time-variant B_0 kernels partitioned into time-invariant subsets. arXiv. Preprint posted online January 9, 2026:arXiv:2601.05469. doi:10.48550/arXiv.2601.05469. To submit, *Magn Reson Med*.
2. **Tian R**, Scheffler K. Group-Patch Joint Compression: Compressing Dynamic B_0 and Static RF Spatial Modulations Across k-Space Subregion Groups for Highly Accelerated MRI. *Magn Reson Med*. 2026; 0:1–20. doi: 10.1002/mrm.70439.
3. **Tian R**, Uecker M, Zaitsev M, and Scheffler K, “Overlap-Kernel EPI: Estimating MRI Shot-to-Shot Phase Variations by Shifted-Kernel Extraction From Overlap Regions at Arbitrary k-Space Locations,” *Magnetic Resonance in Medicine* 95, no. 4 (2026): 2131–2152, <https://doi.org/10.1002/mrm.70196>.
I. I. Rabi YIA Finalist, ISMRM, 2026.
4. H Shaoying, et al. Experience of how to build an MRI machine from scratch, *Progress in Nuclear Magnetic Resonance Spectroscopy*, 2025, 101574, ISSN 0079-6565, doi: 10.1016/j.pnmrs.2025.101574.
5. **Tian R**, Uecker M, Davids M, et al. Accelerated 2D Cartesian MRI with an 8-channel local B_0 coil array combined with parallel imaging. *Magn Reson Med*. 2024; 91: 443–465. doi: 10.1002/mrm.29799. **I. I. Rabi YIA Finalist, ISMRM, 2024.**
6. **Tian R**, Hennel F, Bianchi S, Pruessmann KP. Superresolution MRI with a structured-illumination approach. *Phys. Rev. Applied*, 19. 034074. March 23, 2023. doi: <https://doi.org/10.1103/physrevapplied.19.034074>.

7. Pinotsi D, **Tian R**, Anand P, Miyanishi K, Boss JM, Chang KK, Welter P, So F T.-K., Terada D, Igarashi R, Shirakawa M, Degen CL, Segawa TF. Distance measurements between 5 nanometer diamonds – single particle magnetic resonance or optical super-resolution imaging? *Nanoscale Advance*, 2023. <https://doi.org/10.1039/d2na00815g>.

Conference abstracts

1. **Tian R**, Fast and high-resolution MRI with nonlinear spatial encoding functions. Oral presentation. **Gorter Award finalist**, DACH-ISMRM 2026, Bamberg, Germany.
2. **Tian R**, Uecker M, Zaitsev M, Scheffler K. Overlap-kernel EPI: estimating MRI shot-to-shot phase variations from overlap regions at arbitrary k-space locations. **I. I. Rabi YIA finalist**. Oral presentation, Digital poster, interview. *In proceedings of the annual meeting of ISMRM*, Cape Town, South Africa, 2026.
3. **Tian R**, Solomakha G, Avdievich N, Zaitsev M, Uecker M, Scheffler K. Nonlinear gradient modulations, multi-shot EPI, parallel imaging in a unified RKHS framework. Abstract #00247. Oral presentation. *ISMRM workshop on data sampling and image reconstruction*. Sedona, USA, 2026.
4. **Tian R**, Scheffler K. k-space subregion-wise joint compression: compressing dynamic B_0 and static RF field modulations for accelerated MRI. Abstract #01000. Oral presentation. *In proceedings of the annual meeting of ISMRM*, Cape Town, South Africa, 2026.
5. **Tian R**, Solomakha G, Avdievich N, Scheffler K. Integrated local B_0 - B_1 array: a portable acceleration platform with nonlinear gradient waves numerically optimized in k-space. Abstract #06735. Oral presentation. *The annual meeting of ISMRM*, Cape Town, South Africa, 2026.
6. **Tian R**, Uecker M, Zaitsev M, Scheffler K. Pulseq implementation of overlapped readout segmented EPI with phase fluctuations corrected by shift-invariant kernel extraction. Digital poster. Poster #18. *In proceedings of the ISMRM Workshop on 40 Years of Diffusion: Past, Present & Future Perspectives*, Kyoto, Japan, 2025.

7. **Tian R**, Uecker M, Davids M, Thielscher A, Buckenmaier K, Holder O, Steffen T, Scheffler K. Accelerated 2D Cartesian MRI with an 8-channel local B_0 coil array combined with parallel imaging. **I. I. Rabi YIA Finalist**. Oral presentation, digital poster, interview. Program #0008. *In proceedings of the annual meeting of ISMRM*, Singapore, 2024.
8. **Tian R**, Uecker M, Scheffler K. Navigator-free multi-shot EPI with shift-invariant kernel extraction in subspace. Power pitch. Program #1081. *In proceedings of the annual meeting of ISMRM*, Singapore, 2024.
9. **Tian R**, Uecker M, Holder O, Steffen T, Scheffler K. Fast volumetric Cartesian MRI with auto-calibrated local B_0 coil array – a reproducing kernel Hilbert space perspective. Power pitch. Program #0954. *In proceedings of the annual meeting of ISMRM*, Singapore, 2024.
10. **Tian R**, Steffen T, Scheffler K. In-vivo image acceleration with an 8-channel local B_0 coil array and parallel imaging in a 9.4T human MR scanner. Digital poster. *In proceedings of the joint annual meeting of ISMRM*, Toronto, Canada, 2023.
11. **Tian R**, Davids M, Thielscher A, Steffen T, Scheffler K. Safety evaluation of an 8-channel local B_0 coil array with sinusoidal modulation in a 9.4T human MR scanner. Power pitch. *ISMRM Workshop on MR Safety: From Physics & Physiology to Policies & Practice*, New York, USA, 2022.
12. **Tian R**, Loktyushin A, Buckenmaier K, Steffen T, Scheffler K. Image acceleration with an 8-channel local B_0 coil array compatible with parallel imaging in a 9.4T human MR scanner. Program #0388, oral presentation. *In proceedings of the joint annual meeting of ISMRM-ESMRMB*, London, UK, 2022.

Patents

1. Uecker M, Scheffler K, **Tian R**. Technique for Processing Magnetic Resonance Imaging Data. WO2025218897 (A1) - 2025-10-23. (WO2024EP60513 20240418). Accessed November 7, 2025. https://worldwide.espacenet.com/publicationDetails/biblio?FT=D&date=20251023&DB=&locale=en_EP&CC=WO&NR=2025218897A1&KC=A1&ND=4

2. Uecker M, Scheffler K, **Tian R**. Technique for Calibrating Magnetic Resonance Imaging Data. WO2025218898 (A1) - 2025-10-23. (WO2024EP60514 20240418). Accessed November 11, 2025.
https://worldwide.espacenet.com/publicationDetails/biblio?FT=D&date=20251023&DB=&locale=en_EP&CC=WO&NR=2025218898A1&KC=A1&ND=4
3. **Tian R**, Scheffler K. Technique for compressing and reconstructing magnetic resonance imaging data. Filed, 15 April 2025. European Patent Office, The Hague. PCT application number: PCT/EP2025/060356.

Publication 1

Tian R, Uecker M, Davids M, et al. Accelerated 2D Cartesian MRI with an 8-channel local B_0 coil array combined with parallel imaging. *Magn Reson Med*. 2024; 91: 443–465. doi: 10.1002/mrm.29799

Finalist of the I.I. Rabi Award in the ISMRM 2024 Young Investigator Awards Competition, Singapore.

The main manuscript is attached below. The supporting information can be found online.

RESEARCH ARTICLE

Magnetic Resonance in Medicine

Accelerated 2D Cartesian MRI with an 8-channel local B_0 coil array combined with parallel imaging

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Funding information

ERC Advanced Grant, Grant/Award Number: No.834940

Abstract

Purpose: In MRI, the magnetization of nuclear spins is spatially encoded with linear gradients and radiofrequency receivers sensitivity profiles to produce images, which inherently leads to a long scan time. Cartesian MRI, as widely adopted for clinical scans, can be accelerated with parallel imaging and rapid magnetic field modulation during signal readout. Here, by using an 8-channel local B_0 coil array, the modulation scheme optimized for sampling efficiency is investigated to speed up 2D Cartesian scans.

Theory and Methods: An 8-channel local B_0 coil array is made to carry sinusoidal currents during signal readout to accelerate 2D Cartesian scans. An MRI sampling theory based on reproducing kernel Hilbert space is exploited to visualize the efficiency of nonlinear encoding in arbitrary sampling duration. A field calibration method using current monitors for local B_0 coils and the ESPIRiT algorithm is proposed to facilitate image reconstruction. Image acceleration with various modulation field shapes, aliasing control, and distinct modulation frequencies are scrutinized to find an optimized modulation scheme. A safety evaluation is conducted. In vivo 2D Cartesian scans are accelerated by the local B_0 coils.

Results: For 2D Cartesian MRI, the optimal modulation field by this local B_0 array converges to a nearly linear gradient field. With the field calibration technique, it accelerates the in vivo scans (i.e., proved safe) by threefold and eightfold free of visible artifacts, without and with SENSE, respectively.

Conclusion: The nonlinear encoding analysis tool, the field calibration method, the safety evaluation procedures, and the in vivo reconstructed scans make significant steps to push MRI speed further with the local B_0 coil array.

KEYWORDS

image acceleration, magnetic field calibration, nonlinear gradient, parallel imaging, reproducing kernel Hilbert Space, wave-CAPI

1 | INTRODUCTION

Typically, MRI scans subjects by spatially encoding the nuclear magnetization with linear magnetic field gradients.¹ This can be mathematically described as sampling the object's representation along a trajectory traversing its Fourier domain, called k-space,^{2–4} with the speed limited by the gradient performance under the safety limits.⁵ Inevitably, sampling data in such a sequential manner makes MRI scans inherently slow, and therefore, fast MRI acquisitions have drawn long-standing interests since its invention in 1973.

Early fast MRI techniques include novel pulse sequence design and k-space sampling strategy, such as FLASH which exploits short TR with low flip angle,⁶ EPI that samples a segment of k-space data within a single shot RF excitation,^{7,8} spiral which uses non-Cartesian trajectory for fast k-space readout,⁹ and UNFOLD which explores spatiotemporal dimension to increase the frame rate in dynamic MRI.¹⁰ From 1990s, with increasing popularity of the NMR phased array,¹¹ distinct sensitivity distributions of multiple RF receiver coils were exploited to disentangle aliased pixels given sub-Nyquist sampling,^{12–14} as another physical source to spatially localize spins. Since then, parallel imaging became a driving force to further accelerate MRI scans in three spatial and one temporal dimensions.^{12–16}

The sampling efficiency by using linear gradients and RF receivers became further optimized, by combining tailored pulse sequences and parallel imaging reconstruction. By traversing the k-space along a zigzag trajectory,^{17,18} or shifting the aliasing patterns along the slice or the phase encoding dimensions as CAIPIRINHA (CAIPI),^{19,20} parallel imaging reconstruction can be further improved. The reconstruction of these more sophisticated encoded scans became straightforward to implement with the forward model,^{13,21} which describes all encoding terms within a linear system. Moreover, such formalism connects MRI reconstruction with the inverse problem, and meanwhile, the progress made in mathematical signal processing, notably the compressed sensing (CS) for under-Nyquist sampling,^{22,23} further boosted up MRI sampling speed in certain applications.^{24–28}

Consequently, to achieve highly efficient volumetric gradient echo scans, wave-CAIPI²⁹ was proposed in 2015 as extended from bunched phase encoding¹⁷ in conjunction with CAIPI,²⁰ and can be optionally combined with CS.³⁰ This technique samples k-space along an oversampled corkscrew trajectory, which maximizes the k-space (Cartesian) sampling area per unit time and increases the variation of RF sensitivity among mixed pixels to reach a higher acceleration factor. To implement this, sinusoidal currents with phase shifts were applied in gradient coils

along the two orthogonal phase encoding dimensions during the gradient echo readout. In the meantime, shim coils driven by sinusoidal currents were also used to accelerate MRI acquisition, which superimposed fast rotary nonlinear gradient fields (here, second order) during signal readout. This technique, called FRONSAC (i.e., Fast rotary nonlinear spatial acquisition),³¹ differs from wave-CAIPI in the sense that, it samples k-space with a “stamp”³² instead of a single point at a time, which can rotate, grow, or shrink when traversing k-space driven by the linear gradients (here, neglecting RF sensitivity modulation).

For decades, although the underlying mechanism for many linear-gradient driven fast acquisition strategies^{29,33} became well explained by the existing MRI sampling theory, the second- or third-order³⁴ magnetic field modulation in FRONSAC, which stems from developments of nonlinear gradient MRI^{31,35–45} (i.e., O-space imaging),⁴⁰ appeared much less investigated. Therefore, a concept called “Spread spectrum MRI”⁴⁶ as a more general technique was proposed in 2019, which aims at accelerating MRI acquisitions based on rapid modulation of localized (rather than global^{29,31}) magnetic fields superimposed with the linear gradients, typically using multiple small-size (and, therefore, local) current loops as adapted from multi-coil shim array.^{47,48} These local coil channels with independently driven current waveforms can produce more flexible modulation fields in spatial-temporal domain, and therefore, impose more localized spin phase evolution on objects as fingerprints, to disentangle overlapped pixels in reconstruction. This name originated from the “spread spectrum” in telecommunication,⁴⁹ in which orthogonal codes modulation on multiple users was exploited to separate signals from users and achieve multiple access. By combining rapid field modulations possibly with diverse nonlinear field shapes,^{34,39,40,43} the local B_0 coil array can be a versatile platform to investigate the optimal B_0 field modulations and further speed up MRI scans.

In this paper, with a newly built 8-channel local B_0 coil array, given its geometry allowing a 16-transceive-32-receive RF array⁵⁰ to fit in, we perform highly accelerated 2D Cartesian FLASH scans jointly with parallel imaging (here, SENSE)^{13,21} in a 9.4 T human MR scanner. A modulation field calibration method based on current monitors in the power amplifier and the ESPIRiT algorithm⁵¹ is proposed to reconstruct scans accelerated by the local B_0 coils. Several sinusoidal modulation waveforms are analyzed and compared with additional noise amplification and approximation error maps analytically computed in time-domain (e.g., k-space). A safety evaluation for our setup is performed to ensure the safety of human subjects during in vivo scans. Finally, reconstruction for ex vivo phantom tests and in vivo scans are performed to validate the encoding efficiency of our

setup, as a significant step for accelerating MRI scans with local B_0 coil array. An early account of the developed methodology was presented in conference abstracts.^{52–54}

2 | THEORY

2.1 | System overview

A new local B_0 coil array consisting of 8 square loops, each in $10\text{ cm} \times 10\text{ cm}$ with 14-turn ($\sim 40\text{ }\mu\text{H}$ measured at 10 kHz), is designed and manufactured in-house. A system overview for this local B_0 coil array with 8-channel waveform control is shown in Figure 1, including a system flow diagram with components connections (Figure 1A), a photo of the installed setup (Figure 1B), power amplifier frequency response (Figure 1C), and simulated magnetic field strength by a local B_0 coil (Figure 1D). A more detailed description with photos of all hardware components is available in Data S1.

2.2 | Signal model for local B_0 field modulation

Mathematically, the presence of the locally modulated B_0 magnetic fields results in another spatial-temporal phase evolution term, on the top of the conventional Fourier and RF sensitivity encoding operator for MRI acquisition. Neglecting spin relaxation effects for simplicity, the acquired time-domain continuous signals obtained by the n^{th} RF receiver coil at the time instant t can be written as:

$$S_n(t) = \int c_n(r) \rho(r) \exp \left\{ -\sqrt{-1} \left\{ k(t)r + \gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^t I_{\lambda}(\tau) d\tau \right] \right\} \right\} dr, \quad (1)$$

with $k(t) = \gamma \int_0^t g(\tau) d\tau$ as the k-space trajectory term (i.e., in rad/m). γ is the gyromagnetic ratio (i.e., in rad/s/T), r is the spin spatial location, $\rho(r)$ is the object spin density, $c_n(r)$ is the n^{th} RF sensitivity distribution, $g(\tau)$ is the shaped pulse of the linear gradients, $B_{\lambda}(r)$ is the B_0 field distribution produced by unit current in the λ^{th} local B_0 coil, $I_{\lambda}(\tau)$ is the programmable current waveform in the λ^{th} local coil.

In this paper, the data from multiple RF receivers are either reconstructed independently and later combined using the sum-of-squares (SOS) method, or reconstructed by solving the forward model incorporating both RF sensitivity encoding and local coils modulations to achieve an accumulative acceleration factor, which is denoted as

“joint reconstruction” in this paper. The RF coils compression is not yet applied to allow a simple examination of the reconstruction quality mainly based on the local B_0 modulations.

When the field modulation in each TR of a 2D line-by-line Cartesian scan remains identical, the image reconstruction and the geometry factor map (G-map) calculation are performed efficiently with a preconditioner of fast Fourier transform (FFT) along the phase-encoded dimensions, similar to wave-CAIPI^{29,46} and is derived in Data S1. However, considering arbitrary waveforms in each channel, images are reconstructed by solving the entire encoding matrix in the forward model, with Moore-Penrose pseudoinverse or iterative least squares methods,^{55,56} which inevitably demands large memory and long computation time, but is alleviated by computing in the high-performance computing servers. For future work, the matrix size for explicit formulation can also be minimized by applying the Kaczmarz method, namely the algebraic reconstruction technique, which takes only a row of the encoding matrix during one iteration step, and was used in O-space MRI⁴⁰ and CT.^{57,58}

2.3 | Sampling efficiency of all “spatial encoders” in arbitrary sampling duration

The encoding efficiency of nonlinear gradients, with respect to image-space noise amplification, can be evaluated by analytically computing the G-map¹³ with an encoding matrix incorporating all spatial encoding terms. This measures the SNR efficiency over the entire acquisition period (i.e., the entire encoding matrix); however, the amount of k-space signal representation resolved within arbitrary time intervals (i.e., any one or a few rows of the encoding matrix) cannot be quantified.

Specifically, applying the convolution theorem to Eq. (1), the additional phase modulation by nonlinear gradients corresponds to the convolution of its Fourier transform term with the object’s k-space representation. Therefore, instead of sampling a single k-space point at a time, the acquisitions become collecting the sum signals contributed by k-space data within a dynamic “stamp”,³² which can change over time according to the nonlinear gradient waveforms. Unfortunately, at arbitrary time instants, the sampling efficiencies by distinct “stamps” corresponding to various nonlinear gradient fields, which parallel imaging acceleration can be optionally combined with, remain difficult to compare quantitatively. Although pixel-wise quantitative analysis^{41,43–45,59–62} for both local spatial resolution and SNR can be obtained by evaluating a single (i.e., local k-space trajectory) or all (e.g., pixel noise variance) columns of the entire

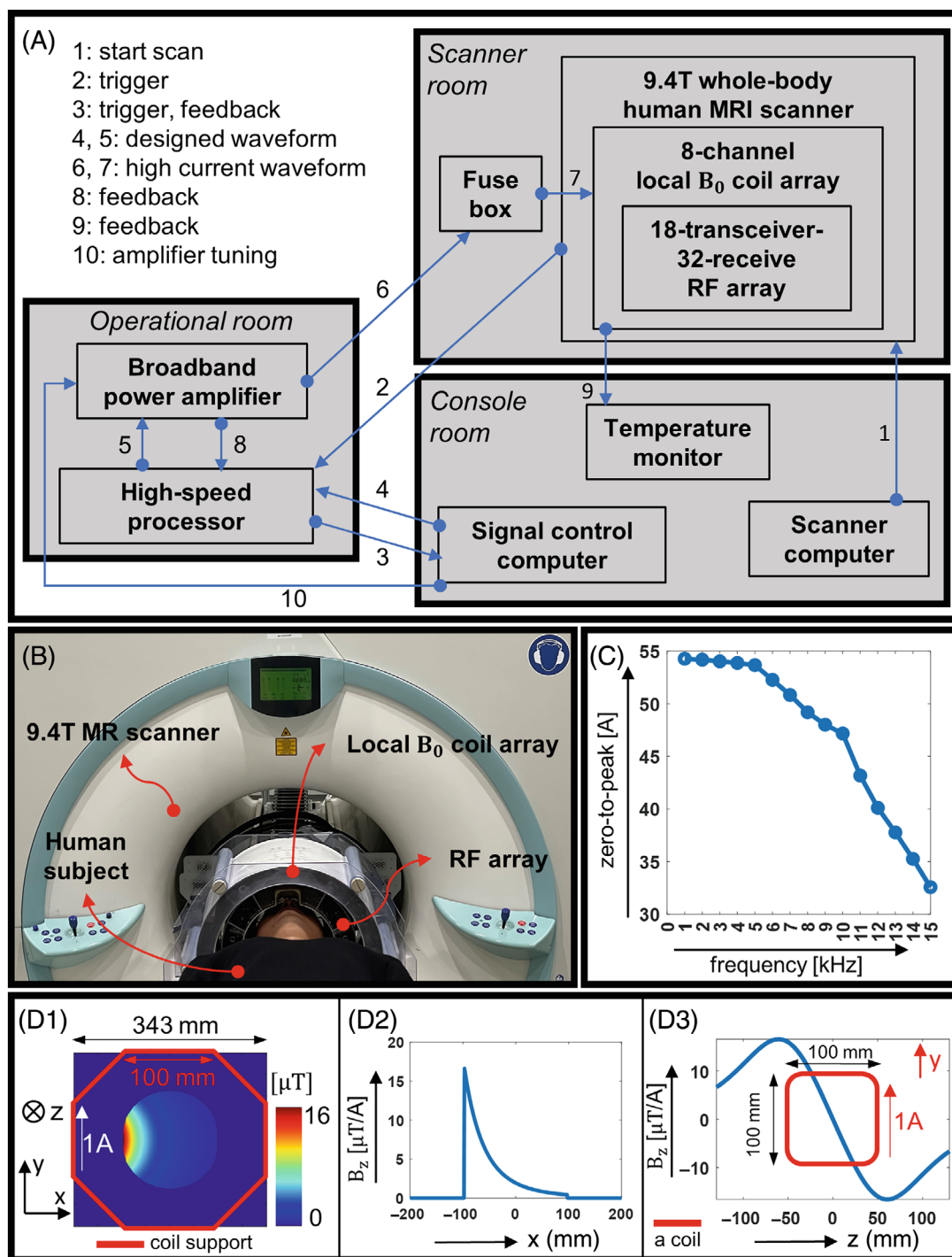


FIGURE 1 System overview. (A) System diagram with the numbered functional flow. The scanner computer starts scans (1), the scanner sends a trigger signal through the high-speed processor to the signal control computer (2, 3), which sends the designed waveforms through the high-speed processor (4) to the broadband power amplifier (5), and outputs high current waveform to the local B_0 coil array (7) protected by the fuse box (6). Simultaneously, the output current is monitored by the power amplifier and recorded through the high-speed processor (8) by the signal control computer (3). The temperature monitor constantly measures the temperature of each local coil during experiments (9), and the signal control computer is used to adjust the tuning setting of the power amplifier once before all experiments. (B) A picture of the installed setup with a human subject. (C) The measured sinusoidal current (zero to peak value) in a frequency ranging from 1 to 15 kHz, output by the broadband power amplifier (averaging between channels). (D1) The simulated magnetic field map given 1A current in a single local B_0 coil. (D2) The 1D plot of the central horizontal line of the map in D1. (D3) The variation of the in-plane maximal field strength generated by 1A in one coil channel (D1) along the z direction, with the relative position to the coil winding.

encoding matrix, how nonlinear gradients at different time points along its “trajectory” (i.e., time integral of gradient waveforms) contributed to the final SNR efficiency of a pixel stays unresolved.

Therefore, in this work, we analytically compute the noise amplification and approximation errors in time-domain (e.g., k-space), to provide more insights over the encoding behavior by the local B_0 coils and guide the waveforms design. To achieve this, a mathematical framework for parallel imaging⁶³ based on the reproducing kernel Hilbert Space (RKHS) theory is extended by incorporating all spatial encoding terms (e.g., linear and non-linear gradients, RF sensitivity) into the signal model. A simplified version of mathematical derivation is provided below.

Essentially, the image reconstruction can be seen as interpolating the continuous time-domain signal representations of objects from the discrete MRI samples acquired by using all “spatial encoders”. The Frobenius norm of the interpolation weights, called the cardinal function, relates to the stability of the interpolation and represents noise amplification as a function of time by the sampling operator. The power function, which yields point-wise bounds of the time-domain approximation error, can also be computed based on this theory.⁶³

From the Riesz representation theorem, the MRI spatial encoding can be described by a vector-valued function $\mathbf{K}^{t_{LG}, t_{NG}, t_{RF}, i}$ as the representer of evaluation, which includes arbitrary switching of linear,^{1,6,7,9} nonlinear gradients^{31,43} and RF transmit/receiver coils,^{64–72} and maps the compactly supported magnetization image to smooth time-domain acquisition signals. It spans the Hilbert Space H of all signals as a RKHS (i.e., a Hilbert Space of the encoding functions where point-evaluation functionals are continuous), and gives ideal acquired samples $f_i(t_{LG}, t_{NG}, t_{RF})$ by the inner product evaluation of continuous signal representation f as the reproducing property:

$$f_i(t_{LG}, t_{NG}, t_{RF}) = \langle \mathbf{K}^{t_{LG}, t_{NG}, t_{RF}, i}, f \rangle_H, \quad (2)$$

where t_{LG}, t_{NG}, t_{RF} represent the sampling time for linear, nonlinear gradients and RF coils with index i , respectively. Generally, the sampling time for each “spatial encoder” can be written as a separate variable as in Eq. (2), to describe arbitrary and independent controls over distinct encoding elements. In this paper, the RF encoding fields (i.e., only receive) remain fixed over time, and the linear and nonlinear gradients share the same time axis (i.e., switched on simultaneously during signal readout). For this special case, the three independent time dimensions are simplified to one without loss of generality:

$$t = (t_{LG}, t_{NG}, t_{RF}). \quad (3)$$

Next, this RKHS can be uniquely characterized by a kernel function (unnecessarily shift-invariant as in the theory)⁶³ obtained by applying the forward signal model to the encoding function based on Eq. (1), which captures the similarity of the two argument encoding functions:

$$\begin{aligned} \mathbf{K}_{ij} & \left(t_{LG}, t_{NG}, t_{RF}, \bar{t}_{LG}, \bar{t}_{NG}, \bar{t}_{RF} \right) \\ & \stackrel{(3)}{=} \mathbf{K}_{ij}(t, \bar{t}) = \left\langle e^{t,i}(r), e^{\bar{t},j}(r) \right\rangle_{L^2}, \quad (4) \\ & = \int c_i(r, t) c_j^H(r, \bar{t}) \exp \left\{ -\sqrt{-1} \left\{ [k(t) - k(\bar{t})]r \right. \right. \\ & \quad \left. \left. + \gamma \sum_{\lambda} B_{\lambda}(r) \left[\int_0^t I_{\lambda}(\tau) d\tau - \int_0^{\bar{t}} I_{\lambda}(\tau) d\tau \right] \right\} \right\} dr, \quad (5) \end{aligned}$$

where the operator $\langle \cdot, \cdot \rangle$ denotes the inner product, r is the image location, the upper bar for sampling time indicates being for the second kernel argument, $e^{t,i}(r), e^{\bar{t},j}(r)$ represent the encoding operator including all “spatial encoders”, $c_i(r, t), c_j(r, \bar{t})$ are the RF sensitivity profiles from receiver index i and j (in this paper, constant in time), the operator $(\cdot)^H$ denotes the complex conjugate operation.

Therefore, by defining a measurement subspace $H_S \subset H$ spanned by $\mathbf{K}^{t,j}$, any continuous function $f^{\parallel} \in H_S$ can be expressed by linear combination of $\mathbf{K}^{t,j}$. Further, the discrete MRI samples $f_i^{\parallel}(t)$ can be obtained by evaluating $f^{\parallel}$ at the sample locations t, i by building the inner product with the representers of evaluation $\mathbf{K}^{t,i}$, and thus, be expressed by the kernel function:

$$f^{\parallel} = \sum_{\bar{t}, j} a_{\bar{t}, j} \mathbf{K}^{\bar{t}, j}, \quad (6)$$

where $a_{\bar{t}, j}$ is the interpolation coefficient.

$$f_i^{\parallel}(t) \stackrel{(2)}{=} \langle \mathbf{K}^{t,i}, f^{\parallel} \rangle \quad (7)$$

$$\stackrel{(4,6)}{=} \sum_{\bar{t}, j} a_{\bar{t}, j} \mathbf{K}_{ij}(t, \bar{t}) \quad (8)$$

Additionally, because the representers can be used to interpolate all functions in H_S it spans, theoretically, the acquired time-domain samples can be exactly interpolated to any samples $f_k^{\parallel}(t)$ in H_S evaluated at locations i, k by an arbitrary encoding function as a reference (e.g., fully sampled Cartesian k-space encoding), with interpolation weights as the cardinal function $u_k^{t,i}(t)$:

$$f_k^{\parallel}(t) = \sum_{i, \bar{t}} f_i^{\parallel}(t) u_k^{t,i}(t), \quad (9)$$

where i and k are the time grid and the RF receiver index of the reference encoding function. Now, the reference

samples $f_k^{\parallel}(t)$ can be expressed either directly by kernel $K_{kj}(t, \bar{t})$, or by kernel encoded acquired samples $f_j^{\parallel}(t)$ and cardinal function $u_k^{t,i}(t)$:

$$f_k^{\parallel}(t) \stackrel{(2)}{=} \langle \mathbf{K}^{t,k}, f^{\parallel} \rangle, \quad (10)$$

$$\stackrel{(8)}{=} \sum_{\bar{t}, j} a_{\bar{t}, j} \mathbf{K}_{kj}(t, \bar{t}), \quad (11)$$

$$\stackrel{(8,9)}{=} \sum_{t, i, \bar{t}, j} a_{\bar{t}, j} \mathbf{K}_{ij}(t, \bar{t}) u_k^{t,i}(t). \quad (12)$$

Eventually, comparing Eqs. (11) and (12), the cardinal function can be intuitively seen as coefficients, which interpolate the representer of evaluation at arbitrary sampling location of a reference encoding function, exactly from their values at the location of the tested sampling function encoded in kernel:

$$\sum_{t, i} \mathbf{K}_{ij}(t, \bar{t}) u_k^{t,i}(\bar{t}) = \mathbf{K}_{kj}(t, \bar{t}). \quad (13)$$

Namely, the time-domain noise amplification factor can be obtained by solving a linear system of equations:

$$\mathbf{M}\mathbf{U} = \mathbf{R}, \quad (14)$$

$$\text{with } \begin{cases} \mathbf{M} = \mathbf{E}_{\bar{t}, j} \mathbf{E}_{t, i}^*, \in \mathbb{C}^{TN \times TN} \\ \mathbf{R} = \mathbf{E}_{\bar{t}, j} \mathbf{E}_{t, k}^*, \in \mathbb{C}^{TN \times TN} \end{cases} \quad (15)$$

where $\mathbf{E}_{t, i}$, $\mathbf{E}_{\bar{t}, j}$ are both the tested encoding matrix, and $\mathbf{E}_{t, k}$ is a reference encoding matrix, for evaluating the kernel. The T, N are the total sampling time points and number of RF receivers, with the upper dot indicating being for the reference matrix. The noise amplification factor is obtained by taking the Frobenius norm of $\mathbf{U}$ along the dimension of t, i (i.e., taking square for each element in $\mathbf{U}$, summing along the vertical dimension, and taking the square root). The asterisk (*) denotes the conjugate transpose operation.

Furthermore, the time-domain approximation error caused by interpolation can be computed with the power function P in the RKHS theory⁶³:

$$\mathbf{P}^{i,k} = \sqrt{\left(\mathbf{E}_{t, k} \mathbf{E}_{t, k}^* - \mathbf{U}^* \mathbf{E}_{t, i} \mathbf{E}_{t, k}^* \right)_{\text{diag}}}. \quad (16)$$

Theoretically, the reference encoding matrix $\mathbf{E}_{t, k}$ can be arbitrarily chosen (e.g., another acceleration factor, spiral, wavelet) to visualize how time-domain acquisition signals gradually lead to the noise amplification and approximation error in the corresponding transform domain. Here, assuming RF receivers' sensitivity constant across the image, the reference is chosen as a fully sampled discrete Fourier transform matrix, to demonstrate errors in k-space

relative to a fully sampled 2D Cartesian scan. Note that, the Fourier transform operation links the signal representation, but not the standard deviation of the noise signal, between image- and k-space.

Finally, Eq. (14) can be easier to solve with less/-compressed RF receiver channels or a higher undersampling factor. Faster computation with pseudo-replica simulation³³ or approximation⁷³ is also worth investigating. Furthermore, limiting the encoding matrix from spanning only a sub-interval (e.g., one TR) of the entire sampling period allows quantifying the contributions of signal encoding during smaller temporal periods in relation to the entire k-space.

2.4 | Calibration of B_0 rapid modulation field

Reconstruction of scans accelerated by local B_0 coils requires additional calibration of the field modulation pattern. The estimation errors in the additional phase evolution imposed by the local B_0 coils can manifest as residue artifacts particularly for high acceleration factors, in a mechanism similar to the SENSE residue artifacts because of calibration errors of RF receivers' sensitivity maps.⁵¹

Theoretically, the additional imposed magnetic field can be measured with a calibration scan modified from the chemical shift sequence, as in O-space MRI.⁴⁰ However, this requires time consuming scans with the entire spatial grid phase encoded, to measure phase evolution resulted from different modulation waveforms, and therefore, does not appear to be the most suitable for testing our setup with various modulation schemes. Additionally, the magnetic field probes, as a third-party monitor, could be used to calibrate the linear gradient insert.^{74–76} However, its feasibility to track high-order nonlinear gradient fields⁶¹ by our setup has not been testified yet.

Therefore, a field calibration method based on current monitors of the power amplifier and the ESPIRiT algorithm is proposed to calibrate our setup, with only one calibration scan for all experiments given various modulation waveforms. Essentially, low-resolution field mapping scans are input to the ESPIRiT algorithm,⁵¹ mimicking data from multiple RF receivers, to extrapolate high-resolution (i.e., the resolution for reconstruction) phase maps with robustness to avoid possible errors during highly accelerated scans. Moreover, the 8-channel current monitors are exploited to resolve phase offset maps contributed by unit current in a local B_0 coil and combined with the monitored modulation currents in later experiments to reconstruct images. This calibration pipeline is illustrated in Figure 2, with detailed procedure and error evaluation documented in Data S1. In this paper,

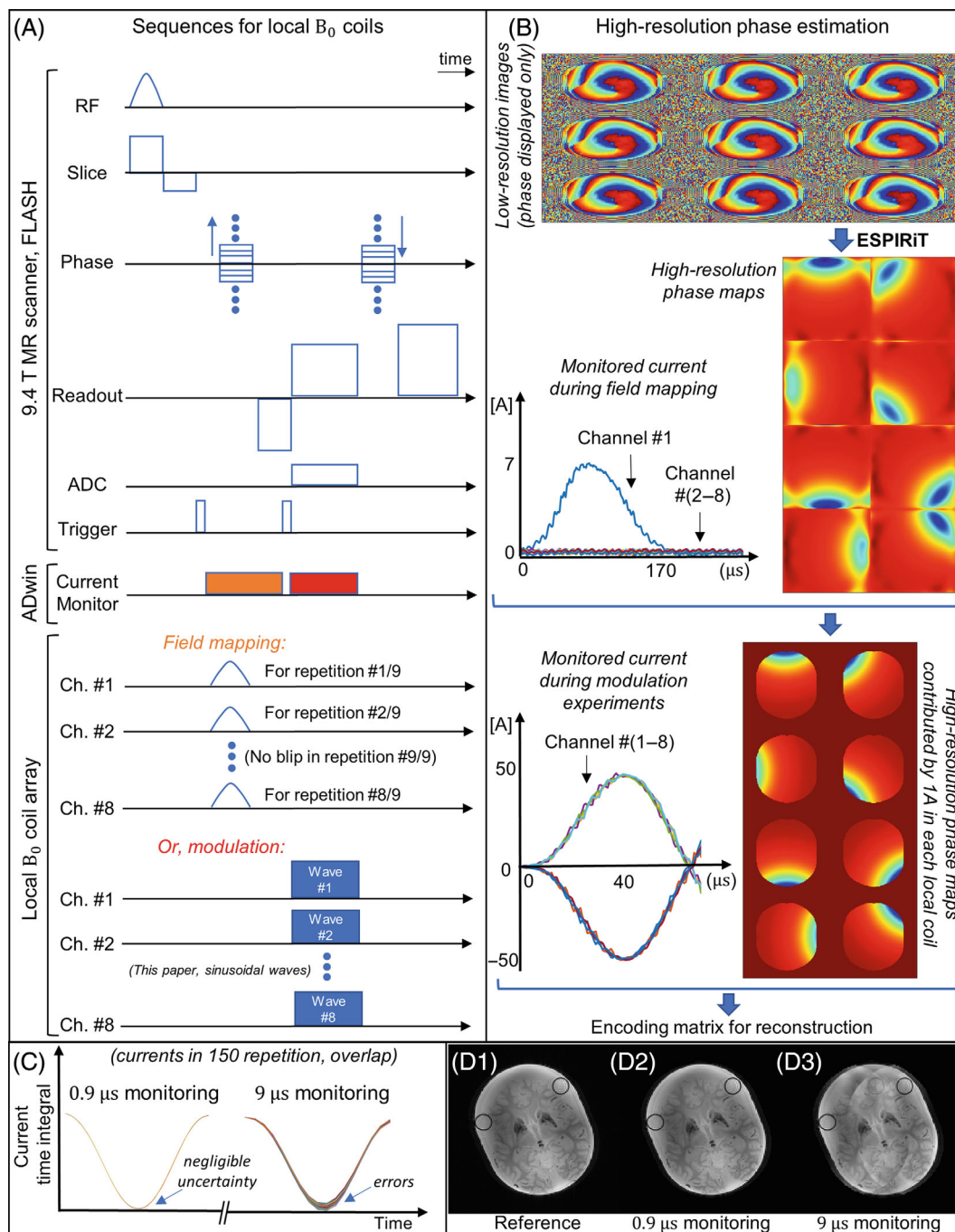


FIGURE 2 Calibration of phase evolution by the local B_0 coil array, with more details for each step and the error evaluation demonstrated in Data S1. (A) The pulse sequences for magnetic field mapping or current modulations of local B_0 coils. (B) Illustration of the phase evolution estimation method, based on low-resolution scans from the sequence shown in (A), current monitors of the power amplifiers, and the ESPIRiT algorithm. Please note that, the low-resolution complex images are displayed only in phase, and the vertical dimension is 40% undersampled in k-space to save scan time. Additionally, the channel coupling between local B_0 coils and linear gradients is included in the monitored currents. (C) The time integral of the current is calculated on an interpolated time grid, based on monitored currents sampled with 0.9 and 9 μ s, respectively. Curves of currents from 150 TR are shown with identical linewidth. (D) Reconstruction based on monitored currents with different sampling rates demonstrates the necessity of fast current monitoring for accurate image reconstruction. All these images are reconstructed by inverting the entire encoding matrix in high-performance computing servers, which contains independently monitored currents in each TR without averaging.

given different objects, the field maps for local B_0 coils require recalibration because of variation of the object's support.

2.5 | Optimized sinusoidal modulation for 2D Cartesian sampling

In previous work of spread spectrum MRI,⁴⁶ sinusoidal waveforms in 5 kHz, 3A zero-to-peak value, and with $\pi/4$ phase increment between adjacent channels were applied to produce magnetic field modulation and speed up scans. This time, different waveforms are compared for the optimal acceleration of 2D Cartesian scans, with respect to the noise amplification in image-space, and the error maps (i.e., noise amplification and approximation errors) in k-space.

Because the acceleration performance does not solely depend on the additional field modulation schemes, but also the k-space trajectory defined by linear gradients, as a starting point, only 2D line-by-line Cartesian sampling is considered in this paper. Moreover, modulation in more than one single sinusoidal frequency can produce more “wiggling” for k-space trajectory,^{77,78} which may provide certain enhancements, but with the underlying mechanism well understood. Therefore, we focus on sinusoidal modulation in only a single frequency for local coils to avoid excessively digressing from scrutinizing characteristics unique to local coils modulations.

2.5.1 | Field shapes—combinations of phase offsets

Sinusoidal currents with distinct combinations of phase offsets are applied to the eight local coil channels and compared. Each scheme leads to a different field patterns oscillating during the readout, including “quadratic,” “quadrupolar,” and “octupolar” field distributions, a circularly rotated nearly linear gradient field formed by $\pi/4$ phase increment between adjacent coils (denoted as “ $\pi/4$ ”), and a nearly linear gradient field oscillated roughly toward the phase encoding dimension formed by applying phase offset 0 and π for each of the four adjacent coils (denoted as “ $0\&\pi$ ”). Additionally, a similar phase arrangement of “ $0\&\pi$ ” is applied in conjunction with a rotation of 22.5° of the linear gradient axis. This allows the nearly linear gradient produced by local coils mostly align along the phase encoding direction, similar to the bunched phase encoding,¹⁷ but with a local gradient setup, and is denoted as “ $0\&\pi$, R22.5°”.

The schemes with various phase offset arrangements are defined below (i.e., “scheme”: [phase offsets in the

eight local B_0 coils]), with the corresponding field/gradient strength distributions and k-space “stamps” calculated from experimentally measured data illustrated in Figure 3. With our array geometry, the “quadratic” scheme produces the strongest magnetic field over the object, the “quadrupolar” produces the maximal gradient field around the edge of FOV, the field produced by the “octupolar” scheme appears much weaker than others, the “ $0\&\pi$ ” scheme, with or without rotation, provides strong gradient fields across the entire FOV.

- “quadratic”: [0, 0, 0, 0, 0, 0, 0, 0]
- “quadrupolar”: [0, 0, π , π , 0, 0, π , π]
- “octupolar”: [0, π , 0, π , 0, π , 0, π]
- “ $\pi/4$ ”: [0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$]
- “ $0\&\pi$ ”: [π , π , 0, 0, 0, 0, π , π]
- “ $0\&\pi$, R22.5°”: [π , π , 0, 0, 0, 0, π , π], with in-plane FOV rotation of 22.5°
- “ $\pi/4$, CAIPI”: [0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$], odd steps; [0, $\pi/4$, $\pi/2$, $3\pi/4$, π , $5\pi/4$, $3\pi/2$, $7\pi/4$] + π , even steps
- “ $0\&\pi$, R22.5°, CAIPI”: [π , π , 0, 0, 0, 0, π , π], odd steps; [π , π , 0, 0, 0, 0, π , π] + π , even steps (both with 22.5° in-plane FOV rotation).

2.5.2 | Aliasing control—sinusoidal phase shifts between phase-encoded steps

Additional sinusoidal phase offsets can be added in all local B_0 coils between consecutive phase-encoded steps to produce a shifted sampling function for signal readout, as defined in the schemes above as well. Interleaving shifted k-space “stamps” can shift aliasing patterns and reduce noise amplification caused by image acceleration in certain scenarios, similar to CAIPI.^{19,20} The reconstruction is performed by inverting the entire encoding matrix, because of the variation of modulation waveforms among TR, in a high-performance computing server for simplicity.

2.5.3 | Frequency selection—k-space coverage for different ranges

Given ~ 10 kHz cutoff frequency of the power amplifier, a broad range of sinusoidal frequencies is tested to reach optimal accelerations. Considering sinusoidal modulation in a single frequency ω (here, for all local B_0 coils) and phase offset φ_γ , the Eq. (1) can be written as:

$$S_n(t) = \int c_n(r) \rho(r) \exp\{-\sqrt{-1}\{k(t)r + \gamma \sum_{\lambda} \left[B_{\lambda}(r) \frac{-\cos(2\pi\omega t + \varphi_{\lambda})}{2\pi\omega} \right] \} \} dr. \quad (17)$$

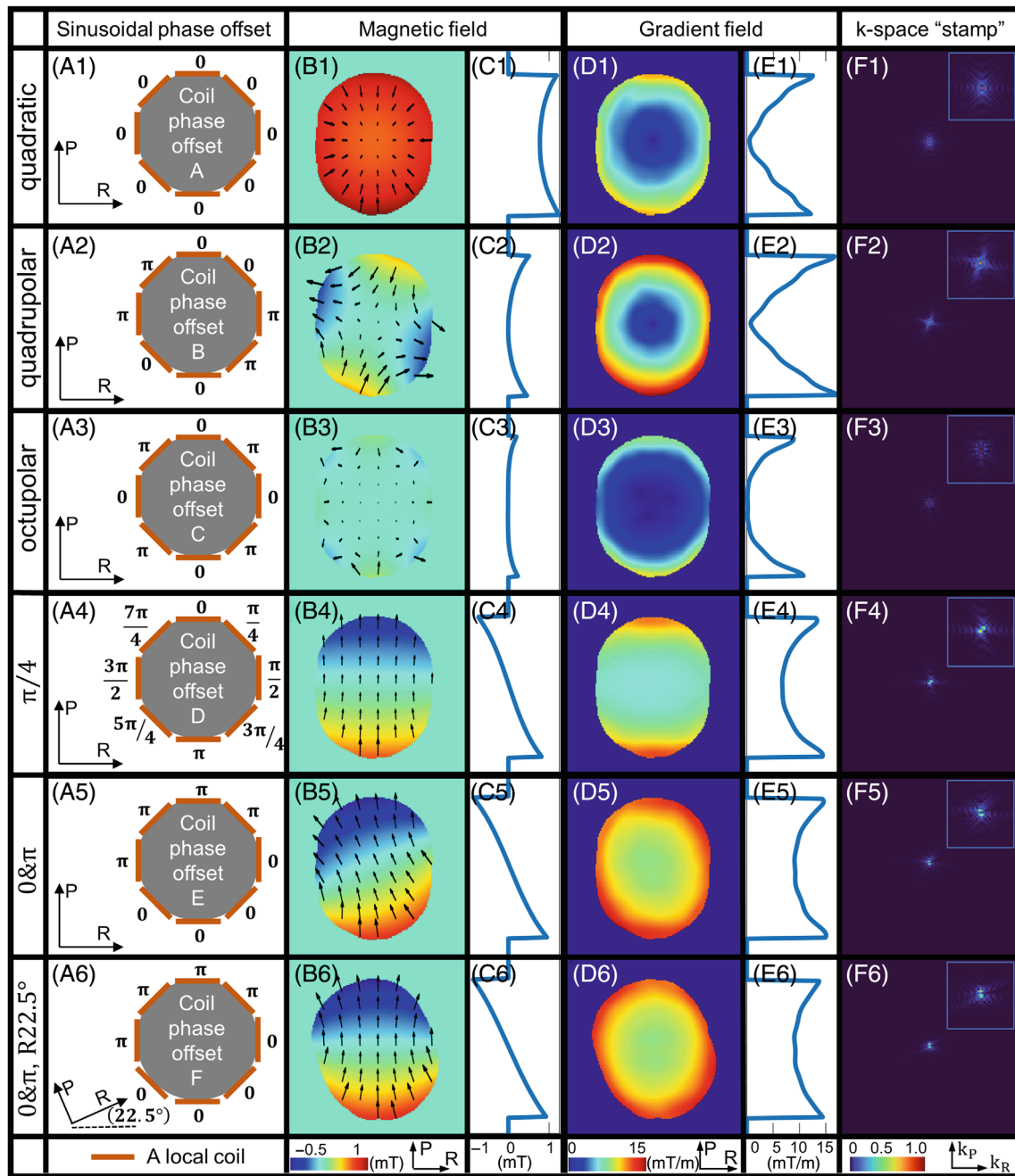


FIGURE 3 The encoding schemes for sinusoidal modulation of local B_0 coils with different phase offset patterns. Rows 1–6: Different sinusoidal phase offset patterns, leading to magnetic field modulation with distinct field shapes. Columns A–F: The 8-channel arrangement for coil phase offset, the 2D magnetic field maps at a time instant during modulation computed from the measured field maps and the coil currents at maximum, the 1D plots along the central vertical line in the 2D magnetic field maps, the 2D gradient field maps at a time instant computed from the measured field maps and the coil currents at maximum, the 1D plots along the central vertical line in the 2D gradient field maps, and the 2D k-space “stamps” (absolute value displayed) calculated by taking the Fourier transform of the 2D spatial encoding terms corresponding to the local B_0 coils at current maximum (note, not the maximum of the imposed phase accumulation). The gradient field in columns D and E are calculated as the net local gradient along a single direction in each pixel, namely, the Euclidean norm of the local gradient in the two orthogonal dimensions. Note that, the k-space “stamps” here, only qualitatively show a coarse sampling coverage at a time instant, whereas the cardinal and the power function maps in later sections can quantitatively compute the k-space sampling efficiency in arbitrary acquisition time duration for rigorous comparisons between different “stamps”.

Obviously, higher sinusoidal frequency inversely scales the phase evolution as the denominator and leads to narrower k-space sampling coverage along the phase encoding dimension, and the k-space sampling “stamp” should appear more localized. Additionally, the linear gradient bandwidth also plays a role in acceleration efficiency by gradient modulations.⁷⁶ In this paper, it is chosen with a scanner analog-to-digital converter dwell time of 3 μ s to allow substantial acceleration by modulation, while not causing excessive fat shifts artifacts during in vivo scans in 9.4 T.

3 | METHODS

All experiments are carried out with the 8-channel local B_0 coil array fixed on the patient table of a 9.4 T whole-body human scanner, equipped with gradients (SC72) with maximum slew rate and absolute strength of 200 T/m/s and 70 mT/m, respectively (Siemens Healthineers). A 16-transceive/32-receive RF coil array modified from the reference⁵⁰ (here, only circularly polarized [CP] mode used for RF transmission) or a plastic support with field probes (Skopec) is inserted inside its support, for imaging or field measurement experiments, respectively.

All imaging experiments are performed with a fully sampled multi-slice 2D FLASH sequence, and undersampled retrospectively to test image acceleration by local coils modulations. The scan sequences are illustrated in Figure 2A. The scan is made with a FOV of 220 mm $\times$ 220 mm, and a matrix size of 150 $\times$ 150 or 192 $\times$ 192 for phantom and in vivo scans, respectively. Sinusoidal currents with 40 A zero-to-peak value (except for 50 A in “field calibration”) are used for local B_0 coils to accelerate scans with scanner readout bandwidth (i.e., linear gradients) $\sim$ 4.5 mT/m. More details about the scan protocols and the information about the ex vivo phantom are documented in Data S1.

4 | RESULTS

4.1 | Phantom experiments with calibrated phase evolution

As shown in Figure 2, the field calibration method described in this paper is used to successfully produce reconstructed images from local B_0 coils modulated scans without additional visible artifacts caused by our setup. In this example, sinusoidal modulation with 7 kHz/50 A_{pk} and the phase offset scheme “0& π , R22.5°” are applied. The fully sampled images in Figure 2D are reconstructed

by inverting the entire encoding matrix, which is constructed with monitored current signals in all TR with a sampling rate of 0.9 μ s or with only a 10-fold undersampled portion (equivalently, sampling at 9 μ s). The reconstruction with the fast current monitor sampling rate (i.e., 0.9 μ s) has no visible errors compared to the reference image with local coils off, whereas the one with the slower rate has additional aliasing artifacts, because of the estimation errors in currents as shown in Figure 2C. Note that, the coupling effects among all gradients (i.e., local B_0 coils, scanner gradients) are included in the monitored current waveforms, and therefore, do not contribute to reconstruction errors. Typical artifacts taking place when not using the ESPIRiT are demonstrated in Data S1.

4.2 | Sinusoidal modulation with different field shapes

Given multiple RF receivers independently reconstructed and combined using SOS, the phantom tests for 7 kHz/40 A_{pk} sinusoidal modulation with distinct phase offset patterns are demonstrated in Figure 4, with respect to the reconstruction with threefold undersampled phase-encoded steps, the G-factor map, and the cardinal and the power function maps given sampling period of a single and all TR, respectively. With our local B_0 array geometry, the scheme “0& π , R22.5°” achieves the best acceleration efficiency for 2D Cartesian MRI, which forms nearly a linear gradient oscillating along phase encoding direction and reduces the mean image-space SNR loss (G-map) by a factor of 3.6 compared to the “ $\pi/4$ ” scheme presented in previous work.⁴⁶ The images by joint reconstruction of multiple RF receivers and local B_0 modulations show similar tendency and are presented in Figure S5, where the additional encoding by RF receivers’ sensitivity does not change the optimal field shape for the 2D Cartesian scan.

For sampling along a straight line in central k-space within one TR, the cardinal function map (column C) generally shows a dark band in the center with high-intensity ripples around its edge and again very dark regions as moving further away. The central dark band corresponds to the sufficiently sampled region by the k-space “stamp,” and the bright ripples indicate the noise amplification originated from some signals around the edge of the “stamp,” which are contained within the sum signal acquired by the “stamp” but remained inefficiently resolved. The large dark regions at two sides of the central dark band with the bright ripples around are completely untouched by the sampling “stamp,” and therefore, no interpolation and the related noise amplification can take place, leading to

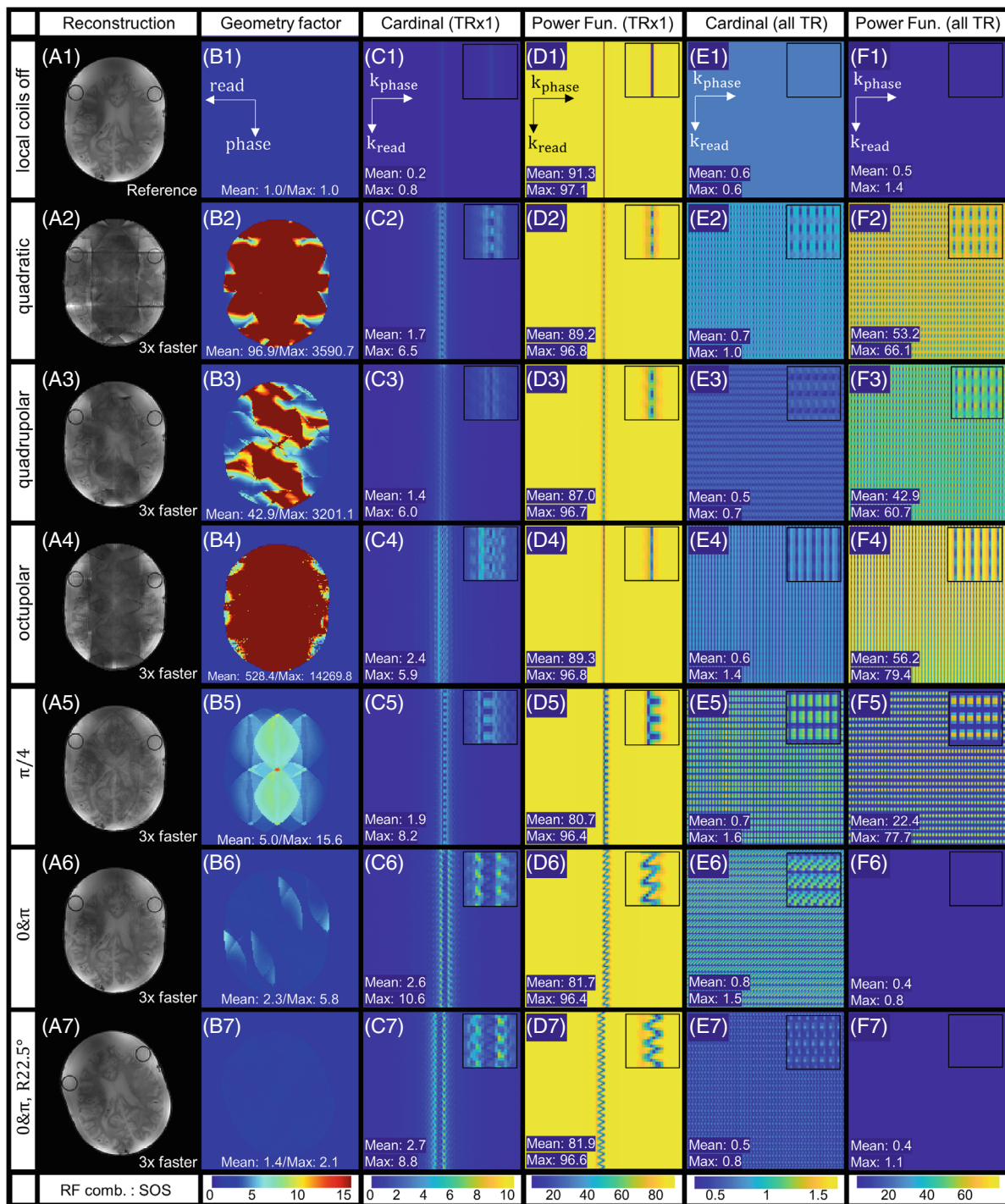


FIGURE 4 Ex vivo scans given local B_0 coils modulations with different sinusoidal phase offset patterns, all with 7 kHz/40A_{pk}. Rows 1–7: The B_0 modulations with different field shapes. Column A to F: reconstruction of a reference and several 3× accelerated FLASH scans by local B_0 coils modulations with RF receiver channels combined using the sum-of-squares method, G-map for noise amplification in image-space, cardinal map for noise amplification in k-space given sampling within one TR, power function map for approximation errors in k-space given sampling within one TR, cardinal map for noise amplification in k-space given sampling overall TR, power function map for approximation errors in k-space given sampling over all TR. The mean and maximum values in columns C and D are calculated from a local region of 20 phase encoding steps around the center of the sampling pattern, and cannot be used to evaluate the overall (i.e., all TR) encoding efficiency for the entire k-space. Note that, the readout and the phase encoding axis in k-space are rotated by 90° relative to image-space, for better visually comparing the k-space coverage widths by different schemes within one TR in a long figure. Additionally, the G-map in B2–B4 contains extremely high values because of the highly unstable inversion of the $E^H E$ term (E: encoding matrix). The reconstruction in A2–A4 computed with the lsqr solver in MATLAB regularizes the solution, but the object texture is severely smoothed out because the signal cannot be recovered regardless of the noise level, corresponding to large mean values of the approximation maps in F2–F4.

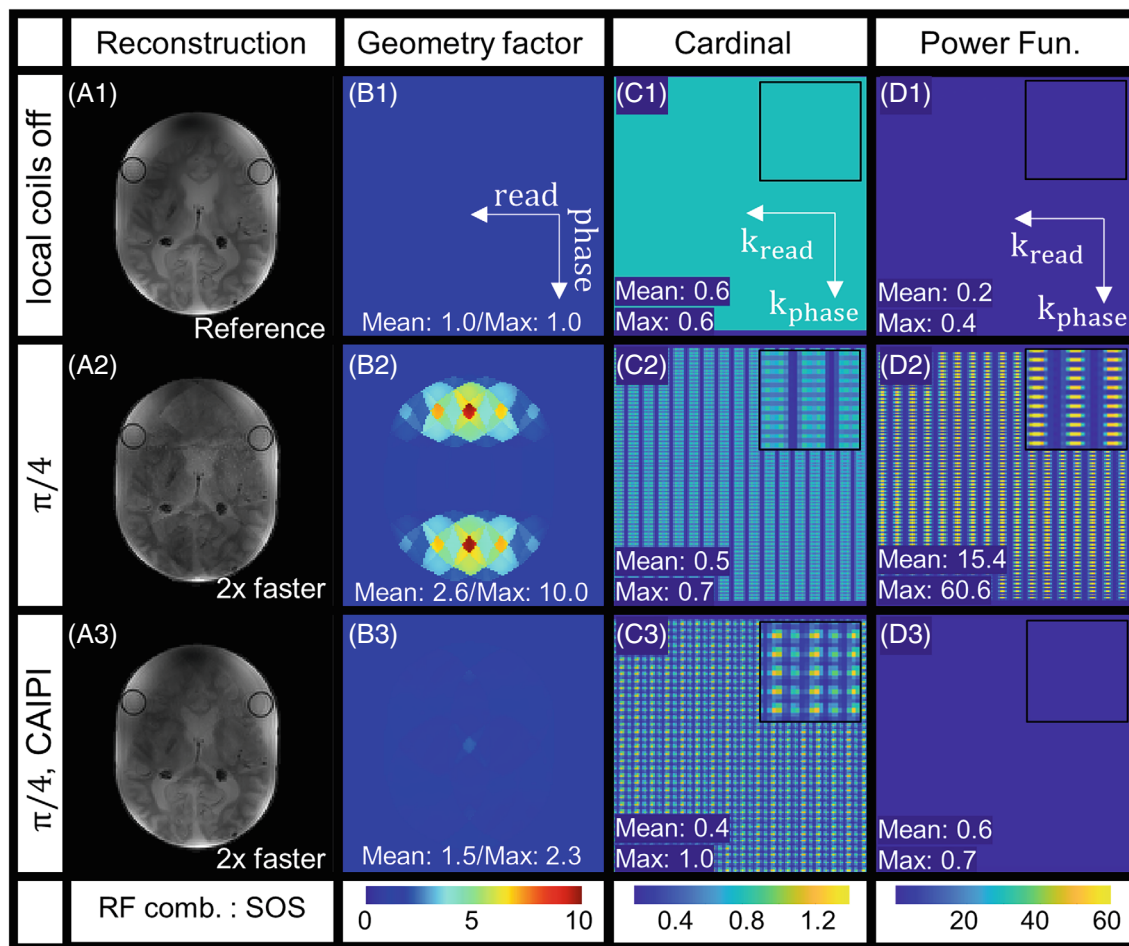


FIGURE 5 Ex vivo scans given 5 kHz/40 A_{pk} sinusoidal local B₀ modulation with π shift in all channels between consecutive 2 $\times$ undersampled phase-encoded steps, manipulating aliasing patterns and mimicking the CAIPIRINHA technique. Rows 1–3: Different modulation schemes—reference fully sampled scan with local coils off, and twofold accelerated FLASH scans with the schemes “ $\pi/4$ ” and “ $\pi/4$, CAIPI”, respectively, given all with RF receiver channels combined with the sum-of-squares method. Columns (A)–(D): Reconstructed images, G-map for noise amplification in image-space, cardinal map for noise amplification in k-space, power function map for approximation errors in k-space. Note that, the advantage of applying CAIPI to local B₀ coils here cannot be simply generalized to other scenarios (e.g., a different undersampling factor or field shape, joint acceleration with RF receivers’ sensitivity).

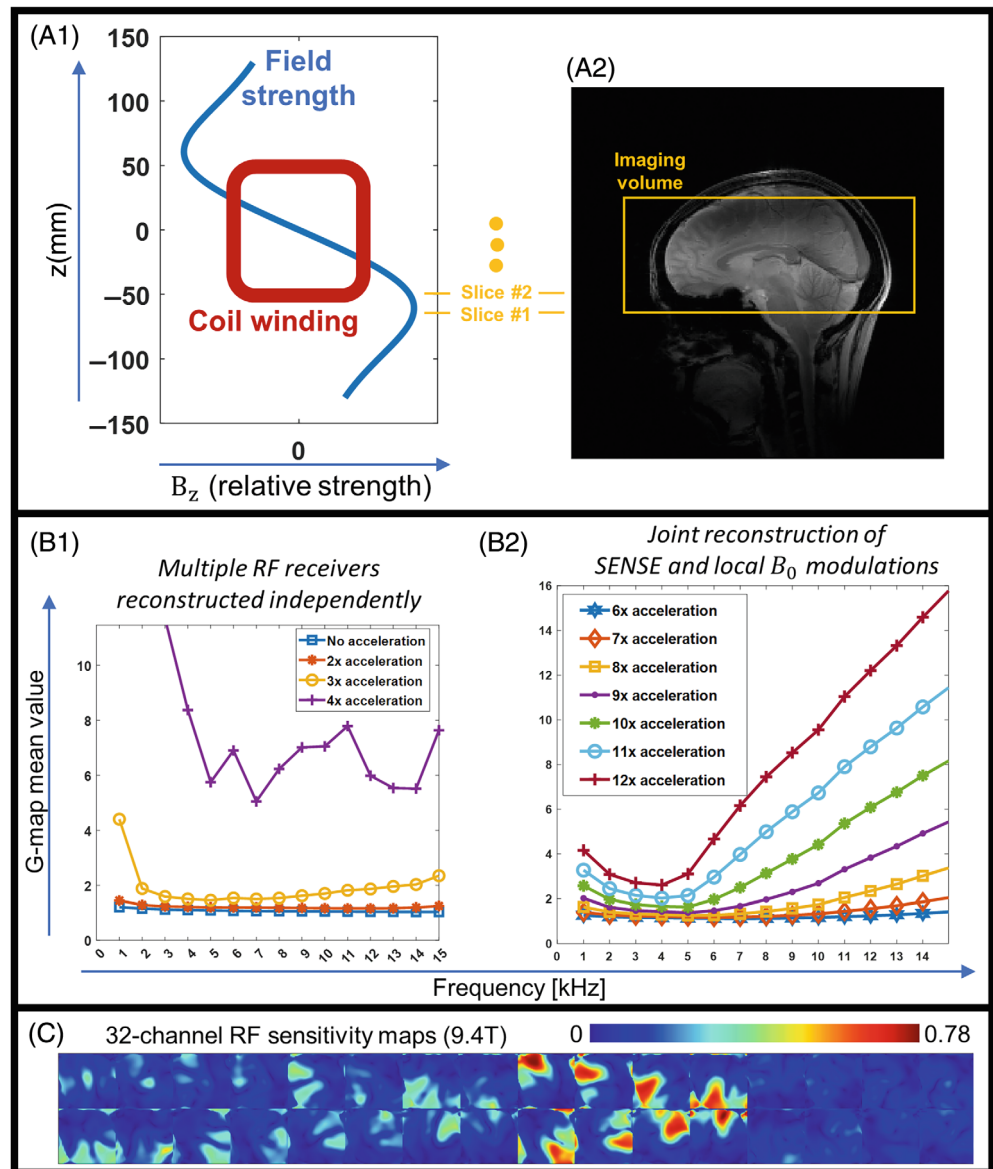
low cardinal function values. Furthermore, the approximation error maps given one TR (column D) indicate high sampling efficiency within the sampling “stamp” and high approximation error as moving further away. Note that, for very localized k-space modulation by local B₀ coils, low cardinal- but high power function can simultaneously take place for regions highly under-sampled.

Therefore, for 2D Cartesian scans with undersampling, the missing k-space data can be optimally sampled by simply applying a linear gradient oscillating along the phase encoding dimension, whereas other schemes gained much less efficiency as their k-space modulation regions are not concentrated along the only one under-sampled dimension. This explains the origin of the tendency in the G-maps (Figure 4B) and the full k-space error maps (Figure 4E,F). In the latter, the scheme “0& π , R22.5°”

shows the most sufficient k-space sampling pattern, whereas others have more “hot spots” in cardinal or power function maps caused by k-space sampling “holes.” Additionally, the schemes “quadratic,” “quadrupolar,” and “octupolar” show relatively low noise amplification, but very high mean values of approximation errors, which may be due to insufficient acquisitions at the undersampled locations given narrower k-space sampling “stamps,” similar to maps by Fourier encoding with a phase-encoded step (Figure 4C1,D1). This also matches the instability of matrix inversion shown in the corresponding G-maps (Figure 4B2–B4). Note that, the statistics (i.e., mean and maximum) for columns C and D are computed based on a local region of the central 20 phase-encoded steps and cannot be used to determine the encoding efficiency of the overall k-space.

FIGURE 6

Demonstration of the acceleration efficiency for the local B_0 coil array (here, take slice 1 as an example). Row A: The relative position between the human subject and the local B_0 coil array during an in vivo scan. Row B: The noise amplification mean value (image-space) for accelerated scans by local B_0 modulations in different single frequencies, without (B1) and with (B2) joint reconstruction of SENSE, simulated with experimentally measured RF receiver sensitivity maps, field maps of B_0 coils, and sinusoidal currents output by the amplifier. Row C: Measured 32-channel RF receivers' sensitivity maps (magnitude) in 9.4 T with visible "standing wave" patterns. Note that these RF receivers' sensitivity patterns differ from the ones assumed in some nonlinear gradient designs at 3 T, such as PATLOC,⁴³ O-space,⁴⁰ and null space imaging.⁷⁴



4.3 | Sinusoidal modulation with aliasing control

Without combining parallel imaging (i.e., here, SENSE), the phantom scans with (5 kHz/40A_{pk}) sinusoidal phase offsets shift (here, π) in all local B_0 coils between consecutive phase-encoded steps after twofold undersampling are illustrated in Figure 5. As explained in wave-CAIPI,²⁹ aliasing effects along the readout direction will be induced by the sinusoidal (linear) magnetic field modulations, which together with the aliasing along the phase encoding dimension, are supposed to be resolved during the reconstruction. However, as in Figure 5B2, the " $\pi/4$ " scheme with 5 kHz modulation frequency suffers from strong noise amplification, particularly at some "hot spots" reaching G-map value ~ 10.0 . This originates from excessive fold-over of object replicates induced along

the readout direction, which can take place when the sinusoidal modulation frequency is too low.

In the local B_0 coil scheme " $\pi/4$, CAIPI", parts of the readout fold-over effects due to 5 kHz field modulation are shifted to the phase encoding direction by varying the phase of sinusoidal currents between phase-encoded steps. This substantially reduces the noise amplification particularly by removing the "hot spots" in G-map, as shown in row 3 of Figure 5. In k-space perspective, given the scheme " $\pi/4$," the power function maps show bright spots as k-space "holes" caused by insufficient sampling. In the " $\pi/4$, CAIPI" scheme, the "hot spots" for approximation errors in k-space are substantially reduced, which indicates more sufficient sampling because of interleaving the shifted "k-space stamps" as in Figure 5C3,D3. However, such CAIPI technique for local B_0 coils might not always lead to significantly improved

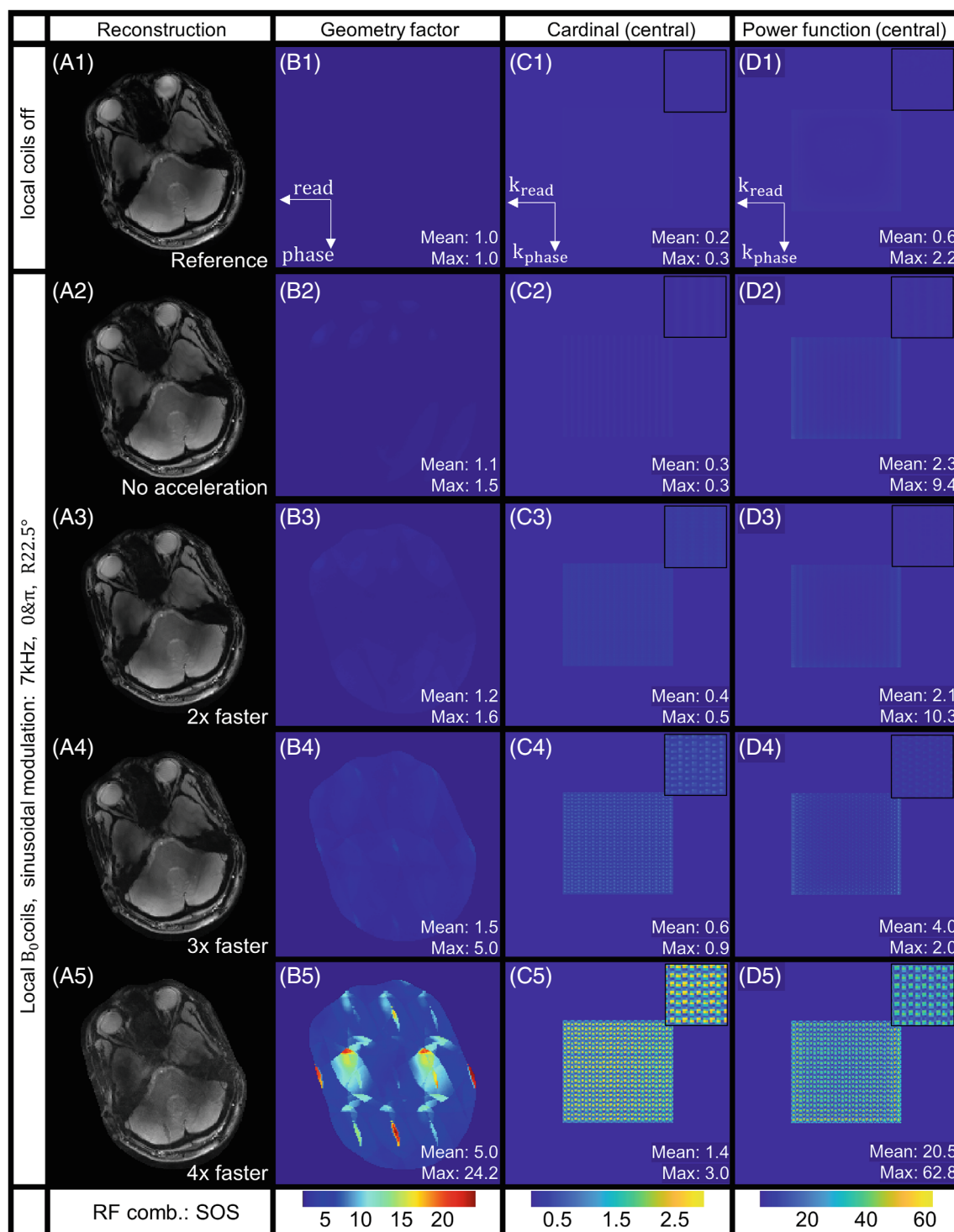


FIGURE 7 Accelerated in vivo FLASH scans with local B_0 coils modulation of 7 kHz/40A_{pk}, given RF receivers' channels combined with the sum-of-squares method. Rows 1–5: A reference scan and several modulated scans with zero to fourfold acceleration. The local coils are modulated with sinusoidal currents with the scheme “0& π , R22.5°.” Columns A–D: Reconstructed images, G-map for noise amplification in image-space, cardinal map for noise amplification in k-space, and power function map for approximation error in k-space. Note that only a central region of 86 × 94 pixels was computed for k-space error maps (columns C and D), to avoid excessively demanding computation.

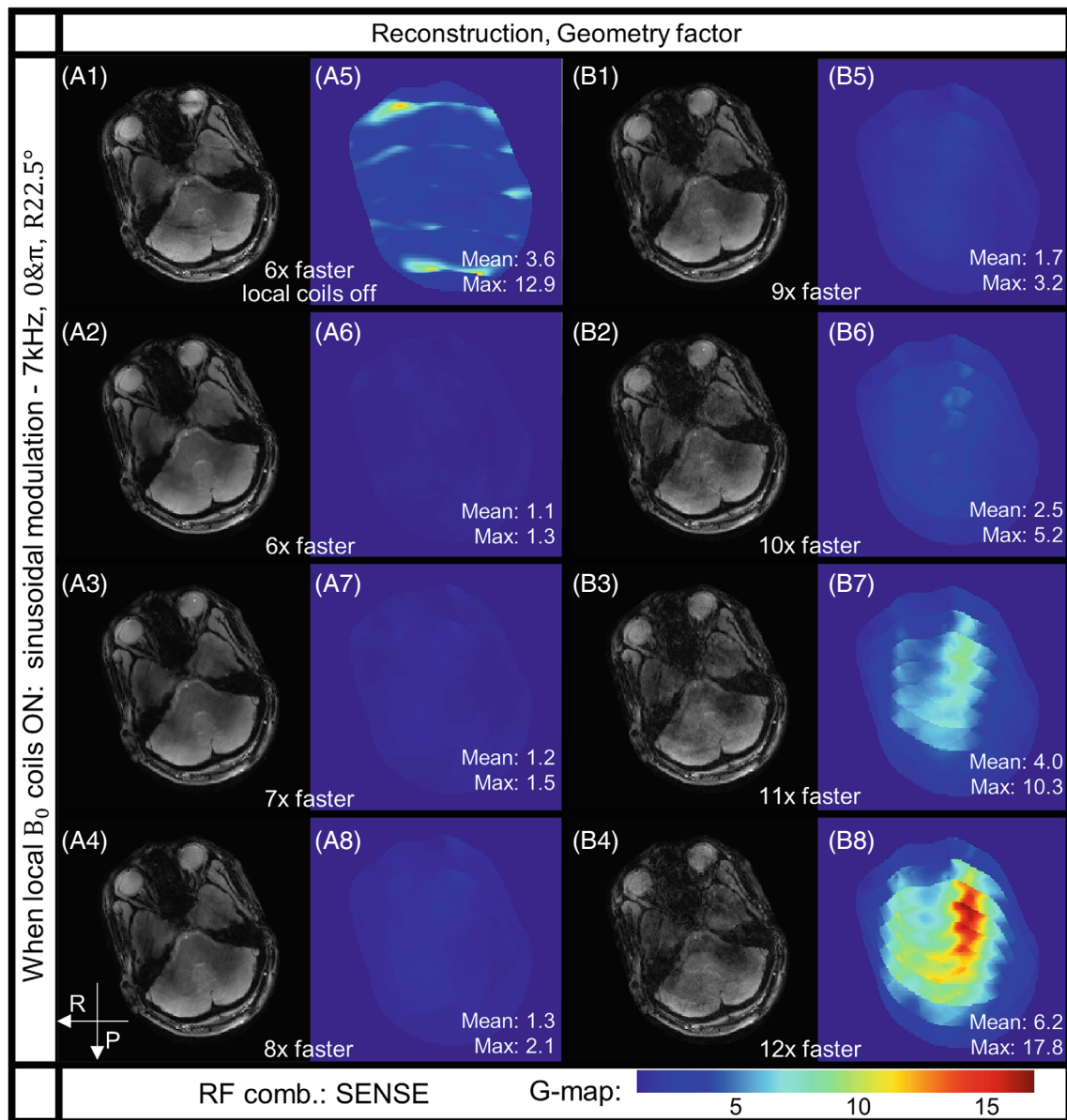


FIGURE 8 In vivo FLASH scans accelerated by sixfold to 12-fold along a phase encoding direction, with joint reconstruction of local coils (7 kHz/40A_{pk}) modulation and SENSE. From A1 to A8: Reconstruction and G-map for accelerated scans ranging from sixfold to eightfold, including a sixfold SENSE reconstruction without local B₀ coils modulation. From B1 to B8: Reconstruction and G-map for accelerated scans from ninefold to 12-fold acceleration factors.

acceleration efficiency in other scenarios, which are shown in Figures S6 and S7.

4.4 | In vivo scans

A safety evaluation of our setup for in vivo scans is performed, which proves our experiments are safe for human subjects with respect to peripheral nerve stimulation and acoustic noise, as documented in Data S1. Therefore, in vivo 2D FLASH scans with local B₀ coil modulations are conducted with healthy volunteers. To maximize encoding efficiency, the “0&pi, R22.5°” scheme is used. A

pre-scan analysis is demonstrated in Figure 6, including field modulation strength along slice direction, G-map analysis, and the RF receivers’ sensitivity pattern as it played an important role in designing other nonlinear gradient methods^{39,40,79} (e.g., relying on the 3 T sensitivity distribution to resolve pixels along the circumferential axis, which might be different for 9 T MRI). Here, frequencies with 7, 10, and 13 kHz are applied during in vivo experiments, given potentially milder relaxation induced artifacts as the frequency increases.²⁹

As in Figure 7, with the local B₀ coils (7 kHz/40A_{pk}, “0&pi, R22.5°”) alone, the in vivo 2D FLASH scan is

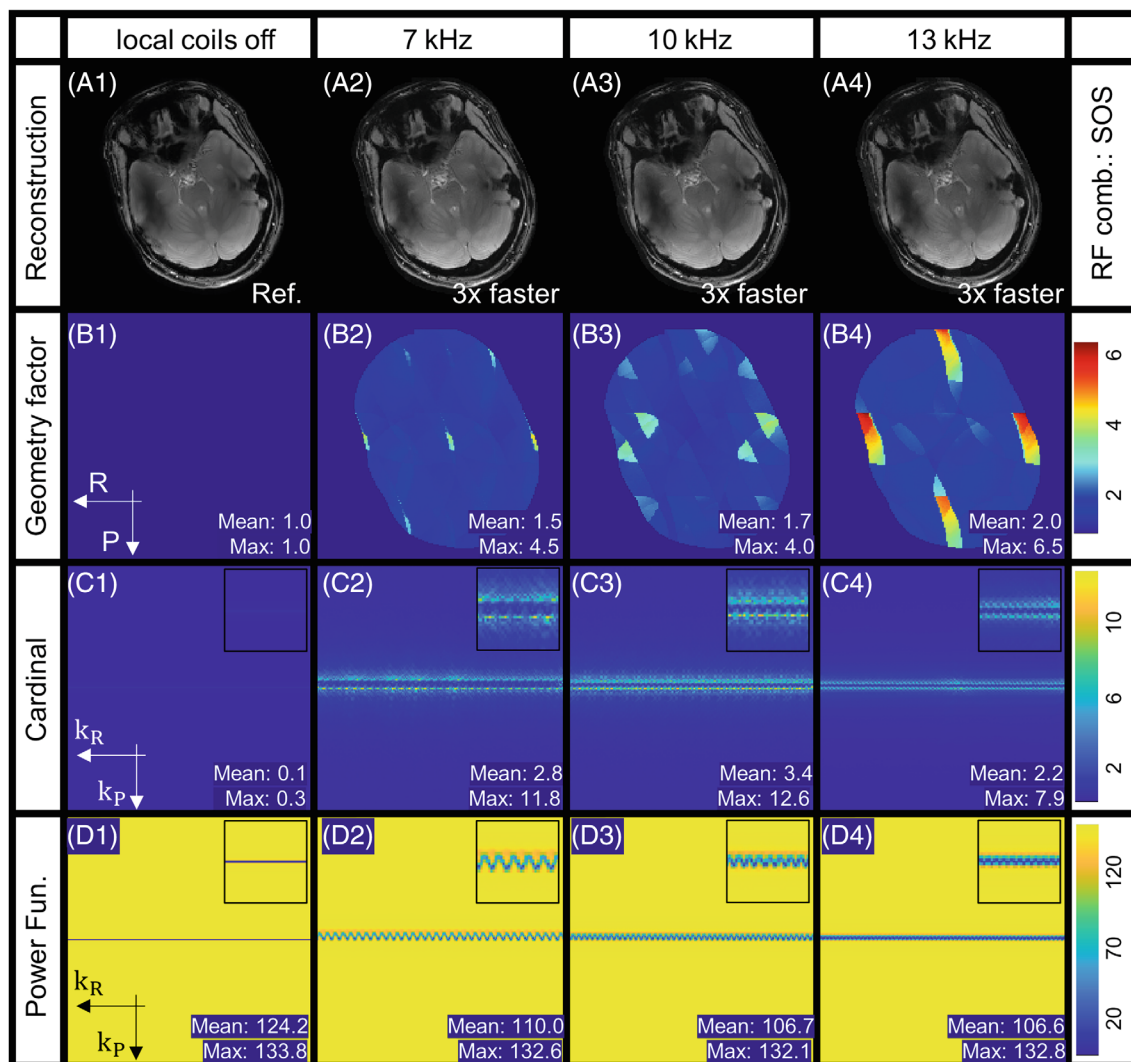


FIGURE 9 In vivo measurement with local B_0 coils modulation, given various modulation frequencies (all in $40A_{pk}$) and multiple RF receivers reconstructed independently. Rows A–D: Reconstructed images, G-map for noise amplification in image-space, cardinal map for noise amplification in k-space, power function map for approximation error in k-space. Columns 1–4: Encoding schemes—reference without local coils, 7, 10, and 13 kHz sinusoidal modulation with the scheme “ $0\pi, R22.5^\circ$.” The mean and maximum values are calculated from the central 16 phase-encoded steps for encoding.

accelerated by threefold with nearly no visible artifacts, and the G-map mean value remains rather low (i.e., 1.5). When reaching acceleration by a factor of four, “hot spots” in the G-map and the k-space error maps appear, because severe aliasing effects take place in image-space corresponding to insufficient sampling coverage visible in k-space error maps as in Figure 7B5,C5,D5.

As in Figure 8, by joint reconstruction of multiple RF receivers and local B_0 coils modulations, image acceleration along single phase encoding dimension is further enhanced. Compared to the reference scan merely accelerated by SENSE, which shows strong residue artifacts, the accumulative acceleration factor can reach eightfold for 2D Cartesian scan with negligible artifacts. Starting from 10-fold to 11-fold undersampled phase-encoded steps, the

artifacts gradually appear protruding, and therefore, the acceleration limit is reached assuming reconstruction in the least squares sense.

Furthermore, the image acceleration with modulation schemes of “ $0\pi, R22.5^\circ$ ” with $40A_{pk}$ in distinct frequencies (7, 10, and 13 kHz) are compared, with multiple RF receivers independently reconstructed (Figure 9) and with joint reconstruction of SENSE and local B_0 modulations (Figure 10), respectively. As in Figure 9, without parallel imaging, the scans accelerated by threefold with modulations in 7 and 10 kHz have negligible artifacts, whereas the one with modulation in 13 kHz has slightly visible errors within very limited support (corresponding to the bottom “bright stripe” in the G-map). From the cardinal and the power function maps given sampling period over one TR,

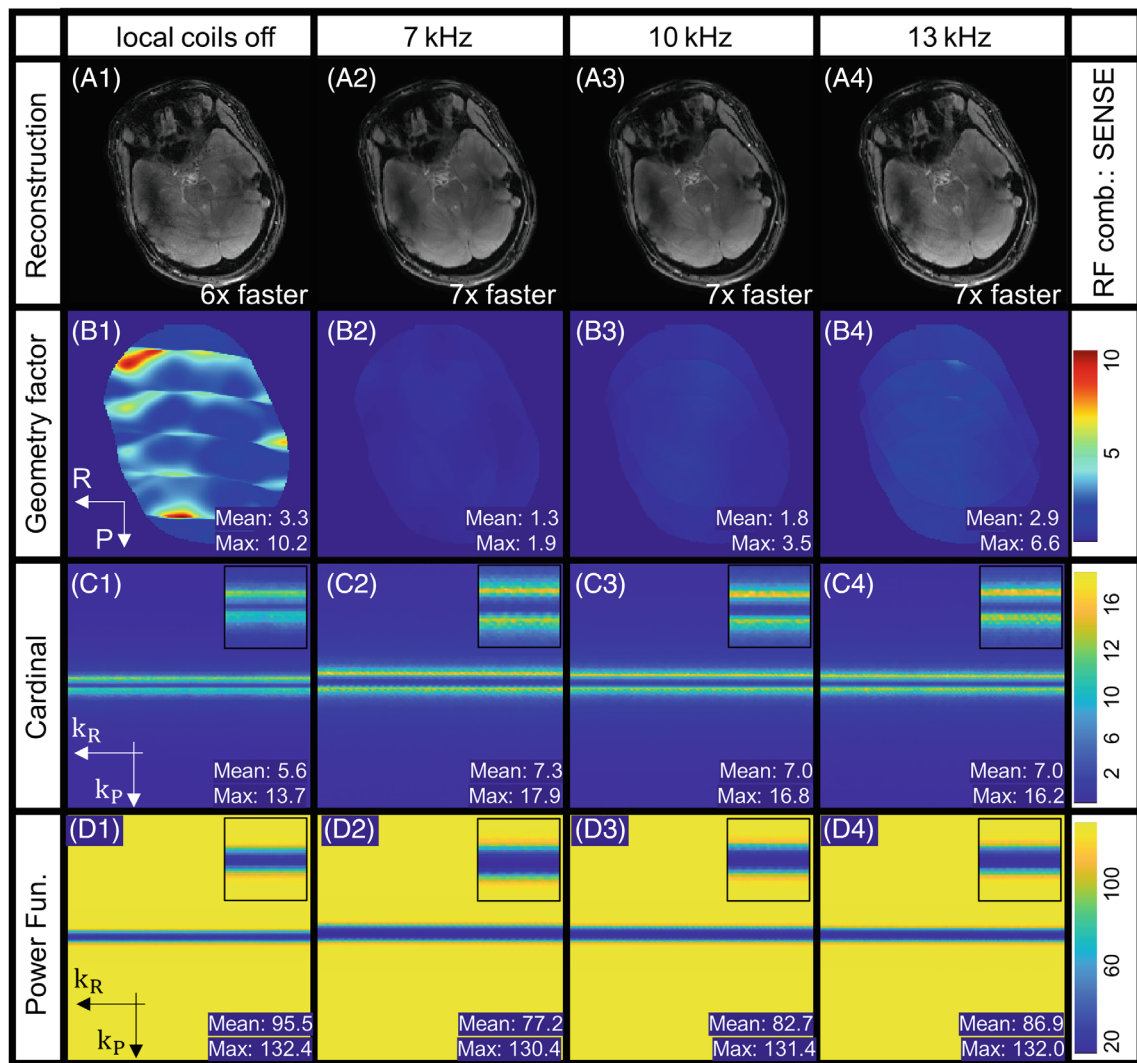


FIGURE 10 In vivo measurement with joint reconstruction of local B_0 coils modulation and SENSE, given various modulation frequencies (all in $40A_{pk}$). Rows A–D: Reconstructed images, G-map for noise amplification in image-space, cardinal map for noise amplification in k-space, power function map for approximation error in k-space. Columns 1–4: Encoding schemes—reference without local coils, 7, 10, and 13 kHz sinusoidal modulation with the scheme “ $0\&\pi$, $R22.5^\circ$.” The mean and maximum values are calculated from the central 24 phase-encoded steps for encoding.

the higher frequency modulation (C4, D4) induces sampling with a narrower k-space “stamp” efficiently resolving data in more localized regions, as indicated by the narrow dark band around the k-space center. The maps for lower frequencies contain wider and brighter regions and imply acquisitions with k-space “stamps” reaching broader range of pixels along the phase encoding dimension, although incapable of resolving all these pixels without more phase-encoded steps, which matches with the prediction by the Eq. (7).

In Figure 10, when combined with SENSE, the 7 $\times$ accelerated scans (A2–A4) outperformed the 6 $\times$ accelerated reference reconstruction with the local B_0 coils off. Moreover, the cardinal and power function maps, compared to the ones without SENSE, quantitatively demonstrated the encoding behavior when combining

the effects by RF receivers’ sensitivity modulation (i.e., GRAPPA kernel)¹⁴ and the dynamic “k-space stamp”³² because of local B_0 modulations, which generally broadens the zig-zag trajectory by B_0 gradients. The lower frequency modulation (i.e., 7 kHz) leads to a wider dark band in k-space error maps (Figure 10C2,D2) and the least image-space SNR loss (Figure 9B2), which matches the prediction of Eq. (7).

5 | DISCUSSION AND CONCLUSIONS

Our proposed local B_0 coil encoding technique is proved to successfully accelerate 2D Cartesian FLASH scans, given the almost artifacts-free threefold and eightfold

undersampled reconstructions, without and with joint acceleration with SENSE, respectively. The quantitative k-space efficiency maps incorporating nonlinear gradients are analytically computed by the extended MRI sampling theory based on RKHS,⁶³ and therefore, the encoding behaviors by various nonlinear gradient modulation schemes can be visually compared along their time-domain “trajectory” within arbitrary sampling duration (e.g., entire k-space, a single TR coverage). The optimal 8-channel modulation waveforms for 2D Cartesian scans turn out to produce a nearly linear gradient oscillating along the phase encoding dimension, leading to substantial SNR improvement compared to the previous work.⁴⁶ The aliasing control technique appears helpful, although limited within certain scenarios. The encoding effects by sinusoidal modulation with different frequencies, optionally combined with SENSE, can also be visually compared in k-space quantitatively. The proposed field calibration technique, the novel encoding analysis tools, the safety evaluation procedure for our custom-built gradient system, and the demonstrated reconstruction examples for 2D Cartesian scans make significant steps toward pushing MRI speed further with local B_0 coil array.

Notably, pushing the nonlinear B_0 gradient encoding into the RKHS theory not only finds another important application for this mathematical framework beyond parallel imaging,⁶³ but also provides an example showing how the k-space efficiency maps computed based on a time segment (e.g., a single TR as demonstrated in this paper) can decouple the entire acquisition duration and quantify individual contributions of encoding within smaller temporal periods. This elucidates the origins of the k-space “holes” observed in many nonlinear field modulation schemes explored within this study, and potentially, could be used to formulate the cost functions in optimization programs to search further enhanced encoding strategies, with less computation time and memory requirements compared to evaluating the entire nonlinear field encoding matrix. Moreover, as pointed by Eqs. (2) and (3), the time-domain coverage and sampling efficiency by arbitrary acquisition strategies given linear B_0 gradients and nonlinear B_0/B_1 fields can also be rigorously analyzed and compared in a “quantitative” k-space by applying the RKHS formalism similarly to the present investigation, such as other nonlinear B_0 gradients methods,^{36,39,40,43,31} RF encoding techniques with the transmit^{64,66–68} and the switching receive fields,^{65,69–72} resolution enhanced scans with sub-pixel spatial modulations within in-plane,^{80–85} and along slice dimension,⁸⁶ just to name a few.

In this paper, the optimal modulation scheme “ $0\&\pi$, R22.5°” for in vivo scans can be seen as a (nearly) bunched phase encoding with a local B_0 coil setup. Such field modulation becomes similar to the one produced

by a local linear gradient insert,^{75,76} with higher energy efficiency and less peripheral nerve stimulation thresholds compared to large-sized scanner gradients. With the current geometry, the magnetic field produced by the local B_0 coils varies slowly across the FOV, as governed by the Biot-Savart law, and is still far less localized to independently modulating just groups of a few pixels. Furthermore, more arbitrary (i.e., spatially and temporally) waveforms than the sinusoidal currents, which remain identical or only vary by π between adjacent phase-encoded steps (i.e., CAIPI) in this paper, can be used for the local B_0 array. However, as a few examples shown in Data S1, for accelerated 2D Cartesian scans, seemingly more flexible B_0 modulations do not necessarily enhance the efficiency in the least squares reconstruction. This is because the linear gradient remains the main “encoder” and leaves a pattern of missing k-space data naturally fit by simply a zig-zag trajectory, which is different than the trajectory optimization in PATLOC (i.e., parallel imaging technique using local gradients)^{42–44,60–62} and O-space⁸⁷ where nonlinear gradients are used for phase and frequency encoding, instead of merely local modulations in k-space.

Nevertheless, our results do not eliminate all possibilities for potential advantages in signal encoding with more arbitrary field modulation, particularly considering scenario beyond 2D Cartesian sampling, such as 3D Cartesian and spiral.⁵² However, in non-Cartesian scans with local coils modulation, system errors (e.g., eddy current, time delay) can manifest as much more severe artifacts, which requires further experimental validation. To facilitate modulation schemes for other trajectories, additional penalty term can be included to remove possible artifacts, although fast forward operator^{88,89} to achieve efficient iterative reconstruction with arbitrary local coil modulation are currently unavailable. In the future, the local B_0 coil acceleration for 3D Cartesian and other non-Cartesian acquisitions remains highly interesting, with open questions remaining for the optimal encoding methods and the potential engineering difficulties.

ACKNOWLEDGMENTS

This study is supported by the ERC Advanced Grant (No. 834940). The ex vivo brain phantom was with courtesy of the Institute of Clinical Anatomy and Cell Analysis, Department of Anatomy, Eberhard Karls University of Tübingen. The first author thanks Dr. Thomas Shiozawa (Institute of Clinical Anatomy and Cell Analysis) for assistance with sample preparation, and Dr. Gisela Hagberg for assistance in scanning this phantom. The first author also thanks Stefan Plappert for guidance in programming the ADwin high-speed processor, Praveen Iyyappan Valsala for assistance in using the Skope field camera,

Dr. Alexander Loktyushin for discussions about image reconstruction, Felix Glang for proofreading this manuscript and discussions about image reconstruction, and Prof. Jason Stockmann and Dr. Sebastian Littin for interesting discussions about nonlinear gradients. M.D. receives research support from Siemens Healthineers. A.T. was supported by the Lundbeck foundation (grant R313-2019-622). Open Access funding enabled and organized by Projekt DEAL.

CONFLICT OF INTEREST STATEMENT

Dr. Mathias Davids receives research support from Siemens Healthineers.

DATA AVAILABILITY STATEMENT

The source code and experimental data can be available on request.

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SUPPORTING INFORMATION

Additional supporting information may be found in the online version of the article at the publisher's website.

Figure S1. Pictures of hardware components of our system for image acceleration. (A) The 9.4 T whole-body human scanner. (B) The 16-transceive-32-receive RF array. (C) The power amplifier. (D) The 8-channel local B_0 coil array. (E) The 8-channel local B_0 coil array during manufacturing. (F) The 8-channel fuse box. (G) The high-speed processor for waveform transmission and current monitor sampling. (H) The 8-channel temperature monitor. (I) The installed setup for in vivo scans with a human subject. (J) The installed setup for field camera measurement of the B_0 field produced by the local coil array.

Figure S2. The joint reconstruction of SENSE and local coils modulation (7 kHz/50A_{pk}, scheme “0& π , R22.5°”), with the B_0 map of each local coil channel extrapolated to the reconstruction resolution with the ESPIRiT algorithm, and a bicubic interpolation technique taking weighted average of 4 x 4 neighboring pixels, respectively.

A: Reference image with the RF channels combined with the sum-of-squares method. No retrospective undersampling is applied. Row B: reconstruction with the high-resolution B_0 map extrapolated by the ESPIRiT algorithm. Row C: reconstruction with the high-resolution B_0 map extrapolated by a bicubic interpolation algorithm (“imresize”) in MATLAB. For row B and C, column 1 to 3 correspond to: Joint reconstruction of SENSE and local coils modulation for the retrospective undersampling factor of 6, 7 and 8, respectively.

Figure S3. The error maps by comparing the reference image (Figure S2A) and the joint reconstruction (Figure S2, row B and C) of SENSE and local coils modulation (7 kHz/50A_{pk}, scheme “0& π , R22.5°”). The B_0 map of each local coil channel is extrapolated with the ESPIRiT algorithm, and a bicubic interpolation technique taking weighted average of 4x4 neighboring pixels, respectively. Row A: the error images for reconstruction with the field maps extrapolated by ESPIRiT. Row B: the error images for reconstruction with the field maps extrapolated by the bicubic interpolation algorithm. Row C: the SSIM images for reconstruction with the field maps extrapolated by ESPIRiT. Row D: the SSIM images for reconstruction with the field maps extrapolated by the bicubic interpolation algorithm. Column 1 to 3: Joint reconstruction of SENSE and local coils modulation, for retrospective undersampling factor of 6, 7 and 8, respectively. Note that, the RMSE for error images is calculated based on a central window in matrix size of 80x60, and the SSIM statistics is made within the object mask.

Figure S4. Safety evaluation of the local B_0 coil array. (A1) Current map simulated in MATLAB. (A2, A3) B field maps simulated with Biot-Savart Law with the current map, given 1A in each coil channel. The net magnetic (B) field (for safety check) and the increased B_z field (for encoding) maps corresponding to a local coil were calculated separately. (A4) The maximal (in-plane) magnetic field distributions along z produced by the local coil array with 50 A in all channels. (A5) The experimentally measured magnetic field (for encoding) map with 50 A in all coils. (A6) The field camera measurements with multiple probe channels, for 50 A (zero to peak) and 1 kHz sinusoidal modulation of all coils with phase offset zero. (A7) The nerve stimulation function for scanner gradients and local coils with 50A_{pk} and 10 kHz sinusoidal modulation. (B1) The PNS threshold calculated by the custom-built software. (B2) The E-field generated by each local coil to predict the magneto-stimulation. (C1) The E-field distribution by each local coil simulated by SimNIBs. (C2) The E-field distribution by linear gradients. (C3) The sum E-field by linearly combining results in C1 and C2. (D) The sound pressure level in dB (A) measured by a sound level meter compatible with the 9.4 T scanner. Note that,

the E-field calculated by the two software are based on different tissue conductivity, which may be responsible for a different scaling factor.

Table S1. The calculation time in HPC server for cardinal and power functions. R is the acceleration factor. The notation “1 TR” and “150 TR” means k-space error maps covering sampling with 1 and 150 phase-encoded steps respectively. The notation “100x100 k-space” means k-space error maps covering only a central region of 100x100 pixels, to avoid excessively computation load.

Figure S5. Joint reconstruction with SENSE, for 7 kHz/40A_{pk} sinusoidal modulation with different field shapes. Row 1 to 7: modulation schemes for distinct phase offset patterns of the 8-channel local coils, which forms different field shapes oscillating during signal readout to accelerate scans. Column A to D: reconstructed 2D FLASH images with 8x undersampling, G-map, cardinal map and power function map for sampling period over one TR. In columns C and D, zoomed in pictures are shown on the upper right corner, and the mean and max. values are calculated from the 20 central phase-encoded steps. Note that, the k-space axis is rotated by 90 degree relative to image space, to better visually compare the k-space error maps in a long figure.

Figure S6. Ex vivo scans given 5 kHz/40A_{pk} sinusoidal local coils modulation with π shift in all channels between consecutive two-fold undersampled phase-encoded steps, manipulating aliasing patterns and mimicking the CAIPIRINHA technique. Row 1 to 5: different modulation schemes—reference fully sampled scan with local coils off, and two-fold accelerated FLASH scans with schemes “45°”, “45°” with CAIPI, “0& π ”, “0& π ” with CAIPI, respectively, given all with RF receiver channels combined with the sum-of-squares method. Column A to D: reconstructed images, G-map for noise amplification in image space, cardinal map for noise amplification in k-space, power function map for approximation errors in k-space.

Figure S7. More examples for aliasing controls with local coil array. Left column A: The G-maps computed from eight-fold accelerated FLASH scans given 7 kHz/40A_{pk} sinusoidal modulation with two phase offset schemes, with and without CAIPI. Joint reconstruction with SENSE and local coils is performed. Additionally, G-map from a reference scan with only SENSE acceleration is compared. Right column B: G-maps computed from two-fold accelerated FLASH scans with 40A_{pk} sinusoidal modulation, showing 2D aliasing control along readout (R) and phase encoding (R) dimension, by changing the modulation frequency and introducing phase offsets (here, π) between consecutive phase-encoded steps, respectively.

Figure S8. Examples of more temporally and spatially arbitrary B_0 modulations by simulation using the experimental data for Figure 4 (local coils’ field maps, phantom

image, all coils with sinusoidal currents up to $40A_{pk}$). Row 1–5: modulation schemes for comparison, consisting of sinusoidal currents with static phase offsets, and those with phase and frequency varying among TR or local B_0 coil channels. Column A to B: the schematics for sinusoidal waveforms in different schemes, the reconstructed images from simulated acquisition data with artificially added noise, the G-map, the Cardinal function, and the power function. In this comparison, no modulation schemes with more variations in currents outperform the

one simply with a nearly bunched phase encoding (i.e., row 1, scheme $0\&\pi$, identical among TR).

How to cite this article: Tian R, Uecker M, Davids M, et al. Accelerated 2D Cartesian MRI with an 8-channel local B_0 coil array combined with parallel imaging. *Magn Reson Med*. 2024;91:443–465. doi: 10.1002/mrm.29799

Publication 2

Tian R, Uecker M, Holder O, Povolni P, Steffen T, Scheffler K. Group-kernel auto-calibration and group-patch k-space reconstruction: Fast MRI with time-variant B_0 kernels partitioned into time-invariant subsets. *arXiv*. Preprint posted online January 9, 2026:arXiv:2601.05469. doi:10.48550/arXiv.2601.05469. To submit, *Magn Reson Med*.

The main manuscript is provided below and the supporting information is available upon request.

Group-kernel auto-calibration and group-patch k-space reconstruction: Fast MRI with time-variant B_0 kernels partitioned into time-invariant subsets

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Abstract

Purpose: Pushing MRI speed further demands more spatially-encoded information captured per unit time, e.g., by superimposing additional field modulations during oversampled readout. However, this can introduce calibration errors that degrade image quality. Thus, we propose a continuous field calibration approach. An efficient k-space reconstruction technique is also explored.

Theory and Methods: Our auto-calibration generalizes GRAPPA kernels to explicitly extract continuous B_0 modulation kernels, solving interpolation relationships between two ACS regions differing only in the extra field modulation. The k-space locations sharing the same instantaneous image-space modulation are grouped, so that subsets of time-invariant kernels can be separately estimated, as a generalized solution for Wave-CAIPI/FRONSAC-type scans. This view further inspires a k-space subregion-wise reconstruction technique, as an efficient alternative to conventional hybrid-space reconstruction. At 9.4T, FLASH accelerated by a local B_0 coil array and Wave-CAIPI were tested with retrospective undersampling.

Results: Artifact-free images were reconstructed, under diverse rapid B_0 modulation schemes, reaching maximum acceleration factors of 8-fold in 2D and 14.6-fold in 3D. Some nonlinear gradients modulation schemes reach similar sampling efficiency as linear gradients modulation. The proposed reconstruction shows potentials in further reducing reconstruction speed.

Conclusion: Rapid B_0 modulations and widely-adopted parallel imaging can share a common mathematical framework, and consequently, achieve similarly-robust reconstructions. Specifically, for scans using dynamic B_0 and static RF kernels, not only signal encoding, but also auto-calibration and reconstruction can be performed in k-space. This paves the way to robustly remove eddy currents, and explore more complex B_0 modulation strategies towards ultimate MRI speed.

Keywords: nonlinear gradients, local B_0 coil array, Wave-CAIPI, auto-calibration, GRAPPA, RKHS.

1 Introduction

Despite decades of progress, MRI remains inherently slow due to the sequential nature of k-space sampling^{1–3}. In a mathematical prospective of the forward model, there are two strategies to fundamentally speed up MRI acquisitions: physically collecting more spatially-encoded information within shorter scan times^{4–14}, or solving the forward model with advanced reconstruction algorithms to suppress undersampling artifacts^{15–19}. Within the broad spectrum of fast MRI strategies, this paper introduces a novel field calibration technique termed group-kernel auto-calibration, and an efficient reconstruction method termed group-patch k-space reconstruction, to facilitate a critical yet currently underutilized image acceleration approach based on rapid B_0 field modulations. These modulations can be linear gradient modulations as in Wave-CAIPI, second- and third-order nonlinear gradient modulations as in FRONSAC, or more general and flexible modulation patterns produced by local B_0 coil array.

For example, in Cartesian MRI, the readout Nyquist sampling rate is determined by the FOV and the gradient strength for frequency-encoding (i.e., k-space traversal speed). When the scanner's ADC can sample faster than this Nyquist-limit, the readout oversampling does not add new and useful spatial information, and is usually removed by averaging adjacent oversampled data. However, this otherwise redundant oversampling can be exploited by superimposing additional rapid magnetic field modulations onto the frequency-encoding gradient, thereby capturing extra phase-encoded information within the same readout duration. Specifically, the k-space line-by-line trajectory can be perturbed into 2D zig-zag (e.g., Bunched-phased-encoding/BPE) patterns^{10,11}, 3D corkscrew patterns (e.g., Wave-CAIPI¹²), or spread into time-varying k-space “stamps” (e.g., FRONSAC^{20–22}, local B_0 coil modulations^{23,24}, temporally-varying RF receiver profiles^{25,26}), thereby boosting information influx to the signal encoding model. According to the convolution theorem, each k-space sample at a given time can be viewed as a weighted sum over the underlying k-space values in a local neighborhood. In this paper, we refer the corresponding set of weights induced by rapid B_0 fields as the B_0 modulation kernel, which is the Fourier transform of the image-space phase modulation pattern at that time. This B_0 kernel provides a unified description of the effective B_0 gradient modulations in k-space at a time, ranging from Dirac delta function in shifts to distributed “stamp”.

By comparison, it is common to speed up acquisition by accelerating “long-range” k-space traversal using high-performance gradients, whereas local “wiggling” the trajectory using additional continuous B_0 gradient modulations remains much less investigated, given a large yet underexplored space of nonlinear gradient modulation fields^{27,28}. Unfortunately, these time- and space-varying modulations require accurate calibration for robust image reconstruction, similar to image-space RF receiver sensitivity maps or k-space GRAPPA kernels (i.e., kernels of RF receive fields) that vary across receiver channels in parallel imaging. It can be even more challenging, because continuous B_0 field modulations can introduce additional complications, such as substantial trajectory deviations due to complex spatial field patterns, time delay and eddy currents, as well as mechanical vibrations – that are usually absent in the time-invariant RF receivers. As a result, this image acceleration strategy could be hindered for broad adoption.

To estimate the additionally-imposed spin phase modulations, standard B_0 field mapping scans for each linear/nonlinear gradient element (e.g., with blip or DC current)^{29,30,24} can be performed, combined with current monitoring and kernel-based extrapolation²⁴. But these maps cannot capture dynamic effects (e.g., the eddy currents during B_0 modulations), possibly ending up with residue fold-over artifacts. Alternatively, a CSI-type sequence can measure a more realistic spatiotemporal model along readout by phase-encoding all spatial pixels^{30–32}. However, it is time-consuming, thermally and mechanically stressful for hardware, requires careful sequence debugging and coil cooling to match calibration-to-imaging conditions, and remains vulnerable to noise and measurement errors. For special case of linear gradient modulations (e.g., Wave-CAIPI), field calibration can be simplified using single-slice projection data¹², with optimization of a few parameters (“partial” auto-calibration)³³ in reconstruction. Nevertheless, robust, widely-deployable field-modulated scans should prefer a complete auto-calibration strategy to estimate a full spatiotemporal model directly from imaging data.

Therefore, we propose group-kernel auto-calibration to generalize self-estimation of time-invariant RF receive (B_1^-) kernels to continuous B_0 modulation kernels, auto-calibrated in separate groups³⁴. Inspired by this view of B_0 kernel groups, we further propose group-patch reconstruction to reconstruct field-modulated data more efficiently via interpolations of undersampled k-space data in patches. Essentially,

both are built on a novel perspective that, the full set of time-variant k -space B_0 kernels can be partitioned into time-invariant subsets across selected k -space locations.

For calibration, from a standard auto-calibration-signal (ACS) region and a field-modulated ACS region, a series of time-invariant B_0 modulation kernels can be separately estimated, by grouping data at all k -space locations that share identical interpolation relationship between the two ACS regions. These kernels together yield the full set of time-varying B_0 kernels, providing a complete spatiotemporal phase evolution model that captures the cumulative effects from field modulations and all undesirable imperfections. This also represents a k -space counterpart to a recent image-space auto-calibration technique that estimates the pixelwise point-spread-function (PSF) for Wave-CAIPI/FRONSAC²². In comparison, when the nonlinear modulations vary slowly across space, k -space kernel extraction benefits from averaging across a large number of linear equations (i.e., due to kernel shifts), and requires only narrow k -space bands rather than full-resolution images as the auto-calibration regions, elegantly inheriting GRAPPA's robustness. Part of our auto-calibration concepts appears similar to a recent nearly-continuous extension³⁵ of dual-polarity GRAPPA³⁶ for EPI, together delineating a more complete picture for linear and nonlinear gradient modulations.

For the proposed group-patch reconstruction, undersampled k -space data can be further grouped into patches, each sampled by a redefined subset of time-varying B_0 kernels. Building on a recent MRI sampling theory describing generalized k -space interpolations based on reproducing kernel Hilbert space (RKHS)³⁷, we compute, for each kernel subset, a single interpolation matrix that can map any undersampled k -space patch in that group to a fully-recovered patch. This interpolation matrix is then efficiently reused to reconstruct all undersampled patches, which are concatenated into a full k -space.

To experimentally test our approaches, 2D and 3D FLASH scans are accelerated by an 8-channel local B_0 coil array as a relatively-arbitrary field-generating hardware, as well as the scanner's linear gradients (i.e., for Wave-CAIPI). Without dense array of field probes³⁸ – which cannot fit between the local B_0 array and the RF coil – our k -space auto-calibration method substantially improves reconstruction quality. It remains robust across linear and nonlinear modulation schemes, along all spatial orientations in a 9.4T human MRI scanner, even under occasional hardware noise. Furthermore, the k -space patch-wise

reconstruction speeds up reconstruction compared to conventional Wave-CAIPI type hybrid-space reconstruction. Overall, our methods enable fast yet robust MRI with rapid B_0 field modulations, while its fundamental acceleration limits depend on the time-integral of B_0 spatial gradients, rather than the RF (B_1) wavelengths³⁹.

2 Theory

2.1 System overview at 9.4T

In this study, 2D and 3D FLASH line-by-line scans are accelerated by superimposing additional rapid B_0 field modulations onto the frequency-encoding gradient during signal readout, using either a custom-built 8-channel local B_0 coil array placed outside the RF coil, or scanner's linear gradient as Wave-CAIPI. In Figure 1, sinusoidal currents are applied to each channel of local B_0 coils or scanner gradients, imposing either local or global spin phase modulations, improving pixel disentanglement in image reconstruction using retrospectively undersampled k-space data. In this paper, the rapid B_0 modulations are identical across phase-encoded steps (i.e., TRs) for simplicity.

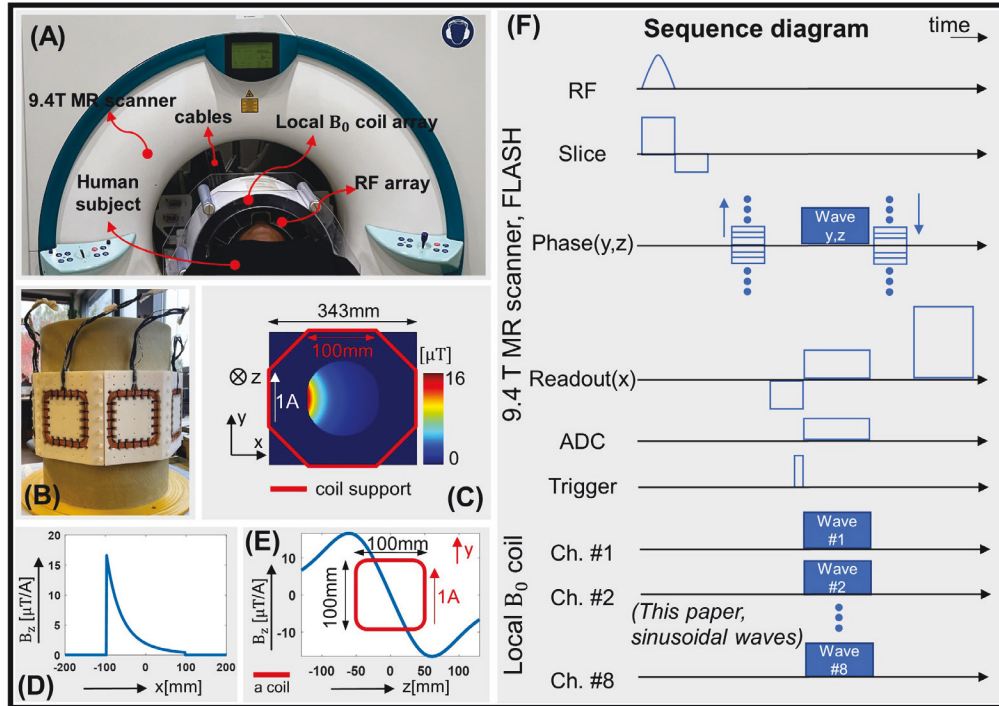


Figure 1. System illustration, see more details in the reference²⁴. (A) The hardware setup in the scanner. (B) The local B_0 coil array under construction (a replica to the reference). (C) Simulated magnetic field map given 1A DC current in a single local B_0 coil, which generate up to around 16 μ T within the effective field of view (FOV) defined by the inner boundary (around 97mm to the FOV center) of a RF array. (D) The 1D plot along the central horizontal line in (C). (E) The variation of the in-plane maximal field strength generated by 1A in a local coil along z direction, with the relative position to the coil winding indicated in red. (F) Pulse sequence diagram for accelerated MRI scans with rapid B_0 field modulations. In a standard FLASH sequence, during signal readout, either the local B_0 coil array or scanner's phase-encoding gradients are switched on with current waveforms. Theoretically, the waveforms in the 8 local B_0 channels can be designed independently. Here, we apply sinusoidal currents for simplicity, to induce additional localized spin phase modulation on the objects to accelerate k-space acquisitions.

2.2 k-space B_0 kernels to model linear and nonlinear gradients modulations

In conventional signal model^{8,40}, each k-space sample $S_n(t)$ acquired at time t from RF receiver channel n is obtained by first multiplying the spin density map by the RF receiver sensitivity map, the phase map induced by rapid B_0 field modulations, the phase map induced by linear gradient for frequency encoding, and then summing all the image-space contributions into a single k-space value at that time.

$$S_n(t) = \int c_n(r) \rho(r) \exp \left\{ -i \left\{ \gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^t I_{\lambda}(\tau) d\tau \right] + k(t)r \right\} \right\} dr, \quad (1)$$

with $k(t) = \gamma \int_0^t g(\tau) d\tau$ as the k-space trajectory term (i.e., in rad/m)

where γ is the gyromagnetic ratio (i.e., in rad/s/T), r is the spatial location, $c_n(r)$ is the n^{th} RF receiver sensitivity map, $g(\tau)$ is the shaped pulse of the linear gradients, $B_{\lambda}(r)$ is the B_0 field distribution produced by unit current in the λ^{th} coil that generates rapid field modulations (local B_0 coil or phase-encoded linear gradient), $I_{\lambda}(\tau)$ is the programmable current waveform in the λ^{th} modulation coil. The spatial distribution of rapid B_0 fields at a time can be linear or nonlinear.

Multiplying by the linear gradient induced phase map and summing over all image-space positions is equivalent to evaluating the Fourier transform at a k-space location corresponding to time t_0 . According

to the convolution theorem, the k-space signal can thus be expressed as the convolution of the Fourier transforms of the three maps, evaluated at that k-space location $k(t_0)$ ¹⁹:

$$S_n(t_0) = \left\{ \underbrace{\mathcal{F}\{c_n(r)\}}_{\text{RF receive kernel}} * \underbrace{\mathcal{F}\{\rho(r)\}}_{\text{original k-space}} * \underbrace{\mathcal{F}\left\{\exp\left\{-i\gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^{t_0} I_{\lambda}(\tau) d\tau\right]\right\}\right\}}_{B_0 \text{ modulation kernel}} \right\}(t_0), (2)$$

where asterisk $*$ is the convolution operation. In this representation, the original k-space object is convolved with a RF receive kernel, so that each k-space sample in channel n becomes a weighted sum of a local neighborhood of the original k-space object. A subsequent convolution with the B_0 modulation kernel further mixes neighboring samples in receiver channel i , so that the final value is a weighted sum of another local neighborhood of the channel-specific k-space data. Note that, the term “GRAPPA kernel”⁹ usually refers to the k-space kernels used to interpolate missing samples from undersampled data in multiple receivers. Here, suggested by the RKHS MRI sampling theory^{24,37}, we use “RF receive kernel” as well as “ B_0 kernel” in a more general sense to denote any k-space interpolation kernel that maps between the original and the acquired k-space in either direction, due to RF receiver or B_0 modulation fields. Across different k-space locations, the RF kernel is shift-invariant and the B_0 kernel is shift-variant, whereas across distinct RF receiver channels, the RF kernel is shift-variant and the B_0 kernel is shift-invariant. Thus, when Equation (2) is evaluated at different time points and for different receiver channels, the corresponding kernel weights might differ.

2.3 Continuous field calibration using kernels in time-invariant groups

The proposed group-kernel auto-calibration technique is based on interpolating k-space data using kernels – which is also a central concept for RF receivers calibration in GRAPPA and ESPIRiT⁴¹. In the original GRAPPA formulation, k-space data are reconstructed by multiplying acquired k-space data neighborhoods with receiver-specific GRAPPA kernels and summing to interpolate missed k-space data. These kernels are estimated from a fully-sampled ACS region. Alternatively, as in Figure 2A, fully-sampled RF receiver kernels with full k-space support (the black, or the red lines) can be estimated by learning interpolation relationships between local neighborhoods of one receiver (the gray, or the

orange block) and corresponding data points from another receiver (used as a reference, the blue block with black outline). These fully-sampled RF kernel coefficients can be Fourier transformed into image-space receiver sensitivity maps, enabling SENSE-like reconstructions similar to ESPIRiT. To estimate the RF kernel weights, a large number of overdetermined linear equations (or training datasets for neural networks) are generated by assuming the kernels are shift-invariant, while the data themselves remain shift-variant across k-space.

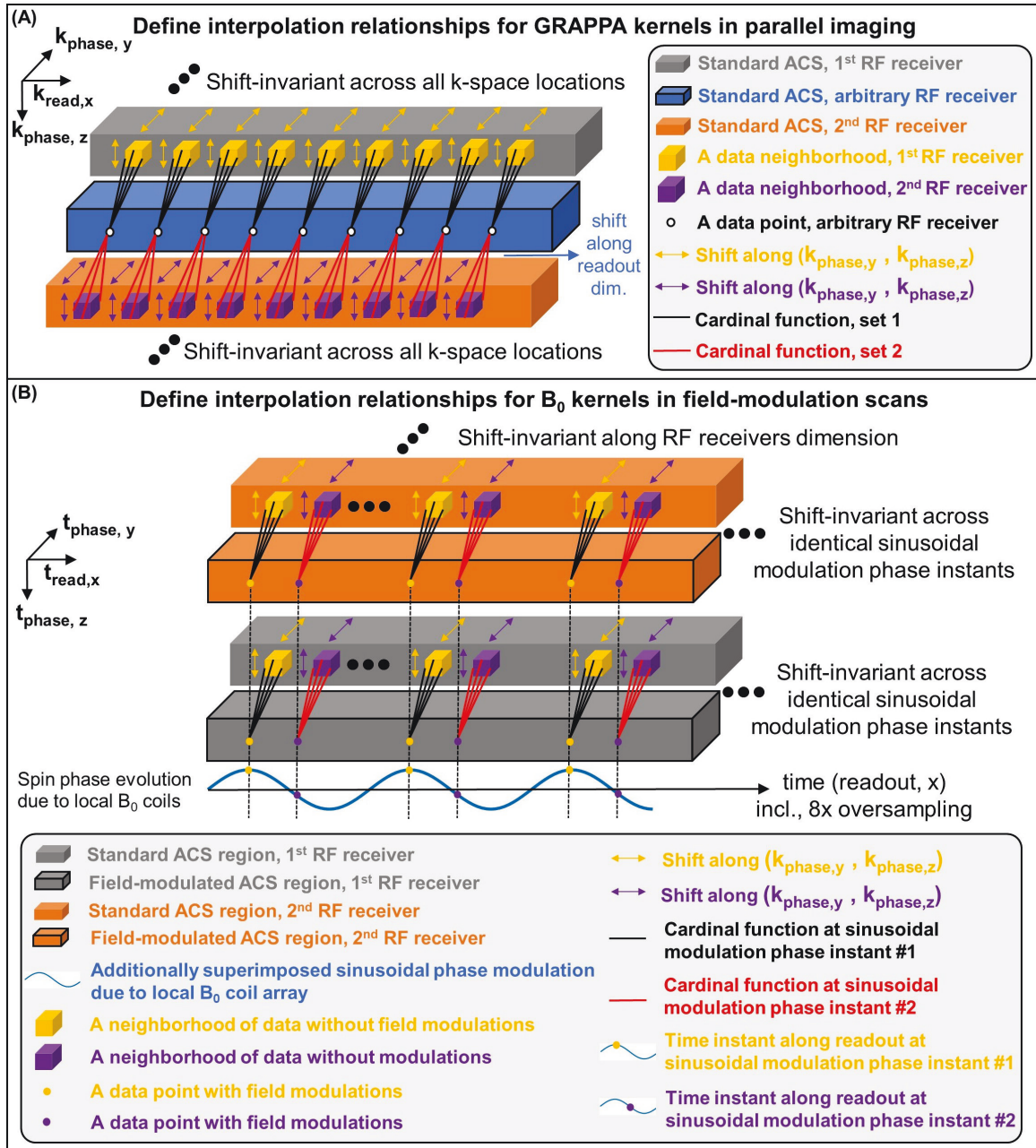


Figure 2. Auto-calibration from parallel imaging to continuous modulations of B_0 fields, both by extracting k-space kernels (RF or B_0) as the Fourier transform of image-space signal modulations. (A). Review of the interpolation relationships governing auto-calibration of GRAPPA weights, interpolating a neighborhood of k-space data encoded by distinct RF receivers to a data point from a RF receiver. A set of GRAPPA weights is shift-invariant along three k-space axes (i.e., k_x, k_y, k_z), unique to specific pair of RF receivers. (B) Auto-calibration of rapid B_0 modulations using e.g., sinusoidal modulations of our 8-channel local B_0 coil array or scanner’s phase-encoding gradients. A B_0 modulation kernel at a particular phase of the sinusoidal oscillation cycle is shift-invariant along two phase encoding axes (i.e., t_y, t_z), the RF receivers’ dimension, and among different readout time instants sharing the same phase of the sinusoidal modulation cycle.

Similarly, as in Figure 2B, the rapidly varying B_0 modulation fields lead to shift-variant B_0 kernels across k-space along the readout direction in Cartesian line-by-line scans, while being shift-invariant across receiver channels. To estimate such time-varying kernels (black and red lines), the proposed group-kernel auto-calibration uses two ACS blocks with readout oversampling: a standard ACS (orange and gray blocks without black outline), and a field-modulated ACS (orange and gray blocks with black outline). Both datasets can also serve as imaging echoes during reconstruction. We partition k-space locations into groups sharing identical spin modulation states, readout timepoints with the same sinusoidal modulation phase across all phase-encoding lines and RF receivers. Within each group, many source-target data pairs $f_i^{i,\hat{t}}(t)$ and $g_i(t)$, supported on neighborhoods $\hat{t} \in D_{\hat{t},i}$, are interpolated using the same kernel $w_i^{i,\hat{t}}(t)$, regardless of different k-space locations t as the neighborhood centers. This defines a subset of interpolation relationships.

$$g_i(t) = \sum_{\hat{t} \in D_{\hat{t},i}} f_i^{i,\hat{t}}(t) w_i^{i,\hat{t}}(t), (3)$$

A small sliding window moves across the ACS data subset in this group to assemble a linear system for kernel estimation:

$$F^H \hat{U} = G^H, (4)$$

$$\text{or, } M_{ACS} \hat{U} = R_{ACS}, (5)$$

$$\text{with } \begin{cases} M_{ACS} = FF^H, \in \mathbb{C}^{D \times D}, (6) \\ R_{ACS} = FG^H, \in \mathbb{C}^{D \times 1}, (7) \end{cases}$$

The matrix $F \in \mathbb{C}^{D \times S}$ consists of S neighborhood vectors (each has D samples) from the standard ACS, while $G \in \mathbb{C}^{1 \times S}$ contains the corresponding target samples from the field-modulated ACS. The kernel $\hat{U} \in \mathbb{C}^{D \times 1}$ can be solved using pseudoinverse, truncated-SVD (tSVD), LSQR, or utilizing neural network. Superscript $(\cdot)^H$ denotes conjugate transpose. Optionally, left-multiplying the Equation (4) by F yields the normal equations in Equation (5) with $M_{ACS} \in \mathbb{C}^{D \times D}$ and $R_{ACS} \in \mathbb{C}^{D \times 1}$.

Repeating this procedure across different kernel groups (i.e., different $w_i^{i,t}(t)$) yields the full set of time-varying B_0 kernels, which after Fourier transform become spatiotemporal spin-phase-evolution maps. These maps capture the cumulative effects of rapid field modulations, eddy currents, gradient delays, concomitant fields, and subtle motions. The standard ACS can be acquired first, so that inversion of Equation (4) or (5) can start earlier. Optionally, receiver channels can be combined before group-kernel auto-calibration. Since kernel calibration per group is independent, the computation is parallelizable.

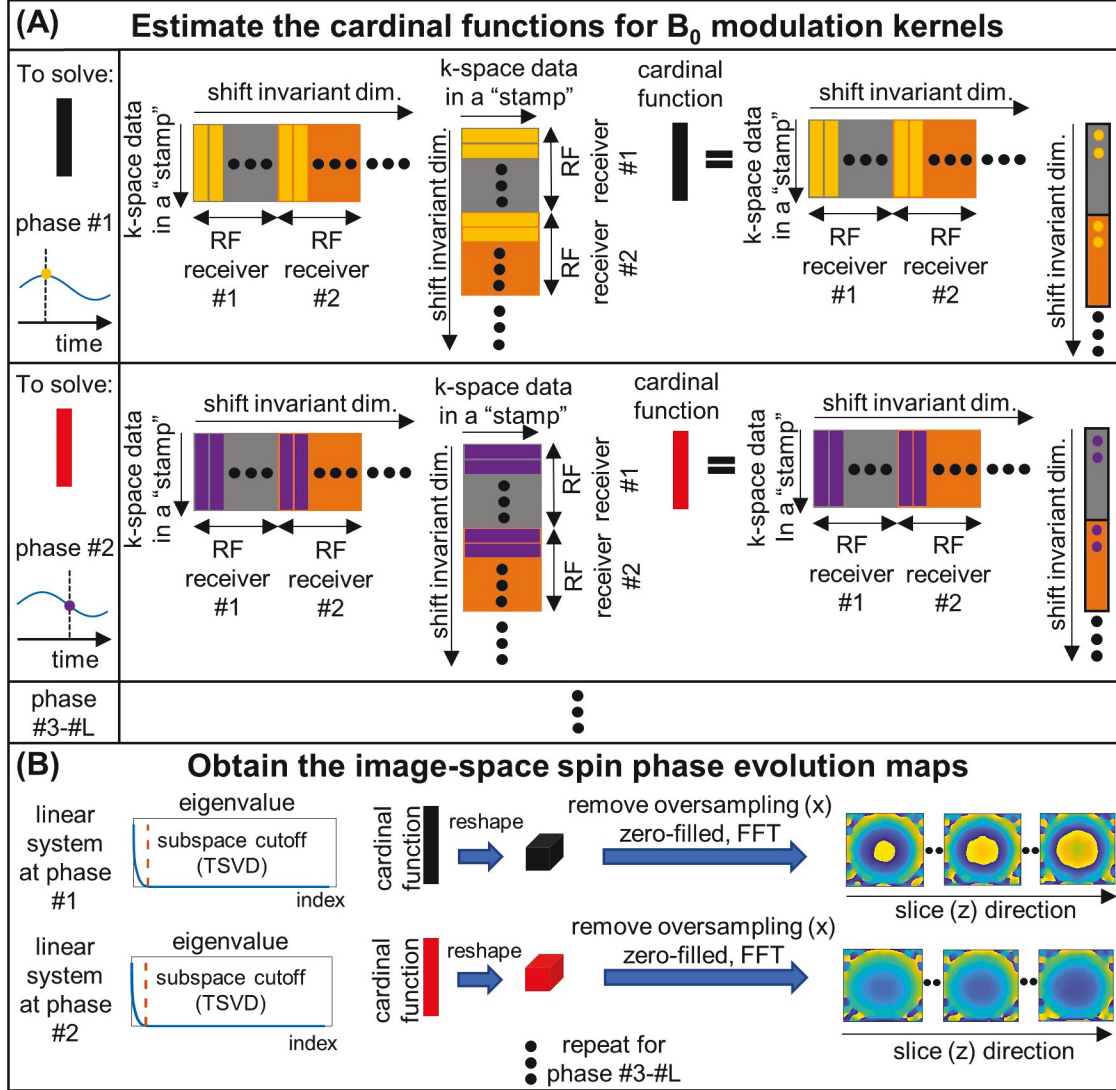


Figure 3. Grouped-kernel auto-calibration based on k-space subregions that span multiple kernel groups. Some symbols are defined in Figure 2. (A) Step 1: Define the kernel interpolation relationships, e.g., a set of linear equations, to interpolate source neighborhoods from the standard ACS to target points from the field-modulated ACS. This interpolation direction can also be reversed. (B) Step 2: Solve the cardinal functions at each phase of a sinusoidal oscillation cycle using a numerical solver, which theoretically, can be linear, nonlinear, or based on neural networks. For example, optional eigenspace filtering can be applied, e.g., using truncated singular value decomposition (TSVD). The interpolation weights are then reshaped, zero-padded and Fourier transformed to yield spin phase evolution maps.

The same principle can be applied in reverse – interpolating from the field-modulated ACS to the standard ACS, similar to classical GRAPPA calibration. This pathway may not easily yield a full spatiotemporal encoding model in a straightforward manner. If extended to EPI as an extreme case of

Wave-CAIPI (i.e., a large k-space “sweep”, one or a few shots), it appears similar to a nearly-continuous extension of dual-polarity GRAPPA³⁵, where two (equivalent) field-modulated ACS blocks with opposite EPI readout polarities that can jointly estimate B_0 and RF receiver kernels for k-space reconstruction.

2.4 Interpolating k-space patches spanning time-invariant modulation segments

The proposed group-patch reconstruction (i.e., image reconstruction by interpolating k-space patches in groups) is based on cardinal functions from the RKHS formalism of MRI. These cardinal functions act as generalized k-space interpolation weights, mapping a patch of undersampled k-space data (source) to another patch of fully-sampled data. In its original form, computing the full k-space cardinal function from the entire signal model of Equation (1) can be computationally prohibitive.

However, previous work has shown that, under nonlinear gradient modulation that is periodic along the readout (and often identical across phase-encoded steps), the cardinal function can be evaluated within a small k-space region, while still reflecting the encoding efficiency of the full k-space. Inspired by this, we partition the undersampled k-space into patches of fixed size, each covering an integer number of modulation periods and sampled by the same subset of B_0 kernels. Consequently, the cardinal function matrix needs to be computed only once for such a kernel subset, since all patches are sampled by that kernel subset and share identical interpolation weights to reconstruct fully-sampled k-space. This reduces the problem to the inversion of a small linear system for a localized k-space subregion, and eliminates repetitive computation at different k-space locations where the underlying field modulation segments are identical across patches.

First, for a pair of source and target patches, we construct two encoding matrices with acquisition indices (t, i) and $(\bar{t}, j)$. Each row represents a spatial encoding map at a time instant, flattened into a 1D vector. The source encoding matrix $E_{\bar{t}, j}$ contains composite spatial encoding maps corresponding to the sampling of the source patch. The target encoding matrix $E_{t, k}$ is a truncated DFT matrix that represents Fourier sampling of the target patch, optionally weighted by a receiver channel sensitivity map $c_k(r)$.

$$E_{\bar{t},j} = c_j(r) \exp \left\{ -i \left\{ \gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^{\bar{t}} I_{\lambda}(\tau) d\tau \right] + k(\bar{t})r \right\} \right\}, (8)$$

$$E_{t,k} = c_k(r) \exp \{ -ik(t)r \}, (9)$$

If reconstructing within a single RF receiver, we set $k = j$. If reconstructing data jointly encoded by RF coils and rapid B_0 modulations, we set $c_k(r) = 1$ to interpolate onto a reference Fourier grid without RF coil weighting.

Second, we compute the Gram (M_{sp}) and cross-Gram (R_{tp}) matrices from these two encoding matrices, and loaded into the RKHS formalism to obtain the cardinal function matrix U , i.e., the interpolation weight matrix that maps the source patch onto the target patch.

$$\begin{cases} M_{sp} = E_{\bar{t},j} E_{t,i}^*, \in \mathbb{C}^{TN \times TN}, (10) \\ R_{tp} = E_{\bar{t},j} E_{t,k}^*, \in \mathbb{C}^{TN \times T}, (11) \end{cases}$$

Specifically, we solve:

$$M_{sp} U = R_{tp}, (12)$$

where $U \in \mathbb{C}^{D \times \hat{D}}$, D and $\hat{D}$ are the sample numbers in the source and target patches respectively. The power function representing k-space approximation errors can also be derived from the estimated cardinal function:

$$p^{t,k} = \sqrt{(E_{t,k} E_{t,k}^* - U^* E_{t,i} E_{t,k}^*)_{diag}}, (13)$$

Third, the calculated cardinal function matrix is used to for reconstructing a target patch, by matrix-vector product.

$$\hat{F}_{tp}^H = F_{sp}^H U, (14)$$

where $\hat{F}_{tp} \in \mathbb{C}^{\hat{D} \times 1}$ is the vectorized reconstructed k-space patch, $F_{sp} \in \mathbb{C}^{D \times 1}$ is the vectorized undersampled k-space patch. In practice, the source patch $(\cdot)_{sp}$, is chosen slightly larger than the target subregion $(\cdot)_{tp}$ to avoid boundary errors.

Since all patches span time-invariant B_0 modulation segments and share identical interpolation weights to fully-sampled k-space, reconstruction can proceed in parallel by interpolating each undersampled source patch to its corresponding fully-sampled target patch. The reconstructed patches are assembled to form the full k-space data, which is Fourier transformed to yield the final reconstructed image.

Unlike GRAPPA/SPIRiT reconstructions, where interpolation weights are tied to the local training kernels, the RKHS framework allows direct interpolation onto Fourier grid regions with arbitrarily selected k-space ranges. This enables flexible, region-by-region reconstruction decoupled from the learned kernels. Compared to global RKHS computation, our approach allows interpolation to k-space subregions tailored for modulation periods of rapid B_0 field, substantially reduces the computational cost. Similar subregion interpolation strategies have been explored for non-Cartesian gridding, but with different implementations.

Compared with the conventional hybrid-space reconstruction (originally developed for Wave-CAIPI) that decouples only the phase-encoding dimensions, our patch-wise reconstruction can decouple all k-space dimensions, including readout with time-varying B_0 field modulations. This can flexibly slice reconstruction data into smaller chunks, and substantially reducing computational burden further.

Theoretically, if applied to EPI, the same principle supports continuous trajectory calibration that can be decoupled from parallel-imaging reconstruction. This avoids embedding the full signal model into SENSE-type reconstructions, which would otherwise become computationally expensive.

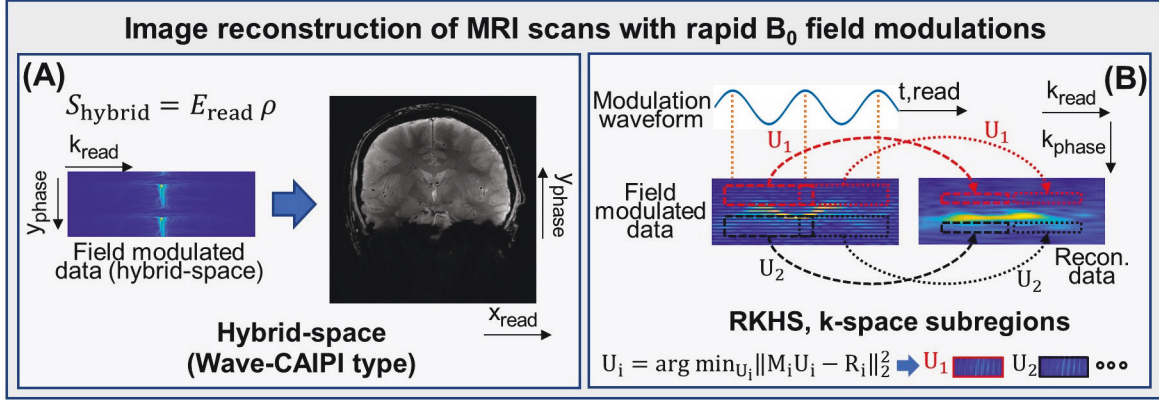


Figure 4. (A) In hybrid-space reconstruction, the phase-encoding dimension(s) are solved by FFT, while the readout forward model is solved by general inversion. Note, the k-space dataset and the reconstructed image only serves as illustration purposes, and size-ratio between their figures is not realistic. (B) In group-patch reconstruction, the k-space data are decoupled in subregions, spanning periodically occurring B_0 kernels, along both phase-encoding and readout dimensions. Cardinal functions for distinct subregions (index i) are calculated based on Equation (10-12), and applied for k-space interpolation based on Equation (14). Appropriately defined subregion can further reduce peak memory and reconstruction time.

3 Methods

Ex-vivo and in-vivo FLASH scans, with and without B_0 gradient modulations, were performed on a 9.4T whole-body human scanner (Siemens Healthineers, Germany), equipped with SC72 gradients with maximum slew-rate and absolute-strength of 200T/m/s and 70mT/m. To compare various rapid B_0 modulation schemes, an 8-channel local B_0 coil array (24.1kg before epoxy fixation; 33.5kg after, eight 10cm x 10cm square loops, 14 turns each, approximately 41 μH measured at 7kHz) was placed outside of a 16-transceive/32-receive RF coil array (modified from the reference⁴², CP-mode transmission, joint acceleration), fixed to the patient table. Several healthy adult volunteers were scanned under institutional ethics approval and informed consent.

During FLASH frequency-encoding, scanner's ADC was 8x oversampled with dwell-time of $3\mu\text{s}$. In B_0 field-modulated scans, either the local B_0 coils or the scanner's phase-encoding (linear) gradients were driven with sinusoidal currents, implemented with a customized IDEA sequence synchronized with external power-amplifier control, and a Pulseseq⁴³ sequence, respectively. For each experiment, fully-sampled k-space dataset with and without (reference) field modulations were acquired. Standard ACS regions, field-modulated ACS regions and undersampled k-space data were then cropped retrospectively, which allowed systematic (different undersampling factors) reconstruction tests with a worst-case guarantee due to possibly stronger subject motions.

For ex-vivo phantom scans in Figure 5-7, multi-slice 2D FLASH in 1.06mm^2 were scanned with FOV $190\times 190\text{mm}^2$, matrix-size 180×180 , slice-thickness 3mm, readout-bandwidth 230Hz/Pixel, readout - duration 4.32ms, TE/TR 4-5ms/30ms. The B_0 coils currents were 7.41kHz/40A_{pk}. In Figure 8, in-vivo 3D FLASH in $1.7\times 1.7\times 1.5\text{mm}^3$ was performed with FOV $220\times 220\times 132\text{mm}^3$, matrix-size $128\times 128\times 88$ (slice oversampling 37.5%), readout-bandwidth 330Hz/Pixel, readout-duration 3.072ms, TE/TR 4ms/10ms. The B_0 coils currents were 9.80kHz/37A_{pk}. In Figure 9, in-vivo 2D FLASH was scanned with FOV $250\times 250\text{mm}^2$, matrix-size 280×280 , slice-thickness 3mm, readout-bandwidth 150Hz/Pixel, readout-duration 6.720ms, TE/TR 6.07ms/30ms. The B_0 coils currents were 7.41kHz/36A_{pk}. In Figure 10 (using local B_0 array), in-vivo 3D FLASH was scanned with FOV $230\times 230\times 88\text{mm}^3$, matrix-size $230\times 230\times 112$ (slice-oversampling 1.27), readout-bandwidth 180Hz/Pixel, readout-duration 5.520ms, TE/TR 6ms/30ms. The B_0 coils currents were 7.41kHz/36A_{pk}. In Figure 10 (wave-CAIPI), in-vivo 3D wave-CAIPI implemented in Pulseseq was designed similarly as the 3D FLASH with B_0 coil array (i.e., identical FOV, matrix-size, readout-bandwidth, readout-duration, TE/TR. Nevertheless, the slew-rates and strengths of the oscillating phase-encoding gradients (y-z) were (3.4-5.3) mT/m, 4.17kHz for non-overlap wave-trajectory with 24 wave-cycles, and (5.3-8.2) mT/m, 2.78kHz for overlapped wave-trajectory with 16 wave-cycles.

The kernel-size ranged from by 6 to 19 along one dimension, with typically 15-60 kernel shifts along each of readout, phase-encoding and RF receivers dimensions, depending on ACS sizes. Removing readout oversampling in the selected source neighborhood before solving interpolation relationships is much more efficient. Typical calibration times are shown in the Figure 8-10.

Most images were still reconstructed in hybrid-space (i.e., FFT resolving phase-encoding dimensions, forward model resolving readout dimension) similar to wave-CAIPI, with optional joint data compression (JCS.) based on B_0 -RF kernels to accelerate compressed-sensing reconstruction. The joint compression will be described in a separate paper. In Figure 11(C), the 2D image was obtained via group-patch reconstruction with k-space subregions interpolations, without compression and parallel imaging, as proof-of-concept implementations.

The linear systems for kernel estimations and least-square (L_2) reconstructions were solved by LSQR, and the CS-reconstructions were solved by FISTA with L_1 regularizations based on total-variation and wavelet. The variable-density undersampling mask was generated by BART⁴⁴. The difference image is calculated based on absolute value of the magnitude image difference, normalized by the range of the reference magnitude image. The normalized-root-mean-square-error (NRMSE) is normalized by the range of the reference magnitude image. The computation was performed offline in MATLAB using HPC-servers (CPU, 2.35GHz, 64 cores), with 1TB RAM option for 3D scans.

4 Results

4.1 Robust field calibration across different spatial axes

In Figure 5, the proposed grouped-kernel field calibration technique is compared with the previous calibration approach²⁴ that combines individual B_0 mapping for each local B_0 coil, current monitoring of sinusoidal waveforms, and ESPIRiT-based extrapolation to high-resolution grid. To evaluate calibration robustness in the presence of potentially complex spatiotemporal eddy current patterns, 2D field-modulated FLASH scans were performed with different slice orientations, based on fully-sampled k-space and least-square reconstruction – ensuring that any image artifacts result from field calibration rather than undersampling. Note, the RF shield between our RF and local B_0 coils was not optimized for cutting eddy currents flowing along z-axis.

In the transverse plane, when the local B_0 coils generated a nearly-linear gradient oscillation along the phase-encoding dimension (i.e., bunched-phase-encoding) with the linear gradient (“LG”) current configuration (with 0-deg and 180-deg sinusoidal phase offsets), both calibration methods produce

artifact-free reconstruction, as in previous study. However, under more complex nonlinear field modulations (e.g., the current configuration “octupolar”), subtle image artifacts emerge when using conventional B_0 -mapping-based calibration, whereas the proposed grouped-kernel calibration continue to yield clean reconstructions. In the coronal plane, significant artifacts appear in reconstructions using the B_0 -mapping-based calibration, likely caused by eddy currents induced on components like the RF coil shield, particularly along the z-axis. In contrast, the proposed auto-calibration method produces consistently clean reconstructed images.

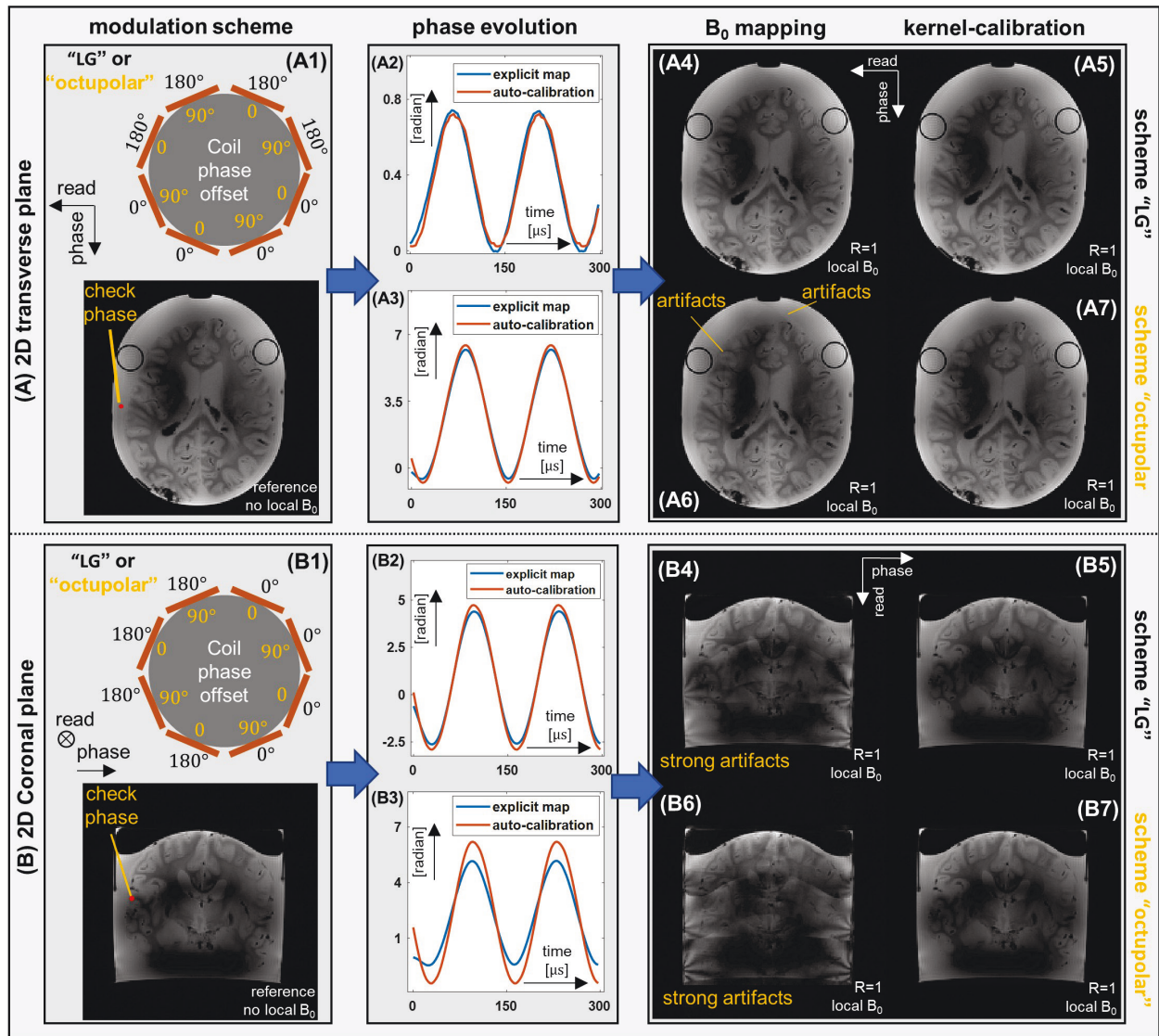


Figure 5. Ex-vivo 2D FLASH with rapid modulation of local B_0 coil array, comparing the proposed auto-calibration method, and a previous technique that obtains phase evolution maps with B_0 mapping, current monitoring and ESPIRiT high-resolution grid

extrapolation. (A) Transverse plane. (B) Sagittal plane. Two modulation schemes “LG” (nearly linear gradient) and “octupolar” (higher-order nonlinear gradient) are shown, with different readout orientations. (A1, B1) the phase offsets of sinusoidal currents in local B_0 coils, and the reference image without B_0 modulations. (A2-A3, B2-B3) The phase evolution of the encoding matrix due to local B_0 modulations at a selected “check-phase” pixel, obtained from the two calibration methods. Linear phase ramp due to DC offsets from coil currents and mean phase differences are removed to visually compare the oscillation of the phase curves only, that is mostly relevant to residue folder-over artifacts. (A4-A7, B4-B7) The reconstruction of fully-sampled k-space using two field-calibration techniques. The B_0 mapping based calibration can lead to artifacts especially along the z axis. The grouped-kernel auto-calibration achieved robust reconstruction across different spatial axes, and complex nonlinear modulation fields.

4.2 Temporally or spatially decoupled local gradients

In Figure 6-7, one important application of grouped-kernel calibration is to enable robust experimental comparisons across distinct field modulations for image acceleration. Without our auto-calibrations, such comparisons can be vulnerable to possible artifacts, as in (A6, B4, B6) of Figure 5. In previous study²⁴, the 8-channel local B_0 coils optimized for 2D Cartesian transverse scans generated a nearly-linear gradient oscillation along the phase-encoding dimension, converging nonlinear gradient modulations to Bunched-phase-encoding. This configuration was proved more efficient than others for the given coil geometry, likely because it concentrated all the “sampling energy” of the modulation functions into the only one undersampled dimension with the induced zig-zag trajectory, thereby avoiding unnecessary modulation along the fully-sampled readout direction.

However, maintaining the global gradient linearity may not be as critical as keeping the proper gradient modulation direction, which thus, provides flexible degrees of freedom in local B_0 encoding. Therefore, leveraging on auto-calibration, the optimized field modulation schemes in transverse and coronal FLASH scans was further investigated, comparing a conventional globally-linear gradient with two local gradients that are either temporally or spatially decoupled. In Figure 6, the two sets of B_0 coils positioned at the opposite sides along phase-encoding axis, can together form a globally-linear gradient when driven by sinusoidal currents with 180-deg phase shifts. However, when temporally alternating their activations with 90-deg phase shifts to break the global spatial-linearity, the resulted local gradient modulation scheme achieved very similar least-square reconstruction quality and minimal difference in mean values of G-map and power function maps. Similarly, in Figure 7, the relative z-position between the ex-vivo phantom and the local B_0 coil array was shifted, to produce a globally linear gradient or

spatially-swapped local gradients with different polarity but similar gradient strengths preserved in local regions (e.g., the four corners of the FOV). Both global and local linear gradient schemes provided similar modulations along phase-encoding dimension within local regions, and can reach similar sampling efficiency and reconstruction quality. Both examples illustrate that, strict global linearity may not be the only option for achieving effective image accelerations, as experimentally verified by our grouped-kernel calibration technique.

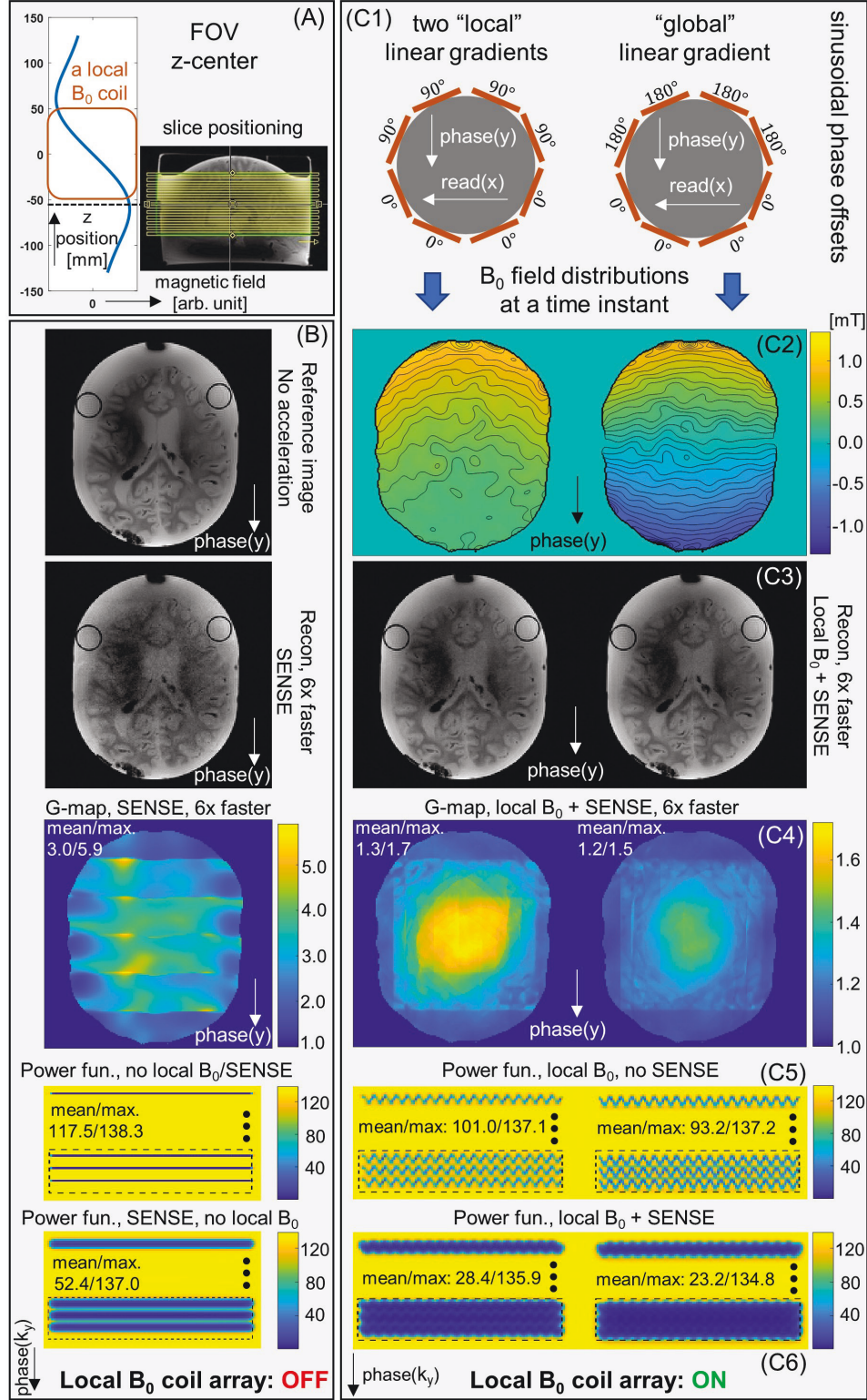


Figure 6. Ex-vivo scan in the 2D transverse plane, comparing temporally-decoupled local linear gradients and a global linear gradient. (A) a slice near the maximum modulation field is shown. (B) The reference image with fully-sampled k-space, the SENSE-only 6x accelerated image and G-map are shown. In k-space, the power function (approximation errors) maps with 6x

undersampling with and without multiple RF receivers are shown. (C1) The phase offsets of sinusoidal currents in local B_0 coils are shown, for synthesizing the local and the global linear gradients along the phase-encoding dimension, respectively. (C2) The magnetic field distributions with iso-contour (i.e., indicating gradient directions) at a time instant (nearly current peak) are shown, for the two modulation schemes. (C3) The reconstruction of 6x joint acceleration of local gradients and SENSE is only slightly noisier than the one using the global gradient, while both are substantially improved than the SENSE-only accelerated image. (C4) The G-map by the local gradients is only slightly worse than the global gradient, while both are substantially reduced than the SENSE-only one. (C5) The k-space power function maps for the two modulation schemes without SENSE are shown, where the global gradient induces a slightly darker zig-zag trajectory corresponding to subtly reduced approximation errors. (C6) The power function maps for the two modulation schemes combining SENSE, where the difference in the effective k-space coverage is little. For both schemes, hot-spots in SENSE-only acceleration are removed by the dynamic B_0 kernels.

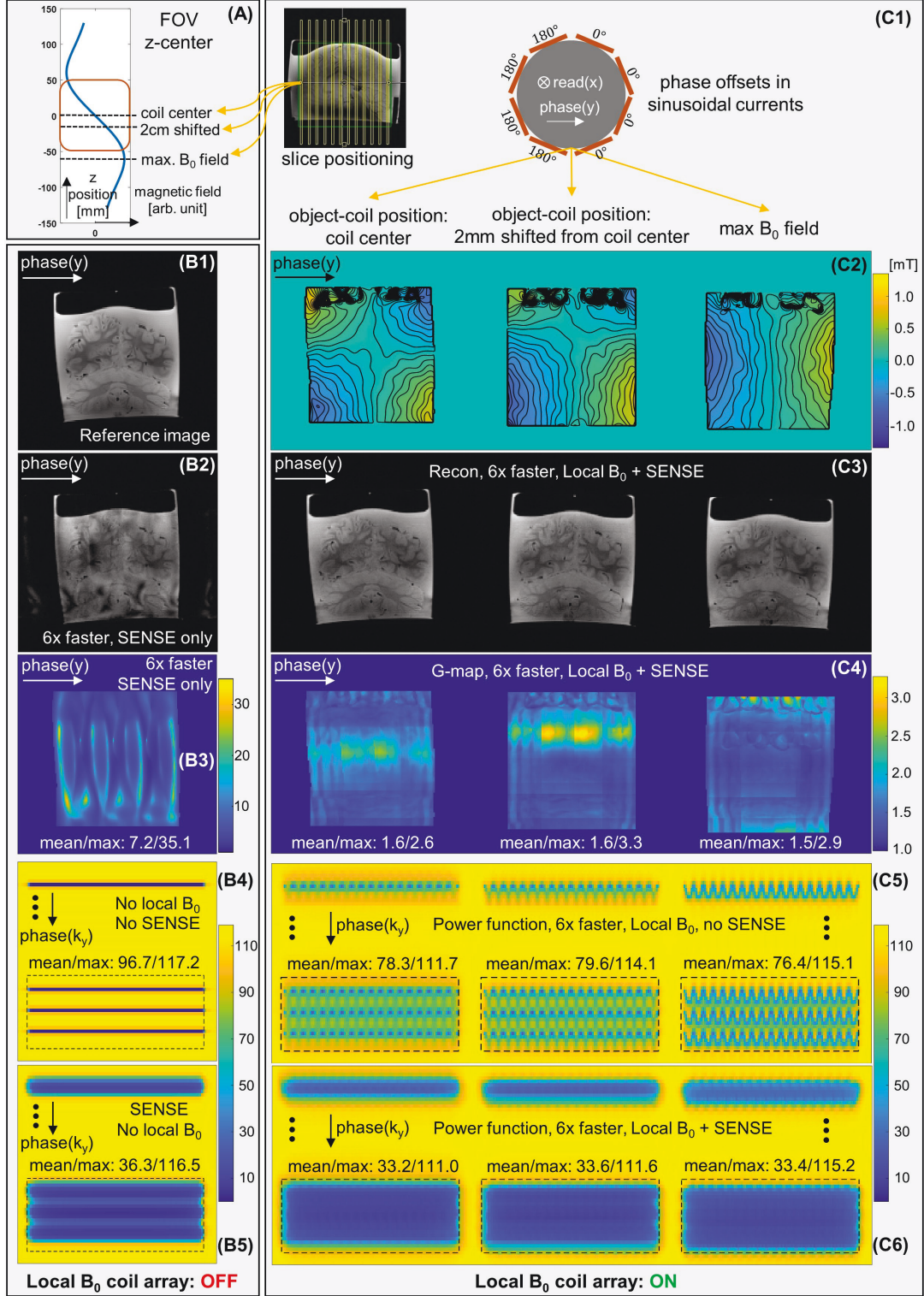


Figure 7. Ex-vivo scan in the 2D coronal plane, comparing sampling efficiency when shifting between spatially-decoupled local gradients and a global linear gradient. (A) The FOV centers along z for generating different B_0 field shapes are shown. (B1-B5) The reference image with fully-sampled k -space, the SENSE-only 6x accelerated image and G-map are shown. In k -space, the power function (approximation errors) maps with 6x undersampling with and without multiple RF receivers are shown. (C1) The

phase offsets of sinusoidal currents in local B_0 coils are shown, for synthesizing the local and the global linear gradients along the phase-encoding dimension, respectively. (C2) The magnetic field distributions with iso-contour (i.e., indicating gradient directions) at a time instant (nearly current peak) are shown, for the three modulation schemes. (C3) The reconstruction of 6x joint acceleration of local gradients and SENSE are similar to the one using the global gradient, while all of them are substantially improved than the SENSE-only accelerated image. (C4) The G-maps by the local gradients are close to the global gradient, while both are substantially reduced than the SENSE-only one. (C5) The k-space power function maps for the three modulation schemes without SENSE are shown, where the global gradient induces a zig-zag trajectory, and when moving to the leftmost case (shifting along z-position), induces sampling using a similarly efficient “stamp”. (C6) The power function maps for the two modulation schemes combining SENSE, where the difference in the effective k-space coverage is little. For all schemes, hot-spots in SENSE-only acceleration are removed by the dynamic B_0 kernels.

4.3 Robust field calibration against hardware spike noise

Figure 8 illustrates the robustness of our proposed grouped-kernel field calibration against hardware noise interference and subtle motions during in-vivo scans. In this case, the field-modulated data were affected by occasional spike noise (visible in RF chains and raw data) caused by malfunctioning local B_0 coils. Additionally, there was slight rotation between the standard and field-modulated scans performed with full k-sampling (each 2mins, >6mins gap between).

Although the field-modulated ACS data were consequently corrupted, the proposed kernel-calibration technique remained accurate in both fully-sampled and 4x3-fold undersampled reconstruction. The solution of each kernel was constrained by a large number of interpolation pairs between the standard and field-modulated ACS regions, across numerous readouts, phase-encoded steps and RF receiver channels, far exceeding the number of unknowns to be resolved per kernel. This kernel-encoded redundancy in k-space data enabled crucial averaging effects and suppressed the influence of such localized spike noise for field calibrations, inheriting the calibration robustness of RF sensitivity kernels/maps in GRAPPA/ESPIRiT.

In contrast, this important noise-resistance feature is absent in the other full-model field calibration techniques, such as the CSI-based calibration scan, and the image-space auto-calibration which estimates the PSF spreading relationships between reference and field-modulated high-resolution images on a pixel-by-pixel basis.

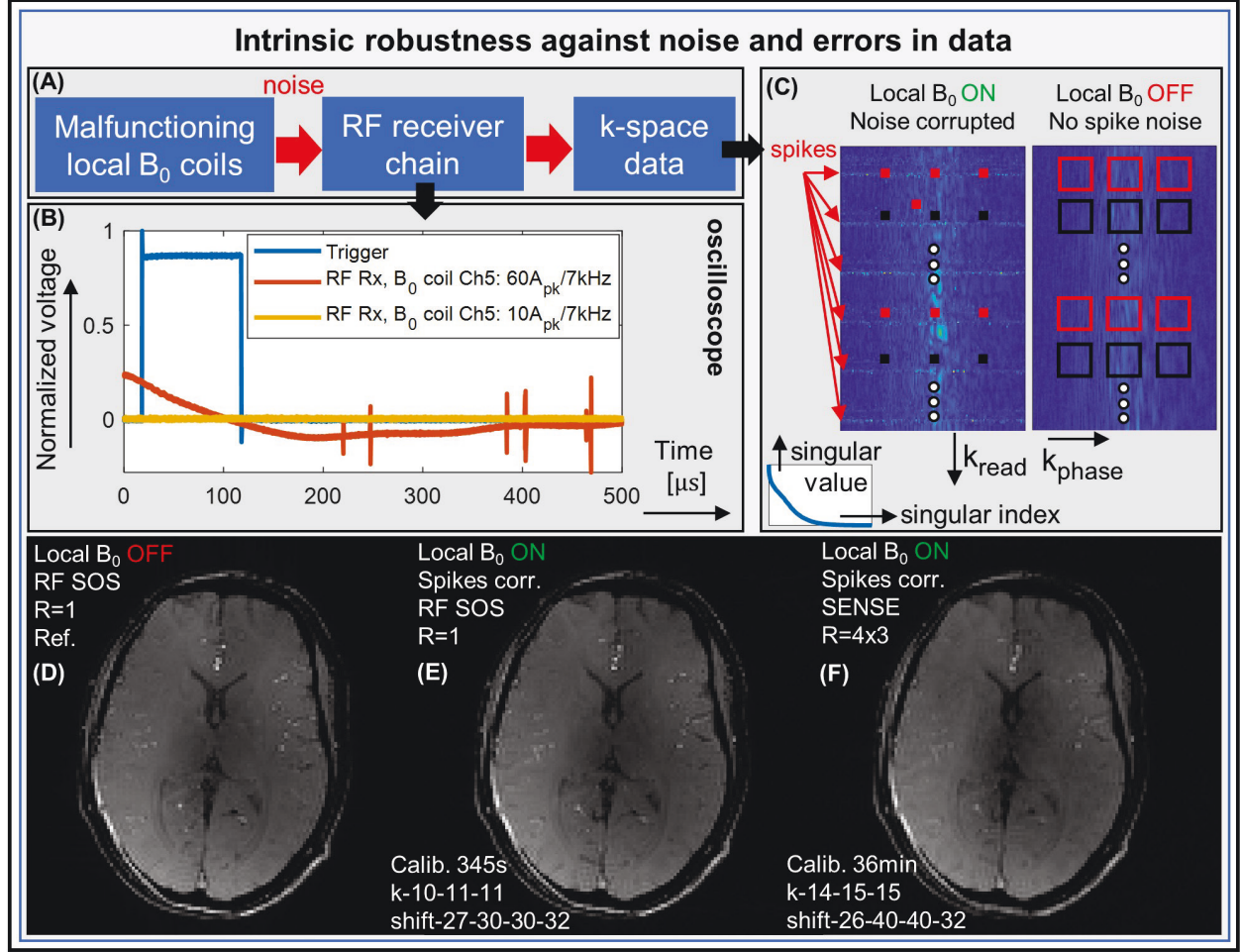


Figure 8. Noise resilience. (A) Malfunctioning local B_0 coils injected additional spike noise into the RF receiver chain. (B) The noise was detected by a high-speed oscilloscope probing the RF receiver chain (after pre-amp), when the sinusoidal coil current increases from $10A_{pk}$ (useless for modulation experiments) to $60A_{pk}$, possibly caused by e.g., electrical sparks. (C) Noise bursts are visible in the raw k-space data (field-modulated ACS), interfering the kernel estimation. (D, E, F) Despite this, both the fully-sampled and 4x3-fold accelerated reconstructions (joint B_0 +SENSE) remain clean without apparent residual artifacts. Subtle inter-scan (between reference and field-modulated) head rotations are also present, which further validates calibration stability.

4.4 In-vivo image acceleration

Figure 9 provides a comprehensive summary of in-vivo 2D FLASH (coronal plane, phase-encoding: head-feet axis) with optimized local B_0 modulation scheme (0-deg), to evaluate in-vivo reconstruction quality and sampling efficiency. Driven by in-phase sinusoidal currents, the local B_0 coils provided a nearly-linear z-gradient along phase-encoding dimension (note, different than Section 4.2), approximately realizing

2D bunched-phase-encoding. Joint acceleration using rapid B_0 modulation and multiple RF receivers (SENSE) yielded clean L_2 reconstruction from 7x undersampled k-space without noticeable residue artifacts but only subtle noise amplification, clearly outperforming SENSE-only acceleration. The G-factor map shows a mean/max of 1.39/2.88 and decreases further towards peripheral slices. Power (approximation error) and cardinal (noise amplifications) functions maps indicates smooth, well-covered k-space without abrupt hot spots. Additional compressed-sensing L_1 reconstruction further optimized the reconstruction quality, which was computed efficiently with joint data compression based on both B_0 and B_1 kernels (approx. total 10s/slice).

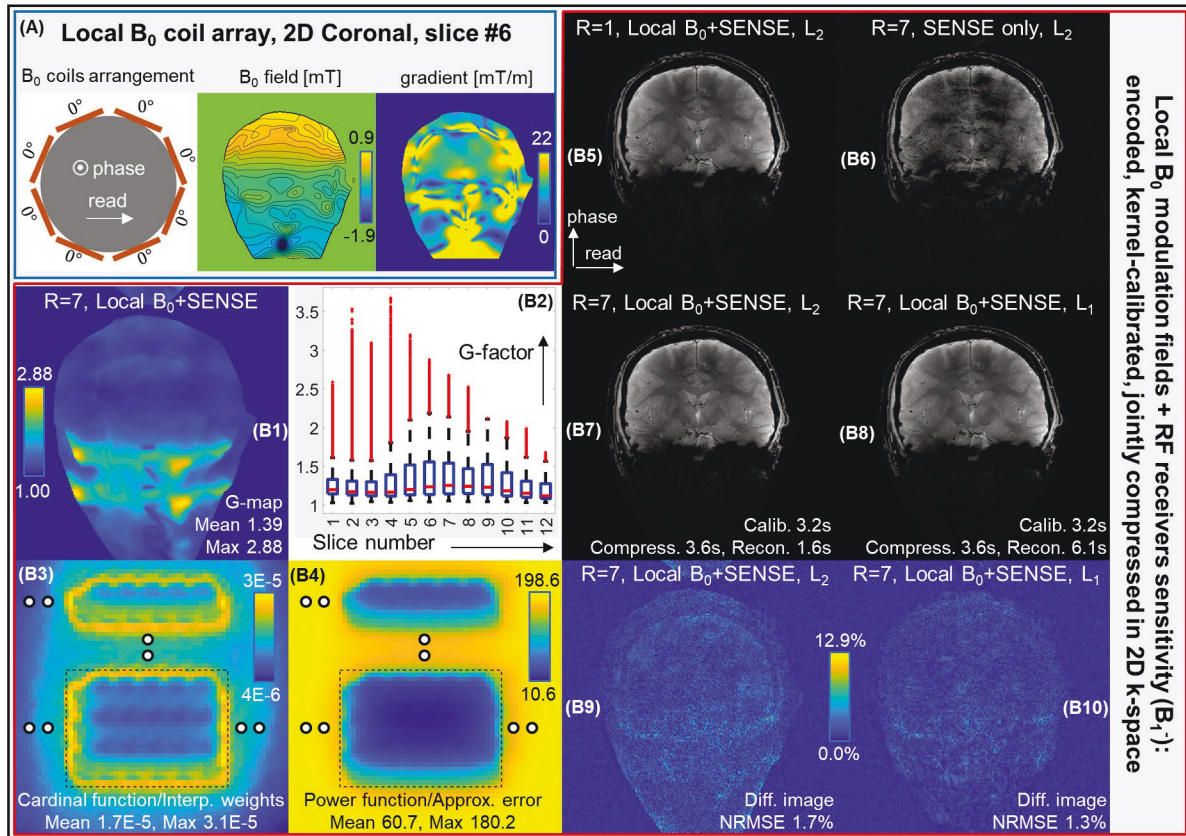


Figure 9. (A): The phase offsets in local B_0 coils, the magnetic field distribution generated at nearly-peak current, and the estimated gradient strength distribution (Euclidean-norm in two orthogonal dimensions) are shown. (B1-B4): The G-map, the G-factor boxplot for each of 12 slices, the k-space cardinal and power function maps are shown, which indicate sufficient sampling to reach 7x joint acceleration factor by B_0 modulations and SENSE. (B5-B10): The reference fully-sampled image with B_0 and SENSE reconstruction is successfully reconstructed. The L_2 reconstruction of 7x undersampled k-space has only subtle noise amplifications without noticeable residue fold-over artifacts. Its L_1 reconstruction further improves the image quality, as also

shown in their difference images, and is very close to the reference image. In comparison, the SENSE-only acceleration has severe artifacts.

Similarly, Figure 10 provides a comprehensive summary of in-vivo 3D FLASH with optimized local B_0 coil array and 3D Wave-CAIPI. The 4x3-fold joint acceleration of local B_0 coils and SENSE produced clean L_2 reconstruction images, with minimal fold-over residue artifacts, and only subtle noise amplification. The B_0 modulation field shape was approximately an off-center quadratic function in the transverse plane, and nearly-linear gradient along z-axis. The compressed-sensing L_1 reconstruction further improved the image quality, and was computed efficiently with joint data compression based on both B_0 and B_1 kernels (approx. 7 mins total/volume).

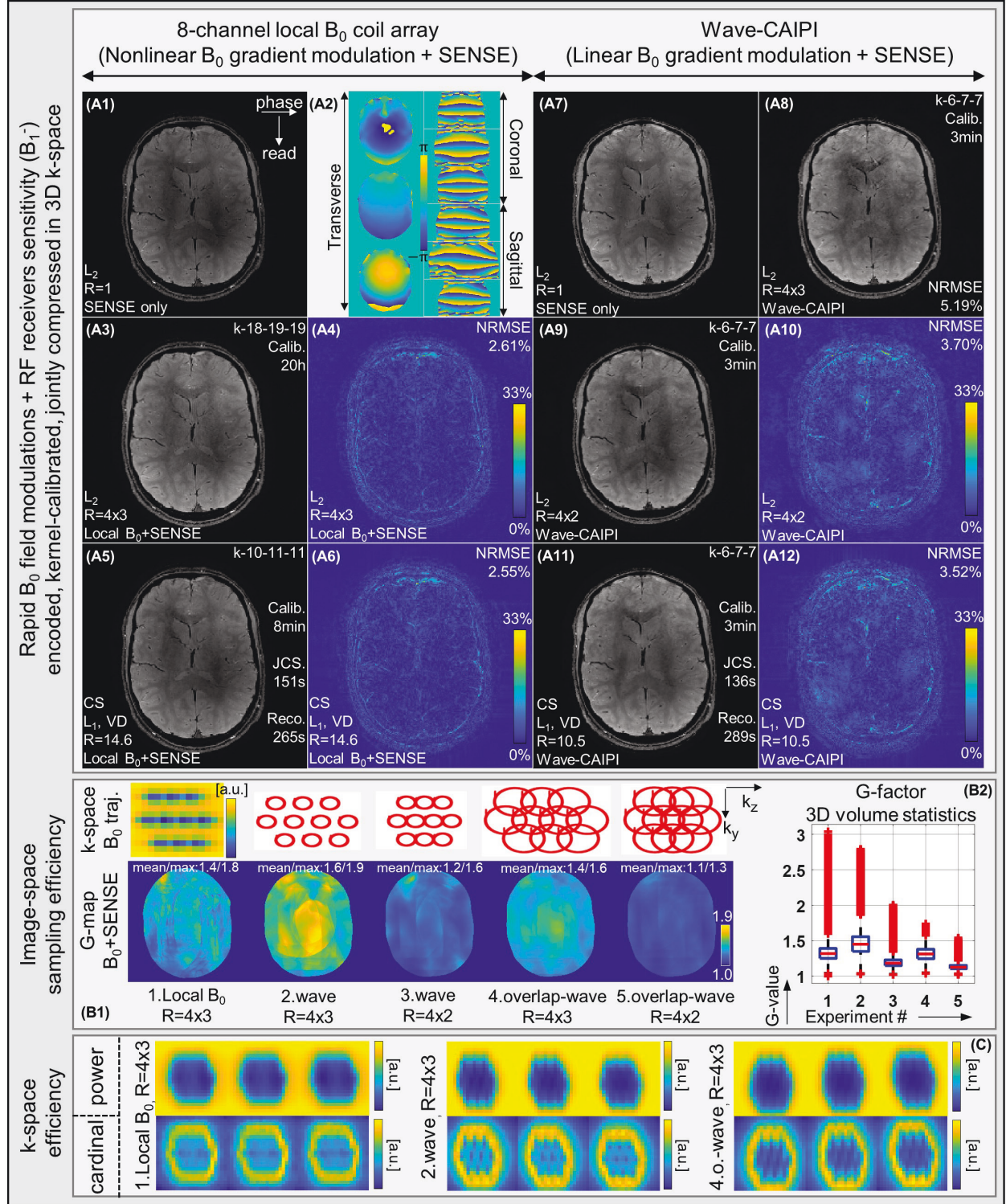


Figure 10. (A) 3D in-vivo MRI accelerated by local B_0 coils (nonlinear gradients) and Wave-CAIPI (linear gradients), together with SENSE. (A1, A7): The reference fully-sampled SENSE images without B_0 modulations. (A2): The additional spin phase evolution maps (near-peak) caused by local B_0 coil array are shown for transverse, coronal and sagittal planes, at the third-dimension locations $\frac{1}{4}$ FOV, $\frac{1}{2}$ FOV, $-\frac{1}{4}$ FOV. They are transverse off-center quadratic functions combined with z-gradients. (A3-A6): The

4x3-fold joint accelerated FLASH has only minor noise amplifications in L_2 reconstructions, with further optimized image quality in L_1 reconstruction with 14.6-fold variable-density undersampling. The difference images are mostly confined to vessels (low signal), edges and fat off-resonance regions. (A8): The 4x3-fold joint accelerated Wave-CAIPI with non-overlapped wave trajectory, with residue fold-over artifacts in L_2 reconstruction. (A9-A12): The 4x2-fold joint accelerated Wave-CAIPI has minor noise amplifications in L_2 reconstructions, and the image quality is further improved in the L_1 reconstruction with 10.5-fold variable-density undersampling. The difference images have errors in the low-frequency regions additional to edges and fat off-resonance areas, possibly due to subject motions. (B1): The k-space B_0 trajectories for five 3D in-vivo experiments are shown, including the k-space “stamp” by local B_0 array, and the pointwise wave-trajectories displayed in Pulseseq, under different undersampling factors. The slice-specific G-maps are shown that, the nonlinear gradients acceleration efficiency is nearly identical to the large “Wave” modulations (overlap-wave, $R=4\times 3$). (B2) The boxplot for the whole-volume G-factor (excluding 10 oversampled edge slices) is shown, where nonlinear gradient encoding (Experiment #1) reaches similar efficiency (25th – 75th percentiles) as the overlapped-wave trajectory (Experiment #4) given 4x3-fold acceleration. (C): The k-space cardinal and power function maps for 4x3-fold acceleration using nonlinear, non-overlapped wave, and overlapped-wave modulations, without displaying in a common scale. The nonlinear gradient modulations produce a smoother and slightly broader k-space coverage; however, an upward scaling is found, which might due to the summed interpolation weights (Frobenius norm) additionally over readout dimensions (3D kernels). The Wave-CAIPI trajectories show more non-uniform k-space coverage. More detailed interpretations are reserved for future investigations.

For comparison, the 4x2-fold accelerated wave-CAIPI with a non-overlapped wave trajectory also yielded clean L_2 and L_1 reconstructions. Although the overlapped wave trajectory with larger gradient currents was nominally capable of 4x3-fold acceleration, the resulting reconstructions were slightly degraded, likely related to strong inter-scan variations (>13mins gap between), and was mainly used for G-map comparisons.

In Figure 10(B1), the local B_0 modulation required larger calibration kernels to achieve artifact-free reconstruction than Wave-CAIPI, which uses a conventional pointwise k-space (B_0) trajectory. Notably, the volume G-factor (25th-75th percentiles) for 4x3-fold acceleration with the nonlinear B_0 gradients was nearly identical as the same acceleration factor achieved with scanner’s modulation gradients (both non-overlapped and overlapped waves). This again suggests that global linearity of the modulation gradients is not always required to achieve comparable acceleration performance.

In Figure 10(C), the power (approximation error) and cardinal (noise amplification) function maps show a smoother, slightly broader k-space coverage by local B_0 coils plus SENSE, than Wave-CAIPI (with

SENSE). However, these maps are shown on different scales. The maps for local B_0 coils appear scaled upward, possibly reflecting additional non-zero interpolation weights from the readout-dimension, which requires further investigations in the future.

4.5 Group-patch RKHS reconstruction: k-space subregion-wise interpolation

In Figure 11, the group-patch reconstruction was implemented and compared with the hybrid-space reconstruction (as in conventional Wave-CAIPI), as proof-of-concept without joint SENSE acceleration, data compression and parallelization.

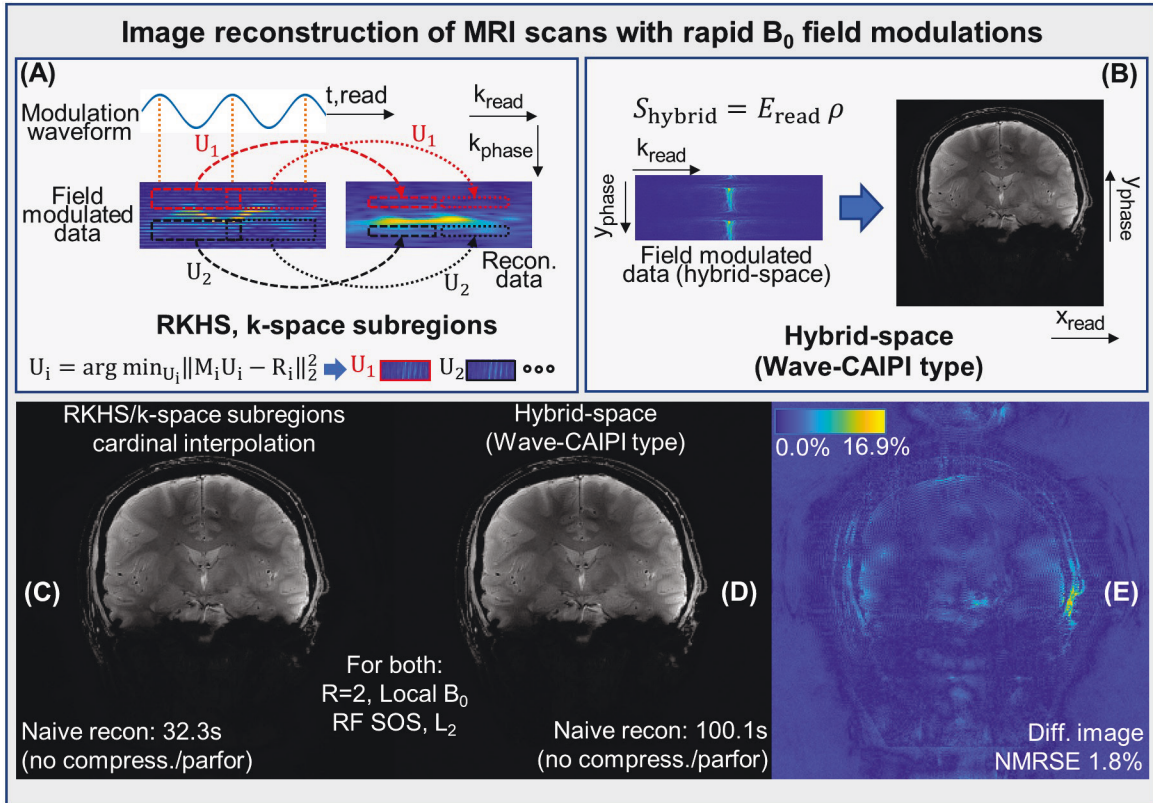


Figure 11. Comparison: recover the k-space data through RKHS interpolations and then yield the image through FFT, or reconstruct the image through hybrid-space as in Wave-CAIPI. (A) In group-patch reconstruction, the k-space data are decoupled in subregions, spanning periodically occurring B_0 kernels, along both phase-encoding and readout dimensions. Cardinal functions for distinct subregions (index i) are calculated based on the RKHS formalism, and applied for k-space interpolation. Appropriately defined subregion can further reduce peak memory and reconstruction time. (B) In hybrid-space reconstruction, the phase-encoding dimension(s) are solved by FFT, while the readout forward model is solved by general

inversion. Note, the k-space dataset and the reconstructed image only serves as illustration purposes, and size-ratio between their figures is not realistic. (C-D) The group-patch and hybrid-space reconstruction images are shown. Such cardinal function interpolation based on k-space subregions achieved about 3x lower computational time than solving forward models in hybrid-space, given 2x undersampled k-space, without compression, parallel computing and SENSE joint accelerations. (E) The difference image shows a subtle fold-over pattern. However, more detailed comparisons after combining joint B_0 - B_1 -kernel compression are reserved for future work. Note, the kernel calibration time, joint compression time, and reconstruction time are shown.

This proposed k-space interpolation-based reconstruction achieved 3x less computational time than solving the forward model in hybrid-space, by interpolating the acquired 2x undersampled k-space onto a readout-oversampled Fourier grid, in multiple subregions spanning several phase-encoded steps and a readout modulation period. A total of 24 cardinal function matrices were computed and reused, while several others were computed once for some k-space edges.

Theoretically, such k-space group-patch reconstruction can further support joint acceleration by rapid B_0 fields and RF receivers modulations, with parallel computing and joint compression of undersampled k-space data, which is reserved for future work. The joint compression technique will be described in a separate paper, although it has been applied in results of Figure 9 and 10 indicated by “JCS.”

Discussions and conclusions

According to the ex-vivo and in-vivo scans with rapid B_0 field modulations, this paper demonstrates that, the grouped-kernel continuous field calibration technique is robust to noise and various system imperfections. Additionally, the RKHS-based, group-patch reconstruction is feasible and efficient.

Such data-driven calibration enabled experimental tests of various B_0 modulation schemes along different FOV axes, which otherwise may become unrealistic e.g., by suffering from complex eddy currents. Global spatial linearity of the modulation gradients is not always required to achieve comparable image acceleration, complementing conclusions in the previous study. Although using our 8 local B_0 coils, nonlinear modulation gradients that can substantially surpass linear ones have not been

found yet, these findings can motivate further hardware designs that can generate more localized B_0 modulation fields.

A 3D nonlinear field (35_{pk} ampere per coil) as a combination of transverse quadratic field⁴⁵ and z-linear gradient delivered similar acceleration performance as Wave-CAIPI. The latter uses scanner gradient modulations, which relies on more elaborate wire patterns and higher electric power to impose linear gradients on the region of interest from the far away magnet bore. While both nonlinear and linear gradients might not be exhaustively optimized (e.g., modulation amplitudes, frequency), the grouped-kernel calibration has shown to be universally-applicable, and may support further searches over different gradient designs (e.g., more localized nonlinear gradients, high-performance head gradients) and different readout trajectories (high-bandwidth GRE, EPI⁴⁶).

All in-vivo data were retrospectively undersampled and cropped for ACS region, so subject motion was inevitable and likely larger than in real-time accelerated scans with integrated ACS. Even under these conditions, the grouped-kernel calibration remained stable, robustly extracting the spatiotemporal B_0 modulations via redundant k-space interpolations. Additionally, receiver-combined ACS data may be explored for faster kernel estimation. As matrix size increases, the conventional 3D Wave-CAIPI reconstruction can be substantially hindered with respect to speed and memory. However, interpolating k-space data stream in small 3D chunks (subregions) using a small reusable set of cardinal functions appears as a promising solution.

Notably, rather than advocating a specific sequence (e.g., BPE, Wave-CAIPI, a particular nonlinear field), the emphasis of this paper is on elucidating the principle of kernel calibration and reconstruction in k-space groups or subregions. To explicitly extract kernels, interpolation relationships can be established anywhere in k-space and grouped by instantaneous image-space modulation, as long as the two carefully-selected ACS regions differ only in the spatial modulations to estimate. With fully-sampled kernels or spin spatial-encoding maps, multidimensional data can be interpolated onto a reconstruction Fourier grid without being tied to the exact form of the learned local kernel.

Beyond rapid modulations (e.g., min. a few kHz) of nonlinear (e.g., local B_0 coils, FRONSAC) and linear B_0 gradients (BPE, Wave-CAIPI), the principle underlying the grouped-kernel calibration or reconstruction techniques might be adapted for other methods with critical necessity to explicitly extract spatial modulation functions (e.g., EPI, radial, fm-bSSFP, RF encoding, shot-to-shot signal variations⁴⁷, subpixel encoding^{48,49}). In future study, while our approaches and other kernel-based MRI methods may be extended to broader scenarios, their connections to other statistical learning algorithms backboneed by the RKHS theory is also interesting to explore.

Essentially, inspired by the RKHS view, nonlinear and linear B_0 modulations can now be encoded²⁴, kernel-auto-calibrated³⁴, jointly compressed, and reconstructed entirely in k-space, unifying the mathematical foundations of GRAPPA (B_1 kernels) and nonlinear gradients (B_0 kernels) encoding. Namely, the oversampled readout induces an additional acceleration dimension that is mathematically-equivalent to parallel imaging acceleration, pushing towards the ultimate MRI speed in a practical and reliable manner.

Appendix - Algorithm implementation examples

A1 Grouped-kernel continuous field calibration

Step 1: Acquire ACS pairs. Obtain two low-resolution fully-sampled ACS datasets, e.g., a standard ACS and a field-modulated ACS, either by integrated acquisition or by retrospective cropping of fully-sampled scans.

Step 2: Roughly estimate trajectory and kernel size. Make an initial guess of the field-modulated trajectory (e.g., peak amplitudes of the drifted B_0 kernels) to guide grouping of k-space points by their instantaneous spatial-modulations.

Step 3: Construct interpolation systems in groups. For each spatial-modulation state (group) at all readout and phase-encoding k-space locations, and all RF receiver channels, slide a mathematical window to collect source neighborhoods in the standard ACS and target points in the field-modulated ACS, e.g., to form a system of linear equations. Reverse direction interpolation may also be applicable.

Step 4: Solve each group's interpolation relationships with a numerical solver, e.g., pseudo-inverse, tSVD, LSQR, or neural networks, to obtain a shift-invariant kernel for that spatial-modulation state.

Step 5: repeat for all groups separately – possible with parallel computing – to recover a complete set of dynamic B_0 kernels. If full Fourier-grid kernels are required, they can be zero-filled and Fourier transformed to derive spatial phase evolution maps.

Step 6: The estimated time-varying kernels or a full spatiotemporal model of phase evolution maps can be used for image reconstruction, either through hybrid-space as in Wave-CAIPI, or via RKHS-based k-space interpolation. Theoretically, if interpolating kernels from field-modulated ACS to standard ACS, the SPIRiT-type reconstruction is also possible.

A2 Group-patch reconstruction with subregion-wise k-space interpolation

Step 1: Partition undersampled k-space data and reconstruction Fourier grid. Divide the full k-space grid into subregions, each span one or a few periodically occurring B_0 kernels, maximizing the reuse of interpolation weights later.

Step 2: Compute cardinal functions. For each subregion, formulate the RKHS interpolation equations as Equation (1-3) using the spatial-encoding maps at the source-subregion's k-space timepoints, and the reference spatial-encoding maps (e.g., DFT matrix) at the target-subregion's k-space timepoints. Solve a cardinal function matrix that maps a slightly larger (with margins) "source" region in undersampled k-space data, onto a target subregion on reconstructed Fourier grid. These cardinal function matrices can be reused for any other subregions sampled by the same set of B_0 kernels, sharing identical interpolation relationships between the source and target regions.

Step 3: Apply the computed cardinal function matrices to map each source subregion in undersampled data onto its target subregion in the reconstruction Fourier grid.

Step 4: Obtain the reconstructed image by Fourier transforming the reconstructed k-space data.

Data availability

The codes and example data will be available in public once the paper is accepted.

Acknowledgement

This study is supported by the ERC Advanced Grant (No. 834940).

The ex vivo brain phantom was with courtesy of the Institute of Clinical Anatomy and Cell Analysis, Department of Anatomy, Eberhard Karls University of Tübingen. The first author thanks Dr. Thomas Shiozawa (Institute of Clinical Anatomy and Cell Analysis) for assistance with sample preparation, and Dr. Gisela Hagberg for assistance in scanning this phantom.

The first author also thanks Stefan Plappert for guidance in programming the ADwin high-speed processor, Dr. Felix Breuer and Prof. Berkin Bilgic for helpful discussions about CAIPIRINHA reconstructions, Dr. Georgiy Solomakha, Dr. Nikolai Avdievitch, Dario Bosch and Dr. Yi Chen for assistance in debugging the hardware noise injected into the RF receiver system, Dr. Anagha Deshmane for discussions about GROG kernels.

We are grateful to the late Mirsat Memaj (Fine mechanical workshop, Max Planck Institute for Biological Cybernetics) for manufacturing the coil support of the 8-channel local B_0 coil array.

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Publication 3

Tian R, Scheffler K. Group-Patch Joint Compression: Compressing Dynamic B₀ and Static RF Spatial Modulations Across k-Space Subregion Groups for Highly Accelerated MRI. *Magn Reson Med*. 2026; 0:1–20. doi: 10.1002/mrm.70439.

The main manuscript is provided below.

RESEARCH ARTICLE OPEN ACCESS

Group-Patch Joint Compression: Compressing Dynamic B_0 and Static RF Spatial Modulations Across k-Space Subregion Groups for Highly Accelerated MRI

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Received: 13 October 2025 | **Revised:** 5 May 2026 | **Accepted:** 6 May 2026

Keywords: compressed sensing reconstruction | compression | FRONSAC | local B_0 coil array | parallel imaging | Wave-CAIPI

ABSTRACT

Purpose: To accelerate MRI further, rapid B_0 field modulations can be applied during oversampled readout to capture additional physical information, as in Wave-CAIPI/FRONSAC/local B_0 coils modulation techniques. These methods, however, turn the Fourier readout into a non-Fourier-encoded dimension that cannot be reconstructed by FFT, posing significant reconstruction challenges especially in compressed-sensing or neural-network frameworks.

Theory and Methods: Because the rapid B_0 modulations still vary slowly relative to the oversampled ADC dwell time, we exploit this encoding redundancy by compressing k-space patch-by-patch across subregions, each of which is jointly encoded by a distinct subset of B_0 and RF (receive) spatial encoding functions. For each subset, a compression matrix is computed once and reused to compress all patches encoded by the same B_0 -RF spatial modulations. This can be implemented by feeding subsets of B_0 and RF spatial encoding maps into an adapted conventional RF array compression algorithm, mimicking an expanded set of virtual receiver channels. This approach was evaluated on human brain scans at 9.4 T/3 T.

Results: The proposed group-patch joint compression achieves substantially higher compression factors than conventional RF-only compression, while minimally compromising encoding efficiency. Typically, joint compression factors of $11\times$ – $20\times$ led to negligible encoding loss, dramatically reducing reconstruction time and peak memory usage. For example, compressed-sensing reconstruction took 1.4–5.1 s/2D slice, 177 s–10.1 min/3D volume on a high-memory CPU node.

Conclusion: Given joint encoding of dynamic B_0 and static RF fields, compressing multidimensional k-space patches in separate groups outperforms compressing RF receivers alone. This substantially mitigates a fundamental computational bottleneck when combining rapid B_0 and RF-receiver modulations.

1 | Introduction

MRI [1] with conventional k-space [2–4] sampling is sequential and thus, inherently slow. Two basic strategies have emerged to effectively reduce scan time: capturing more physically-encoded

information within a shorter time window [5–16], and reconstructing high-quality images from undersampled data using advanced numerical methods [17–25]. These can be mathematically described by the classical forward model $y = Ex$ [10, 26, 27] with additional regularizations.

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When pushing k-space information throughput to its limits by utilizing both rapid B_0 and RF-receive field modulations, such as in Wave-CAIPI [16]/FRONSAC [28, 29]/local B_0 coils modulations [30], the Fourier readout will be turned into another oversampled (e.g., $8\times$), non-Fourier-encoded dimension [14–16, 28, 30–32] in addition to the RF receivers dimension [9–11, 33]. This leads to a reconstruction bottleneck [34], hindering their widespread adoption. Therefore, we propose a novel MRI compression strategy, called group-patch joint compression [35–37], to compress k-space data patches encoded by different B_0 and RF receive (B_1^-) spatial encoding functions (represented by image-space maps, or k-space kernels) in separate groups, each spanning a selected k-space subregion and all receiver channels. This generalizes compressions [38–42] of RF receiver array [43–46] (i.e., RF sensitivity maps [24] or GRAPPA kernels [11, 47]) to further compress data encoded by rapid B_0 field modulations (i.e., dynamic B_0 phase evolution maps [48] or kernels [49]). Note that, for the latter, the k-space coverage is determined by the time integral of gradient modulation currents, rather than by the maximum meaningful [47] GRAPPA kernel size that is constrained by the RF wavelength [50, 51].

Specifically, to surpass the parallel imaging acceleration limits, one could employ a faster k-space trajectory using high-performance gradients [52–57], or introduce rapid local wiggles [14–16, 28, 30, 32, 55, 58] to broaden [30, 47, 59] the trajectory's coverage. Both enrich the physical information acquired by the encoding operator, and are restricted by merely practical limits [60–62] due to peripheral-nerve-stimulation (PNS) and hardware capability. Currently, while high-performance gradients are increasingly available, applying rapid B_0 fields modulations for trajectory wiggles remains much less accessible. Although system imperfections (e.g., eddy currents, concomitant fields) induced by stronger gradient modulations [63] can be captured by field camera [64–66] or B_0 kernels' auto-calibration [67], a remaining fundamental challenge is that, fully capturing the spin signals modulated by rapid B_0 fields demands comparably fast ADC oversampling. This considerably increases the encoding matrix size along non-Fourier encoded dimensions. Although the Siemens ICE environment with parallel CPUs can accelerate the least-squares reconstruction for 1 mm^3 volumetric Wave-CAIPI scans to within a few minutes [41, 68, 69], higher resolution acquisitions and the compressed-sensing (CS) or neural-network-assisted iterative reconstructions remain critically constrained by computational burden [34]. Namely, the scan time becomes shorter, but consequently, the reconstruction time becomes longer.

Despite growing computational power, aggressively modulated and oversampled scans demand commensurately aggressive data compression to avoid prohibitive reconstruction time. Unfortunately, using brute-force SVD operation [70] to compress the entire k-space in a single step is impractical. Here, a key insight is that, the growth factor of the non-Fourier encoding dimensions (e.g., $8\times$ oversampling, 32 RF receivers) usually far exceeds reasonable acceleration factors (e.g., 4×3 for human brain), leading to a substantial increase in data volume despite the substantially reduced acquisition time. This mismatch indicates significant encoding redundancy among neighboring readout samples along wiggled trajectories, since the additionally superimposed

rapid B_0 fields still vary slowly (typically 2–20 kHz, sinusoidal) relative to the oversampled ADC dwell time (e.g., $3\text{ }\mu\text{s}$).

Thus, the proposed group-patch joint compression technique compresses the entire k-space patch-by-patch, based on predefined k-space subregion groups. Its novelty lies in leveraging the periodicity of the rapid B_0 field modulations, which is typically sinusoidal along readout and identical across TRs. Specifically, because the B_0 modulation pattern repeats, we compute a set of compression matrices for small-size k-space patches (e.g., each with 9 readout points, 32 receivers) whose samples fall within specific intervals of a B_0 modulation cycle, corresponding to groups spanning different adjacent intervals within one sinusoidal period. These group-specific compression matrices are computed once and reused to compress all k-space patches assigned to the same group indices that share identical B_0 -RF encoding functions across different k-space subregions and phase-encoding steps. Consequently, the full k-space compression reduces to patch-wise compression using only a small set of reusable group-specific compression matrices. To compute a compression matrix for a subregion group, we adapt a conventional RF array compression algorithm [38], by feeding it with all spatial encoding maps (instead of RF receiver maps only) induced by scanner gradients, local B_0 coils, and multiple RF receivers that encode this k-space subregion.

To characterize and experimentally test our method, we perform 2D and 3D Cartesian scans accelerated by the local B_0 coils modulations [30] (9.4 T) or by the scanner's linear gradient modulations (both 9.4 T and 3 T) in human MRI scanners. When using scanner's gradient modulations, the technique is categorized as Wave-CAIPI [16] if a regular undersampling mask with CAIPIRINHA pattern shifts [71] is used, or CS-Wave [72] if a random mask is used. In the broader context, this work extends our recently proposed MRI framework for nonlinear gradient modulations, where the entire acquisition pipeline can be understood and formulated in k-space: signal encoding [30], field calibration [67], and now, joint data compression to reduce unnecessary memory expansion and enable compressed-sensing reconstructions. Together, these building blocks unify MRI spatial encoding by dynamic B_0 fields and static RF receive fields, turning the oversampled readout axis coupled with rapid B_0 field modulations into another acceleration dimension that could be mathematically-similar to parallel imaging. Generally, the proposed group-patch joint compression is universally-applicable to image acceleration based on rapid magnetic fields modulation techniques, regardless of the hardware used to generate the modulations (e.g., scanner gradients [16, 34, 41, 68, 69, 73–76], gradient inserts [55, 59], FRONSAC [28, 29], large matrix gradient coils [77], local B_0 coils [30, 32], and switched RF coils [78, 79]).

2 | Theory

2.1 | Overview: Hardware, Pulse Sequence, and Data Processing

As illustrated in Figure 1, FLASH line-by-line scans (2D and 3D) are accelerated by superimposing additional rapid B_0 field modulations onto the frequency-encoding gradient during

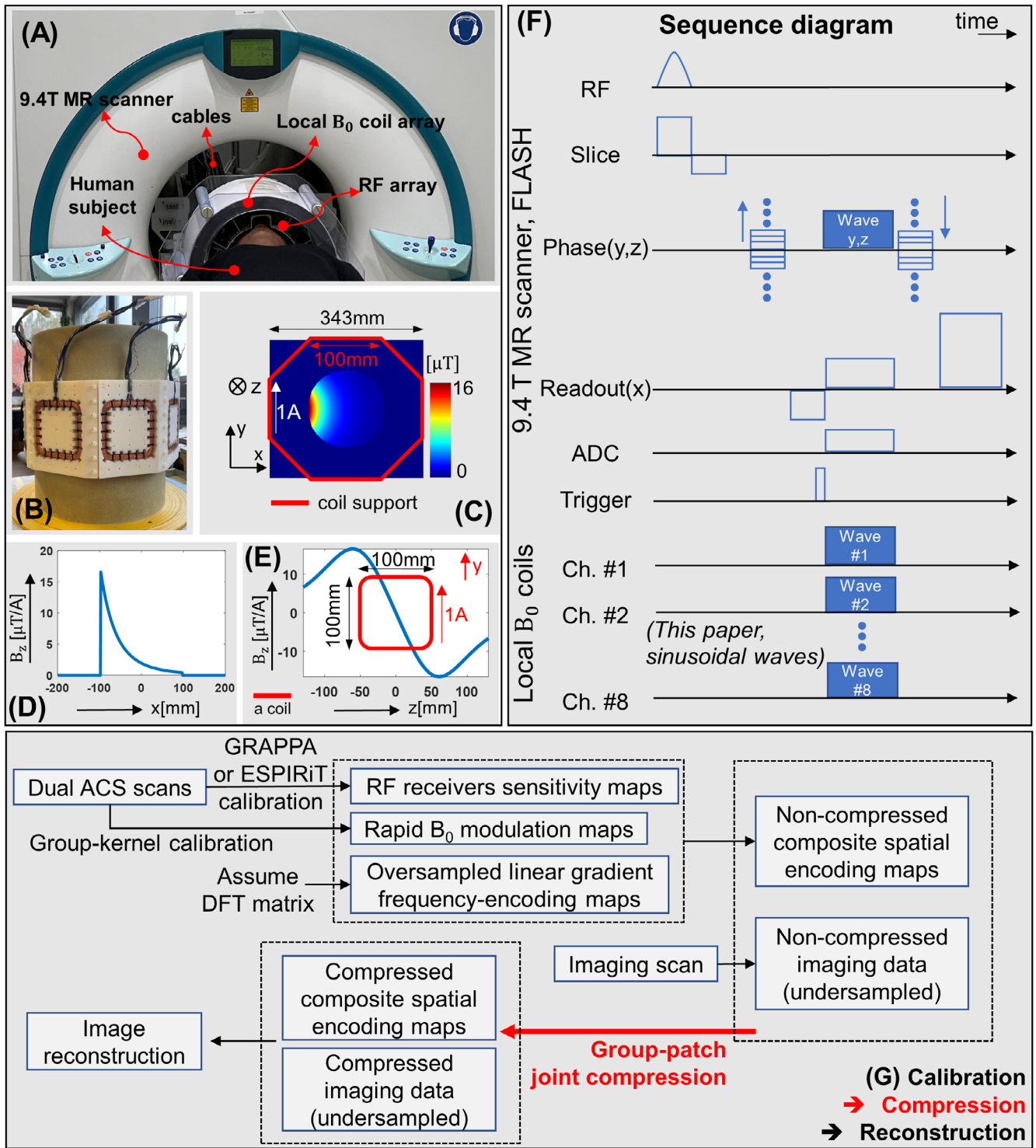


FIGURE 1 | System illustration. (A) The hardware setup in the scanner. (B) The 8-channel local B₀ coil array. (C) Simulated magnetic field map given 1A DC current in a single local B₀ coil, which generates up to around 16 μT within the effective field of view (FOV) defined by the inner boundary (around 97 mm to the FOV center) of an RF array. (D) The 1D plot along the central horizontal line in (C). (E) The variation of the in-plane maximal field strength generated by 1A in a local coil along the z direction, with the relative position to the coil winding indicated in red. (F) Pulse sequence diagram for accelerated MRI scans with rapid B₀ field modulations. In a standard FLASH sequence, during signal readout, either the local B₀ coil array or the scanner's phase-encoding gradients are switched on with current waveforms. Theoretically, the waveforms in the 8 local B₀ channels can be designed independently. Here, we apply sinusoidal currents for simplicity, to induce additional localized spin phase modulation on the objects to accelerate k-space acquisitions. (G) The data processing pipeline consists of calibration, compression and reconstruction steps. The group-patch joint compression proposed in this paper is highlighted in red.

signal readout (i.e., conventional frequency-encoding), using either a custom-built 8-channel local B_0 coil array placed outside the RF coil, or scanner's linear gradient for rapid modulations. Sinusoidal currents are applied to each channel of local B_0 coils or scanner gradients, imposing either local or global spin phase modulations, improving pixel disentanglement in image reconstruction using retrospectively undersampled k-space data. Prior to image reconstruction, the spatial encoding maps from dual auto-calibrating signal (ACS) scans [80] and undersampled imaging data are compressed using the proposed joint compression technique.

2.2 | Signal Encoding in Image-Space and k-Space

In Cartesian MRI (in this paper, FLASH) accelerated by rapid B_0 field modulations and multiple RF receivers, the signal encoding model that maps image-space to k-space can be written as follows:

$$S_n(t) = \int c_n(r) \rho(r) \exp\{-i[k(t)r + k_m(t, r)]\} dr, \quad (1)$$

$$k(t) = \gamma \int_0^t g(\tau) d\tau, \quad (2)$$

$$k_m(t, r) = \gamma \sum_{\lambda} \left[B_{\lambda}(r) \int_0^t I_{\lambda}(\tau) d\tau \right], \quad (3)$$

$S_n(t)$ is the time-domain signal obtained by the n th RF receiver coil at the time instant t . $k(t)$ is the k-space trajectory term (i.e., in rad/m) imposed by scanner's frequency and phase encoding, $k_m(t, r)$ is the phase evolution term imposed by the rapid B_0 gradient modulations (i.e., in rad). γ is the gyromagnetic ratio (i.e., in rad/s/T), r is the spin spatial location, $\rho(r)$ is the object spin density, $c_n(r)$ is the n th RF receiver sensitivity distribution, $g(\tau)$ is the shaped pulse of the linear gradients for the main trajectory (excluding rapid modulations), $B_{\lambda}(r)$ is the B_0 field distribution produced by unit current in the λ th local B_0 coil or scanner gradient element, $I_{\lambda}(\tau)$ is the current waveform in the λ th local coil or the scanner's linear gradient for rapid modulations only.

After discretization over space and stacking time samples and receiver channels:

$$S = E\rho, \quad (4)$$

$S \in \mathbb{C}^{N_t N_c \times 1}$ is the acquired k-space data vector with N_t samples from each of the N_c receivers. $E \in \mathbb{C}^{N_t N_c \times L}$ is the composite encoding matrix incorporating B_0 and RF encoding functions. $\rho \in \mathbb{C}^{L \times 1}$ is the image vector with L pixels. The total sample number of each receiver is the product of the sample counts along the three dimensions, that is, $N_t = N_{t_x} N_{t_y} N_{t_z}$.

For a fixed acquisition time instant t_0 , by the convolution theorem, the k-space signal in Equation (1) can also be seen as the convolution of the Fourier transforms of the three image-space maps, evaluated at a k-space location $k(t_0)$ or a sampling time instant t_0 . This signal model maps between k-space data acquired without and with field modulation, via interpolation with time-varying B_0 and time-invariant RF kernels.

$$S_n(t_0) = \left\{ \underbrace{\mathcal{F}[c_n(r)]}_{\text{RF receive sensitivity kernel}} * \underbrace{\mathcal{F}[\rho(r)]}_{\text{original k-space signal}} * \underbrace{\mathcal{F}\{\exp[-ik_m(t_0, r)]\}}_{\text{rapid } B_0 \text{ modulation kernel}} \right\}(t_0), \quad (5)$$

where asterisk $*$ is the convolution operation. Thus, the modulated sample $S_n(t_0)$ can be interpreted as a weighted sum of local neighborhoods of the original k-space data. The RF sensitivity induces a receiver-specific but time-invariant kernel, while the rapid B_0 modulation induces a receiver-invariant but time-varying kernel. For auto-calibration and interpolation reconstruction in parallel imaging, the multiple RF receive kernels with optionally omitted samples reduce to the well-known GRAPPA kernels.

The two-domain interpretations are illustrated in Figure 2. Here, two sets of k-space ACS regions are acquired without and with field modulations, which are used for B_0 field auto-calibration [67, 80]. Assuming that the ADC can sample faster than the Nyquist rate (determined by the FOV and gradient strength), the signal readout can be oversampled along time axis t (specifically, t_x in Figure 2). Without rapid B_0 modulations (i.e., $k_m(t, r) = 0$), this oversampling does not add new information; instead, if not averaged onto a non-oversampled grid, it simply increases the FOV along readout, introducing signal-free empty background. When switching on the B_0 modulation gradients during oversampling, the neighboring readout samples become informative because the modulation additionally mixes the phase-encoding dimensions. In Figure 2C–E, this is visualized by the kernel “cubes” in the standard ACS block, which depict the local support of the B_0 modulation kernels (reducing to an off-center point in the special case of purely linear gradient modulation). The kernel support—that is, the cube width or off-center displacement—reflects the extent of B_0 -modulation-induced encoding along phase-encoding dimensions that speeds up acquisitions. Thus, rapid B_0 field modulations—regardless of whether linear (e.g., Wave-CAIPI) or nonlinear (e.g., FRON-SAC, local B_0 coils modulations) gradients are used—modify an otherwise redundant oversampled readout to capture useful phase-encoding information.

2.3 | Multidimensional Compressibility and Periodic Redundancy

For reconstruction, although keeping the modulation scheme identical across phase-encoded steps allows efficient use of FFT for reconstructing phase-encoding dimensions [16, 32, 81], the remaining non-Fourier readout encoding model to be inverted scales with the product of the readout oversampling factor (e.g., $\times 8$) and the number of RF receiver channels (e.g., $\times 32$). This becomes even more computationally demanding in compressed-sensing with variable-density-Poisson-disc (VD-PD) [82] undersampling patterns, since the readout forward model becomes difficult to decompose into small groups of aliased phase-encoded pixels [74, 83].

However, the period of the oscillating B_0 fields typically ranges from 500 to 50 μ s. As in Figure 2A,B, although the additional B_0 field modulation varies rapidly relative to the scanner's readout gradient (DC) used for conventional frequency-encoding,

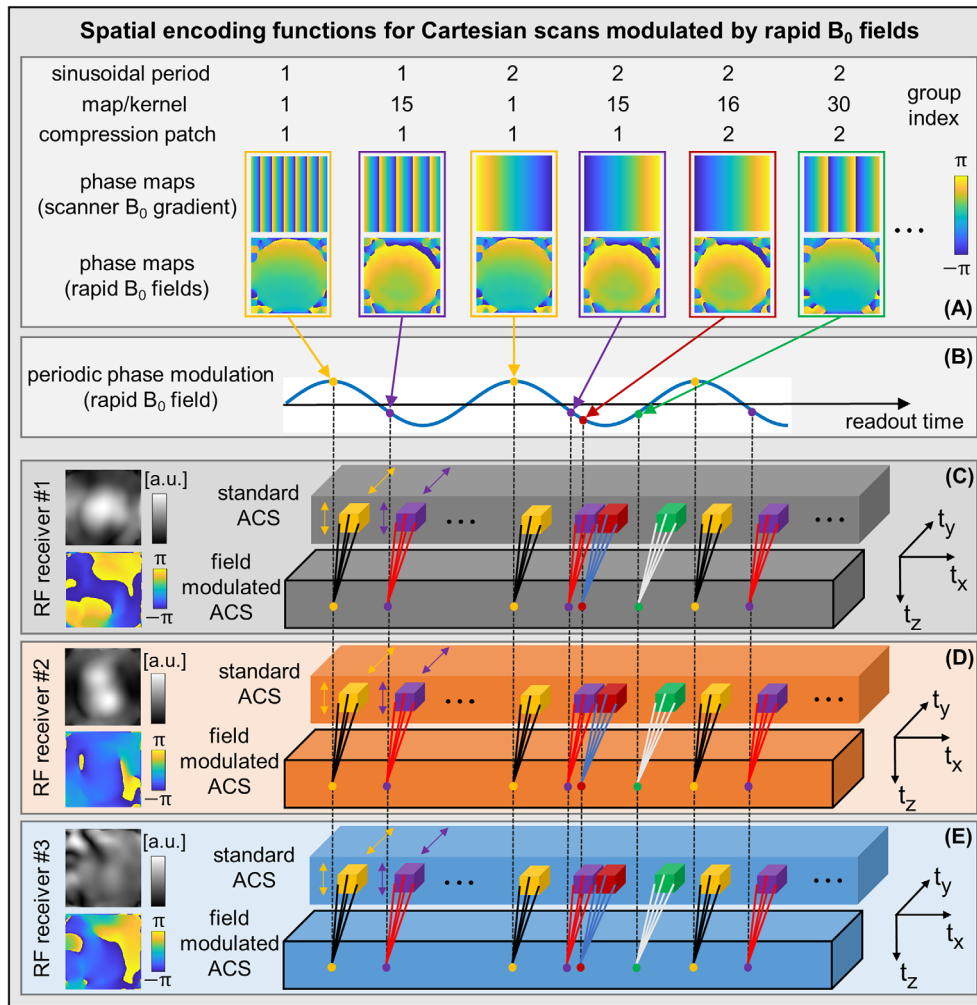


FIGURE 2 | Signal encoding in image-space and k-space, accelerated by rapid B_0 modulation and static RF receivers sensitivities. (A) The spin phase maps induced by static (i.e., scanner frequency-encoding) and dynamic (i.e., additional modulation) B_0 fields, and the group indices for the B_0 kernels (auto-calibration purpose) and k-space subregions (compression purpose) along the readout time axis. (B) Sinusoidal modulation waveform illustrating the modulation phase of the oscillating B_0 fields, regardless of whether their spatial patterns are linear or nonlinear. (C-E) The k-space view of rapid B_0 field modulation, given three RF receiver channels. In this example, two fully-sampled ACS datasets are acquired to auto-calibrate the B_0 modulation kernels using GRE line-by-line scans. The standard ACS (no black outline) can also be used separately for parallel imaging calibration. The field-modulated ACS block (black outline) is acquired under additional rapid B_0 modulation; consequently, each sample can be expressed as the sum value of a weighted neighborhood (kernel) of the original k-space data in the non-modulated standard ACS block. The time-varying kernels are depicted by small cubes in different colors, indicating different modulation phases in a sinusoidal period. Different colors for ACS blocks denote different RF receive channels, with corresponding sensitivity maps shown. Auto-calibration of the phase evolution maps imposed by rapid B_0 modulation is performed by solving these k-space interpolation relationships within each kernel group. Note that, both the auto-calibration and the joint compression exploit the periodicity of rapid B_0 modulations, but with different ways of grouping. Generally, the time-varying B_0 kernels can be partitioned into time-invariant subsets—either time-invariant kernels or time-invariant encoding subregions—by grouping samples that share the same sinusoidal modulation phase across the entire k-space.

it still changes slowly relative to the scanner's oversampled ADC dwell time of a few μs . To exploit the resulting encoding redundancy across adjacent oversampling timepoints, the proposed group-patch joint compression technique jointly compresses small patches of neighboring readout samples across all RF receivers in groups. Leveraging the sinusoidal periodicity, we partition the readout axis into sub-intervals (corresponding to k-space different subregions) within one B_0 modulation period and assign each sub-interval a compression group index; the same set of indices repeats for every subsequent B_0 modulation period, so that samples at the same relative position within another

modulation cycle can be compressed using the same group-specific compression matrix.

Separately, B_0 modulation kernels can also be assigned kernel group indices for the auto-calibration purpose [67, 80]. These kernel indices are defined per readout time instant and repeat across sinusoidal cycles, and therefore differ from the compression grouping that aggregates multiple neighboring readout samples into a patch. Nevertheless, for a given modulation phase instant or segment, the B_0 encoding functions are shift-invariant across modulation cycles.

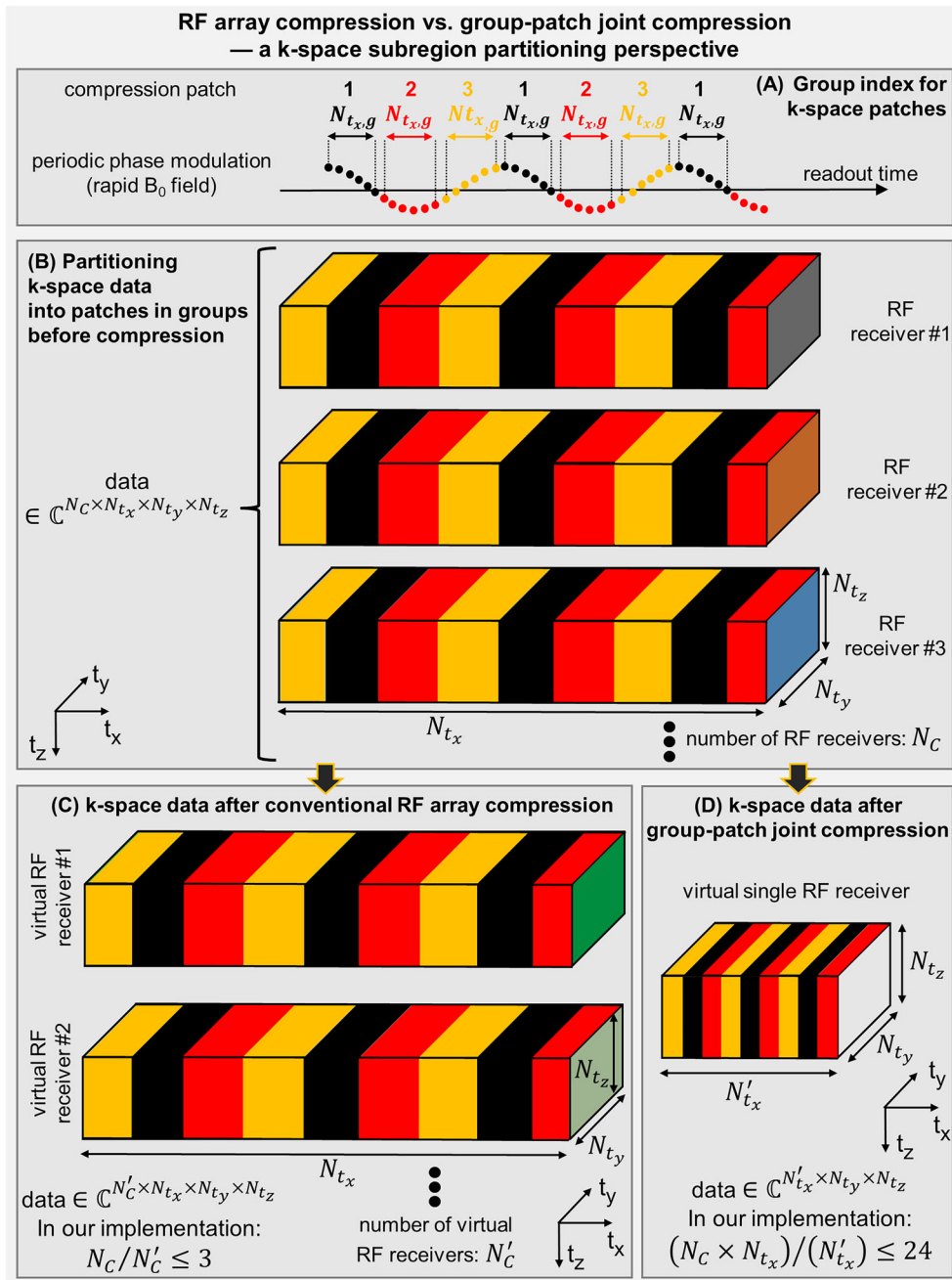


FIGURE 3 | Comparison between conventional RF array compression and the proposed group-patch joint compression, from the perspective of k-space subregion partitioning. (A) Group indices defining k-space compression patches/subregions. The readout time axis is divided into segments spanning, for example, $N_{t_x,8}$ timepoints/samples. Because of the periodicity of rapid B_0 modulation, each sinusoidal period can be partitioned into a small number of segments governed by distinct B_0 encoding functions, which repeat across consecutive sinusoidal periods. (B) Visualization of the k-space data volume prior to compression, showing readout oversampling in different k-space subregions (black, red, yellow) and multiple RF receiver channels (gray, orange, blue). (C) Data volume after conventional RF array compression. The receiver dimension is reduced while all other k-space dimensions remain unchanged. In our implementations, to avoid loss in encoding efficiency, the compression factor (RF-only) may need to stay below 3×. (D) Data volume after the proposed joint compression. The RF receive channels and the readout subregions are compressed jointly, yielding dimensionality reduction along both receiver and readout dimensions. In our implementation, up to 24× compression factor (joint compression) could be possible without noticeable loss in encoding efficiency.

2.4 | From RF Array to Joint B_0 -RF Compression

To explicitly visualize how the compressed spatial encoding maps evolve with increasing compression factors, we compute

group-specific compression matrices by feeding composite spatial encoding maps into a conventional sensitivity-based RF array compression method [38]. Now, these maps incorporate both B_0 (frequency-encoding, rapid modulation) and RF (receive) fields that jointly encode each subregion group. By treating the joint

B_0 and RF spatial encoding maps as an expanded set of virtual receiver maps, this implementation also inherits the advantage of this array compression algorithm based on explicit receiver maps and yield the optimized compression matrices computed from the encoding maps within the region-of-interest (ROI). In contrast, as in Figure 3, the RF-only compression reduces only the receiver-channel dimension, whereas our joint B_0 -RF compression compresses both the receiver and segmented-readout dimensions using a group-specific operation. Theoretically, joint compression may also be performed using k-space data alone by adapting alternative receiver-compression algorithms [39, 40, 42] to the same subregion-wise formulation.

While a full derivation of the original RF array compression method can be found in reference [38], we summarize our compression steps for each k-space subregion group g as below. These steps are repeated for different compression groups, that together span full modulation cycles across the entire k-space.

1. Form an ROI-aggregated square matrix P_g from the composite spatial encoding maps.
2. Compute a compression matrix A_g from U_g^H and the receivers noise-decorrelation matrix T . U_g^H is obtained via singular-value decomposition (SVD) or eigenvalue decomposition of the matrix P_g .
3. Apply A_g to compress the corresponding k-space patches and composite spatial encoding maps, which are then used for image reconstruction.

First, we define a composite encoding vector at a pixel $r \in ROI$, cropped from E in Equation (4):

$$E_g[r] = E_{LG,g}[r] \odot E_{NG,g}[r] \odot E_{RF}[r] \in \mathbb{C}^{N_s \times 1}, \quad (6)$$

where $N_s = N_C N_{t_x,g}$, $N_{t_x,g}$ is the number of oversampled readout timepoints within one B_0 modulation cycle in the group g , N_C is the total receiver channels. $E_g[r]$ is constructed by elementwise multiplication $\odot$ of all spatial encoding vectors at this pixel, induced by the scanner readout gradient ($E_{LG,g}[r] \in \mathbb{C}^{N_s \times 1}$), local B_0 coils ($E_{NG,g}[r] \in \mathbb{C}^{N_s \times 1}$) and RF sensitivity maps ($E_{RF}[r] \in \mathbb{C}^{N_s \times 1}$) (optionally noise-decorrelated, repeated across k-space locations). For RF-only compression, the Equation (6) reduces to $E_g[r] = E_{RF}[r]$.

Consequently, we form the square matrix $P_g \in \mathbb{C}^{N_s \times N_s}$, by taking the outer product between the composite encoding column vector and its pseudo-inverse row vector, and summing over L spatial pixels within the ROI (e.g., image mask).

$$P_g = \sum_{r \in ROI} E_g[r] E_g^\dagger[r], \quad (7)$$

$$= U_g F_g U_g^H, \quad (8)$$

where $(\cdot)^\dagger$ is the pseudo-inverse operation that leads to the optimized compression due to orthonormalization of the encoding vectors [38]. $(\cdot)^H$ denotes conjugate transpose. The unitary matrix U_g^H contains the eigenvectors of P_g , which can be obtained via SVD/eigenvalue decomposition in practice.

By the definition of pseudo-inverse $E_g^\dagger[r] = (E_g^H[r] E_g[r])^{-1} E_g^H[r]$, since $E_g[r]$ is a column vector, $(E_g^H[r] E_g[r])^{-1}$ is a scalar. Thus, we can compute P_g efficiently by rewriting it as direct matrix multiplication operations, without the time-consuming explicit per-pixel vector inversion and the subsequent spatial summation:

$$P_g = \sum_{r \in ROI} \frac{E_g[r]}{E_g^H[r] E_g[r]} E_g^H[r], \quad (9)$$

$$= (E_g N) E_g^H, \quad (10)$$

$$\text{with } N = \text{diag} \left[1 / \left(\sum_{N_s} E_g \odot E_g^* \right) \right], \quad (11)$$

where $E_g \in \mathbb{C}^{N_s \times L}$ is the composite spatial encoding maps for group g , that span N_s k-space positions and L pixels, $(\cdot)^*$ is the complex conjugate operation, $N \in \mathbb{C}^{L \times L}$ is a diagonal matrix with a scaling vector $1 / (\sum_{N_s} E_g \odot E_g^*) \in \mathbb{C}^{1 \times L}$ at its diagonal for pixel-dependent normalization. This scaling vector is obtained by taking elementwise square operation of E_g , summing over along k-space and receiver dimensions as $\sum_{N_s} E_g \odot E_g^*$, and taking the inverse.

Second, a compression matrix $A_g \in \mathbb{C}^{N'_s \times N_s}$ can be obtained:

$$A_g = C_g U_g^H T, \quad (12)$$

where T is the receiver noise-whitening transform (e.g., obtained from Cholesky factorization of the noise covariance matrix and inversion of the decomposed Q , as $\Psi = Q Q^H \rightarrow T = Q^{-1}$), U_g^H is the unitary matrix obtained via e.g., SVD or eigenvalue decomposition of the matrix P_g as in Equation (8), and C_g is a selection matrix that retains the first N'_s rows of U_g^H . The singular-value (or eigen-value) spectrum from F_g in Equation (8) can quantify the compressibility of the spatial encoding functions, indicating a reasonable value of N'_s to preserve the essential information.

Third, the compression matrix A_g is used to compress the k-space data patches $S_g \in \mathbb{C}^{N_s \times G_{t_x,g} N_{t_y} N_{t_z}}$ across all sinusoidal modulation cycles along readout and phase-encoded steps, and the corresponding composite spatial encoding maps E_g belonging to this compression group g . $G_{t_x,g}$ is the repetition time of the group g along readout time axis. N_{t_y} and N_{t_z} are the phase-encoded steps along two dimensions, respectively. This compression matrix can be efficiently reused across different modulation cycles, since these k-space subregions are encoded by the identical spatial modulations caused by rapid B_0 and RF receivers fields, and only differs in the k-space locations.

$$S'_g = A_g S_g, \quad (13)$$

$$E'_g = A_g E_g, \quad (14)$$

The resulted compressed k-space data $S'_g \in \mathbb{C}^{N'_s \times G_{t_x,g} N_{t_y} N_{t_z}}$ and the virtual spatial encoding maps $E'_g \in \mathbb{C}^{N'_s \times L}$ in different groups are combined into a reduced forward model as below, used for least-squares [16, 30, 32, 81] or compressed-sensing [34] image

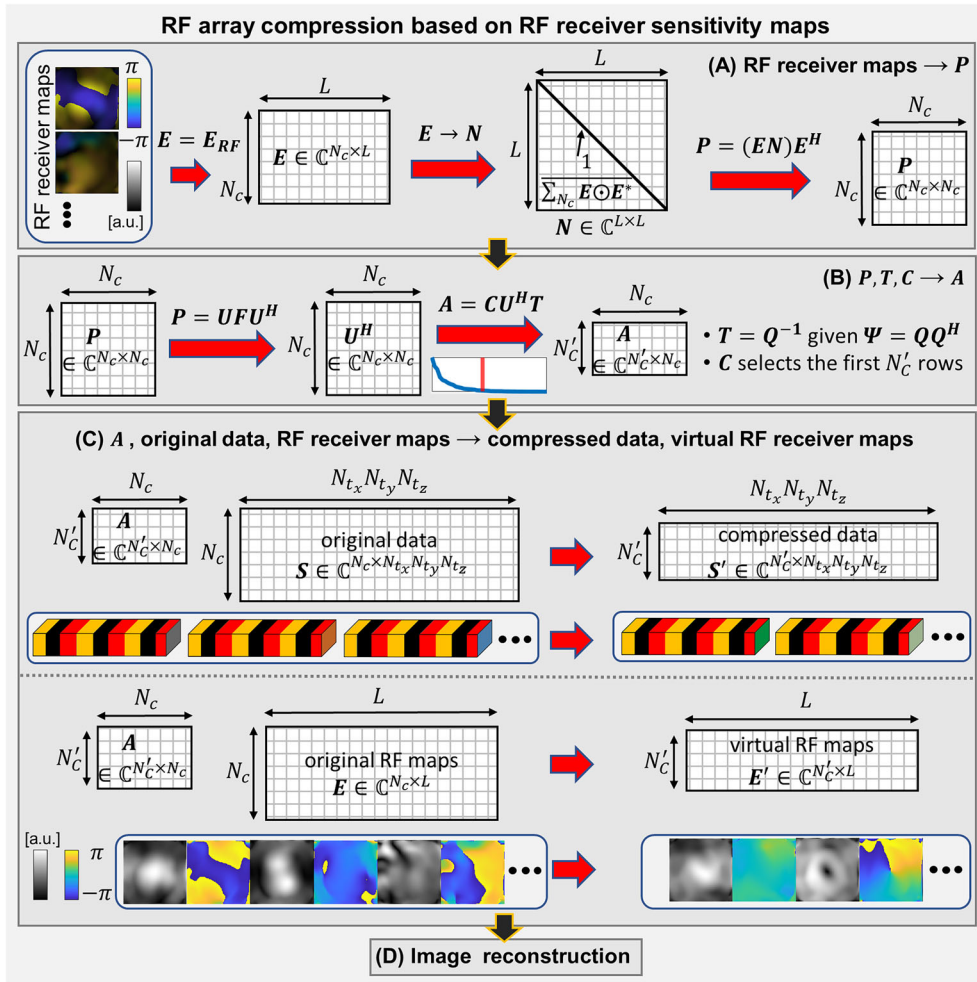


FIGURE 4 | Visualization of the conventional RF array compression, using similar notations for the joint compression but without group index g . (A) Construction of the square matrix P from RF receiver maps E_{RF} , with N_c receivers and L spatial pixels. Note that the RF sensitivity maps are displayed with overlaid amplitude (grayscale) and phase (parula) to save figure space. Amplitude variations are indicated by the non-uniform darkness within the maps. (B) Calculation of the compression matrix A from P , the noise decorrelation matrix T , and the selection matrix C . Namely, an SVD on P with subspace thresholding by C is utilized to yield the compression matrix for RF sensitivity compression alone. N_c receivers will be compressed to N'_c virtual receivers. (C) Application of A to compress the k-space data and sensitivity maps along receiver dimension only. Colored k-space subregions are shown to emphasize that no dimensionality reduction is performed along the readout axis. (D) The compressed k-space data and the compressed RF receiver maps are loaded into the image reconstruction pipeline to reconstruct the final image with less computational time.

reconstruction. The RF array compression and the group-patch joint compression are visually illustrated in Figures 4 and 5.

$$S' = E' \rho, \quad (15)$$

where $S' \in \mathbb{C}^{N'_{tx} N'_{ty} N'_{tz} \times 1}$ and $E' \in \mathbb{C}^{N'_{tx} N'_{ty} N'_{tz} \times L}$. $N'_{tx} = N'_s G_{t_x, g}$ is the compressed virtual dimension of the joint readout-RF receivers space.

3 | Methods

3.1 | Hardware Setup and Coil Configuration

MRI experiments were performed on a 9.4 T whole-body human scanner equipped with SC72 gradients (max. 200 T/m/s, max. 70 mT/m), and a 3 T whole-body Prisma^{fit} human scanner equipped with XR gradients (max. 200 T/m/s, max. 80 mT/m)

(Siemens Healthineers, Erlangen, Germany). At 9.4 T, for non-linear gradient modulation scans, an 8-channel local B_0 coil array [30] was fixed on the patient table, surrounding the image region of interest. A 16-transceive/32-receive RF coil array, modified from the reference [84] and using circularly polarized (CP) mode for RF transmission, was placed inside the local B_0 coil array's hollow cylindrical support. For Wave-CAIPI/CS-Wave using scanner gradient modulations, the local B_0 coil array is switched off. At 3 T, 34 or 44 elements of a 64-channel RF receive array were used.

3.2 | Pulse Sequence and Data Acquisition

Line-by-line 2D/3D FLASH-based sequences (modified IDEA sequence for simultaneously driving local B_0 coils [30], Pulseseq sequence for Wave-CAIPI/CS-Wave) were used for ex vivo and in vivo scans, with $8\times$ readout oversampling and $3 \mu s$

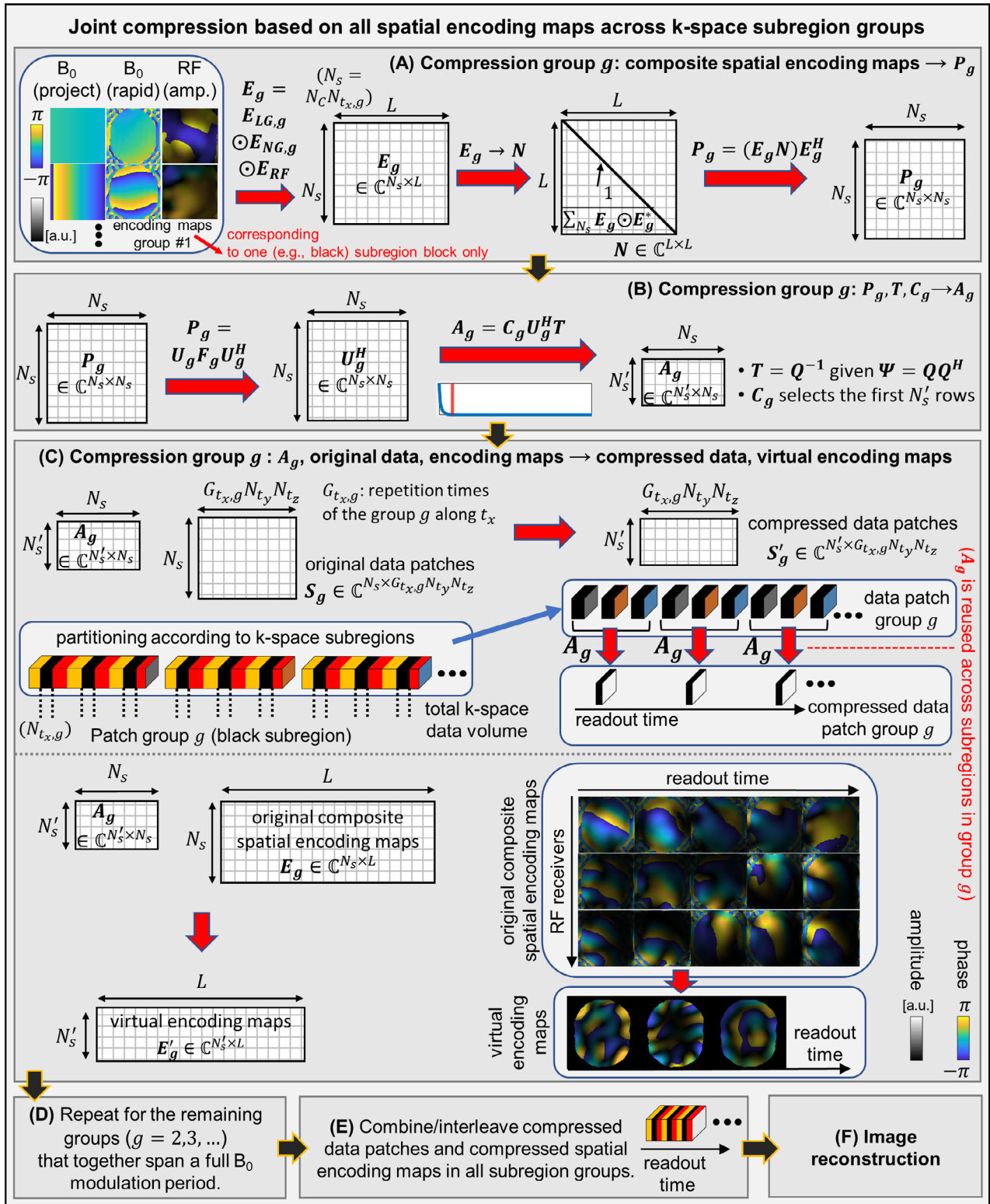


FIGURE 5 | Visualization of the proposed group-patch joint compression. (A) Construction of the square matrix P_g from all spatial encoding maps associated with the subregion group g , with N_s readout samples from all receivers and L spatial pixels. (B) Calculation of the group-specific compression matrix A_g from P_g and the noise decorrelation matrix T . An SVD on P_g with subspace thresholding by C_g is applied to compress the composite signal encoding space, from N_s samples to N'_s samples. (C) Application of A_g to jointly compress the k-space data and composite spatial encoding maps along both receiver and readout dimensions. For a given sinusoidal modulation cycle, k-space patches belonging to the group g (here, the black subregion) from different receiver channels (gray/orange/blue) are compressed together. Patches from the same group occurring in other modulation cycles and across phase-encoded dimensions are compressed using the same compression matrix, avoiding the need to compute a separate compression matrix at a different k-space location. (D) This procedure is repeated for all subregion groups. (E, F) The resulting compressed patches and compressed composite encoding maps are then interleaved/combined to form a reduced signal encoding model for faster image reconstruction.

scanner ADC dwell time. Several healthy adults volunteers were scanned, in agreement with the institution's ethics policy. During the signal readout (i.e., scanner's frequency-encoding), the local B_0 coil array or scanner's phase-encoding gradients were driven by independent sinusoidal modulations of each channel. Identical rapid B_0 modulations were used across TRs for simplicity. The optimized modulation schemes in previous studies [30, 67] were used to maximize acceleration capability of local B_0 coils. The nonlinear gradient strengths and the least-squares encoding efficiency maps (e.g., G-map, power function maps) of the in vivo scans are presented in [Supporting Information](#) and reference [30]. In 2D coronal scans, the local B_0 coils produced a nearly-linear z-gradient as bunched phase encoding (BPE). In 3D scans, they produced a quadratic function in transverse plane coupled with a nearly-linear gradient along z, as a form of nonlinear gradient modulations. Fully-sampled low-resolution FLASH with and without rapid B_0 modulations were acquired as dual ACS, to calibrate B_0 field modulations via B_0 kernel model [80]. Fully-sampled imaging datasets were acquired and retrospectively undersampled. Note that, this helps evaluating reconstructions using various k-space undersampling masks conveniently based on the same dataset, but can increase susceptibility to motions during lengthy scans.

Essential sequence protocols are summarized below; additional details are in the [Supporting Information](#). In Figures 7 and S1, the 2D multi-slice ex vivo scan at 9.4T has a matrix-size of 180×180 ($1.1 \times 1.1 \text{ mm}^2$). In Figure S2, the 3D ex vivo scan at 9.4T has matrix-size of $180 \times 180 \times 80$ ($1.1 \times 1.1 \times 1.0 \text{ mm}^3$). In Figures S3 and S4, the 2D multi-slice in vivo scan at 9.4T has a matrix-size 280×280 ($0.89 \times 0.89 \text{ mm}^2$). In Figures 8 and S4, the in vivo 3D scan at 9.4T has a matrix-size $230 \times 230 \times 112$ ($1.0 \times 1.0 \times 1.0 \text{ mm}^3$). In Figure 9, the in vivo 3D scan at 3T has a matrix-size $230 \times 230 \times 144$ ($1.0 \times 1.0 \times 1.0 \text{ mm}^3$). The TE/TR is 4–6.07/30 ms at 9.4T, 20/35 ms at 3T. The readout bandwidths are 150–230 Hz/Pixel. The ADC dwell time is $3 \mu\text{s}$. The sinusoidal currents in local B_0 coils are $36A_{\text{pk}} - 40A_{\text{pk}}$ per channel, 7.41 kHz. The Wave-CAIPI/CS-Wave has 24 wave-cycles with wave-gradients in y-3.4-z-5.3 mT/m and 4.17 kHz at 9.4T, and 19 wave-cycles with y-6.4-z-7.7 mT/m and 3.33 kHz at 3T. The 2D slice-thickness is 3 mm. The 3D slice oversampling is 26.3%–33.3%.

3.3 | Two Data Compression Approaches

For comparison, the conventional RF array compression [38] was performed as the baseline, based on the RF sensitivity maps estimated from standard ACS data (GRE) via ESPIRiT [24]. In the proposed joint compression approach, the oversampled readout axis was subdivided into subregions, each spanning 5–16 neighboring data points with all receiver channels. For each subregion, the composite encoding matrix is obtained from the synthesized DFT matrix, the rapid B_0 phase evolution maps auto-calibrated via k-space kernels [67], and the RF sensitivity maps used for RF array compression. Typically, one B_0 sinusoidal modulation period spans 45 readout timepoints using local B_0 coils, and 80–100 timepoints using the scanner's modulation gradients as in Wave-CAIPI/CS-Wave. Since identical rapid B_0 modulation was used across phase-encoded steps, k-space data acquired in

different TRs can be compressed by the same group-specific compression matrices.

3.4 | Two Image Reconstruction Algorithms

As illustrated in Figure 6, we implemented two reconstruction algorithms offline in MATLAB, on a CPU node (2× AMD EPYC 7452, 2.35GHz, 64 cores, 2 hyper-threads/core, with 1 TB RAM option). Both algorithms can reconstruct an image $\hat{\rho} \in \mathbb{C}^{N_x \times N_y \times N_z}$ from the compressed samples $S' \in \mathbb{C}^{N'_x \times N'_y \times N'_z}$ (or the original samples $S \in \mathbb{C}^{N_x \times N_y \times N_z}$ for the uncompressed baseline). Because of the identical rapid B_0 modulations across phase-encoding steps, the phase-encoding dimension(s) can be reconstructed efficiently using FFT in both pipelines.

In the reconstruction algorithm #1 (hybrid-space reconstruction [16, 30, 32, 81]), the phase-encoding undersampling is restricted to regular/CAIPI-type patterns that yield structured aliasing (i.e., an R -fold superposition of discrete pixels in the image-space of phase-encoding dimensions). Consequently, the global forward model in Equation (15) (mapping $\mathbb{C}^{N_x \times N_y \times N_z} \rightarrow \mathbb{C}^{N'_x \times N'_y \times N'_z}$) decouples into a set of small readout forward models (i.e., time axis only includes readout) that can be solved as independent unfolding subproblems. Each readout forward model maps all readout pixels and one aliased set of phase-encoding pixels as unknowns, to a single k-space readout line acquired from each RF receiver within one TR (mapping $\mathbb{C}^{N_x R} \rightarrow \mathbb{C}^{N'_x R}$). Our implementation steps are summarized as follows to solve:

$$\hat{\rho} = \underset{\rho}{\operatorname{argmin}} \|E' \rho - S'\|^2, \quad (16)$$

1. Apply inverse FFTs along phase-encoding dimensions to the k-space data.
2. For each aliased set of phase-encoding pixels, solve the corresponding readout forward model. These subproblems can be solved in parallel (e.g., MATLAB “parfor” with multi-threading). The LSQR [85] was used as the numerical solver.
3. Assemble the solutions from all aliased sets to form the full image.

In the second reconstruction algorithm #2, arbitrary phase-encoding undersampling patterns (PD-VD masks generated by BART [82]) can be used, which consequently enables advanced iterative reconstructions (e.g., compressed-sensing [19]) for higher acceleration factors. In this setting, aliasing in the phase-encoding dimension(s) can be incoherent and therefore may not yield separable aliased pixel sets after inverse FFT. As a result, the reconstruction must solve the global forward model in Equation (15) (mapping $\mathbb{C}^{N_x \times N_y \times N_z} \rightarrow \mathbb{C}^{N'_x \times N'_y \times N'_z}$), with all phase-encoded pixels estimated simultaneously. This increases the problem size and makes the reconstruction algorithm #2 computationally slower than the reconstruction algorithm #1. For PD-VD masks, the nominal undersampling factor is defined as the ratio between the number of sampled k-space locations and the total number of grid points on the full square Cartesian grid (without circular cropping). For the reconstruction algorithm #2, FISTA [86] was used as the numerical solver that

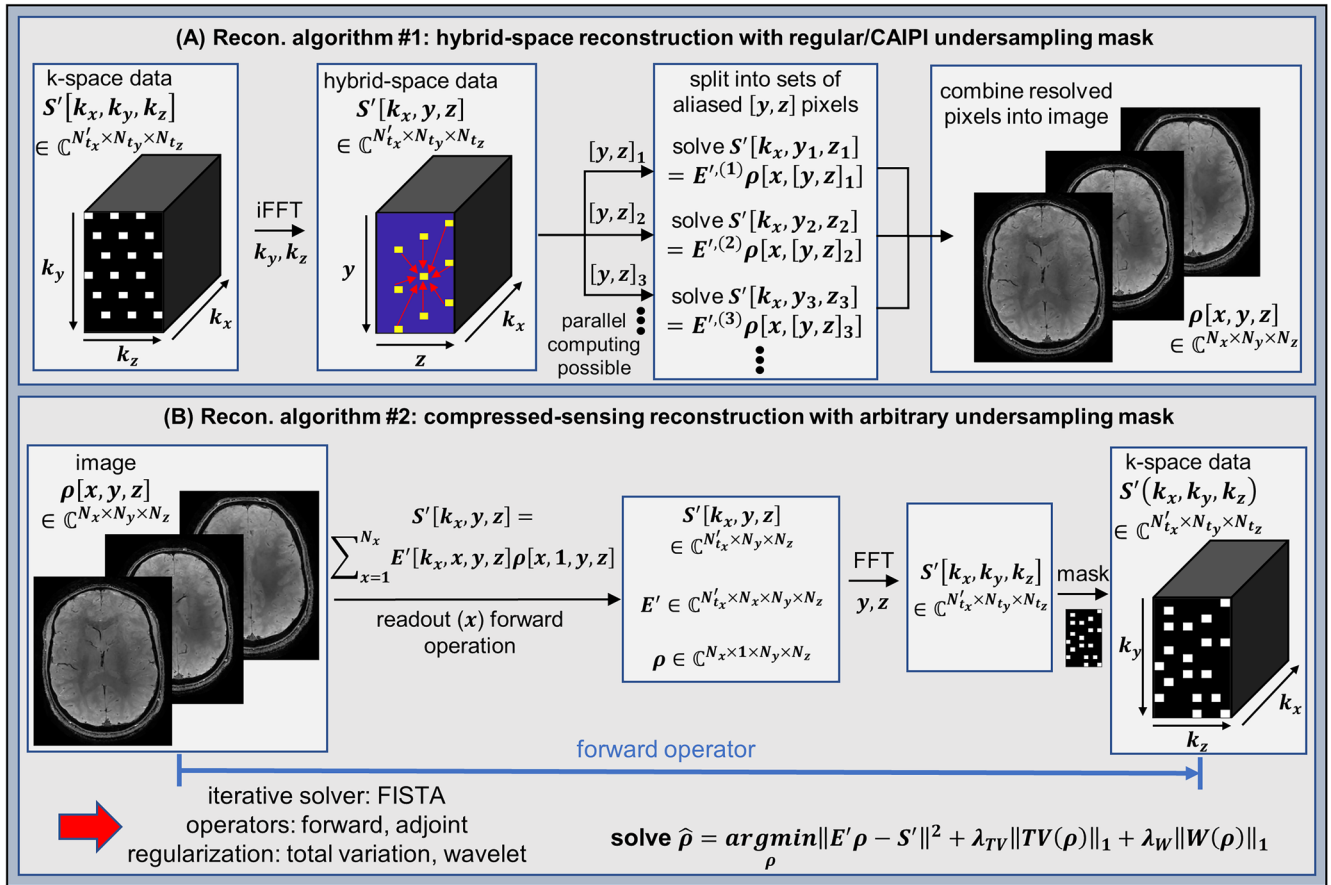


FIGURE 6 | Reconstruction strategies. (A) Reconstruction algorithm #1. For regular/CAIPI undersampling, the global forward model decouples into independent small subproblems that can be solved efficiently in hybrid-space. $[y, z]_1, [y, z]_2, [y, z]_3$ are the superimposed phase-encoded pixels to be separated in the reconstructed image, which will be estimated separately in each set of the subproblems. (B) Reconstruction algorithm #2. For arbitrary undersampling (including regular/CAIPI as a special case), the global forward model is solved directly using FISTA. The corresponding forward operator is illustrated. Because this formulation generally supports arbitrary sampling masks, we implement our compressed-sensing reconstruction with total variation and wavelet regularization based on this scheme.

solves the objective:

$$\hat{\rho} = \underset{\rho}{\operatorname{argmin}} \|E' \rho - S'\|^2 + \lambda_{TV} \|TV(\rho)\|_1 + \lambda_W \|W(\rho)\|_1, \quad (17)$$

where $TV(\cdot)$ denotes the total variation operation, $W(\cdot)$ denotes a wavelet transform. As a special case, algorithm #2 can also support regular/CAIPI undersampling pattern as a special case, where setting $\lambda_{TV} = \lambda_W = 0$ can reduce Equation (17) to least-squares reconstruction. The wavelet regularization was implemented by the soft-thresholding, and the total variation regularization was implemented by smooth approximations in practice. Theoretically, the auto-calibrated B_0 modulation kernels/maps should be robust to system imperfections (e.g., gradient timing delay, gradient nonlinearity, and eddy currents), thereby minimizing estimation errors in encoding matrix and reconstruction artifacts. However, in strongly-modulated, highly-undersampled scans, regularization is usually helpful for attenuating residue artifacts arising from subtle model mismatch, noise and undersampling.

To evaluate the reconstruction quality, the reconstructed image was subtracted by the reference image without compression and undersampling, and then divided by the reference

image to yield a difference image in percentage. Further, the normalized-root-mean-square-error (NRMSE) was calculated comparing the reconstructed image and the reference image, normalized by the range of the reference image (i.e., maximum–minimum). Lastly, structural similarity index (SSIM) was computed comparing the reconstructed image and the reference one using a MATLAB built-in function.

4 | Results

4.1 | Visualization of Compressed Spatial Encoding Functions

Using ex vivo brain scans (eliminating concerns about motions), we first evaluate the compressibility of static RF receive and dynamic B_0 encoding functions, based on singular-value distributions of square matrix P_g that spans the subspace of the composite encoding functions to be compressed, as Equations (7) and (8). First, the subspace of conventional RF array compression using only RF sensitivity maps is shown in Figure 7A. Second, Figure 7B shows the subspace of joint compression based on all spatial encoding maps across a k-space readout subregion within one B_0 modulation period, as assigned to a compression group.

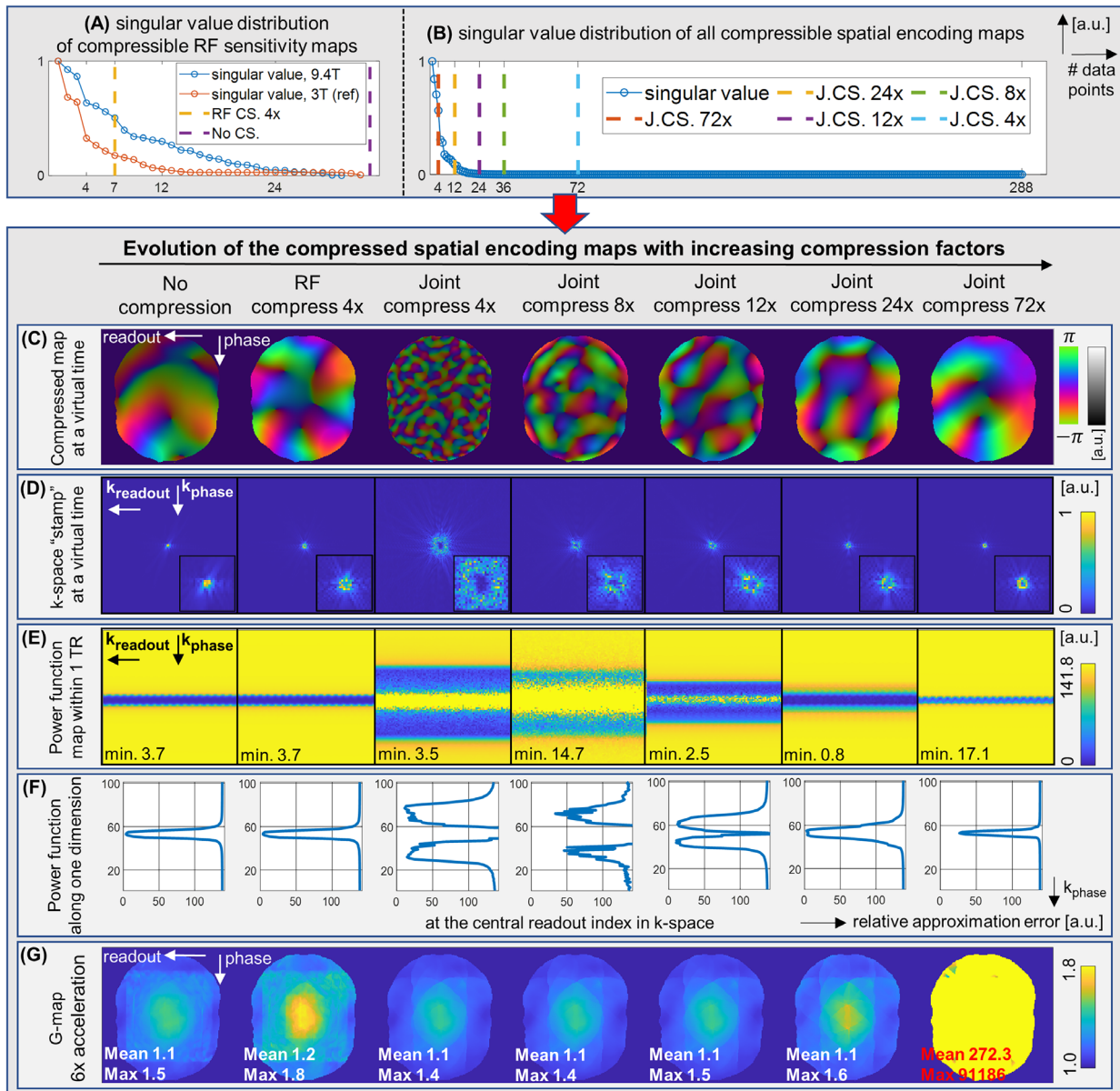


FIGURE 7 | Visualizing the data compressibility of the RF-only and the joint compression approaches. (A) The singular value distribution of the SVD for the square matrix P formulated based on RF sensitivity maps at 9.4 T and 3 T. Note, the plot excludes the first singular value which is way larger than others. (B) The singular value distribution of the SVD for the matrix P_g constructed based on a composite encoding matrix including all spatial encoding maps across a readout duration. (C) The non-compressed or compressed maps at a (virtual) time, around the k-space center. (D) The k-space "stamps" or kernels as the Fourier transform of the non-compressed or compressed spatial encoding maps around the k-space center. (E) The power function maps as k-space approximation error for arbitrary spatial encoding functions. (F) The 1D plots of the power function maps at the central readout pixel in k-space. (G) The G-maps showing image-space noise amplifications given the non-compressed or compressed readout encoding matrix, with 6x phase undersampling.

For nearly-lossless RF array compression, these receive coils [84] at 9.4 T can be compressed from 32 to ~24 virtual coils, and the ones at 3 T can be compressed from 34 to ~15 virtual coils. By contrast, the more complex standing waves at 9.4 T improve the acceleration capabilities, but meanwhile, reduce the compressibility.

At 9.4 T, when extending the same compression algorithm to all spatial encoding maps, the total data can be compressed nearly-lossless by about 12x–24x folds. This surprisingly huge compressibility originates from the encoding redundancy introduced by oversampling the readout (8x) for capturing k-space

signals modulated by B_0 fields that vary slowly relative to ADC dwell time.

Figure 7C–G illustrates the progression of compressed spatial encoding functions (image-space maps or k-space kernels) with increasing compression factors from left to right columns, based on a 2D ex vivo scan on the transverse plane. Additionally, it shows the corresponding k-space B_0 modulation kernel [49] at a (compressed) "time instant," the 2D power function maps representing k-space approximation errors within one phase-encoded step [30], the 1D distribution of the power function maps, and the

G-map representing image-space noise amplification [10]. Here, the power function maps are used to evaluate the cumulative k-space sampling efficiency of arbitrary spatial encoding functions within a single TR, namely, the cumulative encoding effects given the dynamic kernels across one readout line. Such efficiency maps are derived from non-compressed or compressed signal encoding maps spanning one TR (e.g., cropped from E or E'), utilizing Equations (14–16) in our recent nonlinear gradient sampling theory based on reproducing kernel Hilbert space (RKHS) [30]. From this view of RKHS (a Hilbert space of encoding functions), the arbitrary spatial encoding functions under investigation can be mathematically interpolated onto a fully-sampled Fourier function baseline. The power function characterizes the time-domain approximation error of this interpolation, obtained from the subtraction between the Gram matrix of the ideal fully-sampled DFT baseline and the cross-Gram matrix between the investigated encoding functions and this DFT baseline. Consequently, lower power function values reflect a smaller approximation error during interpolation (yielding higher encoding efficiency), whereas higher values indicate the opposite.

The 4× RF-only compression minimally perturbs the encoding functions, but raises the maximum G-factor to 1.8, indicating compromised sampling efficiency. In contrast, the joint compression can abruptly reshape the virtual spatial encoding maps or their equivalent k-space kernels (“stamps”), far more than RF-only compression, yet preserve overall encoding efficiency across phase-encoded steps (G-map) up to about 24× compression. As the joint compression factor increases, the effective k-space coverage (power function) per phase-encoded step evolves from a single trough to two troughs (reflecting redistributed k-space support), then gradually returns toward a single trough at higher factors, as the SVD numerically selects the most informative encoding modes. Note, some compressed spatial encoding functions cannot be physically generated by our current hardware directly, but are merely numerical consequences.

Note, for 8× joint compression in Figure 7E,F, larger k-space approximation errors with wider support per TR do not necessarily lead to degraded overall sampling efficiency considering all phase-encoded steps.

In Supporting Information, Table S1 summarizes the transitioning characteristics for different compression types and increasing compression factors in Figure 7C–G. In Figures S1 and S2 and Table S2, this ex vivo brain phantom was also used to investigate the speed and quality for each of the compression-reconstruction steps, given different compression factors, undersampling masks, and reconstruction algorithms.

4.2 | RF-Only Versus B_0 -RF Compression for In Vivo 3D Scans at 9.4 T

In Figure 8, with reasonable compression factors at 9.4 T, reconstructions of 3D in vivo FLASH with rapid modulations of local B_0 coils (least-squares L_2 recon with regular/CAIPI patterns, CS recon with random patterns) or scanner’s gradients (least-squares L_2 Wave-CAIPI/CS-Wave) are shown, comparing the RF-only compression and the proposed group-patch joint compression. A similar study but for 2D FLASH with rapid B_0 modulations

is documented in Supporting Information (Figure S3), showing 20× joint compression that substantially accelerates 2D compressed-sensing reconstruction.

As in Figure 8A–H, for nonlinear-gradients-modulated scans, least-squares reconstruction (using reconstruction algorithm #1) with 4×3 undersampling and compressed-sensing reconstruction (using reconstruction algorithm #2) with 14.6× PD-VD undersampling are shown. For L_2 reconstructions, 18× joint compression took noticeably more compression time than 4× RF compression, while the total time (compression + reconstruction) was similar.

However, as in difference images, apparent artifacts appear at the bottom of the image given 4× RF compression, while 18× joint compression only incurred slight noise amplification relative to the non-compressed least-squares reconstruction and thus indicates higher compression efficiency.

Moreover, for compressed-sensing [34], joint compression cut reconstruction time to about 4.5 min only, while the peak memory required by 4× RF compression prevented CS reconstruction using the same reconstruction implementation (the reconstruction algorithm #2). The CS reconstruction based on joint compression achieved further improved image quality, close to the non-compressed, fully-sampled reference with identical field modulations.

As in Figure 8I–P, for linear-gradients-modulated scans, namely Wave-CAIPI/CS-Wave, least-squares reconstruction with 4×2 undersampling and compressed-sensing reconstruction with 10.5× undersampling are shown. For L_2 reconstructions, 16× joint compression incurred a long computational time given our current implementation. As a result, even though its reconstruction was 4.6× faster, the total time (compression + reconstruction) was 1.2× slower than the 4× RF compression. Note, the total computational time based on joint compression and L_2 reconstruction can also become less for other matrix sizes (e.g., $180 \times 180 \times 80$ with 32 receivers in Figure S2, $230 \times 230 \times 144$ with 44 receivers in Figure 9), compared to RF-only compression.

Nevertheless, the 16× joint compression leads to reduced artifacts, as indicated by difference maps. Moreover, it allowed compressed-sensing reconstruction (impossible with RF-only compression) in 4.58 min, achieving improved reconstruction quality compared with L_2 , much closer to the non-compressed, fully-sampled reference with identical field modulations.

In Supporting Information (Figure S4), under identical compression factors (i.e., 8×, 16×, 32×), reconstructions using the RF-only compression show clear loss of image quality than the proposed joint compression techniques, for both 2D and 3D in vivo FLASH with local B_0 coil modulations at 9.4 T.

4.3 | RF-Only Versus B_0 -RF Compression for 3 T In Vivo Scans

In Figure 9, reconstruction of 3 T Wave-CAIPI/CS-Wave in vivo scans is shown, comparing 2.2× RF-only compression (20 virtual RF coils) and 11× joint compression.

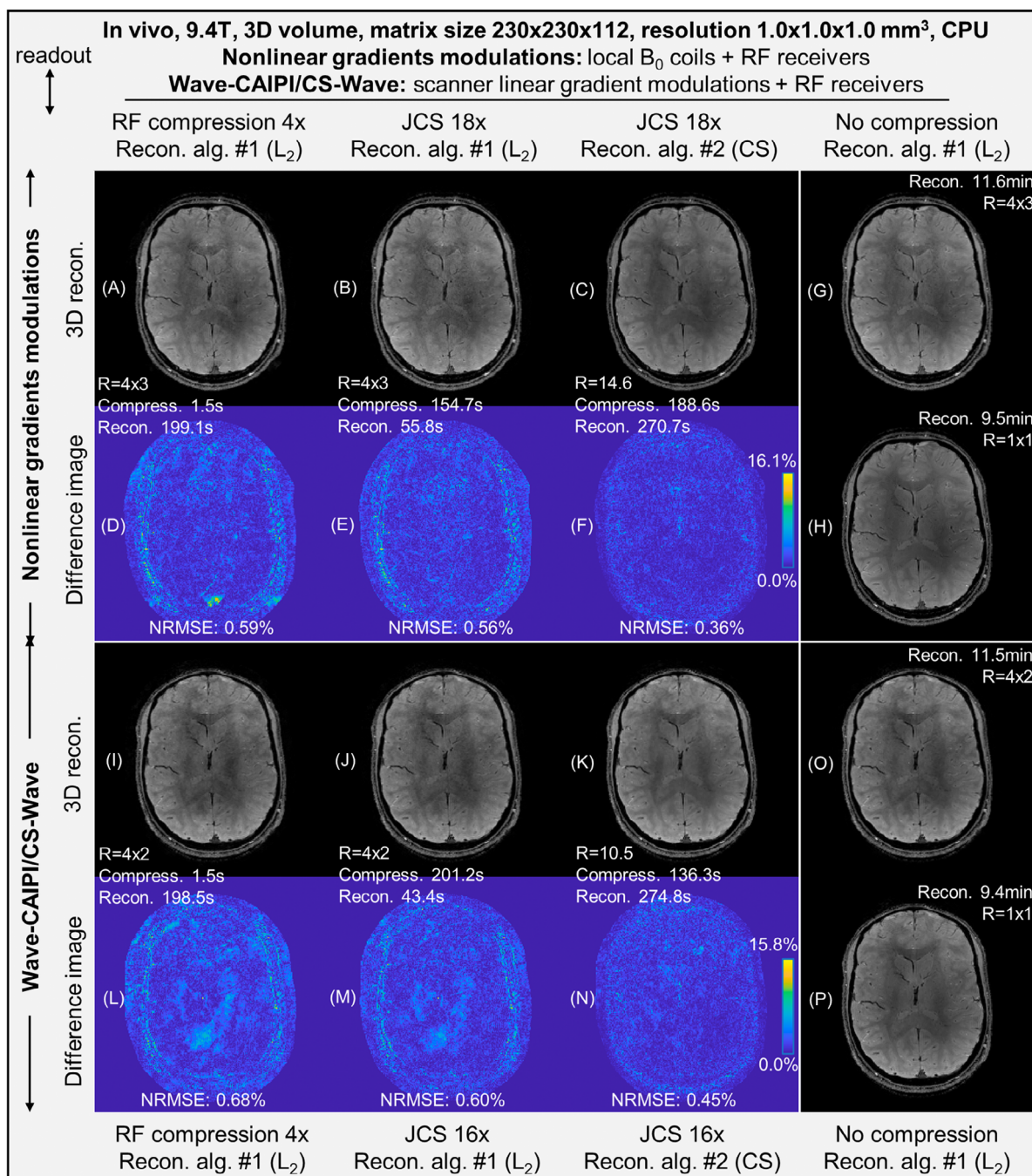


FIGURE 8 | 3D least-squares and compressed-sensing reconstructions with reasonable compression factors for RF-only and joint compression algorithms at 9.4 T. The rapid field modulations were provided by local B₀ coils (A–H) and scanner’s linear phase-encoding gradients (Wave-CAIPI L₂/CS-Wave, I–P). (A) L₂ reconstruction with 4× RF compression. (B) L₂ reconstruction with 18× joint compression. (C) CS reconstruction with 18× joint compression. (D–F) Corresponding difference maps relative to the non-compressed, fully-sampled reference. (G) L₂ reconstruction without compression. (H) Fully-sampled L₂ reconstruction without compression. (I–P) Similar organizations as in (A–H) but for Wave-CAIPI/CS-Wave. Note, (H) and (P) were also reconstructed from the same field-modulated (local B₀ or Wave-CAIPI) scans, and thus, the difference maps here cannot fully reflect field-calibration artifacts, nor subject motions. The compression and reconstruction time are shown. In L₂ reconstructions, MATLAB tool “parfor” was used.

For L₂ reconstructions with 3×3 undersampling, both moderate compression factors (empirically chosen for lower SNR at 3 T) yielded almost identical image quality compared to the non-compressed least-squares reconstruction. However, they reduced reconstruction time from 40.5 to 20.4 min (2.2× RF compression) and to 127.6 s (11× joint compression). For this

dataset, the total time (compression + reconstruction) for joint compression is much less than that for RF compression.

For compressed-sensing with 9.2× PD-VD undersampling, reconstruction time was reduced to 10.1 min by joint compression, whereas 2.2× RF-only compression could not be run due to

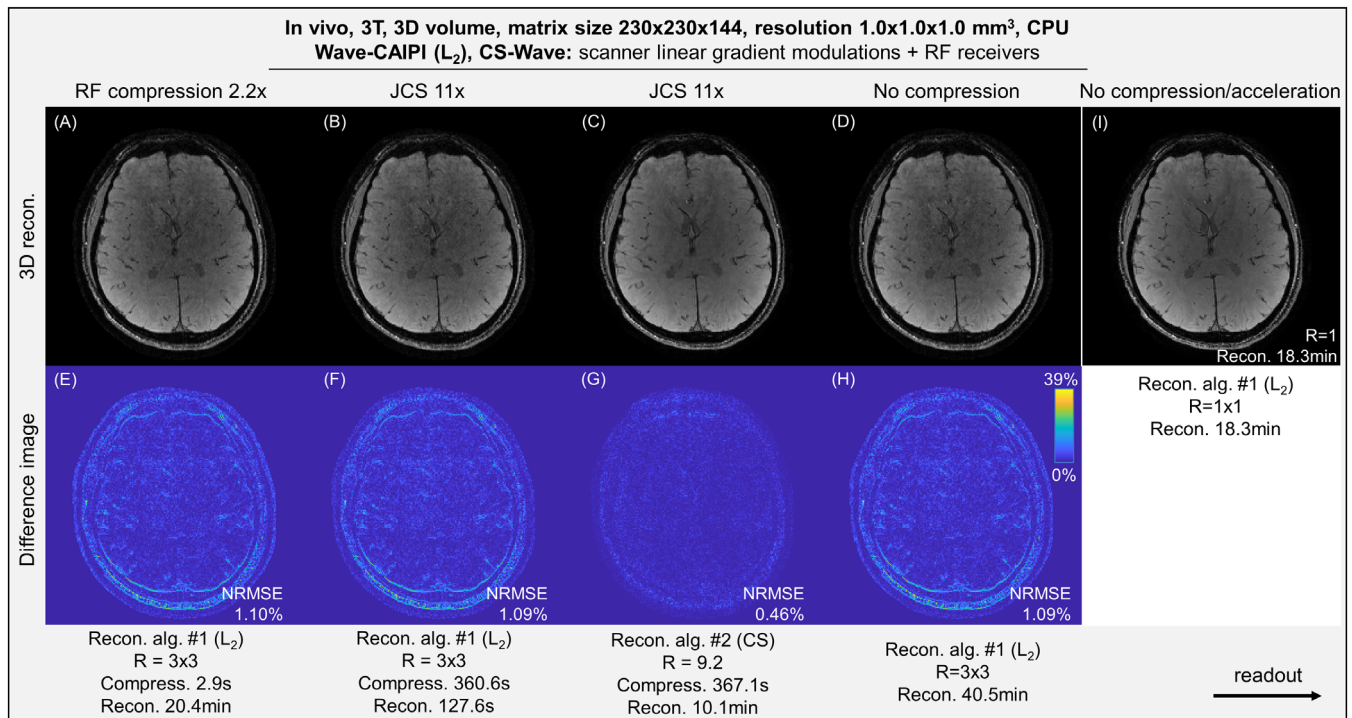


FIGURE 9 | 3D least-squares and compressed-sensing Wave-CAIPI/CS-Wave reconstructions with reasonable compression factors for RF-only and joint compression algorithms at 3T. (A) L₂ reconstruction using 2.2× RF compression with 3×3 undersampling. (B) L₂ reconstruction using 11× joint compression with 3×3 undersampling. (C) Compressed-sensing reconstruction using 11× joint compression with 9.2× PD-VD undersampling. (D) L₂ reconstruction without compression, with 3×3 undersampling. (E–H) Corresponding difference images relative to the non-compressed, fully-sampled least-squares reconstruction with the same field modulations. (I) Non-compressed, fully-sampled L₂ reconstruction of Wave-CAIPI. Note, (I) was also reconstructed from the same Wave-CAIPI scan, and thus, the difference maps here cannot fully reflect field-calibration artifacts, nor inter-scans subject motions. The compression and reconstruction time are shown. In L₂ reconstructions, MATLAB tool “parfor” was used.

peak-memory limits using our reconstruction implementations. The CS reconstruction quality was further improved than L₂, and approached non-compressed, fully-sampled reference with the same linear gradients modulations.

4.4 | Visualization of Compressibility for In Vivo Scans at 9.4 T and 3 T

Figure 10 evaluates the singular-value distributions and compression thresholds obtained during RF array and joint compression algorithms for in vivo scans at 9.4 and 3 T (those in Figure S3, Figures 8 and 9, respectively), visualizing the signal subspaces on which the SVD operated, and their compressibility. They are calculated based on Equation (8).

RF-only compression discarded about 50%–75% of singular vectors in the receiver dimension (44 or 32 coils). In contrast, joint compression was far more aggressive, discarding 91%–95% of singular vectors in a larger joint space (5–16 oversampled readout points × 44/32 coils). Note that, although the RF sensitivity maps at 3T could be generally more compressible than the maps with more complex standing-wave patterns at 9.4T, we empirically applied less aggressive compression to the 3T datasets. This is because of the lower intrinsic SNR at 3T, which might make the encoding model more sensitive to over-compression.

5 | Discussion and Conclusions

We have demonstrated that 2D and 3D MRI scans accelerated by rapid B₀ field modulations and multiple RF receivers can be jointly compressed by about 11×–24×, dramatically exceeds the limits of conventional RF receiver array compression [38]. In our proof-of-concept MATLAB implementations on a high-memory CPU node in HPC, the proposed joint compression substantially reduced both reconstruction time and peak memory for 3D scans, enabling L₂ reconstruction in 19–127 s, CS reconstruction in 177 s–10.1 min, depending on the matrix-size and compression factors. Although 3D joint compression may require longer pre-processing time to compress data and maps (e.g., 47.2–367.1 s), this may start before scans finish and could be further optimized in future study (codes, GPU, programming languages [41, 68, 69]). By contrast, the CS reconstructions are largely hindered under RF-only compression, and, even at moderate compression factors, could suffer from degraded encoding efficiency compared to the ones using joint compression. For 2D scans, our proposed technique took 1.21–3.5 s for compression, enabling CS reconstruction within 1.4–5.1 s.

These improvements arise from the substantially reduced readout forward model, previously enlarged due to oversampled readout and multiple RF receivers—now becomes comparable in size to the data undersampling factors. The subregion-wise operation plays a crucial role: SVD acts for small, decoupled k-space

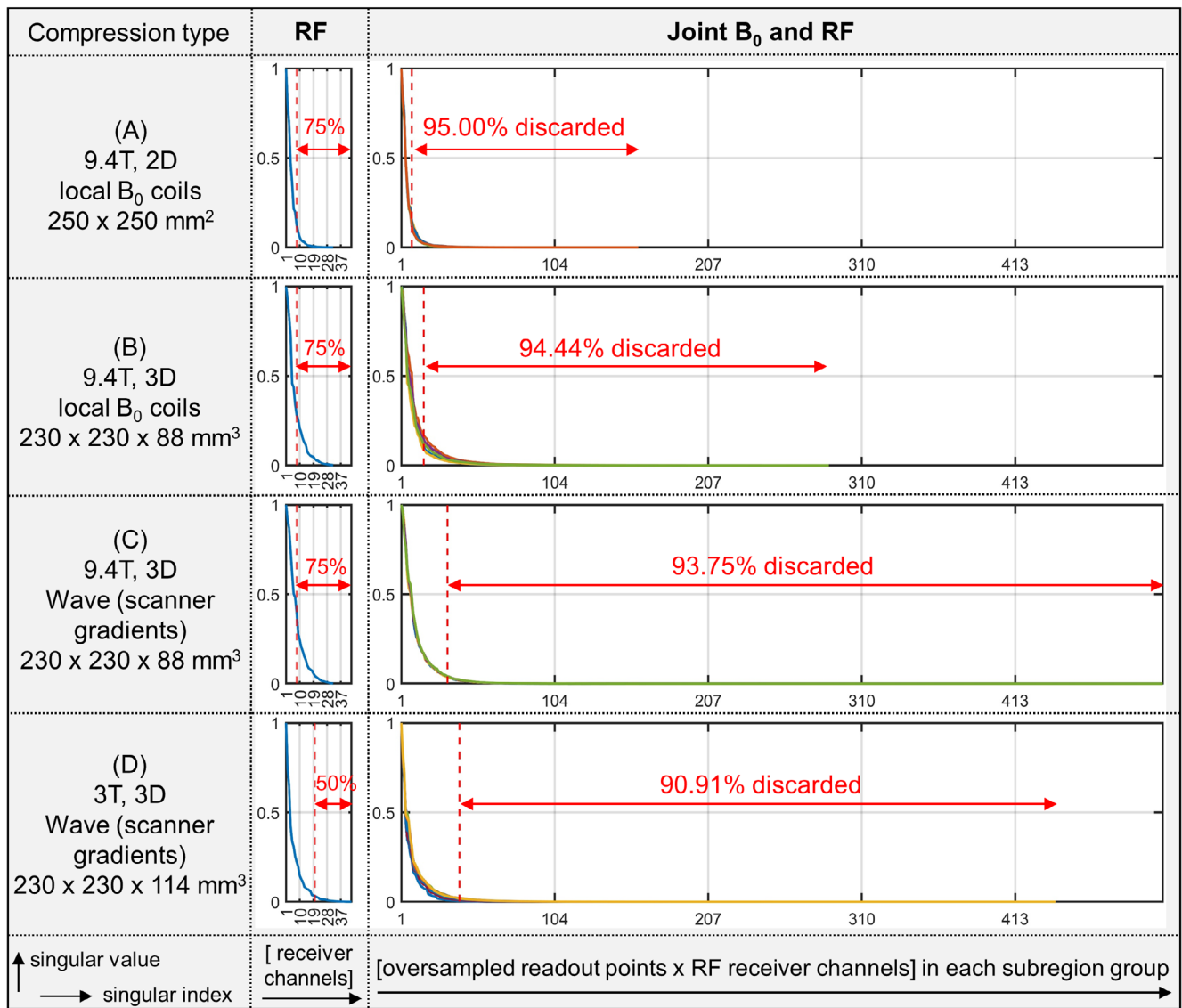


FIGURE 10 | Singular-value distributions obtained in RF array compression and joint compressions for all in vivo scans at 9.4 T and 3 T in this paper. (A) 2D scans with local B_0 coils modulations at 9.4 T. (B) 3D scans with local B_0 coils modulations at 9.4 T. (C) 3D Wave-CAIPI/CS-Wave at 9.4 T. (D) 3D Wave-CAIPI/CS-Wave at 3 T.

patches, and the resulting set of compression matrices can be efficiently reused across other subregions sampled by the same subset of B_0 and RF kernels. Notably, our calculation steps to compress a composite spatial encoding model for a group is similar to the conventional algorithm to explicitly compress the conventional SENSE model [38], but our strategy to partition k-space in subregion groups and compress it in multidimensional patches is novel. “Pushing” these minimally compressible RF sensitivity maps into the highly redundant readout oversampled space with joint compression allows us to surpass the maximum achievable RF compression factor (usually $1.5\times-3.0\times$) for the entire dataset. Therefore, the rapid B_0 modulations during oversampled readout eventually yield an information-enriched but meanwhile compact readout encoding model. Note, the significantly smaller forward model due to joint compression can theoretically make further GPU-acceleration much easier, as G-RAM is usually much more limited relative to CPU memory.

Currently, in our proof-of-concept MATLAB scripts, the entire readout encoding matrix is explicitly stored in memory during compression and reconstruction, which is not the most memory-efficient implementation. Further optimization can mitigate this by exploiting the periodicity of the rapid B_0 modulation. Specifically, the readout encoding matrix only needs to be explicitly computed and stored for a single B_0 modulation period during compression, and repeatedly used to synthesize the entire readout duration during reconstruction in a memory-efficient manner.

In principle, our group-patch joint compression strategy can be beneficial when periodic B_0 -RF encoding redundancy is present. For example, this can happen when rapid B_0 field modulations vary across TRs (e.g., readout CAIPI [30]), or contain multiple (e.g., 2.5–5–10 kHz) temporal frequencies along the readout time axis [29, 87]. As long as the entire k-space can still be partitioned into a small number of subregion groups that share

identical B_0 -RF modulation functions, the full data volume can be compressed efficiently based on a patch-by-patch manner in groups. While calculating and applying compressions may also be done entirely in k-space [39], or utilize neural networks [42], this paper's implementation yields virtual compressed maps in a straightforward manner to facilitate visual interpretations of the compression behaviors. Nevertheless, other compression algorithms might also be adapted for joint B_0 -RF compression for performance comparisons in more details.

Theoretically, beyond reconstruction based on image-space to k-space mapping, the joint compression is also compatible with k-space interpolation-based reconstruction [88]. For a very high-resolution 3D dataset with rapid B_0 modulations and readout oversampling, joint compression can be applied prior to a generalized k-space reconstruction algorithm based on RKHS cardinal-function interpolations [47]. Such k-space reconstruction can be adapted in a similar subregion-wise manner [30, 80] and thus further decouples the segments along k-space readout dimension for faster reconstruction and lower-memory requirements. In the future, combining the subregion-wise joint compression and k-space interpolation-based reconstruction can be crucial; otherwise, 3D CS reconstruction may still incur high peak memory for high-resolution field-modulated scans required for certain applications [89].

Acknowledgments

This study is supported by the ERC Advanced Grant No. 834940. The ex vivo brain phantom was courtesy of the Institute of Clinical Anatomy and Cell Analysis, Department of Anatomy, Eberhard Karls University of Tübingen. The first author thanks Dr. Thomas Shiozawa (Institute of Clinical Anatomy and Cell Analysis) for assistance with sample preparation, and Dr. Gisela Hagberg for assistance in scanning this phantom. The first author would like to thank Prof. Dr. Martin Uecker for discussions about compressed sensing reconstruction algorithms, Dr. Felix Breuer and Prof. Berkin Bilgic for helpful suggestions about CAIPIRINHA patterns in forward model reconstructions, Stefan Plappert for guidance in programming the ADwin high-speed processor, and Dr. Felix Glang, Dr. Jonas Bause, and Praveen Valsala for helpful comments about this manuscript. The first author thanks Dr. Francisek Hennel and Prof. Dr. Klaas P. Pruessmann for interesting discussions about rapid magnetic field modulation techniques. Open Access funding enabled and organized by Projekt DEAL.

Funding

This work was supported by the European Research Council (834940).

Data Availability Statement

The example code or data that support the findings of this study is openly available in Zenodo at <https://zenodo.org/records/20533698>, in Github at https://github.com/ruitianwater/GroupPatchJCS_v1.0, and collected at the HarmonizedMRI website at <https://harmonizedmri.github.io/projects/>.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section. **Figure S1:** The reconstruction of a 2D scan accelerated by rapid B_0 modulation and multiple RF receives, with no compression, RF array compression and joint compression. (A) k-space undersampling mask. (B) Reconstructed image. (C) Difference image between the reconstruction and the reference image without compression and undersampling. (D) SSIM comparing the reconstruction and the reference image without compression and undersampling. (E) computational profiling. With no undersampling and no compression, the reference image sets a baseline of 0.00% NRMSE and 1.00 SSIM. **Figure S2:** The reconstruction of a 3D scan accelerated by rapid B_0 modulation and multiple RF receives, with no compression, RF array compression and joint compression. (A) k-space undersampling mask. (B) Reconstructed image. (C) Difference image between the reconstruction and the reference image without compression and undersampling. (D) SSIM comparing the reconstruction and the reference image without compression and undersampling. (E) computational profiling. With no undersampling and no compression, the reference image sets a baseline of 0.00% NRMSE and 1.0 SSIM. **Figure S3:** 2D least-squares and compressed-sensing reconstructions with reasonable compression factors for RF-only and joint compression algorithms at 9.4 T. The rapid field modulations were provided by local B_0 coils. The Poisson-disc variable-density (PD-VD) mask was generated by BART. (A–D): L_2 and CS reconstructions with 7×

undersampling: 4× RF compression for L_2 reconstruction, 4× RF compression for compressed-sensing reconstruction, 20× joint compression for L_2 reconstruction, 20× joint compression for compressed-sensing reconstruction. (E–H): Different images relative to the non-compressed, fully-sampled image. (I) non-compressed, 7×-undersampled L_2 reconstruction. (J) non-compressed, fully-sampled L_2 reconstruction. Note, (J) was also reconstructed from the same local B_0 modulated scan, and thus, the difference maps here cannot fully reflect field-calibration artifacts, nor inter-scans subject motions. The compression and reconstruction time are shown. In L_2 reconstructions, MATLAB tool "parfor" was used. Here, the CS reconstruction incorporates additional wavelet and total variation regularizations, while still using a 1D regular undersampling mask for 2D reconstruction. **Figure S4:** 2D and 3D least-squares reconstructions with same compression factors at 9.4 T, comparing RF-only and joint compression algorithms. The rapid field modulations were provided by local B_0 coils. (A–D): 16× compression for a 2D scan with 7× undersampling. (E–H): 8× compression for a 2D scan with 7× undersampling. (I): reference fully-sampled 2D image without local B_0 modulations and compressions. (J–M): 32× compression for a 3D scan with 4×3 undersampling. (N–Q): 16× compression for a 3D scan with 4×3 undersampling. (R): reference fully-sampled 3D image without local B_0 modulations and compressions. Next to the reconstructions, the difference images were calculated relative to the reference FLASH images, which should be free of any artifacts caused by local B_0 coil or compression errors, but might contain subject motions relative to the field-modulated scans. The compression and reconstruction time are shown. Hybrid-space least-squares reconstruction with MATLAB tool "parfor" was used.

Figure S5: Supporting Information for Figure S3: Accelerated data sampling enabled by the rapid B_0 modulations generated by the local B_0 coil array at 9.4 T. (A) The phase offsets in local B_0 coils, the magnetic field distribution generated at nearly-peak current, and the estimated gradient strength distribution (Euclidean-norm in two orthogonal dimensions) are shown. (B–E) The G-map, the G-factor boxplot for each of 12 slices, the k-space cardinal and power function maps are shown, which indicate sufficient sampling to reach 7× joint acceleration factor by B_0 modulations and SENSE. (F–K) The reference fully-sampled image with B_0 and SENSE reconstruction is successfully reconstructed. The L_2 reconstruction of 7× undersampled k-space has only subtle noise amplifications without noticeable residue fold-over artifacts. Its L_1 reconstruction further improves the image quality, as also shown in their difference images, and is very close to the reference image. In comparison, the SENSE-only acceleration has severe artifacts. Here, the CS L_1 reconstruction incorporates additional wavelet and total variation regularizations, while still using a 1D regular undersampling mask for 2D reconstruction.

Figure S6: Supporting Information for Figure 8 of the manuscript: Accelerated data sampling enabled by the rapid B_0 modulations generated by the local B_0 coil array, or the scanner's phase-encoding gradients (i.e., Wave-CAIPI/CS-Wave), at 9.4 T. The k-space B_0 trajectory for local B_0 coils modulations is visualized using power function (effective k-space coverage) at a readout timepoint, considering a single RF receiver. The k-space B_0 trajectory for Wave-CAIPI is plotted from Pulseseq built-in functions. The k-space power and cardinal function maps across three readout timepoints incorporate encoding by both rapid B_0 modulations and multiple RF receivers. The volumetric G-factors across the entire 3D volume are shown in the boxplots for the two different experiments, respectively. Note, the k-space efficiency maps based on the RKHS formalism are relative values and unitless. In both experiments, they show effective expansion of k-space coverage (thickened trajectories) by rapid B_0 modulations and multiple RF receivers. However, the detailed comparison between different experiments in the cardinal and power function maps are reserved for future study. **Figure S7: Supporting Information**

for Figure 9 of the manuscript: Accelerated data sampling enabled by the rapid B_0 modulations generated by the scanner's phase-encoding gradients (i.e., Wave-CAIPI/CS-Wave) at 3 T. Left: least-squares reconstruction. Middle: G-map for the corresponding slice position. Right: G-factor boxplot across the entire 3D volume. **Table S1:** Summary of transitioning characteristics of different compression types and factors in Figure 7. **Table S2:** Computational profiling of different reconstruction algorithms. The dataset is a 3D GRE ex vivo scan accelerated by rapid B_0 field modulations and 32 RF receivers, given an image pixel size of $180 \times 180 \times 80$ and a k-space readout oversampling factor of 8. The “time” considers only the reconstruction time duration of the iterative algorithms, excluding the time for loading, saving and preparing the data and encoding matrix which were fed into the iterative reconstruction algorithms. Note the wavelet function in MATLAB was internally double-precision.

Publication 4

Tian R, Uecker M, Zaitsev M, and Scheffler K, “Overlap-Kernel EPI: Estimating MRI Shot-to-Shot Phase Variations by Shifted-Kernel Extraction From Overlap Regions at Arbitrary k-Space Locations,” *Magnetic Resonance in Medicine* 95, no. 4 (2026): 2131–2152, <https://doi.org/10.1002/mrm.70196>.

Finalist of the I.I. Rabi Award in the ISMRM 2026 Young Investigator Awards Competition, Cape town, South Africa.

The main manuscript is provided below and the supporting information is available upon request.

RESEARCH ARTICLE OPEN ACCESS

Overlap-Kernel EPI: Estimating MRI Shot-to-Shot Phase Variations by Shifted-Kernel Extraction From Overlap Regions at Arbitrary k-Space Locations

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Received: 7 July 2025 | **Revised:** 10 November 2025 | **Accepted:** 11 November 2025

Keywords: diffusion | ESPIRiT | GRAPPA | multi-shot EPI | self-navigation | shot-to-shot phase variations

ABSTRACT

Purpose: In multi-shot EPI, shot-dependent phase fluctuations can introduce ghost artifacts, undermining advantages for enhancing resolution or reducing distortion, particularly in diffusion scans. Here, a novel self-navigation strategy based on shift-invariant kernel extraction is proposed, enabling robust estimation of phase inconsistencies from overlap bands at any k-space locations. It is inspired by RF sensitivity auto-calibration in parallel imaging, and extends traditional self-navigation trajectories beyond oversampled k-space center.

Theory and Methods: Our shot-to-shot correction method only requires shot-to-shot sampling trajectories to slightly overlap at arbitrary k-space locations. Such k-space overlaps can be defined as shot-dependent auto-calibrating-signal regions, where a GRAPPA/ESPIRiT-type operation can be applied to explicitly extract shot-to-shot signal variation maps in one step. This inherits GRAPPA/ESPIRiT's robustness, and offers much greater flexibility in designing self-navigated multi-shot sequences.

Results: Our kernel extraction techniques in four self-navigated 2D SE-EPI and GE-EPI sequences are successfully demonstrated, as well as minor applications in extracting maps from navigator data and B_0 maps. Given sufficient SNR, self-navigation EPI can reach identical/better reconstruction quality compared to navigator corrections, applicable to various trajectories. Additionally, it is particularly suitable for the two-shot and four-shot mosaic EPI, where stronger shot-dependent eddy currents can also be removed, and the self-navigated ACS has higher SNR than the refocused navigator echoes.

Conclusion: This paper rigorously analyzes subspace algorithms for phase-stabilized multi-shot MRI, significantly broadens the self-navigation concept, and provides a more complete picture of the connections between signal fluctuations calibration in EPI and RF sensitivity calibration in parallel imaging.

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1 | Introduction

In MRI, multiple shots of k-space [1–3] acquisition can enhance spatial resolution and reduce geometric distortion [4]. This, however, can be undermined by the notorious shot-dependent signal instability primarily manifested as spatially varying phase fluctuations [5, 6], ending up with ghost artifacts. This problem mostly originates from motions (e.g., respiration, heartbeat, mechanical vibration) and system imperfections (e.g., shot-dependent eddy currents), and can become prominent in diffusion-weighted [5] and long gradient-echo [6] scans. Numerous correction methods [7–26] have been developed to mitigate the shot-to-shot phase variations. Nevertheless, inherent limitations persist. Among these methods, self-navigation [12, 14, 21–24] (i.e., phase navigation from imaging data itself) offers an elegant solution, though not yet universally applicable (e.g., unsuitable for mosaic EPI segmentations [27]).

The conventional self-navigated trajectories [14, 21, 22] can directly monitor shot-dependent phase variations, by oversampling a central k-space segment in each shot. However, this technique is only limited to specific trajectories and may not achieve optimal k-space sampling. To the best of our knowledge, existing self-navigated trajectory approaches [28] that can remove diffusion phase fluctuations (note, not translational/rotational motion corrections [29–36]) generally assume that, spatially resolved shot-to-shot nonlinear phase maps can only be directly obtained from the k-space central segments, while high-frequency segments centered at the periphery of k-space are considered insufficient for this purpose.

For multi-shot phase-interleaved EPI, self-navigation can also be achieved by either estimating phase maps from intermediate parallel imaging (PI) reconstruction (e.g., inverse reconstruction method [19], and MUSE [23, 37]), or reducing shot-inconsistency between k-space lines or image-space pixels via post-processing (e.g., low-rank matrix theory [24–26, 38, 39], e.g., MUSSEL [24] and LLR [26]/NLLR [40]). However, these methods face similar limitations: each shot should avoid excessive undersampling beyond the parallel imaging acceleration limit for recovering a full image from that shot alone, so that the combined coverage of multiple RF receivers in that shot still provides a nearly complete k-space especially around the center, thereby avoiding errors in the shot-to-shot corrections. This inherently restricts the original PI acceleration benefits when considering all shots together. Additionally, iterative low-rank regularizations may incur substantially increased computational complexity, or difficulties in interpreting converged results (e.g., residue blurring), especially for highly undersampled k-space with strong diffusion-weighting.

For sequences currently incompatible with the above-mentioned self-navigation techniques, such as readout-segmented mosaic EPI [17, 18, 20, 41] (e.g., RESOLVE [42] in Siemens), navigator echoes can be acquired in separate scans alongside imaging echoes to estimate shot-to-shot phase fluctuations [7–11, 13, 15–18, 20]. Unfortunately, the navigators prolong the total sampling duration, may not perfectly capture imaging echoes' phase (e.g., due to motions, eddy currents), could suffer from T_2 or T_2^* signal decay, and increase SAR if acquired with additional refocusing RF pulses. These disadvantages may be

more pronounced in ultrahigh field (7T or above) with shorter transverse magnetization lifetimes.

For multi-shot mosaic EPI, there is also a magnitude-scan strategy based on phaseless encoding MRI [43–46] without need of phase navigation, which reconstructs a high-resolution k-space in “chunks,” from several “low-resolution” scans with high-frequency signals mixed into the k-space center [44, 46–48]. The shot-dependent k-space aliasing can be introduced by sub-pixel tagging modulations prior to imaging acquisition, and resolved by tagging shifts, which inherits elements of super-resolution microscopy [49–54]. In this way, only the magnitude of the “low-resolution” images is needed for super-resolution reconstruction, removing sensitivity to inter-shot phase fluctuations. Its main drawback is the inevitable SNR loss caused by the modulation of longitudinal magnetization, which suppresses signals from different subpixel compartments and may be unfavorable in scans using strong diffusion-weighting with inherently low SNR.

Given these limitations, we re-evaluate the long-standing idea of self-navigation MRI [14, 19, 21–24, 26], and propose a novel strategy to directly extract shot-to-shot signal variations from imaging data in a single step, extended from GRAPPA [55] and ESPIRiT's [56] theory. It only requires k-space region in separate shots to slightly overlap at arbitrary k-space locations, instead of the overlap area being confined to the k-space center [14, 21, 22]. Thereafter, a GRAPPA-/ESPIRiT-type operation can be applied to overlap areas to explicitly extract phase fluctuation kernels/maps that are later incorporated into the final image reconstruction. This fundamentally broadens the scope of self-navigation and enables much greater flexibility in multi-shot sequence design [27]. So far, non-central k-space overlaps have not been considered in self-navigated trajectory designs [14, 21, 22, 28, 57] for phase extraction/correction in multi-shot diffusion-weighted scans yet.

Essentially, our approach is grounded in Fourier theory. The shot-to-shot phase variations, as the image-space phase maps multiplied by the object's spin distribution, can be seen as k-space kernels convolved with the original k-space data, which are shift-variant along shot dimension, but shift-invariant across k-space locations and RF receiver channels. This signal modeling is also central to the low-rank matrix completion MRI techniques (e.g., MUSSEL [24]), and was earlier applied in calibrationless PI reconstruction [38, 58] encoded by GRAPPA [55] kernels (shift-variant along receiver channels, shift-invariant across k-space locations). Here, a key insight of our work is, given the phase fluctuation kernels convolve with data in the entire k-space rather than merely the center [24], similar autocalibrating-signal (ACS) regions, however, between shots, can be obtained by only slightly overlapping shot-dependent sampling trajectories at arbitrary k-space locations. The resultant explicit phase kernels/maps extractions might also inherit the robustness of GRAPPA/ESPIRiT due to averaging effects of kernel shifts. This study is inspired by 2D phaseless encoding [46], where non-central overlap segments [27] were used to estimate inter-segment phase inconsistency, that was assumed as constant across image-space. It is also inspired by the auto-calibration technique [59] for linear and nonlinear gradient encoding [60], where even much stronger continuous phase modulation

(rather than modulations only between shots) can be robustly self-estimated via shifted kernel extraction in k-space.

In this paper, we demonstrate the broad application of our proposed strategy termed “overlap-kernel EPI” [61] (i.e., obtaining kernels from arbitrary k-space overlaps), in self-navigation using four distinct 2D EPI trajectories, and its utility to also serve as a simple and practical tool for filtering navigator and magnetic field maps [60, 62]. To emphasize the generality of shifted kernels [24, 38, 39, 55, 56, 59, 60, 63–66] in order to stimulate future follow-up work, we encapsulate our method within the generalized k-space interpolation framework [67] and ESPIRiT theory [56], to mathematically bridge kernel-based, explicit auto-calibration of RF receivers and shot-to-shot phase variations. Furthermore, coefficient-of-variation (CV) maps and kernel subspace are inspected to examine the kernel/map extraction quality for the proposed algorithms. In vivo EPI scans conducted in standard and high-performance-gradient 3T scanners were performed to validate our theory. An early account of this work was published in conference abstracts [68, 69].

2 | Theory

2.1 | Four Multi-Shot EPI Sequences, With Optional 2D Navigators For Comparison

Four multi-shot EPI sequences [27] implemented in Pulseseq [70] were used to evaluate our approach to estimate inter-shot phase fluctuations, including mosaic (readout- and/or phase-segmented) and phase-interleaved EPI, with an option of additional 2D navigator scans for comparison. To explicitly extract phase fluctuation maps/kernels from data, at least a small (typically about 15 pixels along the shorter-axis) k-space overlap region between shots was required as shot-dependent ACS regions for constructing a calibration matrix [55, 56], which needed merely subtle adjustments in pulse sequence.

In Figure 1, the k-space sampling trajectories for the four EPI in-plane segmentation schemes are presented. Additionally, a multi-shot readout-segmented mosaic EPI (rs-EPI) sequence is shown as an example, with consecutive shots overlapped by, for example, 6–27 pixels along readout axis (2%–9% overlap considering a full pixel-size of 311 pixels), and hundreds of pixels overlapped along phase-encoding axis. Typically, traditional rs-EPI (e.g., RESOLVE [18, 20, 42]) has only a few (e.g., 2–6) overlapped pixels along readout to avoid k-space holes, although acquiring several more readout pixels isn’t always time-consuming and may better utilize the gradient’s amplitude. Moreover, the readout gradient polarity between overlapped k-space segments is alternated between shots, to minimize difference in transverse spin relaxation and signal dephasing, for ensuring the shot-dependent signal differences in the overlapped k-space originate from inter-shot phase fluctuations only. Multiple RF receiver channels, if available (e.g., after GRAPPA reconstruction [55]), can be used jointly for phase errors estimation, as the phase fluctuation kernels are shift-invariant across RF channels, unlike the GRAPPA kernels being unique to each receiver.

Another two multi-shot mosaic [27] EPI sequences (center-out, short-TE) were designed similarly with small overlap regions,

minimized differences in signal decay/dephasing between overlapped trajectories, and also equipped with an optional 2D navigator. A phase-interleaved EPI sequence with optional navigator was also implemented for testing MUSE, while our kernel extraction approach can replace the original total-variation image-space phase smoothing and improve the phase map quality estimated from intermediate PI reconstruction for each shot [23]. This is similar to JULEP [71] but with only single-step direct phase map extraction, rather than MUSSEL (iterative k-space denoising) followed by MUSE (image-space phase-smoothing).

2.2 | Phase Fluctuation Kernels Meet GRAPPA from a generalized k-space interpolation perspective

We describe shot-to-shot phase fluctuations using similar notations in a reproducing-kernel-Hilbert-space (RKHS) sampling theory [67] to stress a unified framework with the well-established PI. This theory [67] universally characterizes the k-space signal interpolations, as a concept widely used in re-gridding [72–77]. It can yield effective k-space coverage and noise amplification maps for arbitrary spatial encoding functions (i.e., either maps or kernels), such as PI [55] with distinct sampling patterns (e.g., CAIPI [78], Poisson-Disc [79]), nonlinear gradient encoding [59, 60]. While standard GRAPPA theory remains sufficient for understanding auto-calibration of diffusion phase kernels, future work may exploit the cardinal and power function maps, unique to the RKHS formalism, to evaluate and understand phase fluctuation encoding efficiency [66] directly in k-space.

In overlapped data segments at any k-space locations, the time-domain samples in shots can be written as

$$f^{\parallel} = \mathcal{F}\mathcal{P}m, \quad (1)$$

where $f^{\parallel}$ is k-space signal acquired from shot number 1 to N ($N \geq 2$), $\mathcal{F}$ denotes Fourier transform operator, phase fluctuations can be represented by matrix $\mathcal{P} = [\mathcal{P}_1 \mathcal{P}_2 \cdots \mathcal{P}_N]^T$ as a vertically concatenated stack of diagonal matrices $\mathcal{P}_i$ with diagonal elements corresponding to the relative phase map values in each shot, m is the object signal representation seen by RF receivers.

From Fourier theory, the multiplication of phase fluctuation maps with spin distribution can be interpreted as convolution with phase fluctuation kernels in k-space [24, 67]. Namely, within inter-shot overlap region C seen by all RF receivers, a neighborhood of k-space (t) data in shot (i) can be multiplied with a kernel (t, i) and summed up to approximate a sample point i in another shot j :

$$f_j^{\parallel}(i) = \sum_{t,i} f_i^{\parallel}(t) u_j^{t,i}(i), \quad (2)$$

with $t, i \in C$.

$f_i^{\parallel}(t)$ and $f_j^{\parallel}(i)$ are discrete MRI samples interpolated by phase fluctuation kernel $u_j^{t,i}(i)$, which is called cardinal function in the RKHS-MRI theory to generally represent interpolation weights in k-space for arbitrary spatial encoding functions [55, 59, 60, 78].

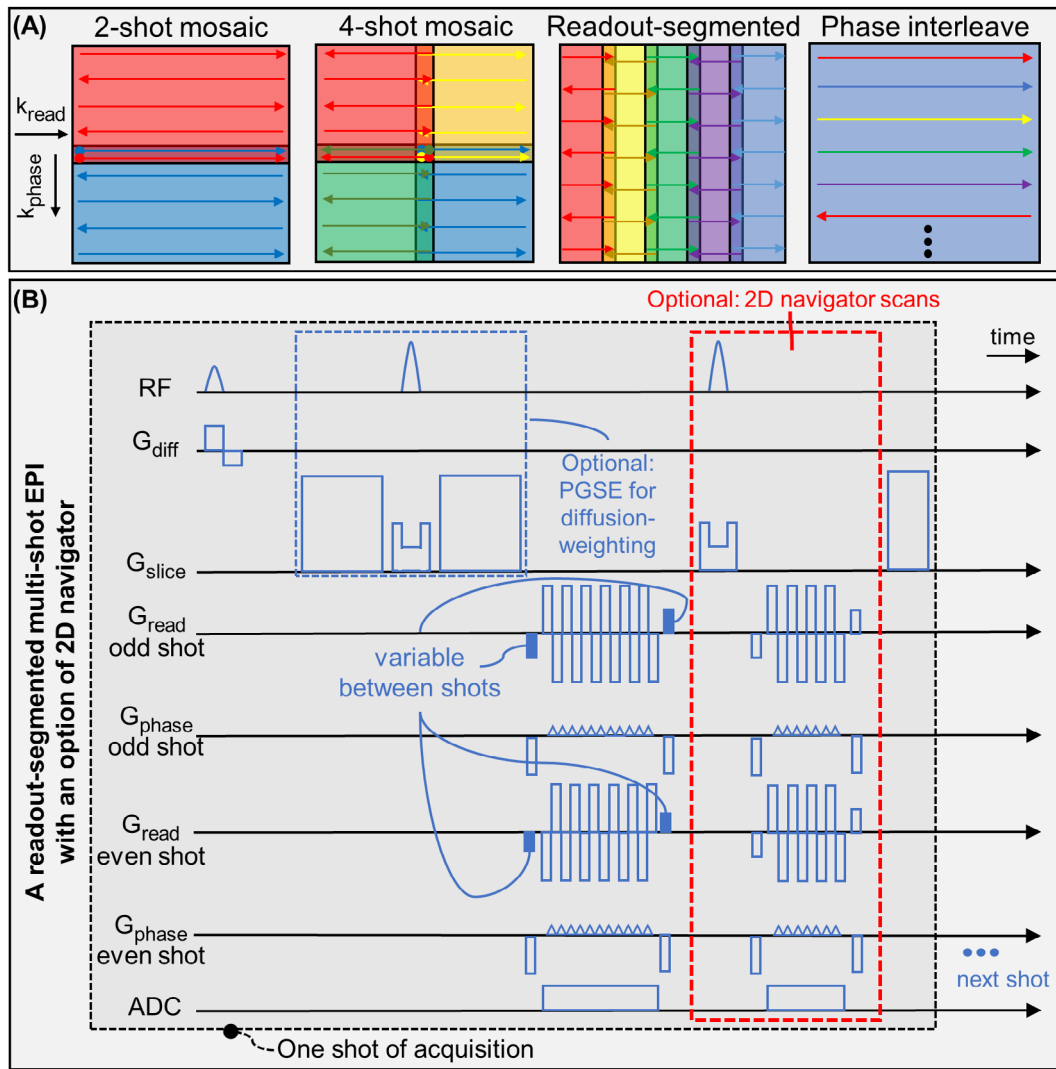


FIGURE 1 | The k-space sampling strategy and sequence. (A) The k-space trajectory of the four EPI sequences: two-shot mosaic EPI, four-shot mosaic EPI, five-shot readout-segmented mosaic EPI, and five-shot phase-interleaved EPI. In the two-shot and four-shot mosaic EPI, the trajectories employ center-out directions and start from the solid circles in the illustration, to minimize differences in T_2 or T_2^* decay in overlapped regions. The density of the trajectories is drawn for illustration purposes only, and cannot reflect the realistic Nyquist sampling. In this paper, the readout-segmented scheme was also adapted for three-shot and seven-shot, and the phase-interleaved scheme was also adapted for two-shot, three-shot, four-shot, and six-shot. (B) A multi-shot EPI sequence with optional PGSE excitation, and optional 2D navigator acquisitions. It is an example of readout-segmented mosaic EPI, but was modified for two-shot mosaic EPI, four-shot mosaic EPI, and phase-interleaved EPI. In readout-segmented trajectory, the readout gradient polarity is flipped between odd and even shots. The readout gradient phase blips (solid blue) are adjusted between shots to sample different k-space locations. The PGSE block used for diffusion preparation can be replaced by gradient echo excitation. The areas of gradient waveforms are drawn for illustration purposes only, and cannot exactly reflect the actual experiments.

The notations t and i indicate time-domain nature for k-space. In our implementations, EPI readout oversampling is removed in this interpolation model for simplicity, without loss of generality. Note, continuous gradient errors (e.g., eddy currents) within each shot can disrupt this model for shot-to-shot signal variations, and may requires additional calibrations.

The kernel width usually spans several pixels (e.g., 6–20, 7 mostly used in this paper), depending on the spatial phase smoothness of the corresponding image-space maps (e.g., Figures 2A and S7). Therefore, a calibration matrix A is constructed by applying a sliding mathematical window throughout the calibration region C , taking each k-space block in $f_i^{\parallel}(t)$ and vectorized as a row

in the matrix, as denoted by operator R_i , similar to Equation (4) [56]. The superscript $(\bullet)^T$ denotes nonconjugate transpose operation. In this way, a linear system with the calibration matrix can be established, linking acquired k-space data and the unknown phase fluctuation kernels, as in Figure 2. Given operation R_i within the calibration region C , Equation (2) can be adapted to describe this linear system:

$$f_j^{\parallel}(i) = \sum_{t,i} \left(R_i f_i^{\parallel}(t) \right)^T u_j^{t,i}(i) = A u_j^{t,i}(i), \quad (3)$$

Based on Equation (3), we test two approaches to estimate the phase fluctuations—directly estimating fully-sampled k-space kernels through a GRAPPA-type operation [55], or calculating the

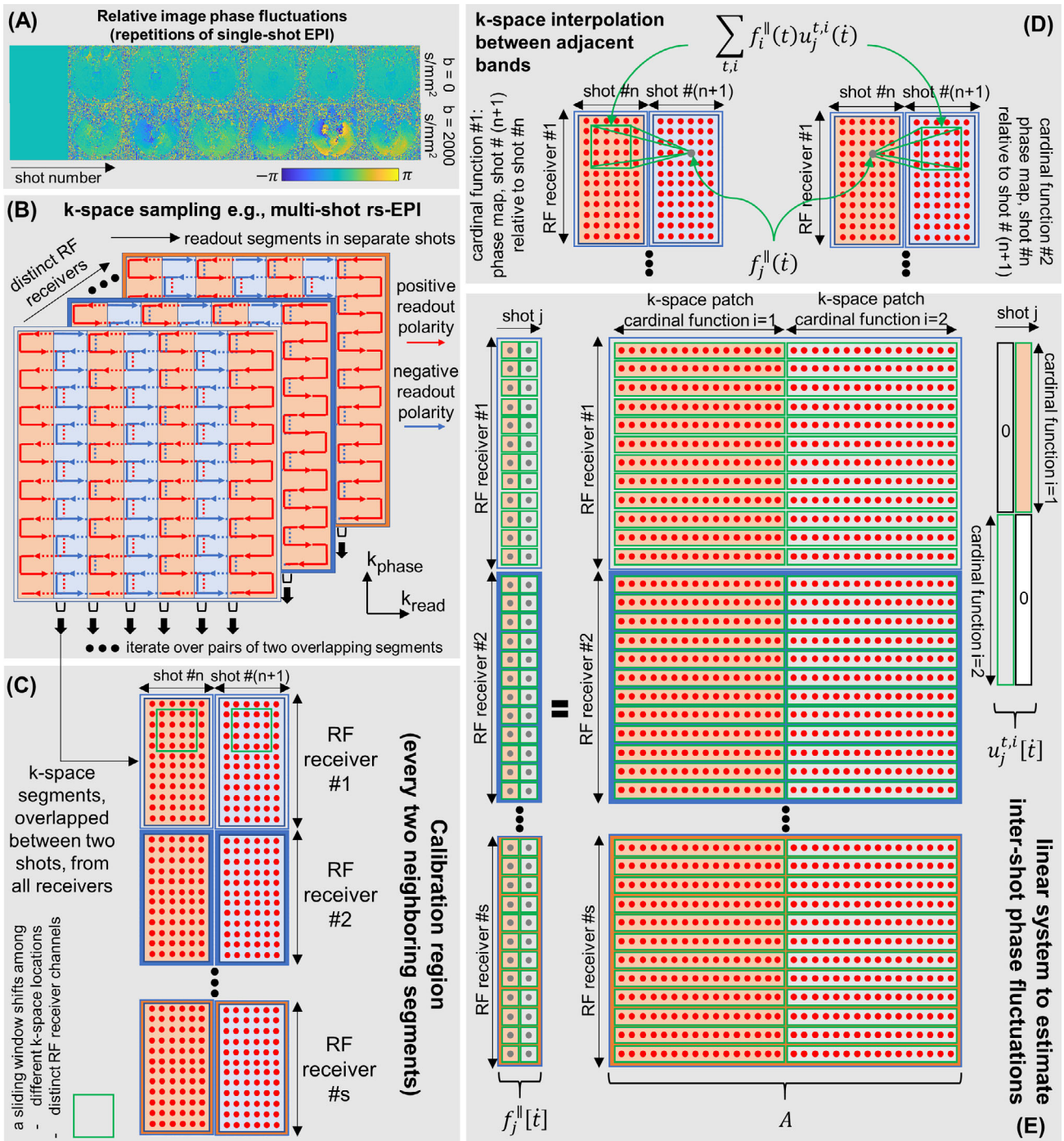


FIGURE 2 | Signal processing perspective for k-space diffusion kernel interpolations. (A) The target unknown—phase variations are demonstrated in image-space. Their Fourier transform representations are the diffusion phase kernels. A single-shot EPI sequence with and without diffusion-weighting was repeated to yield these phase maps. More examples for higher b -values are in Figure S7. (B) As an example, k-space sampling trajectory for readout-segmented EPI is shown, to illustrate the origin of the proposed shot-dependent ACS regions (overlapped segments), for shot-to-shot phase extractions. (C) The calibration region constructed from every two neighboring and overlapped k-space segments. The interpolation kernel is shift-invariant across k-space locations and RF receiver channels, but shift-variant across shot numbers. (D) The k-space interpolation relationships in the calibration region, where a neighborhood from one shot is used to interpolate a single k-space point at a similar location in another shot. (E) The linear system representation. This system involves two cardinal functions, with the width determined by the sliding window size. In practice, only one cardinal function needs to be solved to estimate a relative phase fluctuation map between two shots, for interpolation in one direction. But both cardinal functions are shown here for a complete and general mathematical representation. Theoretically, these unknown kernels can be solved using different algorithms, e.g., pseudo-inverse, tSVD, LSQR, or neural-network based methods.

corresponding image-space phase fluctuation maps via an eigenvalue approach, as an ESPIRiT-type operation [56]. Note, we do not directly call them “GRAPPA” or “ESPIRiT,” to stress they are not used for reconstructing multi RF-receivers’ undersampled data. Theoretically, both operations benefit from the averaging effects based on a large number of shifted kernels, inheriting the robustness of GRAPPA/ESPIRiT.

2.3 | Phase Fluctuation Maps Meet ESPIRiT

For direct estimation of phase fluctuation kernels, the linear system in Figure 2E can be tailored for one-way k-space interpolation to estimate phase fluctuation of one shot relative to another. Alternatively, an eigenvalue approach can be used, and consequently, only calibration matrix A is needed to derive image-space phase fluctuation maps, adapted from Equations (5–18) in ESPIRiT [56] for calibration of RF sensitivity maps.

To see that, the calibration matrix structure suggests the existence of null space, implying all the interpolation relationships are redundantly contained within A . Namely, the shifted k-space neighborhoods reshaped into rows can be interpolated to approximate a data value (e.g., the neighborhood center) within that row, using the same set of cardinal functions/kernels which are shift-invariant across rows.

$$Au_j^{t,i}(i) - f_j^{\parallel}(i) = A(u_j^{t,i}(i) - e_{i,j}) = 0, \quad (4)$$

where $e_{i,j}$ is a matrix with a “1” in appropriate position that chooses the data of $f_j^{\parallel}(i)$ in the calibration region, and “0” elsewhere.

Therefore, the calibration matrix can be decomposed using singular-value decomposition (SVD), given the right singular-vectors (i.e., columns of V) provide a basis for the rows of A , from which phase fluctuation kernels/maps will be extracted. The superscript $(\cdot)^H$ denotes conjugate transpose operation. Moreover, thresholding between its potential signal-space (i.e., row space) $V_{\parallel}$ and noise-space $V_{\perp}$ based on the singular-value magnitudes allows a subspace “denoising” for the calibration data with respect to shift-invariant kernels signals.

$$A = U\Sigma V^H, \quad (5)$$

Consequently, a subspace signal encoding perspective $\mathcal{W}f^{\parallel} = f^{\parallel}$ can be formulated with a operation $\mathcal{W}$, representing rearranging k-space neighborhoods into calibration matrix $R_t f^{\parallel}$, projecting onto signal-space ($V_{\parallel} V_{\parallel}^H$), and reshaping back to the k-space neighborhood ($M^{-1} \sum_t R_t^H$, for all shifted locations t).

$$\mathcal{W}f^{\parallel} = \underbrace{M^{-1} \sum_t R_t^H V_{\parallel} V_{\parallel}^H R_t}_{\mathcal{W}} f^{\parallel} = f^{\parallel}, \quad (6)$$

where $f^{\parallel}$ represents general k-space signals in Equation (1), M represents $\sum_t R_t^H R_t$ as the sample number in each k-space neighborhood selected by R_t . Next, after inverse Fourier transforming the k-space signals in Equation (1) with subspace projection,

which can be seen as part of a image-space operator $\mathcal{G}$, the calibration consistency (6) can be expressed in image-space:

$$\mathcal{G}\mathcal{P}m = \underbrace{F^{-1}\mathcal{W}F}_{\mathcal{G}}\mathcal{P}m = \mathcal{P}m, \quad (7)$$

Similar to equation (16) in ESPIRiT [56], at every non-zero pixel q with phase fluctuation $\bar{p}_q$, we have image-space operator $\mathcal{G}_q$ that can be written as matrix multiplication of $G_q^H G_q$

$$F^{-1}\mathcal{W}F|_q = \mathcal{G}_q, \quad (8)$$

$$\mathcal{G}_q \bar{p}_q = G_q^H G_q \bar{p}_q = \bar{p}_q, \quad (9)$$

Such formulation transforms the problem into calculating the image-space eigenvector of the operator $\mathcal{G}_q$ with eigenvalue of 1, which is exactly the vector of phase fluctuation values. Thus, it is called an eigenvalue approach. Further, the eigenvector of $\mathcal{G}_q$ can be efficiently computed by performing SVD of matrix G_q^H , and selecting its left singular-vector (i.e., from V_G) with the largest singular-value, since

$$\begin{cases} G_q = U_G \Sigma_G V_G^H \\ G_q^H = V_G \Sigma_G U_G^H \\ \mathcal{G}_q = G_q^H G_q = V_G \Sigma_G^2 V_G^H \end{cases} \quad (10)$$

Namely, only G_q^H is needed, obtained via Fourier transform of k-space filters reshaped from the truncated calibration matrix subspace, corresponding to merely the left-half part of the Equation (7) as $(F^{-1}M^{-1}\sum_t R_t^H V_{\parallel})|_q$. Optionally, to further reduce memory usage, V_G can also be obtained via eigenvalue decomposition of $G_q^H G_q$. In practice, the primary singular- or eigen-vector associated with the largest singular- or eigen-value is selected to represent shot-to-shot signal fluctuation maps. The eigenvalues of the (normalized) operator $\mathcal{G}_q = G_q^H G_q$ generally lie between 0 and 1, because in $F^{-1}\mathcal{W}F$, the Fourier transform is unitary and the operator $\mathcal{W}$ representing an average of projections has eigenvalues smaller or equal to one. In this way, the diffusion fluctuation maps can be estimated pixelwise as the eigenvectors of the operator $\mathcal{G}_q$ associated with the eigenvalue of 1. These steps are graphically illustrated in Figure 3.

Note, if phase estimation takes place after combining RF receivers, it is not straightforward to have the “soft” forward model [56] in PI, since there is no encoding redundancy to estimate multiple sets of phase fluctuation maps. Additionally, the residue null-space images [56] cannot be easily produced since EPI segments overlap only in small regions.

2.4 | Reconstruction to Combine all Shots

Given undersampled k-space data, PI reconstruction can be applied to at least overlap k-space region for yielding shot-dependent ACS. This can be called self-navigated ACS for imaging echoes, or separate navigators’ ACS if phase is extracted from 2D navigators via the kernel-based operations.

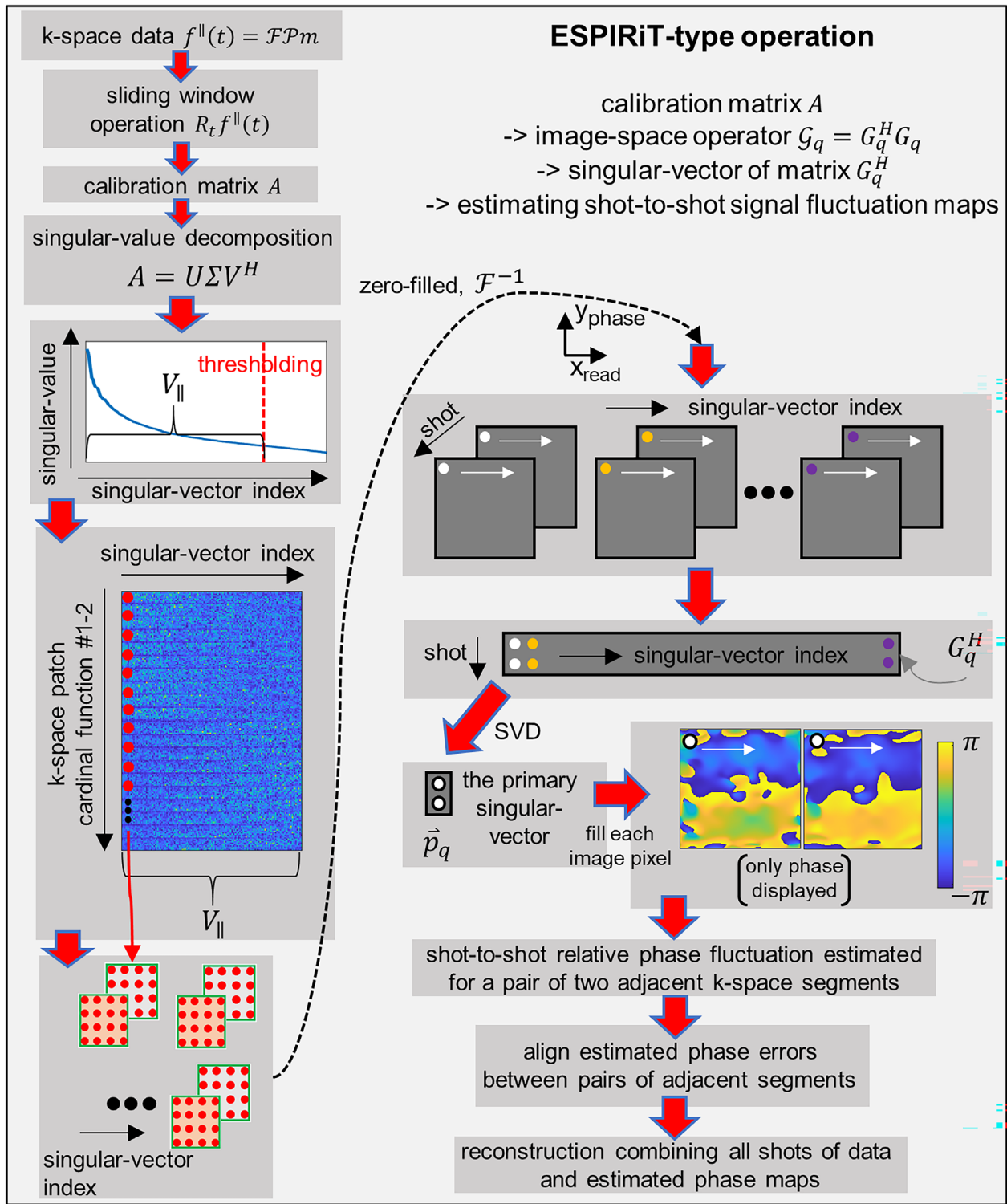


FIGURE 3 | A graphical illustration of the eigenvalue approach (ESPIRiT-type operation) for estimating shot-to-shot phase fluctuations. The k-space data from the overlapping regions of two EPI segments acquired in separate shots are used to construct a calibration matrix A . Through SVD with singular value thresholding, the signal subspace $V_{\parallel}$ is extracted, reshaped into k-space patches, zero-filled, and (inverse) Fourier transform to image space. At each image-space pixel location, a matrix G_q^H across different shot numbers and singular index can be obtained. Performing another SVD on G_q^H , and selecting the primary singular vector corresponding to the largest singular-value, the relative shot-dependent signal variations at each pixel can be estimated. The phase of the estimated these variations is taken, aligned between different pairs of overlap segments, and used to correct the entire EPI segments in all shots during image reconstruction.

Afterwards, there are two ways for reconstructing the final image combining all shots.

For the first approach [80] (here, used for mosaic EPI), if fully-sampled partial k-space segments are obtained after PI (e.g., ESPIRiT/GRAPPA) reconstruction, they can be simply multiplied by the complex conjugate of estimated phase maps in the corresponding shots, and summed up into a final k-space image, with an inverse filter [45] to remove modulation-transfer-function (MTF) due to the partial k-space overlaps. Optionally, an apodization window [45] can be applied on k-space segments to reduce PSF ringing before phase corrections, and will be eventually removed by the inverse filter also.

In the second approach [19] (here, used for phase-interleaved EPI), the k-space data, RF receivers sensitivity maps/kernels and estimated phase maps/kernels can be fed into a forward model to reconstruct a final image, which includes phase fluctuation encoding [66] in this multi-shot PI reconstruction.

3 | Methods

3.1 | MRI Sequence

The 2D multi-slice, multi-shot EPI sequences with optional 2D navigators were implemented in Pulseseq [70], with slight adaptations for the four different trajectories. The experiments were conducted on two commercial 3T human MRI scanners (Prisma^{fit} with max. 80 mT/m and 200 T/m/s, Cima.X with max. 200 mT/m and 200 T/m/s, Siemens Healthineers, Erlangen, Germany), including ex vivo brain phantom scans [81, 82] and in vivo brain scans (several healthy adults volunteers, in agreement with the institution's ethics policy), all using Pulseseq sequences with off-line image reconstruction. The scan protocol included pulsed-gradient-spin-echo (PGSE) diffusion-weighted (b -values from 0 to 3000 s/mm²) or gradient-echo (GE, TE 30 ms) EPI scans, reference ACS scans for PI reconstruction (i.e., low-resolution GRE), EPI projection pre-scans (i.e., phase-encoding gradient disabled) for odd-even-echoes correction, and optionally, B_0 mapping scans consisting of two GRE acquisitions with 3 ms difference in TE.

The in-plane spatial resolution is denoted as “mm (readout dim.) × mm (phase-encoding dim.)”. For sequences with a navigator acquisition, the echo time is specified as “TE imaging/TE navigator.” The FOV was 220 mm², with the slice thickness of 3 mm. The volumetric one-shot TR (one shot for acquiring all slices) was 4 s for SE-EPI, and 1 s for GE-EPI. The partial Fourier acquisition was disabled in two-shot mosaic, four-shot mosaic, two-shot phase-interleaved EPI and was 6/8 for all other sequences.

The scan-dependent sequence protocols (e.g., resolution, shot, ESP, gradient, overlapped pixels, PI acceleration factors, slice number, b -value, diffusion directions) are documented in [Supporting Information](#). The ESP is defined as the time between the centers of two consecutive EPI readout lines, and the effective ESP further considers the undersampling and phase-interleaved factors to reflect distortion levels.

3.2 | Image Reconstruction

The images were reconstructed offline in MATLAB (CPU, 2.35GHz, 64 cores) from raw data, consisting of the following steps. Detailed implementation for Step 4 (shot-to-shot phase extraction) is documented in [Supporting Information](#) Section 9.

1. EPI re-gridding based on nominal trajectory, with readout oversampling eliminated.
2. N/2 ghost corrections based on separate FID scans.
3. SENSE reconstruction [83] with pre-whitening and ESPIRiT map [56]. Distortion correction with B_0 maps was incorporated into the forward model, except for Figures 6, 7, S6, and S8. This step can be replaced by applying GRAPPA reconstruction over overlap regions only.
4. Estimation of shot-dependent phase fluctuation maps using one of the four presented techniques in Figure 6. The computational complexity for kernel extraction is similar to GRAPPA/ESPIRiT, but with much less shot number than RF receiver number. A fast-prototyped MATLAB implementation took below 1 s for GRAPPA-type operation, about 1–2.5 s for ESPIRiT-type operation, to compute 2–5 relative shot-to-shot phase maps of one slice, in one of the mosaic EPI sequences or 2D navigators. The fast-prototyped ESPIRiT-type operation took within 12 s for one-slice-five-shot phase maps in phase-interleaved EPI.
5. Combining all shots of data and phase maps, using one of the two approaches in Section 2.4.
6. Partial-Fourier reconstruction [84]. The factor is defined as the ratio between actual- and fully-sampled lines within the first half k-space.
7. Calculation of tSNR or its inverse (coefficient-of-variation map, CV), or export to FSL-DTI toolbox [85].

4 | Results

4.1 | Robust and Versatile Self-Navigation for Different Trajectories

In Figure 4, our shot-to-shot phase estimation method (here, the eigenvalue approach) enables robust and versatile self-navigated multi-shot EPI on a standard 3T scanner, using four trajectories—two-shot mosaic (center-out), four-shot mosaic (center-out), five-shot readout mosaic, five-shot phase-interleaved (MUSE with kernel extraction). Their sum images consisting of 16 diffusion directions show no visible differences to the corresponding reconstruction corrected with separate 2D navigators. Furthermore, at matched shot number and final resolution, the two-shot and four-shot mosaic EPI show substantially higher SNR due to shorter TE, and improved four-shot self-navigation quality due to fully-sampled (after PI), high-SNR shot-dependent ACS, compared with phase-interleaved EPI. A full-version figure with difference maps between self- and external-navigation, and subspace thresholds is shown in [Supporting Information](#) (Figure S2).

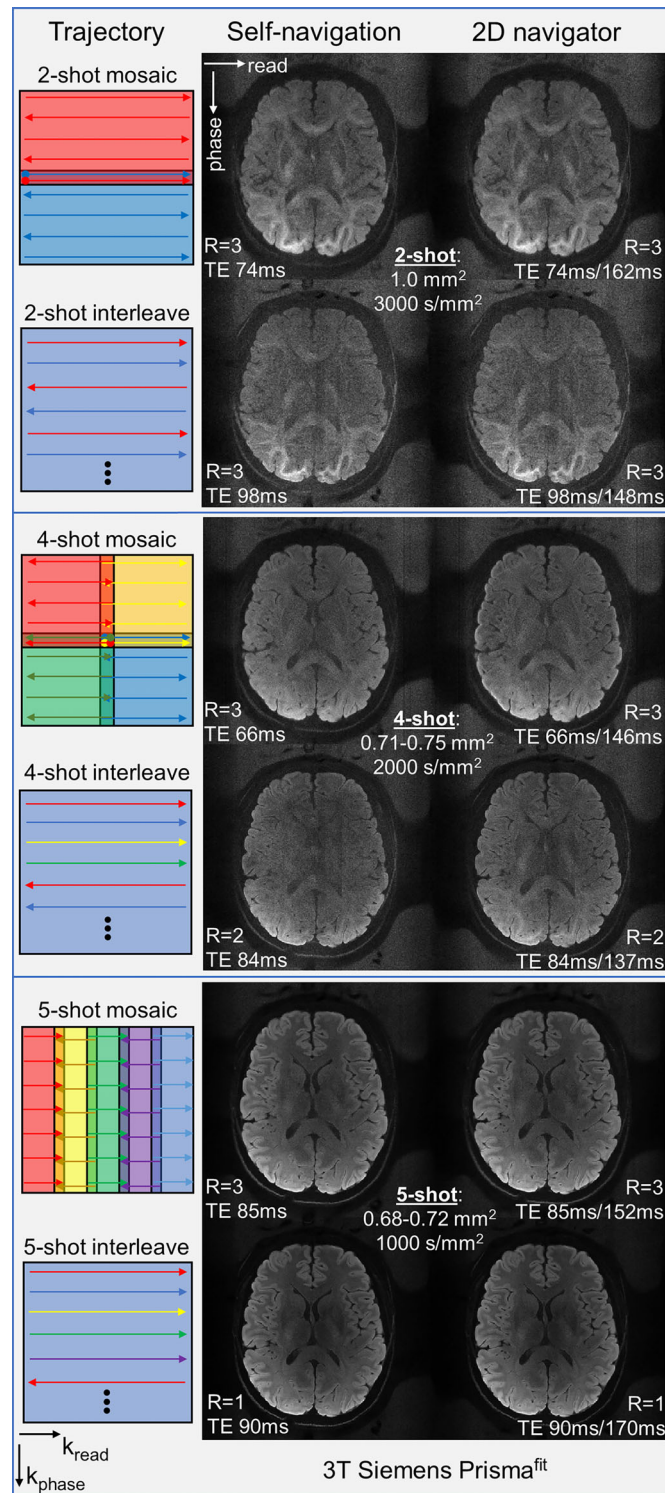


FIGURE 4 | Multi-shot EPI sequences, comparing self-navigation and 2D navigator corrections, mosaic and phase-interleaved (in-plane) segmentation schemes. Here, reconstructions for self-navigation and 2D navigator corrections are based on the same scan, equipped with both k-space band overlap and 2D navigator acquisitions. The overlap regions are located at k-space center, a bit away from center, or far away from center in different sequences, as in the trajectory graphs. The sum images consisting of 16 diffusion-encoding directions are shown, with a full-version figure with difference maps (between self-navigation and 2D navigator corrections) and subspace thresholds in [Supporting Information](#) (Figure S2). The eigenvalue approach (ESPIRiT-type operation) with 7×7 kernel size was utilized for extracting phase maps from self-navigated ACS regions and separate navigator's ACS region. Identical shot number and similar resolution was employed to compare mosaic and phase-interleaved schemes, where the latter can suffer from low-quality intermediate reconstruction for each shot. The parallel imaging acceleration factor R is defined as the undersampling factor along phase-encoding considering all shots of data relative to a full k-space. Thus, in phase-interleaved EPI, the k-space undersampling factor in intermediate PI reconstruction is the multiplication of R and shot number. Typically, 2D navigator acquisition cost a few tens of milliseconds per shot. In comparison, the additional sampling time due to k-space overlapping is negligible. More detailed sequence comparisons should be based on more considerations in acoustic resonance, PNS, distortion, gradient capabilities [86–88], wave-encoding [89], and is beyond the scope of this paper.

Specifically, the two-shot mosaic and phase-interleaved EPI had effective ESP of 0.36 ms/0.18 ms, b -value 3000 s/mm², and no partial-Fourier. The two-shot mosaic sequence incurred two additional EPI readout lines (3× undersampled before PI) per shot for k -space overlap (overlap-size 222 × 13), corresponding to extra 5.4 ms per 2D acquisition, as negligible. However, with 24 ms TE reduction, it shows markedly clearer image structure than the phase-interleaved EPI (central overlap 50 × 50, from intermediate PI reconstruction) for strong diffusion-weighting, both with and without 2D navigators.

The four-shot mosaic EPI had effective ESP of 0.23 ms, no partial-Fourier, and increased resolution from two-shot mosaic EPI without additional k -space MTF due to T_2 decay. It required three additional EPI readout lines (3× undersampled before PI) per shot for k -space overlap (overlap-size 13 × 155), corresponding to extra 5.6 ms per 2D acquisition, as negligible. The four-shot phase-interleaved EPI had effective ESP of 0.14 ms, 6/8 phase-partial-Fourier, and 18 ms longer TE. Given b -value 2000 s/mm², the image quality of four-shot phase-interleaved EPI degrades, especially in self-navigation reconstruction (MUSE with kernel) that failed to estimate accurate phase maps from a central 50 × 50 overlap based on the intermediate PI (here, ESPIRiT) reconstruction. This is because, in this case, 8× undersampling per shot severely exceeds (intermediate) PI's capability for image recovery.

The five-shot readout-segmented EPI had effective ESP of 0.18 ms, phase-partial-Fourier 6/8, and overlap regions of 16 × 274. For this protocol (five-shot, 0.72 mm²), the additional 16 readout-overlapped points did not actually cause additional acquisition time, but just slightly raised the under-utilized readout gradients to 22 mT/m and 137 T/m/s. Here, ESP cannot be further shortened because of the prohibited acoustic resonance band (Figure S3). If ignoring this prohibited band, using similarly underutilized gradient strength/slew rate, effective ESP can be further reduced to 0.15 ms. Note, in other protocols using very short ESP (e.g., below 0.4 ms, 0.13 ms effective with $R = 3$) above the prohibited acoustic band, very narrow readout-segments (e.g., spanning 30 k -space pixels, for lower spatial resolution) may be used, which can make such 16 overlapped points non-negligible and require protocol-specific re-evaluation of feasible overlap schemes. The five-shot phase-interleaved EPI had effective ESP of 0.22 ms, phase-partial-Fourier 6/8, and overlap region of 50 × 50 from intermediate PI (here, ESPIRiT) reconstruction. Because of merely 5× undersampling per shot relative to a full k -space, namely, higher gradient amplitude (43 mT/m, 138 T/m/s) to sample more k -space data, five-shot phase-interleaved EPI finally yields identical self-navigated image quality to the reconstruction corrected by 2D navigators.

In Figure 5, as further quantitative measure, coefficient-of-variation (i.e., $1/\text{tSNR}$) maps (10 repetitions, one out of three diffusion directions) demonstrate improved accuracy in our self-navigation approaches based on explicit kernel extraction, relative to 2D navigator corrections or conventional MUSE (with total-variation-phase-smoothing). Here, a 7 × 7 kernel size was used with around half subspace truncation for all kernel extraction algorithms. Particularly, in two-shot mosaic and five-shot phase-interleaved EPI, CV maps for reconstruction corrected by 2D navigators or MUSE reveal additional artifacts or ghosts [90],

which are minimized in our kernel-based self-navigation methods. The artifacts in 2D navigator corrections may partially result from the shot-dependent eddy currents that cannot be captured by navigators at k -space center. Additionally, the two-shot and four-shot mosaic EPI achieved higher SNR relative to more commonly used readout-segmented and phase-interleaved EPI, validated by smaller CV values.

Note, phase maps extraction from central k -space data—2D navigators, phase-interleaved EPI (i.e., MUSE with kernel extraction), and B_0 field scans (Supporting Information) was also implemented via an ESPIRiT-type operation; their comparisons with non-kernel-based phase extraction are provided in Figure 6 and Supporting Information (Figures S4 and S5). Moreover, shot-dependent eddy currents (only captured by self-navigation rather than additional navigators), subspace choices (kernel sizes and thresholds), measured diffusion kernel widths, increasing b -values, quantitative multi-shot reconstruction errors across different slices are analyzed in Supporting Information (Figures S6–S13).

4.2 | Kernel Extraction Accuracy: SNR and Subspace Analysis

Nevertheless, the estimation accuracy of our overlap-kernel approaches can degrade in very low-SNR calibration regions [12]—for example, in peripheral k -space regions for readout-segmented sequences given strong diffusion-weighting. Both explicit kernel extraction approaches (direct estimation of interpolation kernels, and eigenvalue approach) exhibit similar problems under these conditions, which can be much worse than the central k -space, non-diffusion-weighted ACS in PI [91].

Thus, in Figure 6, we evaluate kernel extraction accuracy in terms of visual quality, using readout-segmented EPI sequence as an example. Four different shot-to-shot phase estimation algorithms (Supporting Information Section 9) are compared—GRAPPA-type operation (linear inversion using Moore-Penrose inverse), ESPIRiT-type operation, low-pass-filtered and zero-filled 2D navigator, and kernel-extracted 2D navigator. Multi-shot reconstructions from high- to low-SNR conditions are compared, corresponding to increasing shot number, resolution, diffusion-weighted b -values, TE, all of which can result in low-SNR in non-central overlapped k -space segments. Generally, all phase fluctuation correction algorithms (Rows 2–5) provided significantly higher spatial resolution than the uncorrected scan with strong blurring (Row 1), revealing more detailed image texture.

In high- and medium-SNR, both two self-navigated reconstructions yield equal or better image quality than 2D navigator corrections (e.g., with local ghost). This is possibly because, they can also calibrate shot-dependent eddy currents (e.g., due to shot-dependent EPI pre-phasers, Figure S6), which cannot be captured by navigator echoes that are always at k -space center. However, in low-SNR regimes, 2D navigators provided much more sufficient SNR at central k -space regions for phase estimation, while the self-navigated reconstructions appear slightly blurred.

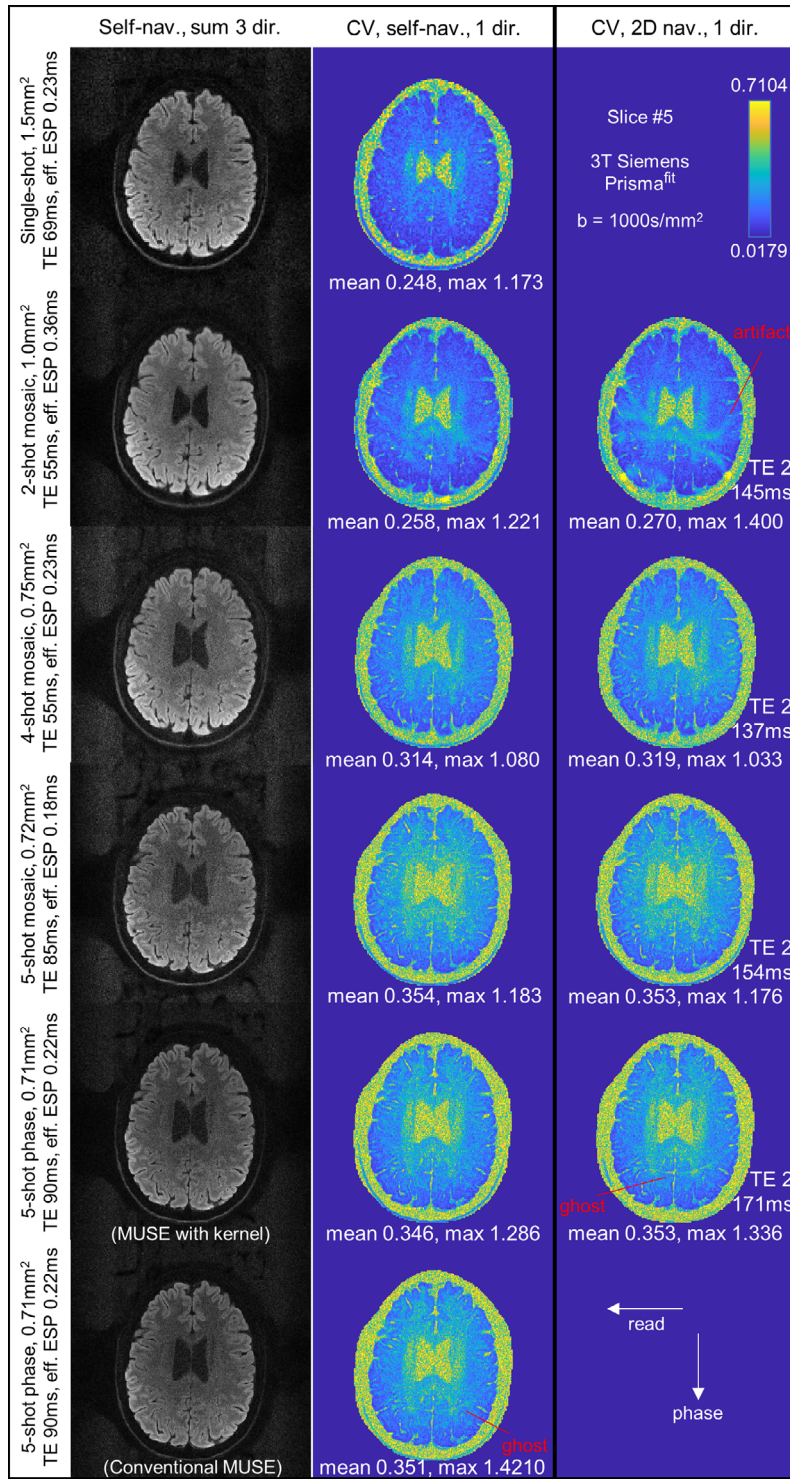


FIGURE 5 | Quantitative measurements of shot-to-shot phase calibration errors, for the multi-shot EPI sequences. Self-navigated sum images consisting of 3 diffusion directions, and coefficient-of-variations (CV) maps for scans with 1 diffusion-direction over 10 repetitions are shown. Different sequences—single-shot, two-shot mosaic, four-shot mosaic, five-shot readout-segmented mosaic, five-shot phase-interleaved with 1000 s/mm² were used. Note, the effective ESP (not ESP) are shown, as the ESP divided by k-space undersampling (including phase-interleaved if available) factor in each shot. Although no apparent artifacts are found in all reconstructed sum images, two-shot and four-shot mosaic EPI has visibly higher SNR, cross-validated by lower CV values, compared to the five-shot readout-segmented and phase-interleaved EPI. Nevertheless, the two-shot and four-shot mosaic EPI also remain more sensitive to B_0 field offsets similar to spiral, which can cause occasional blurring in other slice positions and requires more optimized corrections for trajectory or B_0 inhomogeneity, as shown in [Supporting Information](#) (e.g., Figure S10). Generally, CV maps for self-navigation reconstruction appear lower or similar to 2D navigator corrections, indicating 2D navigators should be removed under similar b -values and B_0 inhomogeneity levels. Notably, in 2D navigator corrections for two-shot mosaic and four-shot phase-interleaved EPI, as well as conventional MUSE (image-space total-variation phase extraction) [90], additional artifact or ghosts can be clearly observed in CV maps, indicating time-course instability in their corresponding shot-to-shot phase estimations.

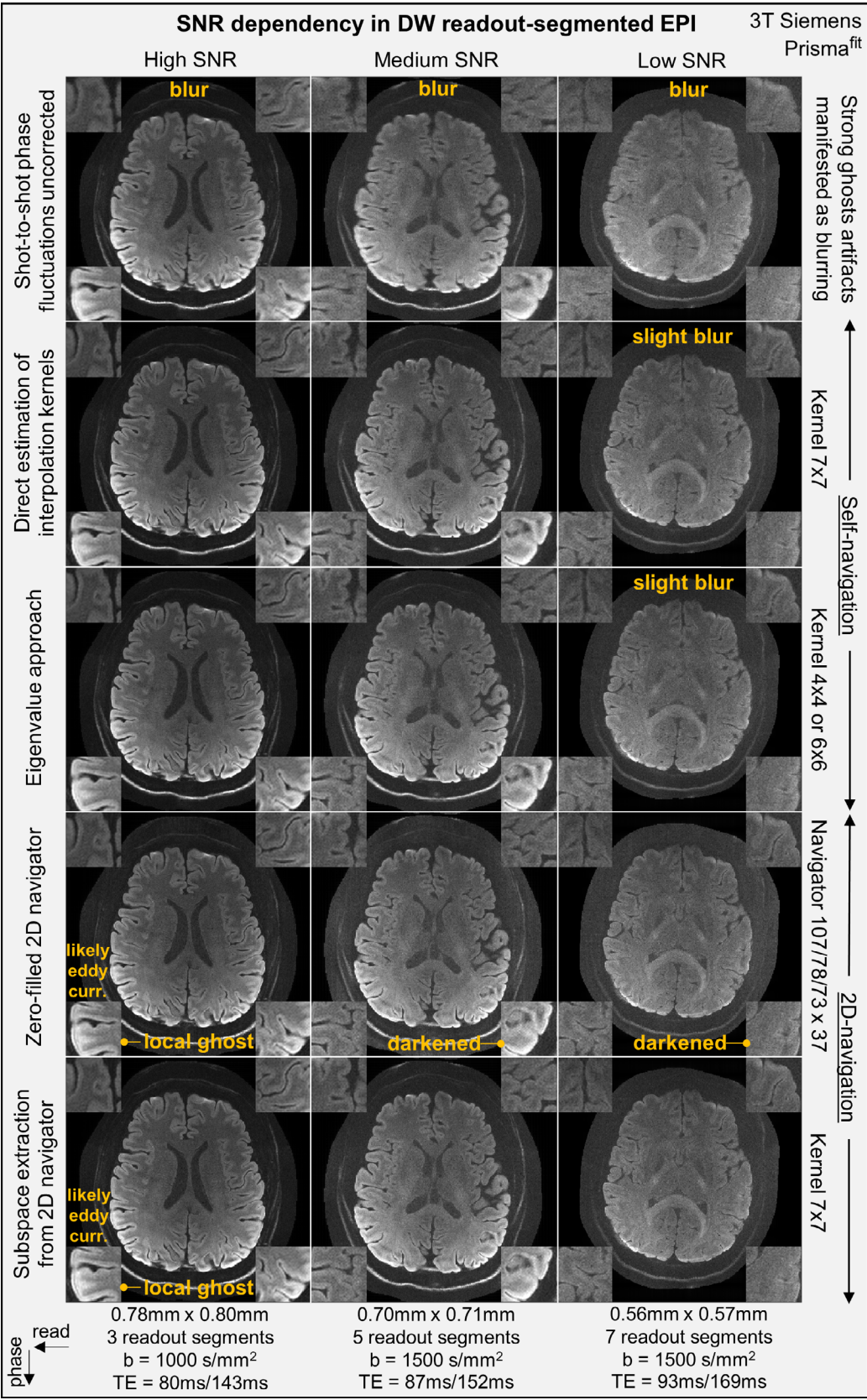


FIGURE 6 | Multi-shot readout-segmented EPI with different methods for correcting shot-dependent phase fluctuations. The diffusion-weighted images are shown as the sum image for a total of 20 or 16 diffusion directions. Column 1–3: High, medium and low SNR scans with increasing resolution, more readout segments, larger diffusion-weighted b -values and longer echo time (TE imaging echoes/TE navigator echoes). Row 1–5: The five approaches without and with shot-to-shot phase fluctuations corrections.

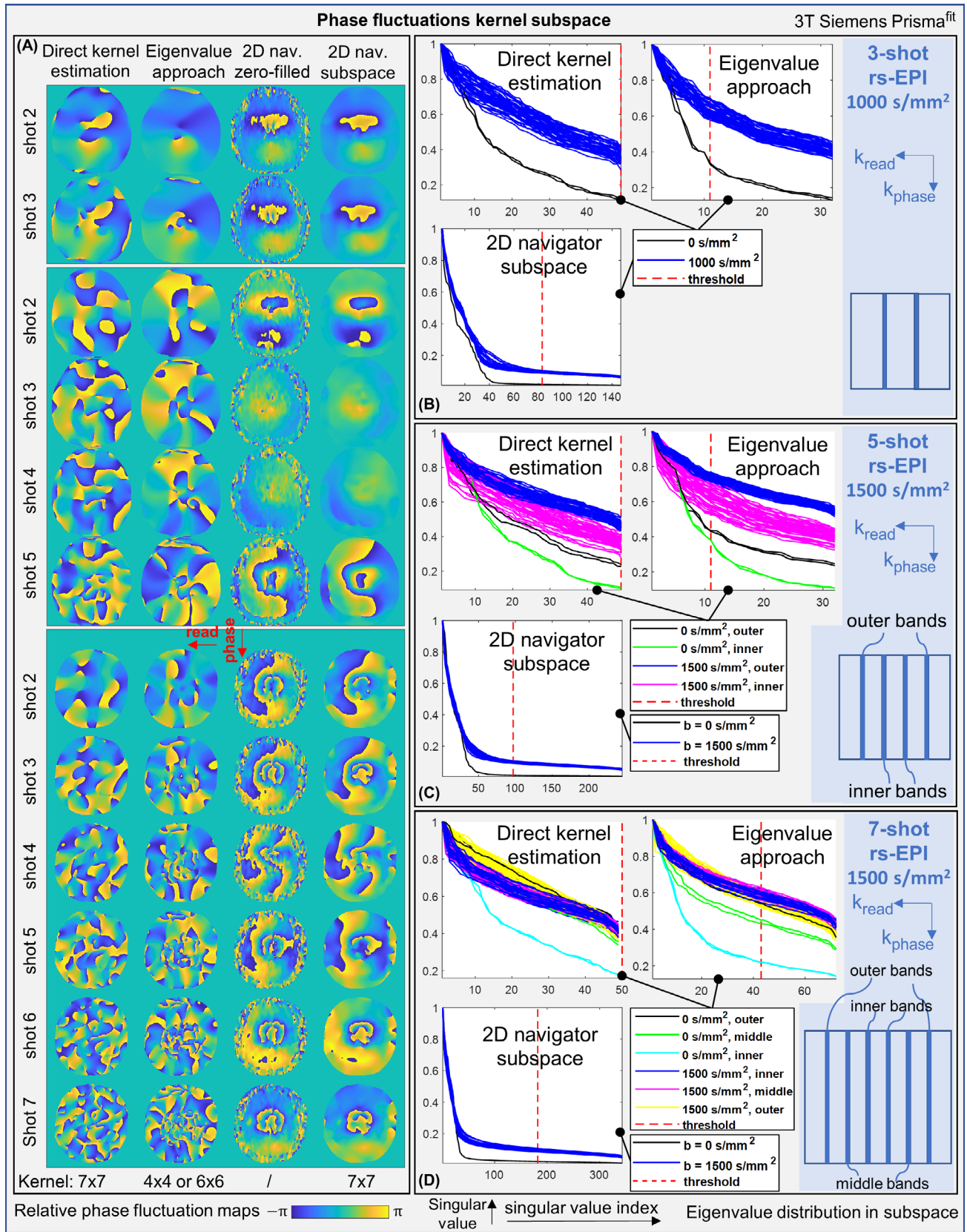


FIGURE 7 | Estimated diffusion phase fluctuation maps by different methods, and the singular-value distributions from the kernel extraction algorithms applying to the overlapped imaging data at different k -space locations, as well as the 2D navigator data at the k -space center. Subspace for different diffusion b -values are also shown.

Additionally, reconstructions using shot-to-shot navigator phases derived from kernel extraction (here, ESPIRiT-type operation) appear more robust than simply low-pass filtering and zero-filling the navigator data (e.g., local darkening).

To further inspect and visualize the extractability of diffusion phase kernels, in Figure 7, estimated relative shot-dependent phase maps and singular-value distributions of kernel subspace are demonstrated for the four phase extraction algorithms in Figure 6. Due to phase wrapping, possibly caused by different kernel extraction algorithms (i.e., numerical consequences rather than physical modulations), a direct comparison between the estimated phase maps is not straightforward.

Nonetheless, a more low-rank (or left-concentrated/flattened) singular-value distribution can imply that, the k-space kernels could be represented with fewer, well-defined singular vectors—leading to higher accuracy in extracting inter-shot signal variations in subspace. Generally, both the direct kernel estimation and the eigenvalue approach had similar singular-value distributions for kernel subspace, indicating their similar multi-shot reconstruction quality (Figure 6). The low-rank property can be substantially reduced by stronger diffusion-weighting, or shifting the k-space locations from inner to outer overlapped bands. Consequently, the most outer bands in the five-shot and seven-shot readout-segmented EPI with 1500 s/mm² exhibit the

least low-rank kernel subspace, indicating largest estimation errors. Meanwhile, as always located at central k-space regions, 2D navigator data had consistent low-rank kernel subspace across different b -values (here, up to 1500 s/mm²), indicating sufficient SNR for robust shot-to-shot diffusion phase extraction.

4.3 | Increased b -Value Limits in High-Performance Gradient Scanners

As in Figures 6 and 7, the phase estimation errors in low-SNR overlapped k-space segments—often observed in very peripheral regions under strong diffusion-weighting—can lead to slightly blurred reconstruction compared to 2D navigator correction. Importantly, this reflects a fundamental limit of DW-MRI signals at outer k-space, which affects all signal processing techniques based on those regions [24], rather than a specific shortcoming unique to our kernel extraction algorithms.

Therefore, in Figure 8, we demonstrate the possibility to address this limitation in specific sequences, by employing a high-performance gradient scanner (3T, Siemens Cima.X). A five-shot readout-segmented EPI with long and short diffusion-encoding gradients was tested for $b = 2000$ s/mm², with identical EPI readout train for comparison.

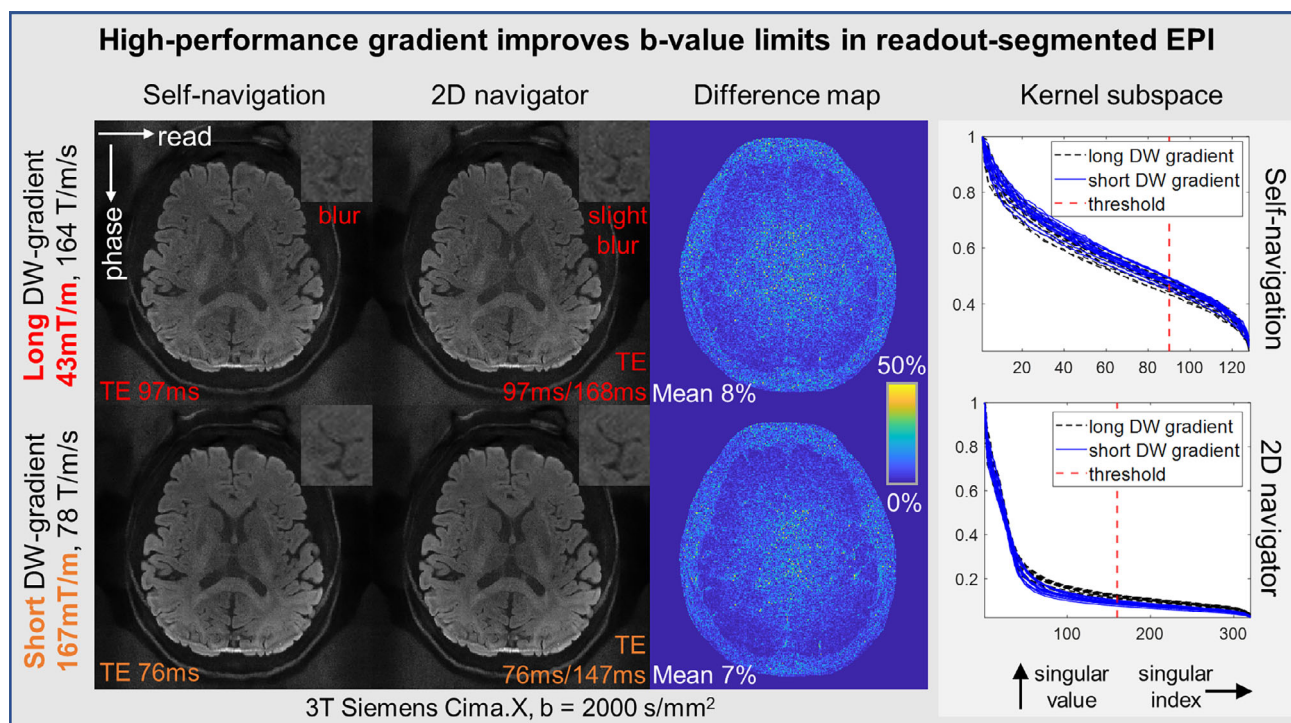


FIGURE 8 | Five-shot readout-segmented EPI in 3T Siemens Cima.X scanner. Long (a protocol to emulate gradient-limits of the Prisma scanner) and short (feasible only with the gradient performance of the Cima.X scanner) diffusion encoding gradients were utilized, comparing the estimation accuracy of shot-to-shot phase variations based on non-central k-space overlap regions. The eigenvalue approach (ESPIRiT-type operation) was used with kernel size 7×7 , for both self-navigated ACS and navigator's ACS. In self-navigated kernel extraction, the singular-value distribution appears slightly more dispersed in the scan with long diffusion gradient, than the short one, which might indicate signal instability due to low-SNR. Note that, due to the dominant cardiovascular nerve stimulation (CNS) limits by whole-body gradient in the Cima.X scanner, the slew rate of diffusion gradients was far below the nominal hardware limit (200 T/m/s). This might be further mitigated [92] in the future to enable self-navigation with higher b -values for the widely adopted RESOLVE sequence.

With a long (slow) diffusion-encoding gradient (comparable to 3T Siemens Prisma^{fit}), the self-navigated EPI reconstruction exhibits stronger local blurring relative to the 2D navigator corrected reconstruction, as in the zoomed region.

With a short (fast) diffusion-encoding gradient, there is no visible difference in resolution between self-navigated and the 2D navigator-corrected images—the zoomed region confirms equivalently resolved local image texture.

4.4 | DTI And GE-EPI

Figure 9 shows the neuroimaging applications of our proposed self-navigated sequences without 2D navigators, including diffusion-weighted imaging, DTI (e.g., the principal diffusion direction maps), and time-series GE-EPI. The artifacts induced by multi-shot phase fluctuations in the uncorrected images, which manifest themselves in both the object and background in diverse forms [12] (Figure S9), were all effectively removed

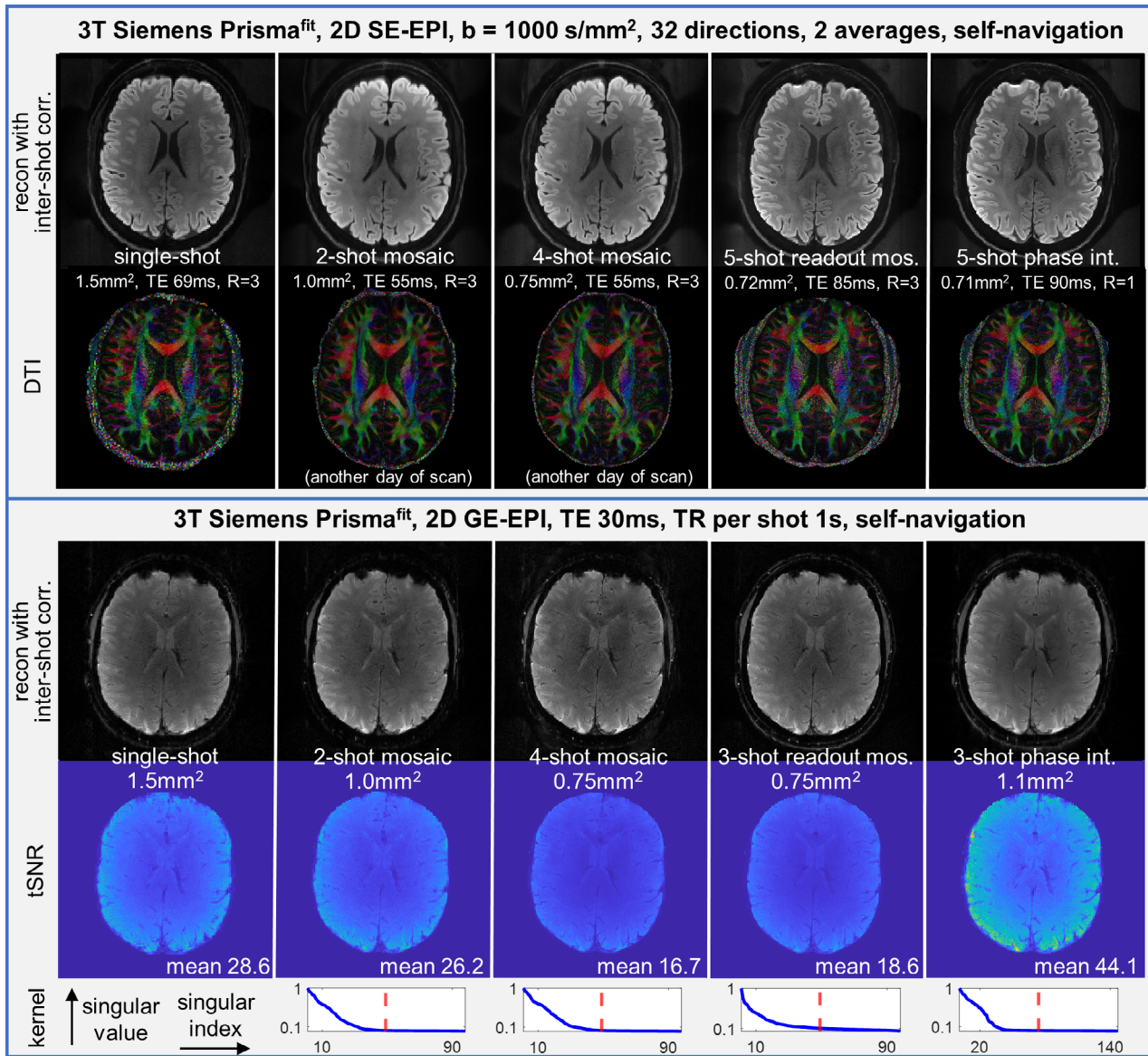


FIGURE 9 | DWI, DTI, GE-EPI images with the four self-navigated EPI sequences. Upper: The DWI/DTI were acquired with 2D SE-EPI with 1000 s/mm^2 , 32 diffusion directions, and 2 averages. Their ESPs are the same as the sequences in Figure 5. The 2D navigator was removed and the time duration of one-shot-one-slice acquisition became shortened by at least a few tens of milliseconds, as documented in Supporting Information. The two-shot and four-shot mosaic diffusion scans were taken on another day of separate scan with smaller slice number and shim volume, due to sensitivity to B_0 inhomogeneity. Otherwise, all other SE scans were performed with 26 slices, and all GE scans were performed with 11 slices. The two-shot and four-shot mosaic EPI yielded much higher SNR due to shorter TE. In GE-EPI, the kernel extractability at non-central k-space regions (e.g., readout mosaic) became much more reliable than diffusion-weighted scans due to sufficient SNR, although shot-to-shot phase fluctuations appear much weaker than in strong diffusion-weighted scans at 3T. More slices positions in these diffusion-weighted scans, as well as artifacts patterns in uncorrected multi-shot reconstructions are shown in Supporting Information. The latter can serve as important indications for protocol selections, when specific artifacts patterns should be strictly avoided (e.g., ghosts which may replicate fake functional activations at a distant brain location due to phase-interleaved trajectories).

by kernel extraction (here, eigenvalue approach) to allow reliable resolution enhancements. Notably, the SNR in two-shot and four-shot mosaic DW-EPI were visibly higher. Furthermore, the tSNR maps for GE-EPI confirm the absence of localized temporal instabilities due to uncorrected multi-shot artifacts, demonstrating that these self-navigated sequences are well-suited for fMRI applications.

5 | Discussion and Conclusions

Our overlap-kernel EPI takes inter-shot overlap bands at arbitrary k-space locations as another kind of ACS regions, for novel shot-to-shot phase extraction in one-step. Particularly, this redefines self-navigation—which traditionally required oversampling or at least recoverable k-space center in every shot—and opens the door to new strategies in designing multi-shot k-space trajectories. Essentially, our method transfers elements in explicit kernel/map extraction techniques in PI (GRAPPA/ESPIRiT) to multi-shot EPI, analogous to how low-rank matrix completion approaches evolved from SAKE [38]/LOARKS [39, 58] into MUSSEL [24], and thus, surprisingly making PI and multi-shot MRI much more thoroughly connected than before. Nevertheless, some conventional GRAPPA's characteristics [91] may not be directly suitable for our scope, since signal quality of the shot-to-shot calibration regions can be much worse than the fully-sampled, non-diffusion-weighted, central k-space ACS in PI.

Rather than advocating a specific sequence, we provide a general mechanism to “interlock” neighboring k-space bands acquired in separate shots. In this paper, this principle successfully enables unconventional self-navigated sequences (two-shot and four-shot mosaic) otherwise hindered by additional shot-dependent eddy currents, upgrades the commonly used readout-segmented and phase-interleaved EPI in certain conditions, improves the use of 2D navigator data, and may provide other minor applications (e.g., extracting B_0 maps given low-to-moderate B_0 inhomogeneity). Note, phase-interleaved EPI with kernel-extracted 2D navigator corrections is similar to realigned GRAPPA [65, 66], but our implementation independently extracts phase kernels, which are loaded into a SENSE/ESPIRiT forward model reconstruction instead of unified GRAPPA interpolations.

The unconventional two-shot/four-shot center-out mosaic EPI combines similar short-TE benefits of spiral sampling [93] and low-trajectory errors in Cartesian sampling, captures both diffusion- and hardware-errors in high-SNR self-navigated ACS over refocused navigators, and appears suitable for multi-shot diffusion scans when maximizing SNR is the priority (e.g., for high-resolution and high b -values applications). The point spread function due to T_2 decay in these center-out mosaic EPI is approximately symmetric about k-space center, yielding apodization characteristics that differ from standard Cartesian EPI. In this case, low-frequency image pixels that usually occupy most of the image support are acquired with minimized T_2 decay, which potentially contributes to visibly higher SNR in reality. In the SNR-limited regimes of our experiments, center-out mosaic EPI produced remarkably clearer images once distortion was adequately corrected, facilitated by our

shot-to-shot phase calibrations. Nevertheless, trade-offs do still exist. The mosaic segmentation along phase-encoding dimension leads to non-minimized ESP, and may incur occasional blurring due to residue trajectory and B_0 inhomogeneity errors (Figure S10) similar to spiral scans. However, while these limitations can still be reduced by high-performance readout gradients [86] or optimized image reconstruction [94], irreversible SNR loss is never correctable. If minimization of ESP is prioritized, a two-shot readout-segmented EPI (center-out along the readout direction) may also be employed with reasonable SNR in the self-navigated ACS near the k-space central region (along readout).

More commonly used multi-shot diffusion sequences [37, 42] can also benefit. Given sufficient SNR, the self-navigated readout-segmented EPI (0.7 mm^2) can reach identical or better (i.e., due to additional shot-dependent eddy current calibration) reconstruction quality than 2D navigator correction (i.e., RESOLVE), with b -values up to 1500 s/mm^2 in 3T Prisma, and 2000 s/mm^2 in 3T Cima.X scanners. The diffusion-weighting limits might be further improved with fully utilized high-performance gradients (e.g., shorter TE) [92] or neural network-based kernel extraction [95]. In self-navigated phase-interleaved EPI, kernel-based phase calibration in a central k-space region [23, 71] always benefits from much higher SNR than in periphery of k-space [24]. Along this path, the overlap-kernel EPI, implemented as MUSE with kernel extraction (one-step GRAPPA/ESPIRiT-type operation applied to intermediate PI reconstruction), further unifies the two steps in JULEP [71] elegantly (structured low-rank constraints and explicit phase mapping), and efficiently removes residue phase estimation errors in conventional MUSE. Nevertheless, in very strong diffusion-weighting ($> 3000 \text{ s/mm}^2$) or ultrahigh-field, both these conventional multi-shot trajectories may suffer from critical SNR drop and increased SAR, for self-navigation or 2D navigators (acquired after refocusing pulses) corrections.

In our implementations, the small overlap regions do not incur substantially increased sampling time, while the refocused navigators typically cost extra time of a few tens of milliseconds (e.g., comparing protocols in Figures S2 and S9). Empirically, a 7×7 kernel size appears sufficient in most of our experiments, while a larger kernel size may be considered for high b -values (e.g., 3000 s/mm^2 , Figure S7), as reserved for future study. Regarding estimation accuracy, our kernel subspace analysis (i.e., singular-value distribution) across k-space overlap locations, diffusion-weighted b -values, and kernel sizes/thresholds confirm limitations of kernel extraction techniques originated from low-SNR calibration regions, mainly due a combination of strong diffusion-weighting, and peripheral k-space locations [12]. This is problematic for outer k-space bands in strong diffusion-weighted readout-segmented EPI, and might pose even stronger challenges for the most outer k-space region in MUSSEL [24]. Therefore, the two-shot/four-shot mosaic EPI with short-TE-overlap regions at/close to the k-space origin deserves more future investigations for high-resolution, strong diffusion-weighting scans using high-performance gradient scanners.

In this work, only nominal trajectory re-gridding followed by 1D-FID-based N/2 ghosts correction has been tested. Our

inter-shot phase calibration only removes relative image-space phase variations or small k-space displacements between shots. Consequently, strong and continuous eddy currents within each shot, if present, may disrupt the kernel interpolation modeling, resulting in shot-to-shot estimation errors. Theoretically, combining more realistic within-shot trajectory corrections based on e.g., even-odd GRAPPA kernels [64], continuous B_0 kernels [59], or field camera [87] can improve quality in inter-shot ACS regions and the resultant estimations. Additionally, the shot-dependent amplitude fluctuation maps are also obtained but discarded for reconstruction, which, however, might become useful in future studies.

For higher sampling efficiency, our demonstrated sequences can be easily extended to SMS [96], or adapted for 3D volumetric scans with tailored overlap regions, to robustly remove primary [5, 6] or residue [90] (e.g., given motion compensation) shot-to-shot signal inconsistencies in different scenarios, well-suited for high-resolution diffusion-weighted [97] or functional [6, 98–100] MRI scans. However, the most optimized multi-shot EPI sequence may strongly depend on applications, involving considerations about achieved resolution, shot number, SNR, tolerable artifact patterns, acoustic prohibited bands, nerve stimulations, field strengths, gradient types, and product availability from the vendor. Generally, any sequences where different shots sample a small common k-space region with sufficient SNR can benefit from this strategy to stabilize inter-shot signals. Similar to PI, the robustness of self-calibration in the form of kernels originates from an elegant property of the Fourier domain: smooth spatial modulation manifests as condensed k-space kernels with a large number of shifts (i.e., averaging and stability). In MRI, these small and computationally efficient kernels can widely represent effects due to B_0 [59, 60, 64, 101, 102] and B_1 [55, 62, 95] magnetic fields, although they may encounter difficulties in cases of unusually abrupt spatial modulations (Figure S5). As such, we believe the proposed kernel extraction techniques [68] encapsulated by the generalized k-space signal interpolations framework [60, 67] or ESPIRiT [56] theory will continue to find more applications [59] in future MRI developments.

Acknowledgments

This study is supported by the European Research Council Advanced Grant (No. 834940). Maxim Zaitsev is co-funded by the National Institutes of Health, USA (R01 EB032378 and U24 NS120056) and the European Union (Project No. 101078393-MRITwins). The ex vivo brain phantom was with courtesy of the Institute of Clinical Anatomy and Cell Analysis, Department of Anatomy, Eberhard Karls University of Tübingen. The first author thanks Dr. Thomas Shiozawa (Institute of Clinical Anatomy and Cell Analysis) for assistance with sample preparation, and Dr. Gisela Hagberg for assistance in scanning this phantom and helpful discussions about diffusion MRI. The first author would like to thank Dr. Franciszek Hennel (ETH Zurich) for numerous discussions about super-resolution MRI with phaseless encoding during the first author's master degree studies, which significantly inspired this paper. The first author would also like to thank Praveen Iyyappan Valsala, Prof. Dr. Felix Glang, and Dr. Sebastian Mueller for their assistance in operating the 3T Prisma scanner, Dr. Qingping Chen for her assistance in operating the 3T Cima.X scanner, and the Pulseseq team for their long-term supports in open-source sequence programming. Open Access funding enabled and organized by Projekt DEAL.

Data Availability Statement

The example codes and data are available on Zenodo (<https://zenodo.org/records/17704660>). The example codes are also available on Github (https://github.com/ruitianwater/OverlapKernelEPI_v1.0) and the HarmonizedMRI website (<https://harmonizedmri.github.io/projects/>). Simplified codes would also be in the demo folder of Pulseseq (<https://pulseseq.github.io/>).

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Supporting Information

Additional supporting information can be found online in the Supporting Information section. **Data S1:** mrm70196-sup-0001-DataS1.pdf. **Figure S1:** Similarity of calibration matrix formulation for auto-calibration in parallel imaging and multi-shot rs-EPI. (A) Low-resolution GRE scans used for calibrating parallel imaging. (B) A sliding mathematical window (green) shifts within the fully-sampled central k-space region, acquired by distinct RF receiver channels (represented by different colors), with each patch or neighborhood reshaped into a row in the calibration matrix. The diffusion fluctuation kernels or cardinal functions remain shift-invariant as the sliding window shifts across different k-space locations and RF receiver channels. (C) k-space sampling trajectory in rs-EPI where two readout segments in consecutive shots overlapped by approximately, for example, 10–20 pixels. The readout gradient polarity is flipped between overlapped segments to ensure consistent T_2 or T_2^* signal decay at the overlapped k-space locations. (D) In the rs-EPI case, a sliding window (green) shifts within the overlapping region between two segments sampled separately in consecutive shots

across all RF receivers, and each neighborhood in a shift is reshaped into a row in the calibration matrix. In this rs-EPI sequence, only two readout segments partially overlap, so each row in the calibration matrix contains two reshaped patches. Data from different k-space locations and RF receiver channels together form a “shift-invariant” dimension for the cardinal functions. (E) A generalized calibration matrix highlights the mathematical similarity between estimating RF receiver sensitivity maps and shot-dependent phase fluctuation maps. The shifts of the window create a “shift-invariant” dimension (the vertical dimension of the matrix), which improves the conditioning of the calibration matrix and makes kernel estimation more robust. **Figure S2:** A full-version of Figure 4 in manuscript. Additionally, the difference maps between self-navigation and 2D navigator corrected reconstruction (normalized by the latter) are shown in percentage. Note, neither reconstruction is the ground-truth, so these difference maps only reflect their reconstruction variations in fine structures. Moreover, the kernel subspaces for self-navigated ACS and separately acquired 2D navigator ACS are shown. The singular-value distributions are normalized with maximum value of 1, with the self-navigation on the left (blue), the 2D navigator on the right (black), and the subspace thresholding indicated by the red dash line. In two-shot and four-shot mosaic EPI, kernel subspaces for self-navigation are more low-rank than the 2D navigators, indicating higher SNR in self-navigated ACS due to shorter TE. In phase-interleaved EPI, self-navigated ACS also exhibit a more low-rank property than 2D navigator, possibly because the intermediate PI reconstruction at central k-space (50×50) has shorter TE than navigator echoes after refocused pulses. However, this does not always translate into improved estimation accuracy, as low-rank distribution in kernel subspace is a too general assumption, and might not specifically capture all errors in self-navigated ACS, for example, resulted from failed intermediate PI reconstructions for individual shots (e.g., residue ghosts, in reconstruction of $8 \times$ under-sampled k-space). **Figure S3:** Acoustic resonance distributions and prohibited bands (rectangles) for 3T Siemens Prisma^{fit}, which shows the readout-segmented EPI (Figure 4 in manuscript) with shortened ESP (by removing k-space overlaps) was hindered by acoustic resonance constraints, not necessary the gradient power. (A) Each shot includes 16 additional overlapped readout points for self-navigation. (B) The k-space shot-to-shot overlaps were removed, and the saved sampling time was converted to a reduction of 0.08 ms in ESP (unfortunately, hits the prohibited bands), while maintaining similar readout gradient performance. In practice, the prolonged ESP in rs-EPI may not result from additional k-space overlaps, but scanner-specific prohibited acoustic bands. The 0.54 ms ESP in (A) can be theoretically shortened by increasing readout gradient amplitude, but is finally prohibited due to potential mechanical damage of gradients. In this protocol, the additional 16 overlapped readout points do not cause extra scan time, but just slightly raises the gradient performance, well below the hardware (80mT/m) and PNS limit. This sequence limitation is common for rs-EPI, but may be mitigated by applying oscillating phase-encoding gradients during low-bandwidth readout as in Wave-EPI. **Figure S4:** The kernel extraction approach can also benefit phase extraction from overlapped data at k-space center. Here, the eigenvalue approach (ESPIRiT-type operation) was used with 7×7 kernel size. (A) Shot-to-shot phase maps can be obtained via kernel extraction from 2D navigators, which was more robust than naive smoothing and zero-filled to high-resolution grid. Sum images with three diffusion directions are shown. This is similar to realigned GRAPPA, but with phase kernels independently extracted and loaded into a SENSE/ESPIRiT forward model, instead of unified GRAPPA interpolations. (B) Comparing different approaches to extract phase maps from intermediate reconstruction of each phase-interleaved shot. Sum images consisting of 12 diffusion directions are shown. Occasionally, errors in

the original MUSE (total variation smoothing of image-space phase) can be observed, which are eliminated by our approach—MUSE with kernel extraction. This is similar to improved extrapolation of RF sensitivity maps by shifted kernels in ESPIRiT, over an image-space operation. **Figure S5:** Four-shot mosaic EPI sequence (B_0 inhomogeneity sensitive) with different approaches to extract B_0 map from the central region of k-space of the GRE data with different TE. The sequence has a moderate ESP 0.7 ms (effective 0.23 ms), but its trajectory remains sensitive to B_0 offset similar to spiral-out trajectory. Compared to naive smoothing with Kaiser filter in k-space, the kernel extraction approach with large kernel size (20×20) appears robust to remove noise around the skull. This worked for low-to-moderate B_0 inhomogeneity at 3T in our tests, but may be difficult to capture very abrupt image-space B_0 variations (e.g., 7T or above) with too large kernels (i.e., computational expensive). The computational time for large (e.g., 20×20) kernel can be substantially increased compared to a usual size (7×7), and also depends on the calibration region size and the finally extrapolated matrix size (if using ESPIRiT-type operation). **Figure S6:** Reconstruction quality with respect to kernel size, subspace threshold, and low-resolution apodization. (A) impact of kernel size and subspace threshold. (B) Effects of apodization on each EPI segment and the corresponding inverse filter. The ex vivo phantom had no physiological phase fluctuations, and can only suffer from shot-dependent system imperfections. The diffusion-weighted images are shown as the sum image for a total of 20 diffusion directions. **Figure S7:** Diffusion phase fluctuations represented as image-space maps and k-space kernels, for b -values 1000, 2000, and 3000 s/mm². Five repetitions (Shot #1–5) of single-shot diffusion-weighted EPI were performed for different b -values. Row 1: The 2D image phase maps for Shot#1. Row 2: The Fourier transform of image-space phase maps, as the diffusion phase kernels (absolute values displayed), for Shot#1. A commonly used 7×7 kernel window (red rectangle) illustrates the portion of diffusion phase information captured by this kernel size across diffusion weighting. Row 3–4: Two 1D distribution of the k-space kernels are plotted to indicate the k-space range of diffusion phases, with the kernel size marked at half of the maximum values. As b -value increases, the k-space diffusion kernels become more spread. Note that, due to sharp image-space truncation from background (finite object support), the k-space kernel distributions might appear more spread than they actually are. **Figure S8:** Readout-segmented EPI sequences with increasing diffusion-weighting, to visualize the SNR dependency on b -values ranging from 0 to 1500 s/mm². The diffusion-weighted images are shown as the sum images for a total of 20 diffusion directions. **Figure S9:** The same diffusion-weighted dataset (1000 s/mm², 32 diffusion directions, and 2 averages) in manuscript's Figure 9, with more slice positions, and uncorrected multi-shot reconstructions to reveal possible artifact patterns which may be helpful for protocol selections. The ESPs are the same as the corresponding sequences with identical trajectories in manuscript's Figure 5. **Figure S10:** The same diffusion-weighted dataset in manuscript's Figure 5, with also non-diffusion-weighted images shown, in another slice position. The self-navigated sum images consisting of three diffusion directions, and coefficient-of-variation maps for one diffusion direction over 10 repetitions are shown. In this slice with potentially stronger B_0 inhomogeneity, the four-shot mosaic self-navigated EPI appears blurred, although its CV maps remain smaller, relative to five-shot readout-segmented and phase-interleaved EPI. Similar to other slice positions, additional artifacts or residue ghosts can be found in the two-shot mosaic and five-shot phase-interleaved EPI with 2D navigator corrections, as well as the conventional MUSE. These are well-minimized in our overlap-kernel self-navigation techniques. Note, the effective ESP (not ESP) are shown, as the ESP divided by k-space undersampling (including possible phase-interleaved) factor

in each shot. **Figure S11:** The same diffusion-weighted dataset in manuscript's Figure 5, with also non-diffusion-weighted images shown, in another slice position. The self-navigated sum images consisting of 3 diffusion directions, and coefficient-of-variation maps for one diffusion direction over 10 repetitions are shown. Similar to other slice positions, additional artifacts or residue ghosts can be found in the two-shot mosaic and five-shot phase-interleaved EPI with 2D navigator corrections, as well as the conventional MUSE. These are well-minimized in our overlap-kernel self-navigation techniques. Note, the effective ESP (not ESP) are shown, as the ESP divided by k-space undersampling (including possible phase-interleaved) factor in each shot. **Figure S12:** The same diffusion-weighted dataset in manuscript's Figure 5, with also non-diffusion-weighted images shown, in another slice position. The self-navigated sum images consisting of 3 diffusion directions, and coefficient-of-variation maps for one diffusion direction over 10 repetitions are shown. In this slice with potentially stronger B_0 inhomogeneity, the four-shot mosaic self-navigated EPI appears blurred, although its CV maps remain smaller, relative to five-shot readout-segmented and phase-interleaved EPI. Similar to other slice positions, additional artifacts or residue ghosts can be found in the two-shot mosaic and five-shot phase-interleaved EPI with 2D navigator corrections, as well as the conventional MUSE. These are well-minimized in our overlap-kernel self-navigation techniques. Note, the effective ESP (not ESP) are shown, as the ESP divided by k-space undersampling (including possible phase-interleaved) factor in each shot. **Figure S13:** The same diffusion-weighted dataset in manuscript's Figure 5, with also non-diffusion-weighted images shown, in another slice position. The self-navigated sum images consisting of 3 diffusion directions, and coefficient-of-variation maps for one diffusion direction over 10 repetitions are shown. In this slice with potentially stronger B_0 inhomogeneity, the four-shot mosaic self-navigated EPI appears blurred, although its CV maps remain smaller, relative to five-shot readout-segmented and phase-interleaved EPI. Similar to other slice positions, additional artifacts or residue ghosts can be found in the two-shot mosaic and five-shot phase-interleaved EPI with 2D navigator corrections, as well as the conventional MUSE. These are well-minimized in our overlap-kernel self-navigation techniques. Note, the effective ESP (not ESP) are shown, as the ESP divided by k-space undersampling (including possible phase-interleaved) factor in each shot. **Table S1:** SE-EPI in Figures 4 and S2. **Table S2:** SE-EPI in Figures 5 and S10–S13. **Table S3:** Readout-segmented EPI in Figures 6 and 7. **Table S4:** SE-EPI with long and short diffusion-weighted gradients in a 3T Siemens Cima.X scanner, as in Figure 8. **Table S5:** Self-navigated SE-EPI in Figures 9 and S9. **Table S6:** Self-navigated GE-EPI in Figure 9. **Table S7:** EPI in Figure S4. **Table S8:** GRE in Figure S5. **Table S9:** EPI in Figure S5. **Table S10:** SE-EPI for tuning kernel, in Figure S6A. **Table S11:** SE-EPI for evaluating filter and inverse filter, in Figure S6B. **Table S12:** SE-EPI in Figure S7. **Table S13:** SE-EPI for evaluating reconstruction with increasing diffusion-weighting, in Figure S8.