Fraction Processing across Domains: Integrating Contributions of Cognition, Motivation, and Emotion

Dissertation

der Mathematisch-Naturwissenschaftlichen Fakultät

der Eberhard Karls Universität Tübingen

zur Erlangung des Grades eines

Doktors der Naturwissenschaften

(Dr. rer. nat.)

Vorgelegt von

Silke M. Wortha (geb. Bieck)

aus Mainz

Tübingen

2021

Gedruckt mit Genehmigung der Mathematisch-Naturwissenschaftlichen Fakultät der Eberhard Karls Universität Tübingen.

Tag der mündlichen Qualifikation:	10.12.2021
Dekan:	Prof. Dr. Thilo Stehle
1. Berichterstatter:	Prof. Dr. Korbinian Möller
2. Berichterstatter:	Dr. Thomas Dresler
3. Berichterstatterin:	Priv. Doz. Dr. Laura Zamarian

To Anneliese

All things are difficult before they are easy. - Thomas Fuller

Summary

The understanding and mastery of fractions provides a critical requirement for individuals cognitive and especially numerical development. However, because of their bipartite structure reflecting the relative relation of numerator and denominator and their different properties compared to natural numbers, fractions can be a particularly challenging learning content for children and even adults. Currently, the integrated theory of numerical development (ITND) is one of the few theories that directly addresses the role of fractions in numerical development. Moreover, the ITND highlights magnitude knowledge as the key mechanism that integrates all numbers and facilitates their handling and, accordingly, their conceptual understanding. Therefore, magnitude knowledge also plays a crucial role for fraction understanding.

However, fraction processing is a complex mechanism that involves many different and not fully understood processes. Therefore, focusing only on the role of magnitude knowledge for fraction processing might not provide a complete picture of all involved processes while dealing with fractions. The present thesis addresses this issue in five studies by employing different methodologies, using different experimental approaches, and testing different age groups to investigate potential predictors of fraction processing. In particular, the core assumption of the ITND was evaluated and extended with additional cognitive and non-cognitive predictors that are relevant for fraction processing. Study 1 confirmed the assumption of the ITND regarding the prominent role of (fraction) magnitude processing for fraction understanding and extended this assumption to numerate adults, complex fractions, and untrained (novel) fractions. Study 2 substantiated the importance of different magnitude-related and unrelated basic numerical skills for fraction processing. Study 3 highlighted the role of magnitude-related and unrelated strategies for fraction processing. Study 4 revealed that motivation might be a relevant predictor of fraction processing but that not all children profit from it. Finally, Study 5 underpinned the importance of negative emotions and emotion regulation for fraction processing. Subsequently, a comprehensive framework of fraction processing is proposed and discussed, integrating cognitive and non-cognitive predictors to provide a fuller picture of what might be relevant predictors for fraction processing. While this framework is only a first approach to comprehensively describe the complex processes involved in fraction understanding and learning, it might provide the basis

for future research and interventions that foster comprehensive fraction understanding and related processes.

Zusammenfassung

Das Verständnis und die Fähigkeit mit Brüchen umzugehen, ist eine essenzielle Voraussetzung für die kognitive und vor allem numerische Entwicklung von Individuen. Allerdings stellen Brüche aufgrund ihrer besonderen bipartiten Struktur, die das relative Verhältnis von Zähler und Nenner widerspiegelt, und ihren im Vergleich zu natürlichen Zahlen unterschiedlichen Eigenschaften, eine große Herausforderung für Kinder und Erwachsene dar. Derzeit ist die sogenannte integrierte Theorie der numerischen Entwicklung (integrated theory of numerical development, ITND) eine der wenigen Theorien, die sich mit der Rolle von Brüchen in der numerischen Entwicklung von Kindern befasst. Darüber hinaus postuliert die ITND, dass die numerische Größe einer Zahl der gemeinsame Faktor ist, der alle Zahlentypen verbindet. Deshalb ist das Verständnis von numerischer Größe der Schlüsselmechanismus, der den Umgang mit Zahlen und das konzeptuelle Verständnis von Zahlen erleichtert. Demzufolge ist das Verständnis von numerischer Größe auch für den Umgang mit Brüchen entscheidend.

Die Verarbeitung von Brüchen ist jedoch ein komplexer Mechanismus, der viele verschiedene und nicht vollständig erforschte Prozesse beinhaltet. Eine ausschließliche Fokussierung auf die Rolle der numerischen Größen bei der Bruchverarbeitung liefert daher möglicherweise kein vollständiges Bild aller beteiligten Prozesse beim Umgang mit Brüchen. Die vorliegende Arbeit widmet sich diesem Problem in fünf Studien. Dabei wurde insbesondere die Kernannahme des ITND evaluiert und die ITND um zusätzliche kognitive und nicht-kognitive Prädiktoren, die für die Bruchverarbeitung relevant sind, erweitert. Studie 1 bestätigte die Annahmen des ITND bezüglich der zentralen Rolle der (Bruch-) Größenverarbeitung für das Bruchverständnis und konnte diese Annahme auf Erwachsene, komplexe Brüche und ungeübte (neue) Brüche erweitern. Studie 2 untermauerte die Bedeutung verschiedener numerischer Basisfähigkeiten für die Bruchverarbeitung. Diese unterschieden sich darin ob für eine Anwendung der entsprechenden Basisfähigkeit Größenverarbeitung benötigt wurde oder nicht. Studie 3 verdeutlichte die Rolle von Strategien für die Bruchverarbeitung, wobei auch hier wieder der Fokus auf Strategien lag, die auf Größenverarbeitung zurückgreifen oder

diese zum Lösen der Aufgabe nicht benötigen. Studie 4 zeigte, dass Motivation ein relevanter Prädiktor für die Bruchverarbeitung sein kann, aber das Motivation allein nicht immer ausreicht, um die Bruchverarbeitung zu verbessern. Studie 5 schließlich untermauerte die Bedeutung von negativen Emotionen und Emotionsregulation für die Bruchverarbeitung. Nachfolgend wird ein umfassender theoretischer Rahmen der Bruchverarbeitung vorgeschlagen und diskutiert, der kognitive und nicht-kognitive Prädiktoren integriert, um ein vollständigeres Bild darüber zu erhalten, was relevante Prädiktoren für die Bruchverarbeitung sein könnten. Dieser theoretische Rahmen ist zwar nur ein erster Ansatz, um die komplexen Prozesse, die am Bruchverständnis und -lernen beteiligt sind, umfassend zu beschreiben, aber er könnte die Grundlage für zukünftige Forschung und Interventionen bilden, die ein umfassendes Bruchverständnis und damit verbundene Prozesse fördern.

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Abbreviations

ACC	Anterior Cingulate Cortex
EEG	Electroencephalography
fMRI	Functional Magnetic Resonance Imaging
IPS	Intraparietal Sulcus
ITND	Integrated Theory of Numerical Development
NCTM	National Council of Teachers of Mathematics
NLE	Number Line Estimation
NMAP	National Mathematics Advisory Panel
PAE	Percentage Absolute Estimation Error
SIMS	Situational Motivation Scale
SRSD	Self-Regulated Strategy Intervention

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Study 2: Wortha, S. M., Klein, E., Lambert, K., Dackermann. T & Moeller, K. (2021). The relevance of basic numerical skills for fraction understanding: evidence from cross-sectional data.[2]

Study 3: Wortha, S. M., Moeller, K., Keuler, M., Nuerk, H.-C., & Bahnmueller, J. (2021). Strategies for Comparing Negative Fractions.[3]

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Study 5: Klein, E.*, Bieck, S. M.*, Bloechle, J., Huber, S., Bahnmueller, J., Willmes, K., & Moeller, K. (2019). Anticipation of difficult tasks: neural correlates of negative emotions and emotion regulation. *Behavioral and Brain Functions*.[5]

*Equal contribution

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[3] Please Note. This is an unpublished manuscript and therefore might not exactly replicate the final published version.

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[5] Reprinted by permission of Springer Nature Customer Service Centre GmbH: Springer Nature, Behavioral and Brain Functions, Anticipation of difficult tasks: neural correlates of negative emotions and emotion regulation. Klein, E., Bieck, S. M., Bloechle, J., Huber, S., Bahnmueller, J., Willmes, K., & Moeller, K. (2019). https://doi.org/10.1186/s12993-019-0155-1; copyright to reuse the content.

PART I: INTRODUCTION

1. GENERAL INTRODUCTION AND THEORETICAL FRAMEWORK

General Introduction

It is widely acknowledged that basic arithmetic and a general understanding of numbers are essential foundations of later mathematics achievement and useful for everyday life (e.g., Claessens & Engel, 2013; Nunes, Bryant, Barros, & Sylva, 2012). Unfortunately, however, many children, adolescents, and adults doubt the relevance of proficient mastery of rational numbers and especially fractions (e.g., Padberg & Wartha, 2017). This seems quite surprising as fractions and the associated concepts of decimals, ratios, and percentages are practically everywhere in our daily lives, for instance, in music (e.g., one half, one quarter, or one-eighth notes), in politics (e.g., 2/3 majority), in the arts (e.g., the *Vitruvian Man* from Leonardo da Vinci), in science (e.g., conditional probabilities or entropy), or in simple ordinary enjoyments like preparing the perfect gin and tonic (e.g., mixing ratio 1:4 or 2:5).

Besides, understanding and using fractions competently is critical for probability judgments (e.g., probability that it will rain tomorrow; Reyna & Brainerd, 1994, 2008), health judgments (e.g., mortality estimation of a disease; Reyna & Brainerd, 2007), and general risk literacy (e.g., consideration of financial investments; Cokely, Galesic, Schulz, Ghazal, & Garcia-Retamero, 2012). Currently, the ongoing global COVID-19 pandemic demonstrates impressively that understanding fractions and proportions is vital, as a lack of sufficient understanding of these concepts can lead to aversive and life-threatening situations (Lau et al., 2021; Thompson et al., 2020). This is because incorrect underlying presumptions about ratios and relative frequencies can lead to underestimating the true contagiousness and mortality of COVID-19 and similar diseases.

All the examples mentioned above illustrate that a lack of fraction knowledge limits not only academic and professional opportunities but also general life prospects on many levels. At an individual level, fractions are essential for grasping higher-order mathematics such as algebra more effectively and successfully (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Wu, 2001). Additionally, fluent processing of fractions promotes a higher degree of abstraction in mathematical thinking (DeWolf, Bassok, & Holyoak, 2016; Empson, 1999; Empson & Levi, 2011; Empson, Levi, & Carpenter, 2011). This is, in turn, important for the development of key mathematical competencies such as logical reasoning, modeling, and for one of the most important competencies for shaping mathematical thinking, pattern recognition (Burton, 1984; Vogel, 2005). At a societal level, a lack of fraction fluency is associated with poorer numeracy skills (e.g., Bailey, Hoard, Nugent, & Geary, 2012). In turn, poor numeracy is associated with severe financial, social, but also health disadvantages (Parsons & Bynner, 1997, 2005), which places a heavy burden on both individuals and entire economies (Gross, Hudson, & Price, 2009).

Therefore, it did not come as a surprise that in 2008, the National Mathematics Advisory Panel (NMAP) in the United States presented their prominent final report, which identified fractions, along with whole number and geometry understanding, as fundamental gatekeepers for achievement in higher mathematics (e.g., algebra; NMAP, 2008). Additionally, a survey conducted with Algebra teachers in this context revealed that one of the greatest weaknesses of school children was solving rational number problems, including operations with fractions and decimals (Hoffer, Venkataraman, Hedberg, & Shagle, 2007).

This marked the beginning of increasing interest in the cognitive processes underlying fraction processing in research areas such as cognitive psychology, developmental psychology, and cognitive neuroscience. Although research on rational numbers was already conducted in the 1980s and earlier in the field of mathematics education (e.g., Kieren, 1976; Novillis, 1976), this research mainly focused on effective classroom instruction and didactic approaches but less on the underlying cognitive processes. For instance, the *Rational Number Project* was a large-scale project that went on for several decades (1979-2002; Behr, Lesh, Post, & Silver, 1983; Bezuk & Cramer, 1989; Cramer & Post, 2002; Cramer & Post, 1995; Post, Behr, Lesh, & Wachsmuth, 1985). One goal of this project was to develop a curriculum in alignment with the United States National Council of Teachers of Mathematics (NCTM) standards to introduce 4th through 8th graders to the world of rational numbers (e.g., fractions, decimals, and ratios).

Given the challenge and the importance of learning fractions, the question needs to be raised: Which (cognitive) requirements are *essential* for mastering fractions? To answer this question, previous research, especially but not exclusively in cognitive psychology, has extensively examined (1) how fractions are processed, (2) what factors make the processing of fractions difficult, and (3) how processing and understanding fractions can be improved. Addressing these questions helped establish a common ground for understanding why fractions are seemingly processed differently from whole numbers (Obersteiner, Dresler, Bieck, & Moeller, 2019; Prediger, 2008; Vamvakoussi & Vosniadou, 2004).

However, understanding how children gain proficiency in mathematics in general and in the subject area of fractions, in particular, requires more than knowledge about how they cognitively process and learn fractions and which content specific numerical factors are important. In fact, *emotional* and *motivational* aspects have repeatedly been shown to profoundly influence human cognition and action (Dolan, 2002). Specifically, emotions have been observed to control our attention (Fredrickson & Branigan, 2005), influence memory processes (Kensinger & Schacter, 2008), influence our motivation and self-regulation (Mega, Ronconi, & De Beni, 2014), and promote or reduce the use of problem-solving strategies (Blanchard-Fields, 2007). Thus, it comes as no surprise that emotions, whether positive or negative, play an important role in learning and are highly relevant for memory performance.

As an extreme, math anxiety is a well-known phenomenon that causes affected individuals to avoid dealing with and thinking about numbers in general (e.g., Ashcraft, 2002) and fractions in particular (Sidney, Thalluri, Buerke, & Thompson, 2019). Additionally, there is evidence that school children and adults seem to have negative attitudes towards fractions without having negative attitudes towards whole numbers in principle, regardless of their mathematical proficiency level (Sidney, Thompson, Fitzsimmons, & Taber, 2019). This finding is crucial as it gives an insight into the differential way individuals deal with numbers and numerical information when that information is explicitly presented as a fraction. Considering this, it is astonishing that theories on the processes involved in numerical cognition in general and fraction processing, in particular, have rarely included affective and motivational factors as well as potential relations between motivation and emotion and fraction understanding.

Given the above-described association between cognition and emotions, two research areas seem to be particularly relevant for evaluating fraction understanding and learning. First, *cognitive* predictors have shown to be of particular importance for individuals' fraction processing and understanding. Second, *motivational* and *affective* predictors seem crucial for learning in general and numerical learning in particular. Accordingly, this dissertation aims to evaluate the role of *cognitive* as well as *motivational* and *affective* predictors of fraction processing.

In my opinion, prior research has largely neglected to jointly investigate these two areas of research to gain a comprehensive understanding of fraction processing, although the two areas appear to be deeply intertwined. In the following sections, I will highlight in more detail, i.) recent key findings on fraction processing and their importance for higher-level education (see Section 1.1), ii.) typical errors in fraction processing made by school children and adults alike and their potential origins (see Section 1.2), and iii.) the integrated theory of numerical development as a novel perspective on the role of fractions for numerical development (see Section 1.3), as well as iv.) the relevance of cognitive (see Section 2) and v.) motivational and affective predictors (see Section 3) for fraction processing. Subsequently, I will present the derived research questions I address in this dissertation (see Section 4).

1.1 The educational problem: Why focus on fractions?

The existing body of literature leaves no doubt about the pivotal role of fractions in children's mathematical development and thinking in secondary school (Lamon, 2020; Litwiller & Bright, 2002; Siegler et al., 2013). Fractions are the first complex, and more abstract content children encounter in the mathematical curriculum and mark the transition to higher-order mathematics. Thus, they bridge the gap between basic numerical skills like arithmetic (e.g., addition, subtraction, multiplication, and division with whole numbers) and more advanced mathematical content like algebra (for a summary of the role of basic numerical skills for fraction understanding, see section 2.2). Accordingly, it is not surprising that proficiency with fractions was found to be fundamental to insight-based learning of algebra (Booth et al., 2014; Empson et al., 2011; Hurst & Cordes, 2018; Rodrigues, Dyson, Hansen, & Jordan, 2016), but also to higher mathematical content beyond that. For instance, many other mathematical topics are dependent on fraction knowledge or a sense of proportional reasoning, such as geometry (e.g., describing a hyperbola with the equation $x^2/a^2 – y^2/b^2 = 1$; NMAP, 2008) and stochastic (e.g., probability theory and distributions which can be expressed as fractions).

After students have been introduced to fractions, they are expected to handle and understand them properly. However, reality shows that this is not the case.

Instead, it is well known that children (Aksu, 1997; Bailey et al., 2015; Behr et al., 1984, 1985; Carpenter et al., 1980; Carraher, 1996; Hasemann, 1981; Lortie-Forgues et al., 2015; Siegler & Pyke, 2013; Stafylidou & Vosniadou, 2004), adults (Lee & Boyadzhiev, 2020; Stigler, Givvin, & Thompson, 2010), and even mathematics teachers (Ma, 1999; Wu, 1999; Zhou, Peverly, & Xin, 2006) struggle in understanding and processing fractions adequately (Kloosterman, 2010; Rittle-Johnson et al., 2001; Siegler & Lortie-Forgues, 2015).

Additionally, among all topics in the school mathematical curriculum, fractions are arguably one of the most difficult and cognitively challenging to teach (Lamon, 2007). These problems in understanding fractions are independent of the time point at which fractions are introduced in the mathematical curriculum[6] (e.g., USA introduction in primary school vs. Germany introduction in 6th – 7th grade; Mullis, Martin, Foy, & Arora, 2012; Reiss, 2004; Schmidt & Houang, 2012). Additionally, problems in understanding fractions seem also independent of culture (Asian vs. European or North American culture; Chan et al., 2007; Liu et al., 2014; Stigler et al., 2010; Yoshida & Sawano, 2002). Thus, this issue with fractions seems universal, persistent, and robust across different learning contexts as children's fraction competence has hardly improved over the last 40 years (Siegler & Lortie-Forgues, 2014).

However, this is detrimental as understanding fractions is essential for many professions such as engineering, medicine, finance, construction, science, and many more (Handel, 2016). Moreover, fraction understanding is a unique predictor of future achievement in higher mathematics, above and beyond several other influential variables like general mathematical knowledge, intelligence, working memory, race, ethnicity, family income, and education (Bailey et al., 2012; Siegler et al., 2012; Torbeyns et al., 2015). From an educational perspective, fractions not only connect basic mathematics with more advanced mathematics topics, but algebraic equations are impossible to solve without knowledge about rational numbers. Additionally, fractions are essential for the entire secondary school mathematics (e.g., Common Core State Standards Initiative, 2010) and even more fundamentally connect and broaden different sets of numbers (from natural numbers to rational numbers) that children must handle.

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[6] For an overview of different mathematics curricula across countries, please see: http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/.

For instance, with the introduction of fraction division, it is possible to have dividends smaller than divisors (e.g., 5 ÷ 6). More generally, with the extension to rational numbers, all basic arithmetic operations (e.g., 7 – 9 or 5 ÷ 2, both operations do not have a solution within the set of natural numbers) and many algebraic equations (e.g., $3 \cdot x = 1$ the value of x cannot be given with natural numbers) are possible without any restrictions (e.g., divide with remainder, arithmetic operations with negative numbers or resulting in a negative number, arithmetic operations and equations including negative or positive fractions are all possible in the set of rational numbers but not in the set of natural numbers).

Given the outlined importance of fraction learning for children's mathematical and more general cognitive development and the fact that fractions are such a challenging content to learn, it is crucial to understand what aspects make fraction processing so difficult. To get to the core of this issue, the following subsection will provide a definition of fractions and describe typical errors that occur because of their usual mode of presentation as a bipartite structure.

1.2 Typical errors and why they arise

In contrast to systematic or careless errors, typical errors are made by many different individuals while solving the same or virtually identical problems (Padberg, 1996). Typical errors that occur during fraction processing and fraction arithmetic have been extensively documented over the past decades (e.g., Eichelmann, Narciss, Schnaubert, & Melis, 2012; Kerslake, 1986). For instance, the NAEP Mathematics Assessment has been tracking student's mathematical knowledge and problem-solving abilities in the U.S since the mid-70s. On the latest NAEP in 2019, only 41% of 4^{th} graders, 34% of 8^{th} graders, and 24% of 12^{th} graders scored above average in math. Closer inspection of the answers to some of the questions that covered fraction understanding revealed severe gaps in fraction understanding. For instance, only 32% of 4^{th} graders correctly identified whether the 6 fractions presented (i.e., 1/3, 2/3, 2/6, 4/6, 2/8, 4/8) were less than, equal to, or greater than 1/2. Of all 8^{th} graders, only 27% could correctly determine the magnitude of two given points on a number line and indicate the magnitude of the midpoint between these points. Finally, only 39% of all 12^{th} graders could correctly interpret an expression with a fractional exponent (i.e., $(16^{1/2})^3$).

These knowledge gaps are persistent in time, although many efforts have been taken to improve school children's fraction knowledge (NMAP, 2008; Siegler et al., 2010), including increasing research on fraction interventions (for a meta-analysis and review on fraction interventions for children with struggles and disabilities see Ennis & Losinski, 2019 and Roesslein & Codding, 2019). One famous example is the comparison between the study results of Carpenter et al. from 1978 and Lortie-Forgues et al. from 2014. In both studies, 8^{th} graders in the U.S. were instructed to decide whether 12/13 + 7/8 was closest to 1, 2, 19, or 2. In 1978, only 24% of the students were able to choose the correct answer (Carpenter et al., 1980). In 2014, 36 years later, only 27% of the students were able to answer this correctly (Lortie-Forgues et al., 2015).

One of the reasons why fractions are so difficult compared to natural numbers is their structure of notation. A fraction consists of a fraction bar, dividing the fraction in a numerator above and a denominator under the bar. This unique and classical bipartite presentation format makes it more difficult to perceive a fraction as one integrated number and grasp its magnitude. As a direct consequence of this bipartite presentation format, fraction magnitude seems to be a particularly challenging concept (Siegler, Thompson, & Schneider, 2011) - resulting in at least two ways of processing fractions: *holistically* or *componentially*. Holistic processing refers to processing overall fraction magnitude as an integrated entity, whereas componential processing denotes that the magnitudes of the fraction's numerator and the denominator are processed separately. The latter processing way is more error-prone and takes longer because it consists of more processing steps.

The existing literature provides accumulating evidence for both processing ways. However, which processing way is chosen by an individual seems to depend mainly on stimulus characteristics (e.g., Schneider & Siegler, 2010), the type of fraction comparison (e.g., Faulkenberry & Pierce, 2011; Huber, Moeller, & Nuerk, 2014; Meert, Grégoire, & Noël, 2010a, 2010b; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013), and specific processing strategies (e.g., Ischebeck, Weilharter, & Körner, 2016; Obersteiner & Tumpek, 2016**;** for instance cross-multiplication vs. visualizing, see also section 2.3). Thus, in magnitude comparison with common components (e.g., 5/7 vs. 5/8; comparing two fractions with the same nominator or denominator), individuals rely more strongly on componential processing, whereas in comparison problems with distinct components (e.g., 5/7 vs. 3/8), individuals tend to process the fractions holistically (Obersteiner et al., 2013).

However, the problems encountered by children and adults are not limited to the way fractions are presented. Instead, they also arise from differences between natural and rational number sets (e.g., Obersteiner, Dresler, et al., 2019; Obersteiner, Reiss, Van Dooren, & Van Hoof, 2019). These are differences in i.) representation, ii.) density, iii.) operations, and iv.) magnitude, which will be explained in more detail in the following. First, unlike natural numbers, fractions can have more than one representation because an infinite number of fractions (i.e., all multiples of a fraction) can refer to the same magnitude (e.g., 1/3, 2/6, 3/9 can be used equivalent; Clarke & Roche, 2009). Moreover, many children have problems with the idea of density, e.g., an infinite quantity of numbers between any pair of rational numbers might be too abstract to understand (Smith, Solomon, & Carey, 2005; Vamvakoussi & Vosniadou, 2010a).

Another challenging difference between natural and rational numbers is that performing multiplication and division with rational numbers might lead to results opposite to those previously known from natural numbers. For instance, multiplying 1/3 x 1/2 will lead to a product smaller than both factors (i.e., 1/6), while multiplying two natural numbers always leads to a product with increased magnitude. Moreover, dividing 1/3 ÷ 1/2 will lead to a result larger than dividend and divisor (i.e., 2/3), whereas dividing two natural numbers always results in a smaller number.

Finally, the magnitude of a fraction does not comply with the base-10 place-value system underlying natural numbers but instead needs an understanding of the multiplicative and inverse relation between numerator and denominator (e.g., *multiplicative relation* for understanding fraction magnitude: 4/9 nine is close to two times four; *inverse relation* for understanding fraction magnitude: the bigger the denominator, the smaller the magnitude of the overall fraction, e.g., 1/4 is smaller than 1/3 although 4 > 3). These problems can stem from applying previously learned properties of natural numbers on fractions (e.g., *natural number bias*; see also section 1.3; Ni & Zhou, 2005).

Additionally, many children struggle to understand the different interpretations of fractions (see Table 1.1; Behr et al., 1983; Lamon, 2020; Lortie-Forgues et al., 2015; Obersteiner, Dresler, et al., 2019). Unfortunately, in these interpretations, the meaning of numerator and denominator can differ. Thus, making it particularly difficult to develop a coherent understanding of a fraction as a number.

Table 1.1: The manifold interpretations of a fraction (i.e., 2/5) and their meanings (modified after Lamon, 2020 and Obersteiner, Dresler, et al., 2019)

Interpretation	Meaning for the example 2/5
Part-whole/ Ratios	*Parts of a whole*: 2 parts out of 5 equal parts; *several parts of several wholes*: 2 out of 5 objects
Measure of quantities	2/5 of any given unit (e.g., meters, liters, miles)
Operator	2/5 x (where x can be anything)
Quotient/ Division	2 divided by 5
Solutions of algebraic equations	the number x that solves the equation $5 \cdot x = 2$

Finally, while many students can perform fraction arithmetic without errors, it does not mean that this is based on a proficient conceptual understanding of fractions. Instead, they may rely on blunt learned procedures without being able to plausibly identify when to apply which operation (i.e., operation errors; Hasemann, 1986; Lortie-Forgues et al., 2015). For instance, Hasemann (1986) showed that school children in 7^{th} grade could not understand that two virtually identical problems (i.e., solving 1/4 + 1/6 in a symbolic and non-symbolic context) could be solved by applying the same procedure.

As a result of all these new aspects that children must encounter for understanding rational numbers and in particular fractions, many children are not able to order fractions with respect to their magnitude, estimate the position of a fraction on a number line with given endpoints or calculate correctly with fractions (e.g., Lortie-Forgues et al., 2015).

In sum, there are many stumbling blocks children must overcome to gain proficiency with fractions. These stumbling blocks can be seen as discontinuities children encounter during their numerical development. So far, some contemporary theories and ideas (e.g., conceptual change; see also section 1.3) focus on these discontinuities to explain how children's understanding of rational numbers develops. However, according to Siegler et al. (2011), numerical development is shaped by discontinuities and continuities. Thus, to gain a more comprehensive view of the development of rational numbers, theories should focus not only on discontinuities but also on continuities between natural numbers and rational numbers. One theory that aims to focus on both discontinuities and continuities is the integrated theory of numerical development which will be discussed in the next section.

1.3 The integrated theory of numerical development: the core role of fractions

The integrated theory of numerical development (ITND) provides a novel perspective on how mathematic proficiency evolves (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014). So far, existing theories on numerical development have focused mainly on natural numbers. Moreover, the development of natural and rational numbers was considered as two distinct processes, with the latter being rarely considered as an important building block for numerical understanding (Geary, 2004, 2007; Gelman & Williams, 1998). On the other hand, theories that try to account for individual difficulties with fractions focused primarily on differences and discontinuities between natural numbers and fractions (e.g., Hartnett & Gelman, 1998; Obersteiner, Dresler, et al., 2019; Vamvakoussi & Vosniadou, 2004).

For instance, Vamvakoussi and Vosniadou (2004) first introduced the *conceptual change theory* to the field of rational numbers. This approach claims that children build their knowledge of new number sets (such as the set of rational numbers) by referring to their already existing knowledge of previous number sets (in this case, natural numbers). For instance, being introduced to the set of rational numbers and in particular to fractions requires children to extend and reorganize their knowledge of natural numbers by including rational numbers as a new concept (e.g., McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2015; Vamvakoussi, Van Dooren, & Verschaffel, 2012; Vamvakoussi & Vosniadou, 2004; Vosniadou & Tsoumakis, 2013). This requires modifying the existing concept by understanding that true principles for natural numbers are not necessarily valid for all number sets in general (Siegler et al., 2011). Ultimately, this means recognizing that each natural number is a rational number, but not vice versa.

When this modification does not occur, individuals tend to overgeneralize their existing knowledge on natural numbers and erroneously transfer it to the case of rational numbers. This, in turn, leads to wrong conclusions when solving rational number problems. This phenomenon is called the *whole number bias* or *natural number bias* (Ni & Zhou, 2005). For instance, many children think that 1/3 is larger than 1/2 because 3 is larger than 2 or conclude that 1/3 + 1/2 is 2/5 because they treat numerator and denominator as distinct natural numbers and add them separately instead of reasoning about the relation between the numerator and the denominator to grasp its underlying magnitude (Alibali & Sidney, 2015). Thus, the conceptual change theory focuses on discontinuities that arise during the transition from natural numbers to rational numbers due to conceptual differences between these number sets (e.g., differences in density or different outcomes for arithmetic operations between both number sets; Lortie-Forgues et al., 2015; McMullen et al., 2015; see also section 1.2).

In contrast, the ITND (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014) focuses not only on differences between natural and rational numbers but also claims that there is a crucial continuity including both numbers sets and, accordingly, focuses mainly on the similarities between them. In particular, the theory highlights that all numbers (i.e., natural and rational numbers, but also other number sets) reflect magnitudes so that all numbers can be placed on number lines. Moreover, it postulates that both natural number and fraction processing are comparably important for numerical development. According to the theory, two insights during mathematical maturation are fundamental for successful fraction understanding: i) comprehending that all numbers (natural numbers as well as rational numbers) represent magnitudes that can be depicted on a (mental) number line, and ii) comprehending that specific characteristics of natural numbers do not necessarily generalize to other number sets, and therefore are not universal to numbers. Those characteristics of natural numbers are, amongst others, having unique successors, a finite number of entities between two numbers, a unique representation via symbols, an increase of magnitude with addition and multiplication processes, and a decrease of magnitude following subtraction and division (see also section 1.2).

Further differences between natural and rational number sets highlighted by the theory are that fraction knowledge is obtained much later in children's numerical development and, therefore, less precise than natural number knowledge. Among other things, this is supposed to contribute to less automated fraction magnitude representations. As a result, strategies should play a greater role in fraction (magnitude) processing compared to natural numbers, where magnitudes are accessed more or less automatically (Berch et al., 1999; Gebuis et al., 2009; Rubinsten & Henik, 2005; Siegler & Braithwaite, 2017; see also section 1.3).

Finally, as a consequence of magnitude understanding being similarly important in natural and rational numbers (i.e., understanding that both number sets reflect magnitudes, Siegler, 2016), the integrated theory claims that both number types should relate to each other and highly correlate with arithmetic and mathematic achievement. In fact, this assumption was substantiated by several studies (Bailey et al., 2012; Siegler et al., 2011; Wong, 2020), indicating that natural numbers and rational numbers are at least comparably important for children’s numerical development and achievement.

In sum, according to the ITND, the role of fraction magnitude processing for proficiency in fraction understanding is undeniable (Hansen, Jordan, & Rodrigues, 2017; Jordan et al., 2013). In this sense, interventions that focus on mapping rational numbers onto number lines could help to improve proficiency with fractions. However, although magnitude processing is one of the most important predictors for fraction understanding, it is certainly not the only predictor that should be considered.

In the following section, I will highlight the role of cognitive predictors (including magnitude processing) that I focus on in this dissertation: the role of i) magnitude processing and number line training, ii) basic numerical skills, and iii) the right strategy choice for successful fraction understanding.

2. COGNITIVE PREDICTORS OF FRACTION PROCESSING

Cognitive abilities cover a wide range of constructs that can describe variations of mental capabilities and skills among individuals. In principle, they can be classified in *domain-general* and *domain-specific cognitive* abilities. Domain-general cognitive abilities cover constructs that are superordinate to specific domains and therefore play a crucial role across domains regardless of their content. These abilities include, but are not limited to, the constructs of working memory and executive functions (e.g., Cowan & Alloway, 2009; Miyake & Shah, 1999), intelligence (e.g., Spearman, 1904), and attention (e.g., Kruschke, 2003).

On the other hand, domain-specific abilities are of primary importance in the single respective content domain. For instance, in numerical cognition, one fundamental ability is to understand and process magnitude information (e.g., Dehaene & Cohen, 1995; Siegler, 2016) which is an irrelevant ability for social cognition and a less relevant ability for language. Thus, identifying domain-general and domain-specific underlying cognitive processes that are important for number processing in general and fraction processing, in particular, is of high relevance to understanding the complex mechanisms involved. Additionally, the identified domain-specific cognitive processes could be starting points for guided interventions to improve proficiency with fractions (see section 2.1 for the importance of number line estimation (NLE) interventions to improve fraction magnitude processing).

Therefore, in the following section, I will focus on a selection of relevant domain-specific cognitive predictors for successful fraction processing. However, this list is not exhaustive. Additional possible domain-general and domain-specific but also meta-cognitive predictors for fraction processing will be discussed in sections 13.1 to 13.3 of this dissertation.

2.1 The Importance of Fraction Magnitude Processing and Number Lines

Understanding and processing numerical magnitude is a universal predictor for mathematical achievement and substantial for mathematical learning (Siegler, 2016; Siegler & Braithwaite, 2017; Torbeyns et al., 2015). Magnitude processing is typically assessed using magnitude comparison tasks or number line estimation tasks (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013; Rousselle & Noël, 2007; Schneider, Thompson, & Rittle-Johnson, 2017). In the magnitude comparison task, individuals are asked to either indicate the larger (or smaller) number out of two or compare the magnitude of a number to a fixed standard. In the classical number line estimation (NLE) task, individuals are asked to locate a given number on a bounded but otherwise empty line (e.g., from 0 to 1, 0 to 100, 0 to 1000). On a neural level, the intraparietal sulcus (IPS) has been frequently identified as a key brain area for processing numerical magnitudes (Dehaene et al., 2003; Fias et al., 2003; Kadosh et al., 2007; for a review see Ansari, 2008). Moreover, Vogel et al. (2013) could show that the number line estimation task activated similar areas within the right posterior IPS than the magnitude comparison task.

For both tasks, it is assumed that they reflect the representation of number magnitude on a mental number line. The most prominent indicator of magnitude processing for the magnitude comparison task is the distance effect (Moyer & Landauer, 1967). The distance effect describes the observation that the further apart the magnitudes of two numbers on the mental number line, the faster and more accurate they are compared. Moreover, the most prominent indicator of magnitude processing for the NLE task is the percentage absolute estimation error (PAE; Siegler & Booth, 2004; Siegler & Opfer, 2003). The PAE refers to the absolute difference between the individuals estimate on the number line and the proper location of the number on that same number line ((abs (estimated position – real position)/ scale of the number line)*100; cf. Siegler & Booth, 2004). The smaller the PAE, the more accurate the given estimates.

Two meta-analyses indicated that performance in both tasks correlated significantly with a wide range of other numerical/mathematical skills (Schneider, Beeres, et al., 2017; Schneider et al., 2018). The strength of the association of magnitude comparison and mathematical competence was *r* =. 278 with a higher effect size for symbolic (.302) than non-symbolic (.241) magnitude comparison (Schneider, Beeres, et al., 2017). For the NLE task, the strength of association of NLE and mathematical competence was *r* =.441. Moreover, this second meta-analysis found that the correlation between mathematical competence and NLE was higher for fraction NLE than for natural number NLE. Finally, correlations of mathematical competence with the NLE task were higher than with the magnitude comparison task (Schneider et al., 2018). This highlights the importance of the NLE task as a measure of (fraction) magnitude processing and as a possible approach for successful fraction interventions to improve fraction magnitude processing.

In fact, there is a variety of domain-general and domain-specific (including basic numerical; see also section 1.2) predictors of fraction processing. However, the ability to accurately estimate magnitudes on number lines has emerged as one of the most important predictors (see also Gersten, Schumacher, & Jordan, 2017). In addition, a wide range of different studies has shown that NLE with natural numbers and fractions has predicted arithmetic skills and general mathematical achievement (e.g., Bailey et al., 2012; Booth & Siegler, 2006, 2008; Holloway & Ansari, 2009; Liu, 2018; Sasanguie et al., 2013; Siegler et al., 2012; Siegler & Pyke, 2013). Moreover, several studies indicated that NLE is a powerful tool to enhance conceptual fraction knowledge by strengthening fraction magnitude understanding in children with or without mathematical disabilities (e.g., Barbieri, Rodrigues, Dyson, & Jordan, 2020; Dyson, Jordan, Rodrigues, Barbieri, & Rinne, 2020; Fazio, Kennedy, & Siegler, 2016; Gunderson, Hamdan, Hildebrand, & Bartek, 2019; Hamdan & Gunderson, 2017; Kiili, Moeller, & Ninaus, 2018; Schumacher et al., 2018; Sidney, Thompson, & Rivera, 2019).

Taken together, the understanding of fraction magnitude and the ability to train this understanding via the NLE task has become one of the centerpieces in research dealing with cognitive processes of fractions. However, proficiency in fraction magnitude processing is not the only predictor for fraction understanding. In fact, there are a variety of domain-general and domain-specific predictors that are also crucial for fraction processing.

2.2 Basic numerical skills as essential Foundations for Fraction Understanding

Mastery of basic numerical skills is an essential foundation in children's development of arithmetic skills and crucial for general mathematical achievement (e.g., Aunio & Räsänen, 2016; Booth & Siegler, 2008; Cowan & Powell, 2014; Dowker, 2005, 2008; Geary, 2000, 2007; Landerl, Bevan, & Butterworth, 2004). Thus, recognizing which basic numerical skills have the largest impact on later mathematical achievement has important implications. However, in general, there is no agreement on what basic numerical skills entail (see Aunio et al., 2004 and Kolkman et al., 2013 for different classifications). So far, a variety of different numerical tasks have been used to assess basic numerical skills like symbolic and non-symbolic numeracy, counting, subitizing, simple arithmetic, a basic understanding of mathematical relations as well as the already mentioned magnitude understanding (e.g., Hirsch, Lambert, Coppens, & Moeller, 2018; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Kolkman et al., 2013; Link, Nuerk, & Moeller, 2014; Martin, Cirino, Sharp, & Barnes, 2014; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Penner-Wilger et al., 2007; Rousselle & Noël, 2007; Schneider, Grabner, & Paetsch, 2009).

An increasing body of literature has shown that basic numerical skills are crucial prerequisites for later fraction understanding (Bailey, Siegler, & Geary, 2014; Hansen et al., 2015; Hecht & Vagi, 2010; Jordan et al., 2013; Liu & Wong, 2020; Mou et al., 2016; Namkung & Fuchs, 2016; Namkung, Fuchs, & Koziol, 2018; Seethaler, Fuchs, Star, & Bryant, 2011; Siegler & Pyke, 2013; Stelzer, Andrés, Canet Juric, Urquijo, & Richard's, 2019; Stelzer, Richard's, Andrés, Vernucci, & Introzzi, 2019; Vukovic et al., 2014; Ye et al., 2016; see also Table 2.1 for all studies including more than one predictor). These studies identified domain-general and domain-specific predictors that are important for proficiency with fractions. However, it is relevant to mention that domain-specific predictors did not only cover basic numerical skills but also more advanced mathematical skills like different fraction and proportional reasoning measures (see Table 2.1). In particular, the *Delaware Longitudinal Study* (for an overview, see Jordan, Resnick, Rodrigues, Hansen, & Dyson, 2017) could determine crucial domain-general and domain-specific predictors of fraction learning. The study was designed as a longitudinal study to monitor students at 8 time points from the end of 3rd/ beginning of 4th to 6th grade. It assessed general cognitive predictors like working memory (assessed in 3rd and 5th grade), language (3rd grade), attentive behavior (3rd and 5th grade), and nonverbal reasoning ability (3rd grade). Moreover, domain-specific predictors like whole number line estimation (3rd and 5th grade), non-symbolic proportional reasoning (5th grade), addition (3rd grade) and multiplication fluency (5th grade), long division (5th grade) as well as general achievement in mathematics (each grade) was assessed over the course of this study. Finally, the study examined four different fraction outcomes in 4th and 6th grade: fraction number line estimation (i.e., with number lines from 0 to 1 and from 1 to 2), fraction concepts (i.e., part whole concept and equivalent fractions), word problems with fractions, and fraction procedures (i.e., all arithmetic operations).

This way, the study was able to identify domain-general and -specific predictors of these fraction outcomes from 3rd to 4th and 5th to 6th grade. In a first study, Jordan et al. (2013) examined the predictive value of students' 3rd grade performance on domain-specific (approximate number system, number line estimation with natural numbers, and calculation fluency) and domain-general predictors (e.g., language, nonverbal reasoning, attentive behavior, working memory, and reading fluency) on their 4th grade performance on fraction concepts and fraction procedures. Results indicated that number line estimation with natural numbers, calculation fluency, reading fluency, attention, language, and nonverbal reasoning were predictors of fraction concepts. Additionally, number line estimation with natural numbers, calculation fluency, attention, and working memory were significant predictors of fraction procedures. In both analyses, number line estimation had the largest predictive value for fraction concepts and fraction procedures.

In a second study, Hansen et al. (2015) investigated 5th graders domain-specific (e.g., number line estimation with natural numbers, non-symbolic proportional reasoning, long division, and multiplication fact fluency) and domain-general (e.g., working memory, attentive behavior, and reading fluency) predictors of students' 6th grade fraction number line estimation, concepts and procedures achievement. This time, the analysis revealed that number line estimation, long division, non-symbolic proportional reasoning, working memory, and attentive behavior were significant predictors of fraction concepts. Moreover, whole number line estimation, multiplication fluency, and attentive behavior were unique predictors of fraction procedures.

Table 2.1 provides an overview of all studies investigating domain-specific as well as domain-general predictors for fraction understanding. To conclude this subsection, I would like to emphasize three points through this table: i.) Most of the studies focused on both domain-general and domain-specific predictors. Only focusing on either domain-general or domain-specific predictors would help broaden the set of possible predictor variables and give a clearer picture of the respective domain. ii.) Moreover, domain-specific predictors focused on basic numerical skills and more advanced mathematical skills like different fraction or proportional reasoning measures. Therefore, emphasizing only on basic numerical skills (including skills dependent and independent of magnitude processing) could help to identify skills that are especially important prior to advanced mathematics. Additionally, most of the investigated basic numerical skills depended on magnitude processing (e.g., different arithmetic skills). However, identifying important skills that are independent of magnitude processing could also be of particular interest, especially for children struggling with magnitude processing (e.g., children and students diagnosed with dyscalculia; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). Therefore, identifying basic numerical skills dependent and independent of magnitude processing might be a more accessible starting point to improve proficiency with fractions. iii.) Finally, in most of the studies, regression analysis was used as a statistical method. However, collinearity between variables is problematic in regression analysis, as it becomes increasingly difficult to determine the independent contribution of each variable to the explained variance in the outcome variable. However, multicollinearity between variables is expected when these variables measure similar (e.g., natural number magnitude and fraction magnitude) or related concepts (e.g., approximate number system and mathematics achievement). Additionally, determining the relative importance of each predictor variable can lead to new approaches to improve fraction understanding by fostering the performance of the most important predictor variables. Therefore, it is necessary to use methodological approaches to better address multicollinearity to understand the relationship and relative importance of variables and avoid underestimating and overestimating predictor variables.

Despite the above-mentioned importance of natural number and fraction magnitude understanding as well as the mastery of basic numerical skills, correct strategy use during fraction processing has also been shown to facilitate fraction understanding and performance.

Table 2.1: Summary of studies investigating domain-specific as well as domain-general predictors for fraction understanding. Please note: Only studies with at least two identified predictors were included.

Study	Design	*N*	Statistical Method	Fraction Outcomes	Identified Predictors: *domain-specific*	Identified Predictors: *domain-general*
Hecht & Vagi, 2010	Longitudinal (from 4th to 5th grade)	181	Mediation analysis	Fraction computation, Fraction estimation, Fraction word problems	Fraction conceptual knowledge, Arithmetic fluency	Attentive behavior, Working memory
Seethaler, Fuchs, Star, & Bryant, 2011	Longitudinal (from 3rd grade to 5th grade)	688	Multiple regression	Rational number arithmetic skills (fraction, decimals, and percentages)	Computational fluency with natural numbers	Language, Nonverbal reasoning, Working memory (numerical), Concept formation

Jordan et al., 2013	Longitudinal (from 3rd to the end of 4th grade)	357	Multiple regression	Fraction concepts, Fraction procedures	Calculation fluency, Number line estimation with natural numbers	Attentive behavior, Language, Nonverbal reasoning, Reading fluency, Working memory
Siegler & Pyke, 2013	Cross-sectional (6th grade and 8th grade)	120 (60 students from each grade)	Hierarchical regression	Fraction arithmetic	Division with natural numbers, Fraction magnitude measures (number line estimation and magnitude comparison task)	Reading, Executive functions

Bailey, Siegler, & Geary, 2014	Longitudinal (3 time points: 1st grade, 7th grade, and 8th grade)	162 – 172 (depending on the analysis)	Multiple regression	Fraction arithmetic, Fraction magnitude knowledge (number line estimation and magnitude comparison task)	Number line estimation with natural numbers, Arithmetic with natural numbers	Working memory
Vukovic et al., 2014	Longitudinal (3 time points: 1st grade, 2nd grade, and 4th grade)	163	Mediation analysis	Fraction concepts	Number knowledge (direct predictor), Number line estimation (mediator), Arithmetic computations (mediator) all three with natural numbers	Attentive behavior, Language, Executive control, Visual-spatial memory

Hansen et al., 2015	Longitudinal (from 5th grade to 6th grade)	334	Multiple regression	Fraction concepts, Fraction procedures	Non-symbolic proportional reasoning, Number line estimation with natural numbers, Long division, Multiplication fact fluency	Working memory, Attentive behavior
Namkung & Fuchs, 2016	Longitudinal (beginning and end of 4th grade)	139	Structural equation modeling/ Path analysis	Number line estimation, fraction arithmetic (addition and subtraction)	-	Language, Attentive behavior, Processing speed, Nonverbal reasoning

Mou et al., 2016	Longitudinal (from 1st grad to 9th grade)	122	Multiple regression/ Bayes factor for regression models	Fraction magnitude knowledge	Fraction procedures, Fraction conceptual knowledge (numerator-denominator relation), Magnitude knowledge of natural numbers	Working memory
Resnick et al., 2016	Longitudinal (from 4th to 6th grade)	517 (472 included in class membership of growth trajectories)	Latent growth curve modeling/ Ordinal logistic regression	Number line estimation, Fraction knowledge	Multiplication fluency, Number line estimation with natural numbers	Classroom attention

Ye et al., 2016	Longitudinal (from 3rd to 6th grade)	536	Mediation analysis	Fraction concepts, Fraction procedures (together fraction knowledge)	Calculation with natural numbers, magnitude reasoning with natural numbers (mediators between fraction outcomes and domain-general predictors)	Attentive behavior, Working memory, Verbal ability, Nonverbal reasoning ability
Stelzer, Andrés, Canet Juric, Urquijo, & Richard's, 2019	Cross-sectional (5th grade)	97	Mediation analysis	Fraction conceptual knowledge	Division (mediator between all three domain-general predictors and fraction knowledge)	Selective attention, Working memory, fluid intelligence

2.3 The Role of Strategies for Fraction Processing

As already mentioned in section 1.3, automated magnitude processing, as it is supposed for natural numbers, is rather unlikely for fractions (e.g., Siegler et al., 2011). In fact, it can be assumed that automatic processing of fractions only occurs for so-called everyday fractions (e.g., 3/4) or some unit fractions (e.g., fractions with 1 as numerator 1/2, 1/3, 1/4, 1/5) because these fractions are so frequent in our daily use that an automatic magnitude representation has been formed (Wortha, Obersteiner, Dresler, accepted). As automatic processing does not occur in most cases, it is more common for individuals to rely on various strategies to help them process fractions or solve any task at hand involving fractions (e.g., a task involving fraction arithmetic or comparing the magnitude of two fractions; Sidney, Thompson, & Opfer, 2019).

In general, a strategy can be defined as a "*procedure or set of procedures for achieving a higher level goal or task"* (Lemaire & Reder, 1999, p. 365). There are multiple ways/procedures for solving any kind of problem. However, these may differ in their correctness and their effectiveness. To account for variability in strategy choice, Lemaire and Siegler (1995) defined four dimensions of strategy changes: a) strategy repertoire, b) strategy distribution, c) strategy execution, and d) strategy selection. The right strategy choice is a developmental process that involves experience and practice, which ultimately leads to improvement in strategy use (Siegler, 1996, 2006).

When it comes to fraction processing, there are various strategies that individuals are using to process them. Moreover, most of the strategies in the literature are magnitude-based (e.g., Clarke & Roche, 2009; Faulkenberry & Pierce, 2011; Fazio, DeWolf, & Siegler, 2016). This means that processing of magnitudes, whether it is componential (e.g., processing the numerator and denominator separately) or holistically (e.g., accessing the magnitude of the whole fraction), is always involved when applying a magnitude-based strategy to solve a fraction problem (for more insights into componential and holistic fraction processing see also section 1.2). In fact, Faulkenberry and Pierce (2011) evaluated strategy use on each trial and found three types of strategies used during magnitude comparison of fractions: i) component-based strategies (e.g., cross multiplication or converting the fractions into decimals), ii) holistic strategies (e.g., visualizing fractions or benchmarking), and iii) simply knowing the answer. Moreover, Fazio, DeWolf, and Siegler (2016) also investigated strategies used by students with different mathematical proficiency levels to solve fraction magnitude comparison tasks on a trial-by-trial basis. Overall, they reported 19 different strategies, which either led to correct or incorrect answers. Interestingly, students with lower mathematical proficiency where more likely to apply strategies that do not lead to a correct solution to 'solve' a fraction problem. However, despite intuition and guessing all reported strategies were again magnitude-based.

Another way to assess strategies in fraction magnitude processing is eye tracking. This method allows to record individuals' eye movements, including eye fixations which is a helpful tool to gain insights into mental processing and problem-solving strategies (Holmqvist et al., 2011; Mock, Huber, Klein, & Moeller, 2016). A large body of eye-tracking studies confirmed that participants strongly relied on either componential or holistical strategies when processing fractions depending on the type of fraction (e.g., Huber, Moeller, & Nuerk, 2014; Hurst & Cordes, 2016; Ischebeck, Weilharter, & Körner, 2016; Obersteiner et al., 2014; Obersteiner & Tumpek, 2016). While fraction comparison problems with common components (e.g., common numerators or denominators) were associated with componential processing strategies, fraction comparison problems with four distinct components were typically solved by more holistic strategies. Therefore, whenever the processing of the overall fraction is not required, simpler componential strategies seem to be used. However, componential strategies can lead to specific error patterns when reasoning about components is overgeneralized (e.g., natural number bias, Alibali & Sidney, 2015; Ni & Zhou, 2005; see also section 1.2). Taken together, the use of magnitude-based strategies seems inevitable, at least for positive fraction magnitude comparisons. Nevertheless, little is known about strategies used for negative fraction magnitude comparison and whether these strategies are also magnitude-based.

To sum up, in section 2, I presented evidence for relevant cognitive predictors of fraction processing based on the literature. Fundamental magnitude processing, proficiency with basic numerical skills, and correct strategy use were identified as crucial prerequisites for fraction understanding. However, it is well known that non-cognitive predictors like motivational and affective variables also play an important role in learning. Consequently, it is reasonable that these processes are also involved in the development of fraction understanding. Therefore, in the following section, I will discuss the role of motivational and affective predictors as potential requirements for proficiency with fractions.

3. MOTIVATIONAL AND AFFECTIVE PREDICTORS OF FRACTION PROCESSING

Like cognitive abilities, non-cognitive factors cover a variety of different constructs. However, in contrast to cognitive abilities, non-cognitive factors cannot be measured by achievement tests but are typically assessed using questionnaires (e.g., surveys that cover trait and state anxiety) or for the case of emotions also using physiological measurements (e.g., pulse rate).

In fact, two of the most important non-cognitive predictors for learning are *motivation* and (negative) *emotions* (Anderman & Dawson, 2011; Bower, 1992; Kusurkar, Ten Cate, Vos, Westers, & Croiset, 2013; Pekrun, 2014). Many studies show that both are important for academic success (Graziano, Reavis, Keane, & Calkins, 2007; Linnenbrink & Pintrich, 2002). In this context, both predictors are referred to as academic motivation (e.g., Vallerand et al., 1992) and academic emotions (e.g., Pekrun, 2016). Additionally, both also play a crucial role in mathematical thinking and problem-solving (Hannula, 2006a, 2006b, 2015; Schukajlow, Rakoczy, & Pekrun, 2017). Therefore, these two non-cognitive predictors could explain not only individual learning progress and achievement in mathematics in general but may also be applied to dealing with fractions in particular - in addition to and beyond cognitive predictors.

For this reason, I will focus on motivation and (negative) emotions as non-cognitive predictors (and obstacles) for successful fraction learning in the following section. But, as mentioned before, this overview is not meant to be exhaustive. For instance, it is known that personality traits play an important role in academic / learning success from both psychological and educational research (e.g., De Feyter et al., 2012; Jensen, 2015). Therefore, additional possible non-cognitive predictors for fraction learning and performance will be discussed in section 13.2 of this dissertation.

3.1 The Importance of Motivation for Fraction Learning and Performance

A key non-cognitive predictor of successful learning is the ability to motivate oneself to learn and to remain motivated during the learning process. Unfortunately, there is no consensus on what motivation is (e.g., Franken, 1998; Kleinginna & Kleinginna, 1981; Morgan, MacTurk, & Hrncir, 1995; but see also Murayama, 2021 for a discussion on whether motivation is a real mental process). In fact, there are various theories on motivation (e.g., content theories, process theories, decision making theories, expectancy-value theory, and self-determination theory; Barbuto Jr, 2006; Maslow, 1943; McClelland, 1987; Ryan & Deci, 2000; Vroom, 1964; Wigfield & Eccles, 2000). Simplified, motivation refers to individuals' reasons for behaving in a certain way. These reasons can be intrinsic (e.g., driven by reasons within the individual) or extrinsic (e.g., driven by external reasons). Moreover, motivation can explain the direction (e.g., avoidance vs. approach behavior; Elliot, 1999), activation (e.g., task execution vs. distraction; Kuhl & Beckmann, 1985), persistence (e.g., invested time; Vollmeyer & Rheinberg, 2000), and intensity (e. g., magnitude of effort; Brehm & Self, 1989) of specific behavior.

Self-determination theory proposes two types of motivation (e.g., extrinsic vs. intrinsic, but see also integrative vs. instrumental for related types of motivation in language learning; Gardner & Lambert, 1972; Ryan & Deci, 2000). For the scope of this dissertation, I will focus on the distinction between *extrinsic* and *intrinsic* motivation.

In the context of education or learning (also referred to as academic motivation; Vallerand et al., 1992), extrinsic motivation refers to the intention of the individual to learn because of external factors. For example, to obtain a better grade, get a university admission, or receive the teacher's praise. On the contrary, intrinsic motivation to learn refers to the individual's drive to learn because of self-determined motives, like personal interest, personally relevant learning content, enjoyment of the learning content, and curiosity (Hennessey, Moran, Altringer, & Amabile, 2015; Ryan & Deci, 2000; Wigfield, Eccles, Schiefele, Roeser, & Davis-Kean, 2007). Both forms of motivation have specific effects on learning outcomes. For instance, intrinsic motivation improves conceptual learning and cognitive flexibility, whereas extrinsic motivation seems to enable competitiveness and reach fast performance goals like better grades (Ryan & Deci, 2000; Streblow & Schiefele, 2006; Wigfield, 1997).

However, both forms of motivation are often intertwined and can occur simultaneously (Amabile, Hill, Hennessey, & Tighe, 1994; Buff, 2001; Cerasoli, Nicklin, & Ford, 2014).

In general, intrinsic motivation seems to have more lasting effects on learning than extrinsic motivation. Research has shown a general positive association between intrinsic motivation and task performance (Schiefele & Streblow, 2005). Additionally, intrinsic motivation seems to lead to more profound processing of the learning content (Walker, Greene, & Mansell, 2006). However, extrinsic motivation is also important for learning as it is particularly relevant for goal orientation and competitive behavior in the learning context (Schiefele, Streblow, Ermgassen, & Moschner, 2003).

Research on mathematical learning has repeatedly shown that motivation is a driving factor in this content area. Motivation is important for engagement with mathematics, problem-solving, performance, and achievement in mathematics (e.g., Hannula, 2006b; Michaelides, Brown, Eklöf, & Papanastasiou, 2019; Schukajlow et al., 2017). In this context, based on the Yerkes-Dodson-Law, the relationship between motivation, difficulty of the learning task, and performance can be described as following an inverted U-shape. According to this approach, motivation decreases when task difficulty is either too high or too low - affecting task performance in turn (Yerkes & Dodson, 1908). Moreover, difficult content is often accompanied by low motivation and subsequently less engagement with the topic (Siegler & Pyke, 2013). Ultimately, this results in knowledge gaps and may lead to a vicious circle because with knowledge gaps, the actual learning content becomes insignificant, and this, in turn, may result in poorer motivation to learn (Reyna, Chapman, Dougherty, & Confrey, 2012; Siegler & Pyke, 2013). Therefore, motivation might play a crucial role in understanding fractions as it is one of the most challenging topics children encounter in the mathematical curriculum (Lamon, 2020). Given the importance of motivation for mathematics and difficult tasks in general, it seems reasonable to examine the role of motivation for fraction learning and performance.

In addition to the role of motivation for learning, it is well known that emotions can influence both handling the learning content, the approach, and individual performance in problem-solving. Therefore, I will discuss the role of emotions for learning in general and fraction learning in particular.

3.2 The Role of (Negative) Emotions for learning

There are many definitions but no clear agreement on what an emotion is (see also Kleinginna & Kleinginna, 1981a). In general, however, most definitions share the view that emotions can be divided into several categories. Overall, emotions can be defined as multidimensional constructs that reflect complex, interrelated categories of *affect* (e.g., happiness or anger), as well as associated *physiological* responses (e.g., increased heart rate and pulse), *cognitive* processes (e.g., interpretation of the situation), *expressive* responses (e.g., verbal or non-verbal reflected for instance by facial expressions), and *behavioral/ motivational* responses (e.g., fight or flight reactions; Scherer, 2009). Moreover, emotions can be classified as *positive* or *negative* along the dimension of valence (Colombetti, 2005; Ekman & Davidson, 1994; Lewin, 1936; Solomon & Stone, 2002).

Research consistently showed that emotions significantly impact learning processes and academic achievement (e.g., Pekrun & Linnenbrink-Garcia, 2014). In relation to learning, so-called achievement or academic emotions are of particular relevance. Academic emotions can be defined as emotions that appear in the context of performance-related activities such as an examination situation but also attending school classes or doing homework (Pekrun, 2016; Pekrun & Stephens, 2012). Like motivation, emotions can be driving forces that can lead to turning towards a task or avoiding a task. For instance, several studies have shown that negative emotions lead to task avoidance and even impaired task performance (for an overview see Pekrun & Linnenbrink-Garcia, 2014). On the other side, positive emotions can lead to focusing more on the learning content and thus foster learning and performance (Pekrun, Goetz, Titz, & Perry, 2002; Pekrun, Lichtenfeld, Marsh, Murayama, & Goetz, 2017; Pekrun & Linnenbrink-Garcia, 2012; but see also Baumeister, Alquist, & Vohs, 2015; D'Mello, Lehman, Pekrun, & Graesser, 2014 for opposite results). Of particular importance in this context is also emotion regulation because not only emotions themselves play a role in learning, but also their regulation. Emotion regulation can be defined as "*processes by which individuals influence which emotions they have, when they have them, and how they experience and express these emotions*" (Gross, 1998, p. 275). However, it is debated whether emotions and emotion regulation are distinguishable (e.g., Campos, Frankel, & Camras, 2004; Werner & Gross, 2010). Emotion regulation plays a crucial role in learning situations and school

(Boekaerts & Pekrun, 2015) as well as in examination situations (e.g., test anxiety; Bradley et al., 2010).

In the content domain of mathematics, math anxiety is the prime example where negative emotions lead to avoidance of mathematics and impaired mathematical achievement (Ashcraft, 2002; Dowker, Sarkar, & Looi, 2016). Especially difficult mathematical tasks are associated with math anxiety and negative emotions (also known as the anxiety-complexity effect; Ashcraft & Kirk, 2001; Suárez-Pellicioni, Núñez-Peña, & Colomé, 2013). As fractions can be considered one of the most challenging topics in the mathematical curriculum, it is reasonable to assume that dealing with fractions can lead to math anxiety. In fact, Sidney, Thalluri, et al. (2019) showed that performance on both fraction magnitude comparison and NLE task was impaired in adult participants with higher math anxiety compared to low math-anxious participants. Moreover, Rayner, Pitsolantis, and Osana (2009) found that math anxiety negatively affected preservice teachers' performance in both conceptual and procedural fraction knowledge tests. This shows that even individuals who should be used to and able to deal with the learning content are not immune from the influence of negative emotions caused by fractions. In turn, math anxiety caused by the learning content might affect students’ performance in mathematical problem-solving (Beilock, Gunderson, Ramirez, & Levine, 2010). However, more research is needed to determine whether the observed association between math anxiety and fraction understanding i) only affects highly math-anxious individuals, ii) is generalizable to rational numbers or specific to more complex rational numbers like fractions, and iii) which role emotion regulation plays during fraction processing.

4. Research questions – Considering cognitive and non-cognitive predictors of fraction processing

The present thesis is based on the assumption that the ability of the human cognitive system to efficiently process complex and challenging learning content can only be fully understood by considering relevant *cognitive* and *non-cognitive* processes, such as motivational and affective processes. Accordingly, a model that intends to account for fraction processing should take a more holistic approach and consider relevant cognitive factors specific for handling fractions or rational numbers and more domain-general factors that are crucial to master fractions. Thus, comparable to other learning content, mastery of rational numbers and especially of fractions should be predicted not only by cognitive but also by motivational and affective variables. However, while existing theories on fraction processing have mainly focused on cognitive variables that facilitate or impede fraction processing (Siegler et al., 2011; Siegler & Lortie-Forgues, 2014; Vamvakoussi & Vosniadou, 2004, 2010b), none of these current theories have considered motivational and affective variables as possible factors contributing to fraction processing yet.

For this reason, the present work aims to empirically extend the scope of the currently most prominent theory of fraction processing (i.e., ITND, Siegler & Lortie-Forgues, 2014) to provide a more comprehensive idea by considering relevant predictors of fraction processing. In particular, I aim to evaluate the role and associations of cognitive as well as motivational and affective predictors of fraction processing.

In line with Siegler's integrated theory (Siegler & Lortie-Forgues, 2014), my starting point will be fraction magnitude processing because there is extensive evidence showing that magnitude processing seems to be the core competence underlying fraction processing and understanding (Jordan, Rodrigues, Hansen, & Resnick, 2017; Mou et al., 2016; Siegler, 2016; see also section 1.3 and section 2.1). Therefore, in a first intervention study, I investigated changes in neuro-functional correlates of fraction magnitude processing following an intensive 5-day NLE training in adult participants. More specifically, I was interested in pre-post comparisons of brain activation (measured using functional magnetic resonance imaging, fMRI) associated with three target tasks (e.g., symbolic fraction magnitude comparison task, line proportion comparison task, and fraction-line proportion matching task) and additional transfer effects.

Building on this, in the second study, I examined the role of domain-specific skills for fraction processing in a cross-sectional sample of 939 German secondary school children from grades 7 to 11. In particular, I assessed a comprehensive battery of basic numerical skills (including skills dependent and independent of magnitude processing; see also section 2.2) while controlling for influences of general cognitive ability, grade level, and sex. To account for multicollinearity and determine each predictor variable's relative importance, I used *relative weight analysis* as a methodological approach. Unlike other studies, this allowed me to prevent over- and underestimating the relevance of predictors. In addition, this might give a clearer picture of the contribution each predictor makes to fraction processing.

The third study then considered the role of strategies for fraction processing not necessarily related to magnitude processing (section 2.3). In this study, a fraction magnitude comparison task with positive and negative fractions was employed to test specific strategies, usually used to compare positive fractions or negative numbers. Additionally, participants' eye-fixation behavior was tracked to evaluate strategy use online. Thus, study 2 and study 3 aimed to investigate potential additional relevant cognitive predictors over and beyond magnitude processing.

In a next step, the influence of motivational (section 3.1) and affective (i.e., negative emotions; section 3.2) predictors for fraction processing were observed. In study 4, different motivation profiles across secondary school students were examined to evaluate the role of intrinsic and extrinsic motivation for proficiency and performance with fractions. For this, 256 7th graders played a computerized learning game over 5 consecutive weeks. Additionally, mathematics and German (as first language) school grades, as well as the Situational Motivation Scale (SIMS; Guay, Vallerand, & Blanchard, 2000) were assessed. Finally, motivation profiles were examined via latent profile analysis to identify possible distinct motivational subpopulations within the school students.

Finally, in study 5, the role of (negative) emotions and emotion regulation for difficult tasks (i.e., symbolic fraction and proportion magnitude comparison) compared to easier tasks (i.e., decimal and pie charts magnitude comparison) in adult participants was explored to get a first idea of the influence of emotion regulation on rational number processing. Therefore, I again used fMRI to examine brain activation following the presentation of a numerical cue that signalized either a difficult or an easy upcoming rational magnitude comparison task.

Taken together, the present thesis aims to shed light on the role of cognitive (i.e., magnitude processing, basic numerical skills, and strategies) as well as motivational and affective predictors (i.e., intrinsic and extrinsic motivation, negative emotions, and emotion regulation) in fraction processing. Thereby, the current thesis adds neurofunctional correlates for magnitude-specific fraction processing to Siegler et al.'s ITND and extends it beyond the role of magnitude processing as the core cognitive predictor for fraction understanding.

However, unlike other theories of fraction processing, it must be considered that this thesis does not intend to investigate fraction processing from a developmental point of view but pursues a more multimodal approach (see Table 4.1) to study fraction processing in different age groups. To this end, I chose not only different study designs (e.g., training study, cross-sectional study) and methods (fMRI, eye tracking) but also investigated fraction processing from adolescence to adulthood, intending to pursue a more general approach to answer the question on what could be relevant factors for efficient fraction processing across domains and how some of these factors are associated with each other.

Table 4.1: Summary of the multimodal approaches for this thesis

	Study Design	Participants	Fraction Type	*N*	Method
Cognitive Predictors of Fraction Processing					
Study I	Training study	Adults	Single- and two-digit fractions/ line proportions	48	fMRI
Study II	Cross-sectional study	School children (7th – 10th grade)	Single-digit fractions	939	Relative Weight Analysis
Study III	Laboratory study	Adults	Single-digit fractions	30	Eye Tracking
Motivational & Affective Predictors of Fraction Processing					
Study IV	Field Study	School children (7th grade)	Single-digit fractions	256	Latent Profile Analysis
Study V	Laboratory study	Adults	Fractions, decimals, and proportions (always 1/4)	25	fMRI

PART II: EMPIRICAL STUDIES

Studies of Section 2:

Cognitive Predictors of Fraction Processing

Study 1: Wortha, S. M., Bloechle, J., Ninaus, M., Kiili, K., Lindstedt, A., Bahnmueller, J., Moeller, K., & Klein, E. (2020). Neurofunctional plasticity in fraction learning: An fMRI training study. *Trends in Neuroscience and Education*, *21*, 100141.

Study 2: Wortha, S.M., Klein, E., Lambert, K., Dackermann, T., & Moeller, K. (2021). The relevance of basic numerical skills for fraction understanding: evidence from cross-sectional data (unpublished manuscript).

Study 3: Wortha, S.M., Moeller, K., Keuler, M., Nuerk, H-C., & Bahnmueller, J. (2021). Strategies for Comparing Negative Fractions (unpublished manuscript).

5. Study 1: Neurofunctional plasticity in fraction learning: an fMRI training study[7]

Silke M. Wortha[1,2], Johannes Bloechle[3], Manuel Ninaus[1,4], Kristian Kiili[5], Antero Lindstedt[6], Julia Bahnmueller[1,7], Korbinian Moeller[1,4,7,8] & Elise Klein[4,9]

[1] LEAD Graduate School & Research Network, University of Tuebingen, Tuebingen, Germany

[2] Department of Neurology, Universitätsmedizin Greifswald, Greifswald, Germany

[3] Department of Psychiatry and Psychotherapy, University of Tuebingen, Tuebingen, Germany

[4] Leibniz-Institut für Wissensmedien, Tuebingen, Germany

[5] Faculty of Education and Culture, Tampere University, Tampere, Finland

[6] Faculty of Information Technology and Communication Sciences, Tampere University, Pori, Finland

[7] Centre for Mathematical Cognition, School of Science, Loughborough University, United Kingdom

[8] Individual Development and Adaptive Education Center, Frankfurt am Main, Germany

[9] Université de Paris, LaPsyDÉ, CNRS, Sorbonne Paris Cité, Paris, France

[7] Reprinted by permission of Elsevier: Trends in Neuroscience and Education, Neurofunctional plasticity in fraction learning: an fMRI training study. Wortha, S. M., Bloechle, J., Ninaus, M., Kiili, K., Lindstedt, A., Bahnmueller, J., Moeller, K., & Klein, E. (2020). https://doi.org/10.1016/j.tine.2020.100141; copyright to reuse the content.

Abstract

Background: Fractions are known to be difficult for children and adults. Behavioral studies suggest that magnitude processing of fractions can be improved via number line estimation (NLE) trainings, but little is known about the neural correlates of fraction learning.

Method: To examine the neuro-cognitive foundations of fraction learning, behavioral performance and neural correlates were measured before and after a five-day NLE training.

Results: In all evaluation tasks behavioral performance increased after training. We observed a fronto-parietal network associated with number magnitude processing to be recruited in all tasks as indicated by a numerical distance effect. For symbolic fractions, the distance effect on intraparietal activation was only observed after training.

Conclusion: The absence of a distance effect of symbolic fractions before the training could indicate an initially less automatic access to their overall magnitude. NLE training facilitates processing of overall fraction magnitude as indicated by the distance effect in neural activation.

Keywords: fraction processing, number line estimation training, flow experience, numerical distance effect, fMRI

5.1 Introduction

Over the last decade, research has shown repeatedly that understanding fractions is a crucial predictor of future achievement in higher mathematic [1–3]. However, despite intense research efforts in this area, children's poor performance when it comes to handling and understanding fractions hardly changed [4–7]. For example, in 2015 Lortie-Forgues and colleagues found that only 27% of 8th graders in the United States were able to choose correctly the number closest to the result of a fraction addition problem out of four given solution probes [8]. A similar result was already reported 1978 by the National Assessment of Educational Progress [9], when only 24% of the 8th graders chose the correct answer to the same question. Importantly, these difficulties in understanding fractions may be persisting regardless of educational efforts because - unlike natural numbers - fraction magnitude processing seems to be more difficult due to its bipartite structure reflecting the relative relation of numerator and denominator [10]. According to the integrated theory of numerical development, magnitude information is the crucial basis for understanding numbers. Moreover, the understanding that all real numbers (e.g., natural numbers, integers, rational numbers) can be represented on a number line is a key assumption for numerical learning. Therefore, promoting fraction magnitude understanding seems crucial for fostering fraction understanding more generally [11–13]. Thus, interventions with the aim to improve fraction understanding and therefore conceptual knowledge of fractions should focus on fostering mastery in processing and representing fraction magnitude. In the context of (fraction) magnitude understanding number lines have been shown to be a beneficial instructional tool [14].

Against this background, we aimed at understanding the neuro-cognitive foundations underlying successful fraction learning and their plasticity. For this purpose, participants had to complete a number line estimation training and a flow questionnaire on five consecutive days. In the following we will highlight the most important research results from the research areas which are relevant for our study. First, we will give a brief overview about the relevance of number line estimation training for fraction magnitude understanding. Second, we will introduce flow as a state which is beneficial for learning and especially for fraction learning. Third, we will summarize present knowledge about the neural correlates of fraction processing and highlight the importance of our study in this context. Finally, we will introduce the aim of the current study including our specific hypothesis.

Number line estimation as predictor for fraction magnitude learning

The mental number line is a metaphor for the nature of the number magnitude representation whereby numbers are represented spatially with their magnitude increasing from left to right (at least in Western cultures [15]). In numerical cognition research, number line estimation (henceforth NLE) is used repeatedly to assess number magnitude understanding – especially in children ([16–18], but see [19] for additional aspects). In the NLE task, participants have to indicate the spatial position of a target number on a given number line for which only start- and endpoint are specified [18].

As magnitude is the semantic core for any type of number, the task can not only be used to assess, but also to train fraction magnitude understanding [12]. For instance, Hamdan & Gunderson [20] conducted a training study with three conditions for fraction learning (i.e., number line estimation training, area model training, and a non-numerical control). They observed that although children in both the NLE training and the area model training improved in the respective tasks, only children completing the NLE training showed transfer effects to a not trained magnitude comparison task with fractions.

Moreover, Barbieri and colleagues [21] used a number line-based intervention to improve fraction understanding in children with poor conceptual knowledge of fractions and compared the number line intervention group to a standard mathematics curriculum group. The number line intervention group showed significantly larger learning gains than the control group. Finally, computerized and game-based versions of the NLE task where used successfully to assess and improve children’s fraction understanding [22–24]. Taken together this substantiates that number lines are a powerful instructional tool and the NLE task can be applied successfully to foster fraction (magnitude) understanding. However, successful fraction magnitude learning might not only depend on improving conceptual knowledge of fractions, but also on a more fundamental ability to be able to reach a beneficial cognitive state for learning which is known as flow experience.

Flow experience as an indicator for optimal learning

Learning is not a pure cognitive process but is affected by motivation and emotions [25]. A beneficial factor for learning that is considered specifically in computerized approaches on learning is the *flow experience* of the learner [26]. Flow was first coined by Csikszentmihalyi [27] and can be described as a positive emotional state [28,29] and as a holistic approach to motivation [30]. In particular, flow is characterized by a combination of factors such as increased concentration, reduced self-consciousness, sense of control, that are experienced as intrinsically rewarding [29]. Flow is usually reached when task demands meet personal skills or resources in a balanced way. Thus, when the skills of the learner are too low for the demands of a given task – for instance at the beginning of a training – flow experience is rather low. The same is true when the skills of the learner are too high for a given task which leads to boredom and reduced flow experience. Therefore, flow experience seems to be an optimal state for intrinsically motivated learning, which helps focus on the given task and can lead to improved performance [31]. This is further supported by studies on the neural correlates of flow experience. These studies could show that flow experience is associated with increased activation in areas of the multiple demand system such as inferior frontal gyrus, putamen and anterior insula and decreased activation in areas typically associated with the default mode network such as amygdala, medial prefrontal cortex and posterior cingulate cortex [32,33].

Flow experience was specifically, but not solely considered in different computer-based learning settings. For instance, in game-based learning (for a review see [34]), collaborative learning of problem-solving in virtual environments [35], hypermedia learning [36], e-learning [37], but also in creative processes like music learning [38,39]. As such, flow experience should also be beneficial for fostering fraction magnitude understanding using a computerized NLE training. However, successful fraction magnitude learning might not only depend on improving conceptual knowledge of fractions and on the learners' flow experience, but also on the successful interplay of certain neural correlates underlying the neuro-cognitive foundations of fraction learning. Therefore, in the following the current state of research on neural correlates of fractional learning is briefly outlined.

Neural correlates of fraction and proportion magnitude processing

Despite above described established relevance of fraction knowledge and longstanding research in educational sciences and psychology, little is still known about the neural mechanisms underlying the processing of proportions and fractions in general and the neural correlates of fraction learning in particular. To date, there are only few neuroimaging studies investigating the neural correlates of processing proportions [40–42] and fractions [43–46] in adults.

One important aspect across most studies is that the numerical distance effect was used as an indicator of automatic processing of overall fraction magnitude. The numerical distance effect [47] describes the finding that two numbers are compared faster and more accurately the larger the numerical distance between them (i.e., the farther apart they are on the mental number line, e.g. 3 and 7 is easier to compare than 3 and 4). For instance, for fraction magnitude comparison, Ischebeck et al. [45] observed that neural activation within the right IPS was modulated by the overall numerical distance between the to-be-compared fractions (e.g., numerical distance between $\frac{2}{4}$ and $\frac{3}{7}$), but not by the numerical distance between numerators or denominators (i.e., numerical distance between 2 and 3 for numerators and numerical distance between 4 and 7 for denominators when comparing $\frac{2}{4}$ and $\frac{2}{7}$). Moreover, Mock and colleagues [41] observed a joint neural correlate of specific occipito-parietal activation including the right intraparietal sulcus (IPS) for the processing of different notations of proportions including not only fractions, but also pie charts, dot patterns and decimals.

Finally, Klabunde et al. [48] conducted a first fMRI training study on proportions in participants with fragile X syndrome and a control group with intellectual disabilities. Participants were trained for two days in 10 min sessions until they were able to have over 80% accuracy on matching fractions to pie charts and pie charts to decimals. Neurofunctional changes from before to after the training indicated significantly increased brain activation in the left inferior parietal lobule, left postcentral gyrus, and left insula for both groups. However, the mechanism of interest in this study was not the distance effect but to investigate neural correlates of stimulus equivalence relations.

In summary, previous studies indicate that the distance effect for overall magnitude of the to-be-compared proportions/fractions seems a good measure reflecting automatic processing of overall fraction magnitude. As such, the numerical distance effect as a hallmark effect for magnitude processing indicated that the (right) IPS seems to play a crucial role in the processing of proportion and fraction magnitude independent of the actual task, which is in line with its involvement in the processing of natural number magnitude [49]. As such, one might argue that the presence of a numerical distance effect seems to indicate automatic processing of overall fraction magnitude in proficient fraction processing (see [50] for a similar argument on the relation between distance effect and arithmetic performance), while absence of a numerical distance effect might indicate that the fractions' magnitude is not automatically accessed.

The present study

Against this background, we aimed at investigating neuro-functional correlates and their plasticity associated with an NLE training of fraction magnitude understanding. In particular, we evaluated changes in fraction magnitude processing on the neural level as reflected by the numerical distance effect for overall fraction magnitude. To the best of our knowledge, this is the first study investigating the neural correlates of fraction learning through a NLE training in healthy adult participants. We assessed neural activation associated with fraction magnitude processing using fMRI before and after a five-day consecutive NLE training on fractions. Additionally, we assessed participants' flow experience in each training session.

Similar to previous studies applying NLE training to children, we expected the training to improve participants' conceptual knowledge of fraction magnitude on a behavioral level [20,21]. In particular, we expected significant improvements in NLE performance over the five-day training sessions. Additionally, we expected participants flow experience to be associated with NLE training improvements over the five-day training. Finally, on the neural level, we expected significant changes of IPS activation associated with the numerical distance effect from pre- to post fMRI session indicating more automatic processing of fractions' overall magnitude after the NLE training. This should become evident by a more pronounced numerical distance effect in behavioral measures but also neural activation in IPS after the training.

5.2 Methods

Participants

48 right-handed adult participants (*M* = 23.73, *SD* = 3.65, female = 32) took part in this fMRI study. All participants were German native speakers with normal or corrected to normal vision and reported no history of psychiatric or neurological disorders or drug abuse. The study was approved by the local Ethics Committee of the Medical Faculty of the University of Tübingen. Participants gave written informed consent and received monetary compensation for their participation.

Study Design

The study was designed as a pre-post fMRI training study with five consecutive days of training between the fMRI measurements. On the first day before the training and on the last day after the training, participants had to complete two different magnitude comparison tasks (i.e., comparison of symbolic fractions and comparison of line proportions, respectively) and a fraction-line proportion matching task (i.e., indicating whether the magnitude reflected by a fraction matched that of a line proportion or not) while their brain activation was measured using fMRI (see Figure 5.1). In addition to these computerized tasks, participants also completed a paper-pencil-based NLE task prior to entering the scanner for pre- and post-test measurement (for more details see below). Due to technical problems, behavioral data of fMRI measurements could only be obtained from 24 right-handed adult participants (*M* = 22.50, *SD* = 3.76, female = 16). Imaging results did not differ substantially between the participant group with and without behavioral data (i.e., no supratheshold clusters at an uncorrected $p < .001$ and cluster size of 10). Therefore, imaging results as well as all behavioral data obtained outside the scanner (i.e., training data, flow experience and paper-pencil-based number line estimation task) will be reported for all 48 participants, while behavioral fMRI results are reported for the respective 24 participants only.

The training consisted of a fraction number line estimation task. It took place outside the scanner and each training session lasted around 15-20 minutes depending on participants individual performance. After each training session participants completed a brief questionnaire on mental flow to evaluate possible changes in flow experience over the training period (for more details on the flow questionnaire see below).

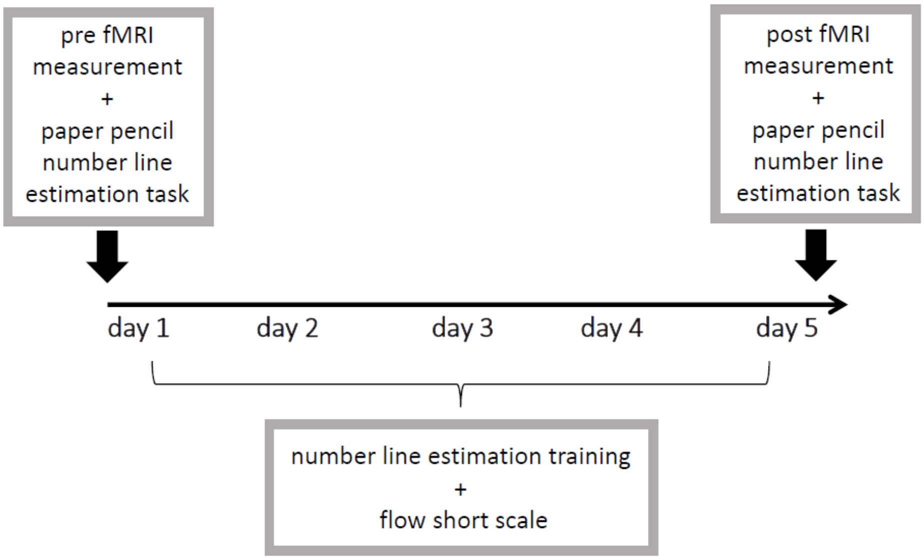

Figure 5.1: Study design. Before and after a five-day number line estimation training fMRI measurements were conducted to evaluate neurofunctional changes in brain activation through the training. After each training session flow was assessed using the flow was assessed using the flow short scale [51]. Prior to entering the scanner for the pre and post measurement a paper-pencil number line estimation task was administered.

Item sets and stimuli

In this study, two different item sets (one to be trained and one not to be trained) were used. The order of sets was counterbalanced across participants so that whichever set a participant did not train on served as the untrained set at the pre- and post-test. However, each participant was tested during the fMRI sessions on both item sets. Each item set consisted of 48 stimuli with items always presented in randomized order. To ensure comparable difficulty of item sets, they were matched on overall problem size and numerical distance between fraction pairs used. All fractions used in the stimuli sets were proper fractions with nominators and denominators ranging from 1 to 30. To evaluate neurofunctional changes in fraction magnitude processing, pre-post-test evaluation tasks performed in the fMRI scanner included both trained and untrained items. Items of the two item sets can be found in the Appendix (Table A5.1).

Training Procedure

For the training, we used two variations of a fraction number line estimation (NLE) training with feedback, which differed in appearance and framing, but not in numerical content and task. In particular, half of the participants were trained using an NLE task set within a *gamified* environment (see Figure 5.2), while the other half were trained in the same NLE task in a *non-game* environment (see Figure 5.2). For the gamified NLE task we used the *Semideus* research engine, which was already applied successfully in previous studies for assessing and training fraction knowledge [23,24,52–54].

In both versions of the NLE task, participants had to indicate the correct position of a given fraction on a number line ranging from 0 to 1 by maneuvering an avatar in the gamified version and moving a white slider along the number line using the left and right arrow keys of a computer keyboard in the non-game version. After reaching the estimated correct position, participants had to press the space bar to select that position. Participants were instructed to indicate the right position as quickly and accurately as possible within a time limit of 10 seconds. They received positive feedback (i.e., cheering avatar plus coins awarded in the gamified vs. green checkmark in the non-game version) when their answer fell within a range ±5% around the correct position. In case they failed to answer or did not answer accurately enough, negative feedback was given (i.e., avatar struck by lightning plus loss of virtual energy in the gamified vs. red cross shown in the non-game version) and participants had to repeat the item until it was correctly solved within the ±5% range. At each training session, participants worked through 96 items in 12 runs containing 8 items each. Each item from the trained stimulus set was encountered twice within a training session. Items were presented in randomized order and were identical in both versions of the NLE training.

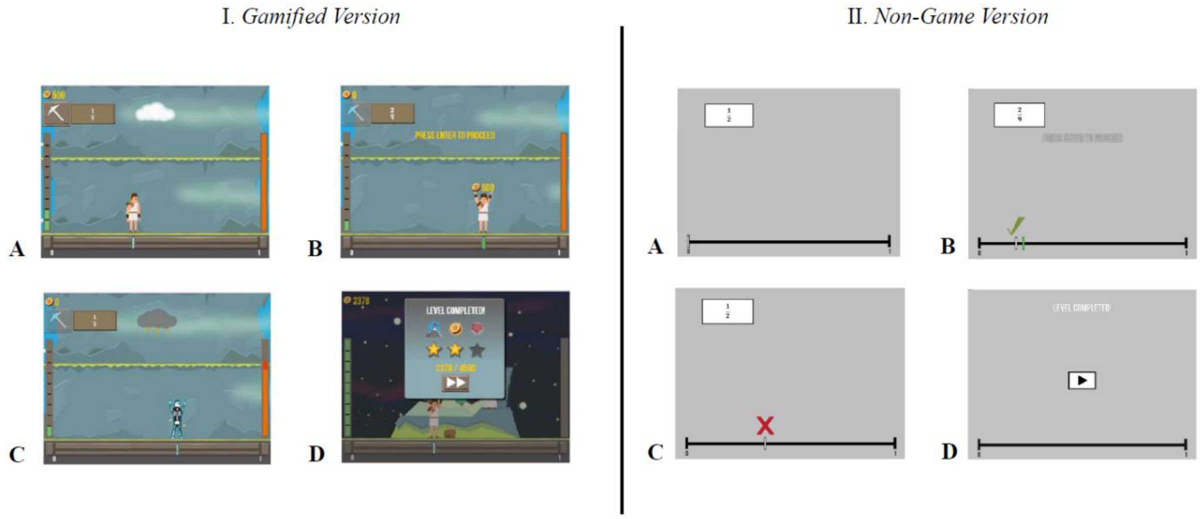

Figure 5.2: Examples of different stages of the gamified (I.) and non-game version (II.) of the NLE training.
I. *Gamified version:* **(A)** Beginning of a new trial. A fraction is shown in the left upper corner and the avatar Semideus has to be moved on a number line between 0 and 1 towards the anticipated position on the number line of the shown fraction. **(B)** The result of a successful trial. If the position of the fraction is estimated correctly (tolerated range: ±5 %) Semideus is rewarded with coins. **(C)** The result of a failed trial. In case the position of the fraction is not estimated correctly Semideus is struck by lightning and the participant needs to try again. **(D)** Completed level with feedback. Mountain: For completing the level, they earned one star; Coin: for collecting at least 2000 points they earned a second star; Heart: for maintaining at least 80% of the life points, they earned a third star. II. *Non-game version:* **(A)** Beginning of a new trial. A fraction is shown in the left upper corner and the participant has to move the white slider on a number line between 0 and 1 towards the anticipated position on the number line of the shown fraction. **(B)** The result of a successful trial. In case the position of the fraction is estimated correctly (tolerated range: ±5 %) a green check mark appears. **(C)** The result of a failed trial. If the position of the fraction is not estimated correctly a red cross appears and the participant must try again. **(D)** Completed level.

Flow Short Scale

Flow was assessed using the German version of the flow short scale [51]. This questionnaire consists of 16 items. Thirteen items are associated with the flow scale (7-point Likert-scale ranging from 1 = totally disagree to 7 = totally agree), which has a three-dimensional structure: The first dimension assessed by the scale is *fluency of performance* (6 items, i.e., “My thoughts or activities run fluently and smoothly”, α = .92). The second dimension is *absorption by activity* (4 items, i.e., “I'm completely focused on what I'm doing right now”, α = .80). Finally, the third dimension is *perceived importance* or concern (3 items, i.e., “I'm worried about failure”, α = .90).

Additionally, the questionnaire includes 3 items of a demand scale (10-point Likert-scale ranging from 1 = easy to 10 = difficult), which aims at assessing how demanding the current task was for the participant (i.e., “For me personally, the current requirements are….”). To monitor flow experience over the course of the training, participants had to complete the flow short scale following each completed training session resulting in five completed flow short scales per participant.

Paper pencil number line estimation task outside the fMRI scanner

In addition to the computerized in-scanner tasks (see below) participants had to complete a paper pencil version of a number line estimation task prior to both fMRI measurements (pre and post training), respectively. The number line ranged from 0 to 1 with only the endpoints specified and participants had to indicate the spatial position on the number line for all items from both item sets (i.e., the trained and the untrained set, thus 96 items in total prior to and after the training). This task allowed to evaluate potential improvements in spatial localization of fractions on the number line through the training.

Tasks performed inside the fMRI scanner

For the fMRI paradigm a block design was used, with four separate blocks for the three different tasks (i.e., fraction-line proportion matching task, line proportion comparison task and symbolic fraction magnitude comparison task). For each item in the fraction-line proportion matching task we presented a matching (i.e., magnitude of symbolic fraction and line proportion were identical) and a non-matching version (i.e., magnitude of symbolic fraction and line proportion were not identical) in the fraction-line proportion matching task. Therefore, this task took twice as long than both comparison tasks and was run in two blocks.

In the fraction-line proportion matching task, participants were shown a fraction and a line proportion (see Figure 5.3a). They had to indicate whether the fraction and the line proportion reflected the same magnitude or not (i.e., identical: left button, different: right button). In half of the items, magnitudes of fraction and line proportion matched. In the line proportion comparison task (see Figure 5.3b), participants were shown two line proportions and had to decide which proportion was the numerically larger one by pressing a corresponding response button (i.e., the right button when the upper and the left button when the lower line proportion was larger). Similar, in the symbolic fraction magnitude comparison task (see Figure 5.3c), participants were shown two fractions and had to decide which fraction was numerically larger again.

Each block consisted of 4 practice followed by 48 critical trials. In both sessions, half of the critical trials consisted of trained stimuli while the other half were untrained stimuli. Stimulus order was random for each participant and each session. Additionally, 22 trails of a scrambled word task were randomly interspersed in each block of each condition to control for eye-movements and to control that participants would stay focused. During these trials, two strings of scrambled letters were shown on top of each other and participants had to decide which of the strings would form a real word (see Figure 5.3d).

To prevent adaptation of the BOLD signal, 6000 ms pauses (white screen, RGB values = 255 255 255) were randomly interspersed between trials. All stimuli were projected on a screen outside the scanner and made visible to participants through a mirror mounted on the head coil of the scanner. Foam pads were used to minimize head movements within the head coil during fMRI acquisition. Stimuli were presented in black font against a white background (RGB values = 255 255 255). The experiment was programmed using Presentation® v16.1 software (www.neurobs.com).

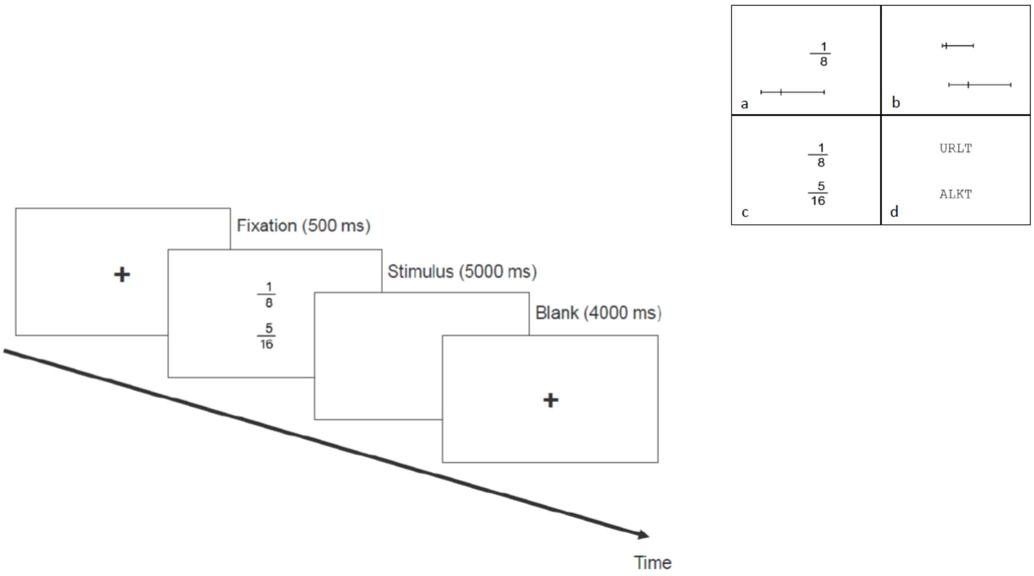

Figure 5.3: Experimental procedure with examples (upper right box) for the different tasks. Example for a) the fraction-line proportion matching task, b) the line proportion comparison task, c) the symbolic fraction magnitude comparison task, and d) the scrambled letters control task. In this example the lower four letters can be unscrambled to form the word “kalt” (German word for “cold”). The upper four letters cannot be formed to any word used in German.

Each trial started with a fixation cross (500 ms), followed by the respective stimulus which appeared for 5000 ms or until participants responded. Subsequently, a blank screen appeared for 4000 ms followed by the beginning of a new trial (see Figure 5.3). Participants responded by pressing one of two MRI compatible response buttons with either their left or their right thumb. Participants were instructed to answer as fast and as accurately as possible.

MRI and fMRI Acquisition

A high-resolution T1-weighted anatomical scan was acquired by a 3T Siemens Magnetom Prisma MRI system (Siemens AG; Erlangen, Germany) equipped with a 64–channel head-neck matrix coil (TR = 2400 s, matrix = 256 × 256, 176 slices, voxel size = 1.0 × 1.0 × 1.0 mm^3; FOV = 256 mm, TE = 2.92 ms; flip angle = 8°). The anatomical scan was always performed at the end of each experimental session.

Functional T2*-weighted images were obtained using multiband gradient-echo Echo planar imaging (EPI; TR = 792 ms; TE = 30 ms; flip angle = 58°; FOV = 192 mm, 64 × 64 matrix; 48 slices, voxel size = 3.0 × 3.0 × 3.0 mm^3). Total scanning time was approximately 80 minutes. A baseline (rest) condition was accomplished by including about 20% null events in the paradigm.

Analysis

Preliminary Analysis

Prior to the analysis of the behavioral and imaging data of the present study, possible differences between the two variants of NLE training (*gamified* vs. *non game-based* training) were examined both on the behavioral as well as the neuro-functional level. However, the behavioral analysis after the training showed no significant differences in reaction times or error rates for all three evaluation tasks performed in the scanner (i.e., symbolic fraction magnitude comparison, line proportion comparison, and fraction-line proportion matching task, all p-values > .05, all $Fs \leq 2.85$). In line with this, the analysis of imaging data revealed no significant difference between the two groups after the training for any of the three evaluation tasks at an uncorrected p-value of .001 with a cluster size of $k = 10$ voxels. Because neither behavioral nor neurofunctional differences were observed for the two trainings, we decided to merge both training groups in order to investigate fraction learning across groups with higher statistical power.

Behavioral analysis

Training and flow data

To evaluate learning outcomes over the five training time points and associations with participants' flow experience during learning we used a latent growth linear mixed-effects model over the five training time points. Regarding dependent variables, we were interested in the mean percentage absolute estimation error (PAE; [55]) and the number of attempts participants needed to estimate a given fraction correctly. Models were fitted in R using 'lmer' from the 'lme4' package [56]. To provide *p*-values we used the 'summary' function of the "lmerTest" R package [57]. Summary statistics were extracted via the 'analyze' function of 'psycho' [58].

Number line estimation task

Mean PAE (cf. [55]) was calculated to reflect performance in the number line estimation task at the two time points. Items without a response were not considered for analyses. To evaluate performance changes in PAE between the pre- vs. post-test, a linear mixed-effects model was fitted using 'lmer' from the "lme4" R package [56]. Again, to provide *p*-values we used the 'summary' function of the "lmerTest" R package [57]. Additionally, summary statistics were also extracted via the 'analyze' function of "psycho" R package [58].

Behavioral fMRI data

For the analysis of the behavioral fMRI data we evaluated reaction times and accuracy as dependent variables. Three separate linear mixed-effects models were run to analyze reaction times for the three different evaluation tasks. Items without a response and items answered incorrectly were not considered for reaction time analyses. To analyze error rates and to include participants' individual performance as random effect, we ran three separate generalized linear mixed-effects models (GLMM) fitted by using the R package 'lme4' [56]. In the GLMMs we assumed a binomial error distribution and used the logit as the link function. For both types of analysis, we provided *p*-values with the 'summary' function of the "lmerTest" R package [57].

Imaging analysis

fMRI data analyses were performed using SPM12 (http://www.fil.ion.ucl.ac.uk/spm). Images were motion corrected and realigned to each participant's mean image. The mean image was co-registered with the anatomical whole-brain volume. Imaging data was then normalized into standard stereotaxic MNI space (Montreal Neurological Institute, McGill University, Montreal, Canada). Images were resampled every 2.5 mm using 4th degree spline interpolation to obtain isovoxels and then smoothed with a 5 mm full-width at half-maximum (FWHM) Gaussian kernel to accommodate inter-subject variation in brain anatomy and to increase signal-to-noise ratio in the images. Data were high-pass filtered (128 s) to remove low-frequency noise components and corrected for autocorrelation assuming an AR(1) process. Brain activity was convolved over all experimental trials with the canonical hemodynamic response function (HRF) and its first time derivative.

For the first level analysis, pre- and post-fMRI training sessions were combined on the subject level in a generalized linear model (GLM). For each participant, we considered the two factors item-set (trained vs. untrained) and session (pre vs post). This resulted in four experimental conditions: trained pre (T1), trained post (T2), untrained pre (UT1) and untrained post (UT2). Additionally, to evaluate the influence of fraction problem difficulty we included the covariate numerical distance between to-be-compared fractions as a parametric regressor in the first level analysis. As a control variable the scrambled word problems (hereafter revered to as "words") were also included in the first level. Finally, the six movement parameters from preprocessing were entered into the model to capture signal variations due to head motion.

These images then entered the second level random-effects group analysis using a flexible factorial design. The SPM Anatomy Toolbox [59], available for all published cytoarchitectonic maps (http://www.fz-juelich.de/ime/spm_anatomy_toolbox), was used for anatomical localization of effects where applicable. For areas not yet implemented, the anatomical automatic labelling tool (AAL) in SPM12 (http://www.cyceron.fr/web/aal anatomical_automatic_labeling.html) was used.

All contrasts calculated reflect the parametric modulation of the fMRI signal by the numerical distance between two proportions presented (distance effect). Simple contrasts (distance effect in each notation) were family-wise error corrected at $p < .05$

at the whole-brain level with a cluster size of k = 10 voxels. Complex contrasts comparing two distance effects (e.g., distance effect in fractions after training vs. distance effect prior to the training) were thresholded at an uncorrected p-value of < .001 at the voxel level with a cluster size of k = 10 voxels and were reported when they remained significant following family-wise error correction (FWE) at the cluster-level with $p_{\text{cluster-corr}} < .05$.

5.3 Results

5.3.1 Behavioral Results

Training and Flow Experience

Percentage absolute estimation error

Differences in PAE and possible associations with flow experience over time were analyzed using a latent growth linear mixed effect model, predicting PAE by flow experience and time (i.e., five training time points) as fixed factors while also including a random intercept to account for participants' individual differences in prior knowledge. The model explained a significant proportion of variance in PAE (R^2 = 75.22%; fixed effects: R^2 = 12.97%) and showed that PAE [β = - 0.34, SE = 0.15, t(193) = - 2.29, p < .05] significantly improved over time (see Figure 5.4 A for PAE changes over time). Moreover, the fixed effect of flow [β = - 0.02, SE = 0.01, t(198) = - 3.13, p < .01] was significant, indicating that flow experience changed over time (see Figure 5.4 C for general changes in flow experience over time). The interaction between time and flow experience was not significant [t(193) = 0.62, p = .54].

Number of attempts

Identical to the first analysis, differences in the number of attempts participants needed to estimate the given fraction during the NLE training correctly over time and possible associations with flow experience were analyzed using again a latent growth linear mixed effect model. Number of attempts needed was predicted by participants' flow experience and time (i.e., five training time points) as fixed factors. Again, we included a random intercept to account for participants individual differences in prior knowledge. The model explained a significant proportion of variance in the number of attempts needed to estimate the given fraction correctly (R^2 = 71.24%; fixed effects:

R^2 = 16.16%) and showed that the number of attempts significantly decreased over time [β = - 0.18, SE = 0.07, $t(194)$ = - 2.70, $p < .01$; see Figure 5.4 B for number of attempts changes over time]. Within this model the fixed effect of flow was significant [β = - 0.01, SE = 0.06, $t(201)$ = - 3.40, $p < .001$], indicating that flow experience changed over time (see Figure 5.4 C for general changes in flow experience over time). Again, the interaction between time and flow experience was not significant [$t(194)$ = 0.95, p = .34].

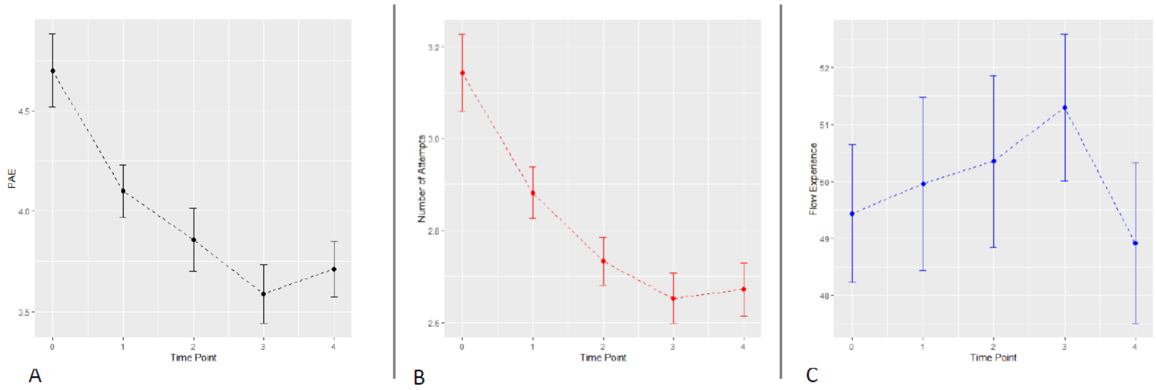

Figure 5.4: Improvement of PAE (A), number of attempts needed (B), and changes in flow experience (C) over the training period (i.e., five time points). Error bars indicate standard errors (SEM).

Number line estimation task

To investigate whether PAE for the paper pencil-based NLE tasks differed between pre- and post-test we ran another linear mixed effect model. We defined the two time points (pre vs. post) as fixed effect and included a random intercept for subjects to account for individual variability. The overall model predicting differences in PAE on the two number line estimation tests (pre vs. post) explained a significant proportion of variance (R^2 = 63.34%, in which the fixed effects explained R^2 = 5.56% of the variance). The effect of session was significant [β = - 0.01, SE = 0.00, $t(47)$ = - 3.80, $p < .001$] indicating that performance was better on the posttest than the pretest (reflected by smaller estimation errors).

Behavioral fMRI results

Fraction-line proportion matching task

Performance changes in reaction time between pre- and post fMRI sessions for the *fraction-line proportion matching task* were evaluated by a linear mixed effect model. We defined session (pre vs. post), itemset used (trained vs. untrained), and numerical distance (i.e., only for the non-matching items) as fixed effects. Moreover, interactions between session and item set as well as session and numerical distance were also included as fixed effects to evaluate whether the possible influence of item set or numerical distance on participants reaction time changed from pre- to post-test. Finally, we included a random intercept to account for participants individual differences in prior knowledge.

The model explained significant proportions of variance on participants reaction times (R^2 = 26.18%; fixed effects: R^2 = 9.74%) and showed that reaction times significantly improved from pre- to post-test [β = - 260.85, SE = 89.37, $t(1068)$ = -2.92, $p < .01$]. Additionally, the fixed effect of numerical distance was significant [β = -1306.84, SE = 210.59, $t(1069)$ = -6.21, $p < .001$], indicating that reaction times improved more strongly for increasing distances. Neither the fixed effect of itemset nor the interactions were significant [all $t \leq 0.48$, all $p_s > .05$].

For the evaluation of performance changes in accuracy between pre- and post-test, we ran a generalized linear mixed-effects model, by using logit as the link function and assuming a binomial distribution of the error rates. To avoid overfitting of the model we only included session, itemset used and numerical distance as fixed effects and a random intercept accounting for individual differences in prior knowledge. The model revealed a significant fixed effect of numerical distance [z = -6.86, $p < .001$], indicating that increasing distances between fractions and line proportions led to less errors. The fixed effects of session and trained item were not significant [all z $\leq$ -0.18, all $p_s > .05$].

Line proportion comparison task

Identical to the analysis of the *fraction-line proportion matching task* we ran the same linear mixed-effects model (including session, itemset used, numerical distance, as well as the interaction session and item set and the interaction session and numerical distance as fixed effects and a random intercept to account for individual differences in prior knowledge) to investigate possible performance changes in reaction time on the *line proportion comparison task* between pre- and post fMRI session. The model explained a significant proportion of variance on participants reaction times (R^2 = 42.71%; fixed effects: R^2 = 8.57%) and showed that reaction times significantly improved from pre to post-test [β = -279.33, SE = 118.19, $t(507) = -2.36$, $p < .05$]. Additionally, the fixed effect of numerical distance was significant [β = -1686.98, SE = 295.05, $t(510) = -5.72$, $p < .001$], indicating that reaction times got significantly faster with increasing distances. Neither the fixed effect of itemset nor the interactions were significant [all t ≤ 0.35, all $p_s > .05$].

Again, to investigate possible performance changes in accuracy for the *line proportion comparison task* between pre- and post-test, we ran a generalized linear mixed-effects model, using logit as the link function and assumed a binomial distribution of the error rates. To avoid overfitting of the model we only included session, itemset used and numerical distance as fixed effects and a random intercept accounting for individual differences in prior knowledge. Analyzing error rates for the line proportion comparison task revealed a significant fixed effect of numerical distance [z = -7.5, $p < .001$], indicating that increasing distances between line proportions led to less errors. However, there were no significant differences for session and item set [all z ≤ -1.10, all $p_s > .05$].

Symbolic fraction magnitude comparison task

Finally, possible performance changes in reaction time for the *symbolic fraction magnitude comparison task* between pre- and post-test were tested identical to the prior two evaluation tasks. We again ran a linear mixed-effects model with session, itemset used, numerical distance as well as the interaction session and item set and the interaction session and numerical distance as fixed effects and a random intercept accounting for individual differences in prior knowledge. Interestingly, in this model only the fixed effect for numerical distance was significant [β = -1282.76, SE = 337.30, $t(499) = -3.80$, $p < .001$], indicating that reaction times were significantly faster for increasing distances. Neither the fixed effects of session and itemset nor the interactions were significant [all t ≤ 0.43, all $p_s > .05$].

Possible performance changes in accuracy for the *symbolic fraction magnitude comparison* task between pre- and post-test, were evaluated again by a generalized linear mixed-effects model, using logit as the link function and assuming a binomial distribution of the error rates. To avoid overfitting of the model we only included session, itemset used and numerical distance as fixed effects and a random intercept accounting for individual differences in prior knowledge. Thus, analyzing error rates for the fraction task revealed a significant fixed effect of session [$z = -2.38$, $p < .05$], indicating that participants performed better in the post-test compared to the pre-test. Additionally, the fixed effect of numerical distance was significant [$z = -9.17$, $p < .001$], indicating that increasing distances between fractions led to less errors when comparing two fractions. However, there was no significant difference for item set [$z = 0.282$, $p = .079$].

5.3.2 Imaging results

Distance effect before the training

Fraction-line proportion matching task

Processing of smaller numerical distances between fractions and line proportions in the fraction-line proportion matching task was associated with stronger magnitude-specific fMRI signal before the training in a bilateral fronto-parietal neural network including areas in the intraparietal cortex (hIP3), the superior parietal cortex (SPL), the inferior frontal gyrus (Areas 44 and 45), bilateral inferior temporal gyri as well as bilateral insula. Further activated clusters were found in the bilateral middle frontal gyri as well as right-hemispheric subcortical areas such as thalamus and caudate nucleus as well as the cerebellum (see Table 5.1, Figure 5.5 A and C depicted in red color).

Line proportion comparison task

Processing of smaller numerical distance between to-be-compared line proportions modulated the fMRI signal before the training in a right-hemispheric fronto-parietal network centered around the right intraparietal sulcus (hIP3). Smaller numerical distance between proportions led to stronger activation in the right IPS and the right anterior IPS reflecting fraction magnitude processing. Additionally, there was a significant cluster of activation in the right inferior frontal gyrus (see Table 5.1, Figure 5.5 B and C depicted in golden color).

Symbolic fraction magnitude comparison task

Activation in no cluster of voxels was modulated significantly by numerical distance at the given threshold in the symbolic fraction magnitude comparison task before the training.

Importantly, there were no significant differences in brain activation observed between the two stimulus sets: when comparing the two sets before the training (set to be trained vs. untrained set) in any of the three conditions (i.e., fraction-line proportion matching task, line proportion comparison task, symbolic fraction magnitude comparison task).

Table 5.1: Distance effect for line proportion comparison and fraction-line proportion matching task at a familywise error corrected p < .05, cluster size k = 10 at the whole brain level.

Contrast	Brain region	MNI (x, y, z)			Cluster size	*t*
Distance effect	RH anterior intraparietal sulcus (hIP2)	43	-36	47	61	5.63
Line proportion	RH intraparietal sulcus (hIP3)	33	-52	53	12	5.04
	RH inferior frontal gyrus (44)	51	8	23	15	5.61
Distance effect	RH precentral gyrus	38	-20	55	2195	11.49
Matching task	RH intraparietal sulcus (hIP3)	26	-57	55		7.15
	RH superior parietal lobe (SPL)	18	-60	58		5.46
	LH intraparietal sulcus (hIP3)	-30	-58	42	531	7.29
	LH superior parietal lobe (SPL)	-25	-57	53		7.01
	LH inferior frontal gyrus (IFG 44)	-42	5	30	167	7.71
	RH inferior frontal gyrus (IFG 44)	53	8	28	179	7.38
	LH inferior frontal gyrus (IFG 45)	-40	28	23	85	6.77
	LH middle frontal gyrus	-20	6	55	58	6.25
	RH middle frontal gyrus	41	41	18	85	5.91
	LH posterior medial frontal gyrus	-7	8	58	271	7.24

RH inferior temporal gyrus	51	-52	-10	22	5.91
LH inferior temporal gyrus	-50	-57	-10	30	5.79
RH thalamus	18	-22	10	68	7.57
RH caudate nucleus	8	16	3	20	6.24
RH insula	33	-20	15	86	6.76
RH insula	31	28	3	54	6.08
RH insula	41	1	10	13	5.75
LH insula	-35	18	3	17	5.55
LH cerebellum	-17	-52	-23	306	10.56

Abbreviations: LH – left hemisphere; RH – right hemisphere; MNI – montreal neurological institute).

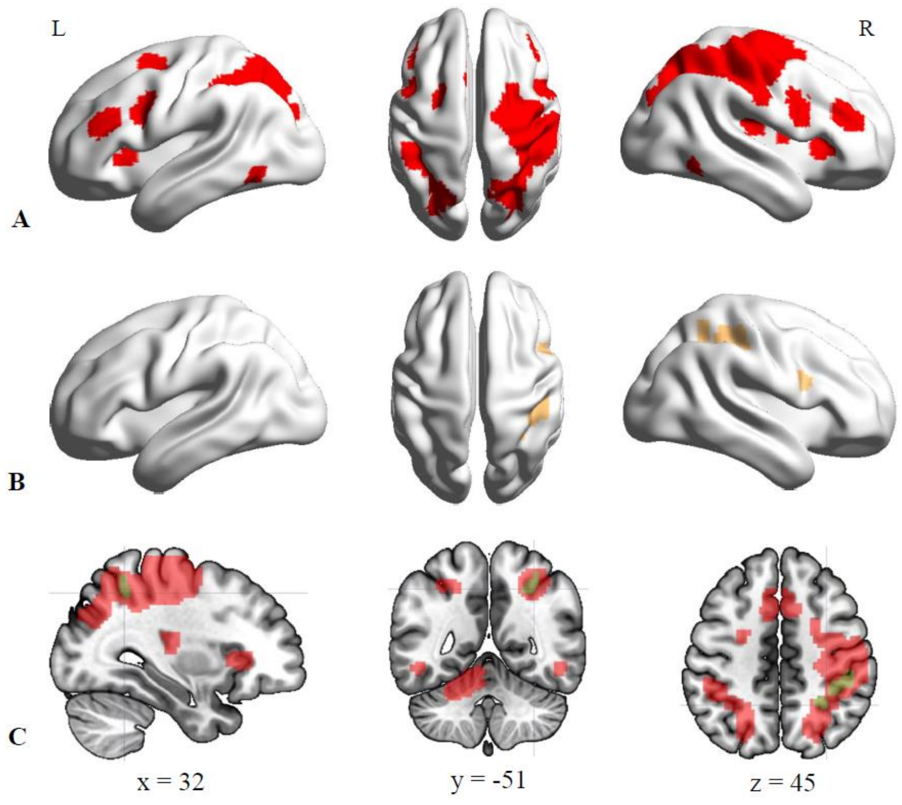

Figure 5.5: Effect of overall magnitude processing in the fractions-lines matching task (Panels A and C: red) and the lines proportion comparison task (Panels B and C: gold) as reflected by the distance effect.

Training and transfer effects

Line proportion comparison / fraction-line proportion matching task

When comparing the distance effect after the training to the distance effect before the training, no suprathreshold clusters of activation were observed for the line proportion comparison or in the fraction-line proportion matching task.

Symbolic fraction magnitude comparison task

Direct comparison of the distance effect after the training to the distance effect before the training in the symbolic fraction magnitude comparison task revealed significant increased activation differences in a bilateral fronto-parietal network centered around the intraparietal sulcus (hIP3; see Table 5.2, Figure 5.6). Further clusters of significant increased activation differences were observed in the right superior parietal lobe (SPL) and the left inferior parietal lobe (PFt), the right fusiform gyrus, the bilateral frontal cortex and the left thalamus.

When comparing distance effects observed for the trained item set to those observed for untrained item set after the training, no suprathreshold clusters of activation were observed, indicating that the training effect was comparably strong for trained as well as untrained fraction items. This means that for fraction magnitude processing it seemed that the training effect generalized to untrained items after the NLE training.

Table 5.2: Effect of training for processing of overall fraction magnitude as reflected by the distance effect for the symbolic fraction magnitude comparison task.

Contrast	Brain region	MNI (x, y, z)			Cluster size	*t*
Fractions post vs. pre training	RH superior parietal lobe (SPL)	26	-45	45	59	4.42
	RH intraparietal sulcus (hIP3)	31	-60	48	21	3.70
	LH intraparietal sulcus (hIP3)	-27	-62	50	15	3.53
	LH supramarginal gyrus (PFt)	-34	-28	33	55	4.56

LH middle frontal gyrus	26	1	40	80	5.36
RH posterior medial frontal gyrus	13	11	53	133	4.08
RH inferior frontal gyrus (IFG)	41	36	15	45	4.05
RH fusiform gyrus	36	-47	-18	15	3.51
LH thalamus	-17	-5	3	18	5.61
LH cuneus	-25	-57	28	15	4.07

Abbreviations: LH – left hemisphere; RH – right hemisphere; MNI – montreal neurological institute).

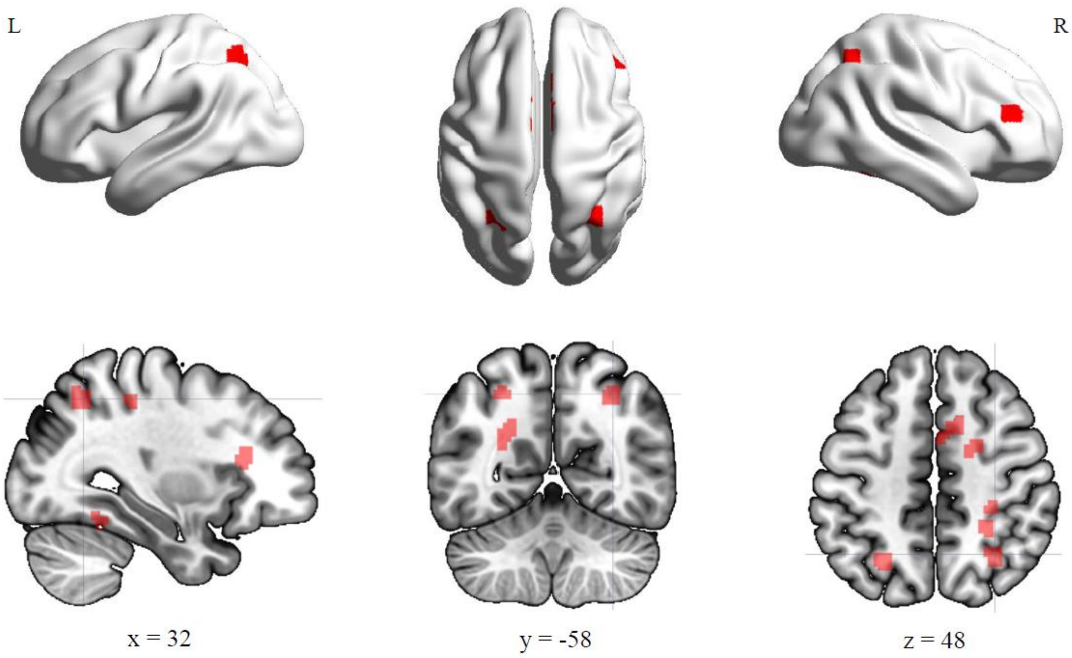

Figure 5.6: Effect of training for processing of overall fraction magnitude as reflected by the distance effect for the symbolic fraction magnitude comparison task (pcluster-corr < .05, k = 10 voxels).

5.4 Discussion

The aim of the present study was to investigate potential neuro-functional changes of brain activation in adult participants through a five-day consecutive NLE training on fraction magnitude. While there already exist a number of studies indicating the effectiveness of NLE training on the behavioral level [20–22,60] and some few studies investigating the neural correlates of fraction and proportion processing [41,42,44,45], little is still known about the neural correlates of fraction learning.

Evaluation tasks included symbolic fraction and non-symbolic line proportion magnitude comparison tasks and a fraction-line proportion matching task. Additionally, learning trajectories over the five time points of the NLE training, participants' corresponding flow experience as well as a paper pencil pre- post NLE task were evaluated. In the following, behavioral as well as neurofunctional results for these measures will be discussed in more detail.

The interplay between training performance and flow experience

Flow experience has been described as an optimal state for intrinsically motivated learning, which helps focus on the given task and can lead to improved performance [31]. Another explanation for optimal learning was first described by Yerkes and Dodson in 1908 and refers to the relationship between arousal and performance (i.e., Yerkes-Dodson-Law; [61]). This law states that learners' optimal performance is achieved on a medium level of arousal reflected by an inverted U-shape relation between the respective parameters. Transferred to flow experience this inverted U-shape suggests that a balance between cognitive demands of the task at hand and individual skill level is the basis for best possible flow experience. Thus, when skills of a learner are too poor for the demands of a given task – for instance at the beginning of a training – flow experience may be rather low. The same is true when skills of a learner are too advanced for a given task. Both non-optimal states can lead to boredom and/or frustration, and reduced flow experience. In turn, this might interfere with learning of the given task [62].

In line with these assumptions, our results for training performance and flow experience over the five day NLE training indicated that at the beginning of the training participants' flow experience was significantly lower and PAE as well as number of attempts needed to solve a trial successfully were significantly higher as compared to later training days. This possibly reflects an imbalance between task demands and individual skills. PAE and number of attempts needed on the first training day suggest that participants' ability on the task seemed to be low in the beginning. With each training day passing flow experience increased while at the same time PAE and number of attempts needed decreased. Thus, participants experienced a more and more optimal learning situation in which demands of the task and individual skill level were in balance.

Interestingly, on the last training day participants' flow experience dramatically decreased again to values lower than on the first training day. This was accompanied by slight increases of PAE and number of attempts needed. Thus, we assume that participants peak of performance was already achieved at the 4th training day. We can only speculate whether this was caused either by boredom or the fact that fraction magnitudes could not be estimated more accurately by our participants after 4 days of training. Importantly, however, this decrease in flow experience was not associated with a general decrease in training performance. Moreover, we observed that participants significantly improved in the paper pencil NLE task from pre to post session.

Transfer effects in fraction and proportion learning

Behavioral data indicated significant performance improvements for all three tasks. Importantly, these improvements did not differ between trained and untrained items, indicating transfer effects of the training to untrained items. Additionally, neurofunctional data showed similar results: before the training no significant differences in brain activation were observed between the two stimulus sets (trained and untrained set) for all three evaluation tasks. After the training, again, no suprathreshold clusters of activation were observed when comparing trained and untrained items for all three evaluation tasks. Thus, indicating that the training effect was comparably strong for trained as well as untrained items and seemed to generalize to untrained items after the NLE training.

Moreover, as discussed in more detail below, for the case of the fraction magnitude comparison task we think brain activation associated with numerical distance after the NLE training indicates that overall symbolic fraction magnitude was not automatically processed before training.

However, one might argue that our results were elicited by the applied drill-like training approach. In the literature, this is often used to investigate numerical learning in terms of arithmetic fact learning [63,64]. Nevertheless, we are confident that participants did not just learn specific fraction magnitudes by heart for at least two reasons: First, if fraction magnitudes were learned by drill no transfer effect from trained to untrained items should be evident neither on the behavioral nor on the neural level. Moreover, fractions that are learned by heart should not show a numerical distance effect especially for untrained items because their overall magnitude should not be processed. In our study induced automated magnitude activation was also present for untrained items. Thus, it is unlikely that our training supported fact learning but rather improved magnitude representation.

Second, previous literature reported different neural correlates for arithmetic fact learning than observed in the current study. In particular, learning arithmetic facts is associated with a shift from bilateral fronto-parietal processing around the IPS to a primarily left hemispheric network in the medial temporary lobe (MTL) involving the hippocampus (cf. [64–66]). However, in the present study, we rather observed a shift towards more activation within the fronto-parietal network of magnitude processing [67–69] – thus, indicating more explicit processing of overall fraction magnitude and not fact retrieval after the training.

This supports the notion that the training indeed resulted in a general conceptual improvement and automatization of fraction magnitude processing, in contrast to training fact retrieval of specific fraction magnitudes (cf. [64] for limited evidence of transfer effects in multiplication fact training).

Differential neural activity patterns before training and possible implications

Surprisingly, and not consistent with the previous literature on neural correlates of fraction processing brain activation before the training for the three tasks of interest revealed different activation patterns in the IPS associated with the numerical distance effect. For the non-symbolic line proportion comparison and the fraction-line proportion matching task we found significant neural activation patterns in the typical fronto-parietal network observed previously for proportion and fraction processing (cf. [40]). In particular, the line proportion comparison task led to increased activation in the right intraparietal sulcus, whereas the fraction-line matching task led to increased activation in the bilateral intraparietal sulcus. This is in line with research on brain activation for symbolic and non-symbolic magnitude processing [70].

Interestingly, for the symbolic fraction magnitude comparison task activation in no cluster of voxels was modulated significantly by numerical distance before the training. This was surprising as previous studies on the neural correlates of fraction magnitude processing consistently reported IPS activity to be modulated by numerical distance for fraction magnitude processing. Importantly, the presented magnitudes did not differ between the three evaluation tasks. This means, that participants of our study were generally able to process the presented magnitudes. However, access to symbolic fraction magnitudes during the respective fraction comparison task might be reduced probably because of the bipartite nature of fractions [10]. Thus, fractions are more difficult to compare than for instance line proportions.

Moreover, our fraction items used in this study where more complex because of two reasons: i) Our fraction pairs did not involve fractions with common components. In this case, reasoning about the natural number components alone (i.e., processing numerators and denominators separately) might often not help to find the right solution when comparing two fractions without common components. However, we used fractions without common components as we wanted to specifically investigate and promote overall fraction magnitude processing. For instance, [71] found that mathematic experts showed a distance effect for overall fraction magnitude while comparing fractions without common components but not for fraction pairs with common components. Moreover, comparing fraction pairs with common components is typically susceptible to what has been called the natural number bias [72]. ii) We used fractions with numerators and denominators ranging from 1 to 30. This had two major reasons: a) the difficulty to match two item sets of fraction pairs on both overall numerical distance and problem size including only fractions without common components and b) the fact that we wanted to make sure that fractions were rather unfamiliar to participants to be able to investigate fraction learning on a neural level. Thus, participants may not have had a specific representation of the magnitude of the presented fractions prior to our study.

Training induced distance effect in the intraparietal sulcus for fraction magnitude processing

In line with previous results of training studies, imaging results after the training showed that the processing of overall symbolic fraction magnitude was improved. In particular, distance related neural activation for symbolic fraction processing became significantly stronger from pre- to post-test in the bilateral intraparietal sulcus. This may indicate that our NLE training helped to establish easier access to the representation of overall fraction magnitude. These results are inconsistent with a NLE training study on natural numbers with pre-post-test fMRI comparing children with and without developmental dyscalculia [73]. After the NLE training both groups showed decreased activation of brain areas involved in number magnitude processing (for instance bilateral middle and superior frontal gyrus and left intraparietal sulcus). The authors interpreted these results as reflection of more automized processing of numerical magnitude after their training.

In this context, however, there is inconsistent evidence in the literature about the distance effect as an indicator for better numerical/ mathematical performance (see Moeller et al., 2011). On the one hand, there are studies showing that a larger distance effect was associated with poorer numerical/ mathematical performance [74,75]. On the other hand, there also are studies observing that a more pronounced distance effect was associated with better numerical/ mathematical performance [50,76]. To accommodate these inconsistent lines of evidence, [50] suggested that the relation between the size of the distance effect and mathematical performance might not be linear but curvilinear instead. In particular, these authors suggested that the size of the distance effect is depended on two factors: i) automated access to processed magnitudes decreases the distance effect whereas ii) increasing task complexity may increase the distance effect while processing magnitudes.

In line with this argument, it needs to be noted that we do not necessarily think that a larger distance effect indicates better number/ fraction magnitude processing. However, in the present study the increase of the distance effect on a neural level in symbolic fraction magnitude comparison might nevertheless indicate more automatic access to overall fraction magnitude as task complexity was very high. Moreover, due to the bipartite nature of fractions [10] overall fraction magnitude may not have been as automatically activated prior to the NLE training. Thus, after the NLE training our participants may have built a more coherent fraction magnitude representation reflected by a larger distance effect. In line with the hypothetical curvilinear model by [50] we think that the distance effect might decrease again after having established the magnitude representation with further training.

Moreover, no significant differences between pre- and post-fMRI were found for the line proportion comparison and fraction-line proportion matching task. This may indicate that improvement towards more automated activation on a neural level was not achieved through training as activation related to the numerical distance effect was already there before training. Again, this might reflect that the bipartite nature of fractions might have hindered automated magnitude processing of the fractions before training. In turn, non-symbolic proportions are further reflected by visual-spatial aspects which may have helped process the actual relative magnitude expressed as compared to symbolic fractions for which this relation needs to be built by participants themselves. Thus, the NLE training might not have changed processing of line proportions and fraction-line matching significantly on the neural level as the respective relative magnitudes may have already been processed before the training due to facilitation by visual-spatial aspects of the presentation format.

Taken together, these results indicate that even well-educated adults benefitted from a NLE based training of fractions aimed at improving conceptual knowledge of fraction magnitude. Importantly, the training did not only induce significant training effects on the bahavioral level but in particular also led to changes in brain activation associated with the processing of symbolic fraction magnitude. This indicates processes of neurofunctional plasticity in fraction learning. In the following, we will discuss implications of these results for education.

Implications for education

The final report of the National Mathematics Advisory Panel states that ‘one key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line’ (p. 28; [77]). However, fraction learning and understanding still is an educational challenge not only in the US but globally. The integrated theory of numerical development, postulates that one core basis of all (rational) numbers is their magnitudes and that these magnitudes can be represented on a mental number line [13]. Students difficulties with fractions often arise from missing conceptual understanding, which among other things but not exclusively involves an understanding of their magnitudes [78]. Therefore, it is a crucial step for students to learn that fractions are numbers with magnitudes that can be represented on a number line as well.

Accordingly, the recommendation to use number lines as an instructional tool to foster conceptual understanding of fractions is given in different guidelines for educational practice in fraction teaching (e.g., *Teaching fractions* [78] or *Developing Effective Fraction Instruction for Kindergarten through 8th grade* [79]). This recommendation is supported by recent evidence from different intervention studies that used number lines as intervention tools and found significant improvements of children's performance and understanding of fractions [20–23,60].

With respect to educational practice, our results support the existing body of literature that processing of proportion and fraction magnitudes can be improved by NLE training. Moreover, to the best of our knowledge this is the first study indicating that such a training improves symbolic fraction processing as reflected by the numerical distance effect on a neural level. In particular, we argue that relative magnitude information of complex fractions may initially not be processed automatically within the IPS as indicated by the missing numerical distance related activation in the IPS before but significant activation associated with numerical distance after the training.

Limitations

When interpreting the results of the current study there are some limiting aspects that need to be considered. First of all, we are well aware that the current study is only a first step towards a better understanding of the underlying neural processes of fraction learning. In particular, this study investigated fraction learning on fractions more complex than those fractions first encounter in school. This was the case for two major reasons: i) Our fraction pairs did not consist of fractions with common components, which limits the number of available fractions when only considering those with numerators and denominators ranging from 1 to 9. Therefore, we used fractions with numerators and denominators ranging from 1 to 30 allowing for proper matching of stimuli sets. ii) We ran our study with adult participants for whom we assumed that they should be more or less proficient with fractions with numerators and denominators up to 9. Thus, to be able to detect training effects we used more complex fractions.

As such, to investigate fraction learning more fundamentally, less complex fractions (i.e., with single-digit numerators and denominators or even unit fractions) should be used. Moreover, our study investigated fraction processing in adult participants. To focus more on the fundamentals of fraction learning the developing brain should be investigated. A first attempt, to investigate developmental differences in fraction magnitude processing on a neural level is a study by [80]. In this study, the authors applied 2 mA bilateral tDCS and found that adults and children benefitted differently during fraction processing by tDCS applied to different areas of stimulation

(IPS vs. dorsolateral prefrontal cortex (DLPFC)). However, imaging studies on the neural correlates of proportion and fraction magnitude processing in children are still missing.

5.5 Conclusion

It is well known that fractions are difficult to learn and understand not only for children and students but even adults and teachers [7,81,82]. Therefore, the ability to foster and improve fraction knowledge is of high educational importance. Apart from beneficial effects of NLE training on the behavioral level, our study provides first insights into the neural correlates of fraction learning. In particular, we did not observe numerical magnitude to significantly modulate brain activation before the training for the processing of symbolic fractions. This might indicate that overall fraction magnitude is not yet processed automatically before training. Thus, through the training participants might have built up more automated processing of overall fraction magnitude. As such, our results indicate a specific improvement of overall fraction magnitude processing through NLE training reflected on the neural level. This case of neuronal plasticity in fraction learning indicates neurofunctional changes elicited by training of educationally relevant content. Therefore, our study supports the importance of NLE trainings for fraction learning on a neurophysiological level.

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Appendix

Table A5.1: Stimuli and corresponding parameters (i.e., fraction magnitudes, numerical distance, problem size, matched distances, and problem sizes within and between stimulus sets) for both item sets.

fraction pairs	magnitude fraction 1	magnitude fraction 2	numerical distance	problem size (PS)	matched distance within and between sets	matched PS within and between sets	Item set
$\frac{1}{8}$ $\frac{5}{16}$	0,125	0,313	0,1875	0,438	0,022	0,078	1
$\frac{2}{3}$ $\frac{8}{11}$	0,667	0,727	0,061	1,394	0,022	0,078	1
$\frac{3}{8}$ $\frac{11}{13}$	0,375	0,846	0,471	1,221	0,027	0,069	1
$\frac{1}{4}$ $\frac{22}{25}$	0,25	0,88	0,63	1,13	0,027	0,069	1

$\frac{5}{12}$ $\frac{10}{17}$	0,417	0,588	0,172	1,005	0,006	0,066	1
$\frac{3}{14}$ $\frac{17}{25}$	0,214	0,68	0,466	0,894	0,006	0,066	1
$\frac{3}{19}$ $\frac{21}{29}$	0,158	0,724	0,566	0,882	0,006	0,066	1
$\frac{1}{24}$ $\frac{16}{25}$	0,042	0,64	0,598	0,682	0,006	0,066	1
$\frac{5}{27}$ $\frac{1}{26}$	0,185	0,039	0,147	0,224	0,008	0,045	1
$\frac{6}{23}$ $\frac{9}{25}$	0,261	0,36	0,099	0,621	0,008	0,045	1
$\frac{23}{27}$ $\frac{13}{22}$	0,852	0,591	0,261	1,443	0,016	0,062	1

$\frac{19}{23}$ $\frac{21}{25}$	0,826	0,84	0,014	1,666	0,016	0,062	1
$\frac{4}{7}$ $\frac{5}{21}$	0,571	0,238	0,333	0,810	0,013	0,087	1
$\frac{3}{7}$ $\frac{9}{22}$	0,429	0,409	0,020	0,838	0,013	0,087	1
$\frac{4}{5}$ $\frac{17}{30}$	0,8	0,567	0,233	1,367	0,022	0,017	1
$\frac{5}{6}$ $\frac{17}{22}$	0,833	0,773	0,061	1,606	0,022	0,017	1
$\frac{9}{14}$ $\frac{15}{26}$	0,643	0,577	0,066	1,220	0,00017897	0,035	1
$\frac{9}{17}$ $\frac{11}{29}$	0,529	0,379	0,150	0,909	0,00017897	0,035	1

$\frac{9}{10}$ $\frac{14}{17}$	0,9	0,824	0,077	1,724	0,00017897	0,035	1
$\frac{9}{13}$ $\frac{11}{17}$	0,692	0,647	0,045	1,339	0,00017897	0,035	1
$\frac{6}{13}$ $\frac{7}{19}$	0,462	0,368	0,093	0,830	0,006	0,033	1
$\frac{6}{11}$ $\frac{9}{26}$	0,545	0,346	0,199	0,892	0,006	0,033	1
$\frac{19}{20}$ $\frac{23}{29}$	0,95	0,793	0,157	1,743	0,001	0,002	1
$\frac{19}{25}$ $\frac{10}{11}$	0,76	0,909	0,149	1,669	0,001	0,002	1
$\frac{1}{9}$ $\frac{6}{25}$	0,111	0,24	0,129	0,351	0,022	0,078	2

$\frac{5}{8}$ $\frac{7}{10}$	0,625	0,7	0,075	1,325	0,022	0,078	2
$\frac{4}{9}$ $\frac{25}{28}$	0,444	0,893	0,448	1,337	0,027	0,069	2
$\frac{2}{9}$ $\frac{13}{14}$	0,222	0,929	0,706	1,151	0,027	0,069	2
$\frac{1}{18}$ $\frac{25}{26}$	0,056	0,962	0,906	1,017	0,006	0,066	2
$\frac{1}{12}$ $\frac{10}{29}$	0,083	0,345	0,262	0,428	0,006	0,066	2
$\frac{5}{19}$ $\frac{17}{20}$	0,263	0,85	0,587	1,113	0,006	0,066	2
$\frac{9}{20}$ $\frac{13}{27}$	0,45	0,482	0,032	0,932	0,006	0,066	2

$\frac{5}{23}$ $\frac{7}{30}$	0,217	0,233	0,016	0,451	0,008	0,045	2
$\frac{7}{27}$ $\frac{1}{22}$	0,259	0,046	0,214	0,305	0,008	0,045	2
$\frac{20}{27}$ $\frac{11}{20}$	0,741	0,55	0,191	1,291	0,016	0,062	2
$\frac{19}{21}$ $\frac{15}{19}$	0,905	0,790	0,115	1,694	0,016	0,062	2
$\frac{2}{5}$ $\frac{3}{29}$	0,4	0,104	0,297	0,504	0,013	0,087	2
$\frac{1}{2}$ $\frac{8}{17}$	0,5	0,471	0,029	0,971	0,013	0,087	2
$\frac{3}{4}$ $\frac{13}{24}$	0,75	0,542	0,208	1,292	0,022	0,017	2

$\frac{8}{9}$ $\frac{22}{29}$	0,889	0,759	0,130	1,648	0,022	0,017	2
$\frac{7}{13}$ $\frac{11}{21}$	0,538	0,524	0,015	1,062	0,00017897	0,035	2
$\frac{9}{11}$ $\frac{21}{26}$	0,818	0,808	0,011	1,626	0,00017897	0,035	2
$\frac{8}{15}$ $\frac{13}{29}$	0,533	0,448	0,085	0,982	0,00017897	0,035	2
$\frac{9}{19}$ $\frac{11}{25}$	0,474	0,44	0,034	0,914	0,00017897	0,035	2
$\frac{8}{13}$ $\frac{9}{23}$	0,615	0,391	0,224	1,007	0,006	0,033	2
$\frac{6}{17}$ $\frac{8}{27}$	0,353	0,296	0,057	0,649	0,006	0,033	2

$\frac{20}{23}$ $\frac{12}{13}$	0,870	0,923	0,054	1,793	0,001	0,002	2
$\frac{15}{22}$ $\frac{14}{15}$	0,682	0,933	0,252	1,615	0,001	0,002	2

6. Study 2: The relevance of basic numerical skills for fraction understanding: evidence from cross-sectional data[8]

Silke M. Wortha[1,2,3], Elise Klein[4,5], Katharina Lambert[2,6], Tanja Dackermann[5], Korbinian Moeller[2,5,7,8]

[1] Department of Neurology, University Medicine Greifswald, Greifswald, Germany

[2] LEAD Graduate School & Research Network, University of Tuebingen, Germany

[3] Department of Psychiatry and Psychotherapy, University of Tuebingen, Tuebingen, Germany

[4] Universite de Paris, LaPsyDE, CNRS, Sorbonne Paris Cite, Paris, France

[5] Leibniz-Institut fur Wissensmedien, Tuebingen, Germany

[6] University of Education, Heidelberg, Germany

[7] Centre for Mathematical Cognition, School of Science, Loughborough University, United Kingdom

[8] Individual Development and Adaptive Education Center, Frankfurt am Main, Germany

[8] Please Note. This is an unpublished manuscript and therefore might not exactly replicate the final published version.

Abstract

Recent research indicated that fraction understanding is an important predictor of later mathematical development. In the current study we investigated the influence of basic numerical skills on students' fraction understanding. We analyzed data of 939 German secondary school students and evaluated the determinants of fraction understanding considering basic numerical skills as predictors (i.e., number line estimation, basic arithmetic operations, non-symbolic magnitude comparison, etc.). Additionally, we controlled for influences of general cognitive ability, grade level, and sex. We found that multiplication, subtraction, conceptual knowledge, and number line estimation, were significant predictors of fraction understanding beyond influences of general cognitive ability and sex. This indicates that specific basic numerical skills acquired in primary school influence mathematical achievement in secondary school in general and fraction understanding in particular. In turn, strengthening these skills should provide children with a broader fundament for fraction learning. As such, the current data indicates that recapitulating basic numerical content in secondary school mathematics education may be beneficial for acquiring more complex mathematical concepts such as fractions.

Word count abstract: 168

Keywords: fraction understanding, basic numerical skills, multiple regression analysis, relative weight analysis

6.1 Introduction

It has been argued that children's numerical development is driven by the acquisition of basic numerical skills (e.g., Dowker, 2005; Geary, 2000). These basic numerical skills are seen as building blocks for later numerical and mathematical achievement. For successful numerical development different basic numerical skills were found to be important: For instance, symbolic and non-symbolic magnitude knowledge (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Siegler, 2016; Siegler & Lortie-Forgues, 2014), as well as a spatial representation of magnitudes as described by the metaphor of a mental number line (e.g., Schneider, Grabner, & Paetsch, 2009; see Fischer & Shaki, 2014 for a review), understanding of the place-value structure of the Arabic number system (e.g., Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; see Nuerk, Moeller, Klein, Willmes, & Fischer, 2011), acquisition of arithmetic fact knowledge (i.e., multiplication tables; Dehaene et al., 2003), as well as skills on procedural and conceptual numerical knowledge (e.g., carry operations, or understanding of the relationship between addition and multiplication, Delazer, 2003; Robinson, Dubé, & Beatch, 2017). Therefore, it comes with no surprise that the mastery of such basic numerical skills predicts not only future numerical skills and mathematical achievement in school (e.g., Moeller et al., 2011; Schneider, Grabner, & Paetsch, 2009), but also more general life prospects (i.e., employment rate; e.g., Duncan et al., 2007; Parsons & Bynner, 1997; Von Aster & Shalev, 2007). As regards educational/ mathematical achievement, several studies showed that mastery of certain basic numerical skills were found to be associated with later numerical achievement: For instance symbolic and non-symbolic magnitude knowledge (e.g., Booth & Siegler, 2008; Kolkman, Kroesbergen, & Leseman, 2013; Link, Nuerk, & Moeller, 2014; Schneider et al., 2017) and understanding the place-value structure of the Arabic number system (Moeller et al., 2011). Therefore, basic numerical skills are seen as highly relevant for children's normal development of later numerical and arithmetical capabilities.

In addition, several studies investigated the relevance of domain-general and domain-specific skills for fraction magnitude knowledge and fraction arithmetic. For instance, Bailey, Siegler, and Geary (2014) observed that knowledge of whole number magnitude and arithmetic in first grade predicted knowledge of fraction magnitude and arithmetic in middle school (i.e., 7th and 8th grade) even after controlling for general cognitive abilities, parental education, parental income, race, and gender. But neither whole number magnitude nor whole number arithmetic predicted reading achievement in middle school. Moreover, in another longitudinal study Hansen and colleagues (2015) observed that number line estimation, non-symbolic proportional reasoning, division, working memory, and attentive behavior contributed to 6th graders' knowledge of fraction concepts, whereas number line estimation, multiplication fact fluency, division, and attention contributed to knowledge of fraction procedures. Moreover, Ye and colleagues (2016) found that basic numerical competencies such as magnitude reasoning and calculation fully mediated the association between general cognitive abilities (e.g., attention, working memory, etc.) in 3rd grade and fraction knowledge in 6th grade. Finally, Jordan et al., (2013) evaluated developmental predictors of fraction concepts and fraction procedures in school children. The authors observed that attentive behavior, language, non-verbal reasoning, number line estimation, calculation fluency and reading fluency predicted conceptual understanding of fractions; while attentive behavior, number line estimation, calculation fluency and working memory predicted procedural understanding of fractions.

Nevertheless, a more comprehensive and systematic analysis of the relevance of basic numerical skills for the mastery of fractions is still missing, which is surprising, given that fraction knowledge is highly important for later mathematical development. Fractions and their related concepts of decimals, ratios, percentages and proportions are omnipresent in algebra, geometry, statistics, and in the sciences (e.g., biology, chemistry or physics). As a result, it is almost impossible to gain a deeper understanding of these disciplines without an understanding of fractions and rational numbers in general. Therefore, mastery of fractions and the ability to successfully calculate with fractions are important steps in children's mathematical development in secondary school (Lamon, 2012; Litwiller & Bright, 2002). Accordingly, there is evidence that students' fraction knowledge is a valid predictor of their actual but also future math achievement. For instance, Booth and Newton (2012) found that children's early understanding of fractions in 5th or 6th grade predicted their later mathematical achievement and knowledge of algebra in high school even when controlling for IQ, reading achievement, working memory, family education and income, and whole number arithmetic knowledge (see also Siegler et al., 2012; Siegler & Pyke, 2013). Therefore, understanding fractions provides a critical foundation for later algebra learning (e.g., NMAP, 2008). At the same time, however, understanding the concept of fractions is very difficult for students, but also adults (e.g., Stafylidou & Vosniadou, 2004; Stigler, Givvin, & Thompson, 2010).

Taken together, basic numerical skills are relevant predictors for later numerical and arithmetical performance while understanding fractions predicts more advanced mathematical and algebraic achievement.

The current study aimed at investigating in more depth which basic numerical skills are associated with fraction understanding as potential building blocks of fraction learning in secondary school students using a cross-sectional design. We hypothesized that basic numerical skills implying the understanding of number magnitudes but also of basic arithmetic operations should be particularly relevant to fraction understanding. As regards other basic numerical skills such as conceptual knowledge or basic geometry our approach was exploratory. However, it was our specific aim to assess a comprehensive set of basic numerical skills to get a more profound picture of influences of basic numerical skills on fraction understanding. As such, the aim of the current study was to identify and differentiate basic numerical skills predicting fraction understanding and to quantify the relative importance of predictor variables.

6.2 Methods

Participants

For this study a subsample of N = 1248 students from German schools in the federal state Baden-Württemberg was analyzed. We included only children from 7th to 11th grade as introduction to fractions only occurs at the end of 6th grade in the state's mathematics curriculum. Moreover, we excluded students older than 18 (likely repeaters; N = 272) and students with missing values on at least one of the considered variables (N = 37, i.e., the tests on basic numerical competences, grade level, sex, or general cognitive ability), resulting in a final sample of 939 students (7th grade N = 200, 8th grade N = 215, 9th grade N = 210, 10th grade N = 136, , 11th grade N = 178) for the analyses (age M = 15.14 years, SD = 1.49; 47% females). The current work presents parts of a larger project in which a standardized test of basic numerical curricular mathematical abilities for secondary school was developed. To test our hypotheses we used a statistical approach similar to Ludewig, Lambert, Dackermann, Scheiter, & Moeller (2019).

Measures

Fraction understanding

Fraction understanding was evaluated by assessing procedural knowledge (eight items, i.e., calculate and shorten as far as possible: $\frac{1}{3}+\frac{5}{8}=?$) and magnitude knowledge (four items, i.e., writing the proportion of given areas as a fraction). The test was restricted to 3 minutes in total. Correctly answered items were considered in a sum score, which served as criterion variable in all analysis.

Basic Numerical Skills

A battery of basic numerical competences was administered including eight subtests (i.e., non-symbolic magnitude comparison, number line estimation, approximate arithmetic, addition, subtraction, multiplication, conceptual knowledge about arithmetic, and basic geometry). All subtests were speeded to assess the level of automatization and because of test economy. Furthermore, all subtests only addressed numerical/mathematical competences of primary school curriculum and examples to ensure task understanding. Unless indicated differently, correctly solved items were considered as sum scores for analyses. In the following, the respective subtests are described separately and in more detail. Additionally, common examples for items of the different tests are given in Appendix A6.1.

Non-symbolic magnitude comparison: 24 pairs of dot clouds (ranging from 30 to 69) were shown, and students had to decide which of the two dot clouds was numerically larger. Dot clouds were matched for overall surface to prevent children from using strategies based on perceptual features: for half of the items the surface with the lower quantity of dots was larger and for the other half the surface with the higher quantity of dots was larger. In order to avoid counting-based strategies, the time limit for this task was 1 minute. The number of correctly solved items served as predictor variable.

Number line estimation: Students had to estimate the correct location of a given number on a number line. Only the endpoints of the number line were defined (e.g., estimate the location of 64 on a number line ranging from 0 to 100). In total, the task included 24 items with changing endpoints (i.e., 6 items on a number line from 0 to 10 and 0 to 100, respectively, and 4 items on a number line from 0 to 1.000, 0 to 10.000 and 0 to 100.000, respectively). In order to avoid counting-based strategies, the task was limited to 1.5 minutes. The mean percentage absolute estimation error (PAE; cf. Booth & Siegler, 2006) served as predictor variable. Items with no response were not considered for analyses.

Approximate arithmetic: Students were presented a problem with two different incorrect solution probes. They had to estimate and choose the solution being closer to the correct result (e.g., "Which result is closer to 347 - 120? solution probes: 215 or 260"). 16 addition and 16 subtraction problems were presented, and difficulty level increased with item number. Students had 2 minutes to solve as many problems as possible. The sum of correctly solved items served as predictor variable.

Arithmetic operations: These included i) *addition*, ii) *subtraction* and iii) *multiplication* with 36 arithmetic problems each, ordered in increasing difficulty. Addition and subtraction problems covered numbers ranging up to 10,000. Multiplication problems covered problems with single-digit, two-digit and three-digit operands (with a maximum problem size each of 72, 4698, 3400, respectively). For each operation, students had to solve as many problems as possible within 2 minutes. In all three arithmetic operation tests, the sum of correctly solved items served as predictor variable, respectively.

Conceptual knowledge about arithmetic: Students were shown two problems of which the first problem was presented with a solution. They had to decide whether the solution of the first problem helped to solve the second problem without having to calculate (e.g., "Does 4 + 8 = 12 help you to solve 12 - 4 = __?"). 40 pairs of arithmetic problems including addition, subtraction, multiplication, and division were presented (i.e., 20 problems in which the first equation was helpful to know for solving the second one and 20 in which this was not the case). Students were instructed to make a correct decision for as many problems as possible within 2 minutes. For this test, students had to correctly identify relationships between numerical/arithmetic operations without actually solving the arithmetic problems by performing the relevant computations. The sum of correctly solved items served as predictor variable.

Basic geometry: Students had to solve 12 mirror image problems. For this, a flipped geometrical form had to be drawn by mirroring it on a presented axis. For each correct line in a drawing, students were given one point and scores for each item varied between 6 and 12 points. The percentage of correctly solved items served as predictor variable.

G*eneral cognitive ability:* Students' general cognitive ability was assessed using two subtests of the German version of the Culture Fair Intelligence Scale 20-revision (i.e., continuation of sequences and completion of matrices; CFT 20-R; Weiß, Albinus, & Arzt, 2006). In the *continuation of sequences* subtest, students are given a sequence of changing shapes and they need to find a logical continuation to this sequence. In the *completion of matrices* subtest, students are given a matrix of changing shapes and they need to find a logically matching shape for the blank cell of the given matrix. Subtests were administered as stated in the manual and the sum of correctly solved items served as predictor variable.

Procedure

All tests were administered during regular school hours in the students' classrooms and testing took a maximum of 90 minutes. In each grade, the same basic numerical skills test and fraction understanding test was assessed. For students below the age of 18, parents received information about the study and provided written informed consent prior to testing, whereas students above the age of 18 provided written informed consent themselves. The study was approved by regional school authorities. Students received instructions by trained student assistants. In order to make sure children understood the individual tasks there were written instructions which were also read out to them. Each task was introduced by examples to make sure students understood the tasks. For all tasks speeded versions were used due to test economy and to assess automatization.

Statistical Analyses

Multiple regression

Multiple regression analysis was used to determine the influence of the measured basic numerical predictors on fraction understanding. We used False Discovery Rate (FDR) controlling the *p*-value for multiple testing (Benjamini & Hochberg, 1995).

Relative weight analysis

Determining the relative contribution of each predictor to the explained variance is usually difficult. For instance, Darlington (1968) pointed out that it is problematic to use multiple regression for assessing the relative importance of correlated predictors as this method is not able to correctly partition variance to the different correlated predictors. As predictors are usually correlated with one another, relative weight analysis is more appropriate to evaluate the relative importance of predictors. Therefore, we used relative weight analysis as suggested by Johnson (2000) to quantify the relative importance of correlated predictor variables in the multiple regression analysis.

Relative weight analysis determines which variables contribute the most regarding explained variance in terms of R^2. For this, the predictor variables are transformed into a set of orthogonal variables in a way that they are maximally related to the original predictors. The resulting relative weights represent the predictors' additive decomposition of the total model R^2. There are two measures of relative weight: raw relative weight and rescaled relative weight. Raw relative weights add up to the R^2 of the model while rescaled relative weights add up to 100% representing the relative importance of a particular variable in the final regression model. Relative weights can be understood as the share of declared variance to which each predictor variable can be appropriately assigned (Tonidandel & LeBreton, 2015). We identified significance of relative weights using the procedure described by Tonidandel, LeBreton, & Johnson (2009). Importantly, relative weight analysis not only helps evaluating how much variance each predictor explains by itself and in conjunction with other predictors. Instead, it is also able to uncover ulterior predictors that regression analyses may miss to detect due to shared variance of correlated predictors (see Stadler, Cooper-Thomas, & Greiff, 2017 for a more detailed introduction and discussion).

Variables

Overall, 29 predictor variables were incorporated into the multiple regression and relative weight analysis. This included the eight basic numerical skills assessed (i.e., non-symbolic magnitude comparison, number line estimation, approximate arithmetic, addition, subtraction, multiplication, conceptual knowledge about arithmetic, and basic geometry) as well as general cognitive ability, grade level and sex. Moreover, interaction terms of grade level and sex with general cognitive ability and the eight basic numerical skills were also included as predictor variables. All continuous variables were centered and sex was effect coded (i.e., female = -1, male = 1) prior to analyses.

Statistical software

Statistical analysis was performed with R (R Core Team, 2017). For the multiple regression analysis, we used the 'lm' function for fitting linear models of the standard R package "state" (R Core Team, 2017). Additionally, we used the 'p.adjust' function with the 'fdr' method to adjust *p*-values. For the relative weights analysis, we applied the syntax provided by Tonidandel & LeBreton, (2015).

6.3 Results

Descriptive Statistics. Mean fraction understanding score was *M* = 3.31 (SD = 2.17, obtained range = 0-11) of 12 possible points. Descriptive statistics for all predictor variables are shown in Table 6.1. For detailed results of descriptive statistics divided by grades[9] (i.e., 5th grad to 11th grade) see Appendix A6.2.

Table 6.1: Descriptive Statistics and obtained range of all basic numerical measures (N = 939).

	M	*SD*	*Range*
Addition	20.06	4.13	2 - 31
Subtraction	16.38	4.71	0 - 32
Multiplication	20.92	4.25	4 - 29
Number line estimation (PAE)	6.52	4.07	1.58 - 47.47
Approximate arithmetic	19.62	5.12	0 - 32
Conceptual knowledge	18.12	6.18	2 - 36
Basic geometry (%)	58.23	21.17	0.00 - 100.00
Non-symbolic magnitude comparison	18.65	3.19	1 - 24
General cognitive ability	18.77	4.28	3 - 29

[9] Please note that results did not change substantially when using age instead of grade level as predictor in the analyses.

The correlation matrix depicted in Table 6.2 displays that all basic numerical skills were significantly correlated with fraction understanding, as well as amongst each other. The highest correlation between two basic numerical skills was r = .70 and was observed between subtraction and addition performance, followed by high correlations between multiplication and addition performance as well as multiplication and subtraction performance (r = .57, respectively). About 70% of correlations were below r = .30. Sex was negatively correlated with fraction understanding indicating that females performed better in the fraction understanding test than males. Moreover, grade level was positively correlated with fraction understanding indicating that students attending higher grade levels performed better than students attending lower grade levels (see Table 6.2).

Table 6.2: Correlations between fraction understanding, basic numerical skills, general cognitive ability, as well as sex and grade level.

Variable	1	2	3	4	5	6	7	8	9	10	11
1. Fraction understanding	1										
2. Addition	.39**	1									
3. Subtraction	.42**	.70**	1								
4. Multiplication	.42**	.57**	.57**	1							
5. Number line estimation	-.25**	-.14**	-.25**	-.16**	1						
6. Approximate arithmetic	.24**	.44**	.48**	.34**	-.13**	1					
7. Conceptual knowledge	.34**	.38**	.36**	.33**	-.12**	.43**	1				
8. Basic geometry	.27**	.20**	.22**	.18**	-.23**	.07*	.20**	1			
9. Non-sym. mag. comp.	.08*	.13**	.12**	.14**	-.06*	.12**	.09*	.08*	1		
10. G. cognitive ability	.41**	.34**	.37**	.31**	-.32**	.20**	.28**	.39**	.20**	1	
11. SexEff[a]	-.13**	.06	.16**	.01	-.08*	.17**	-.09*	-.07*	-.01	-.06	1
12. Grade level	.02	.12**	.11**	-.04	-.14**	.13**	.05	.03	.01	.11**	.19**

Note: **p < .01, *p < .05. N = 939. [a]Code female = -1, male = 1

Multiple regression analysis. The final regression model explained 36% of the variance [R^2 =.35, *adj.* R^2 = .33, $F(29, 909) = 17.08$, p <.001]. Six variables significantly predicted performance in the fraction understanding test: general cognitive ability, number line estimation, subtraction, conceptual knowledge, multiplication, and sex (see Table 6.3). Better general cognitive ability, subtraction, conceptual knowledge, and multiplication performance was associated with better performance on the fraction understanding test, whereas smaller estimation errors in the number line estimation task predicted better performance on the fraction understanding test.

Relative weight analysis: We performed relative weight analysis to reveal the proportional contribution of each predictor variable to the total variance of R^2 while accounting for multicollinearity (e.g., Johnson & LeBreton, 2004; Tonidandel & LeBreton, 2011).

Rescaled relative weights indicated that the proportional contribution of the predictor *multiplication* explained 17.41% of the total variance in the fraction understanding test. Thus, multiple regression as well as relative weight analysis point out multiplication to be the best predictor for fraction understanding. This was followed by *general cognitive ability* (16.52%), *subtraction* (14.44%), *conceptual knowledge* (10.60%), *number line estimation* (7.23%) and *sex* (4.91%).

Although, addition, basic geometry as well as approximate arithmetic were not found to be significant predictors in the multiple regression analysis, relative weight analysis revealed rescaled relative weights that were significantly different from a random variable (addition: *RS-RW* = 11.29%; basic geometry: *RS-RW* = 6.58%; approximate arithmetic: *RS-RW* = 3.84%). Therefore, these three basic numerical skills might be also relevant for the prediction of fraction understanding.

Table 6.3: Multiple regression results. Please note that for reasons of readability only main effects are shown (see Appendix A6.3 for a table including all interaction terms, no interaction was significant[10]).

	B	β	[*L-CI,U-CI*]	*RW*	*t*	*p*	*RS-RW* (%)
Criteria = Fraction understanding [multiple R^2= .35, *adj.* R^2 = .33, $F(29,909) = 17.08$, p <.001]							
Intercept	3.34	.00	[-.04, .08]	.00	52.91	.000	0
Multiplication	0.09	.18	[.11, .25]	.06	5.07	.000	17.41*
G. cognitive ability	0.08	.16	[.10, .23]	.06	4.88	.000	16.52*
Subtraction	0.07	.15	[.06, .23]	.05	3.52	.002	14.44*
Addition	0.04	.07	[-.01, .15]	.04	1.72	.249	11.29*
Conceptual knowledge	0.04	.12	[.05, .18]	.04	3.62	.002	10.60*
Number line estimation (PAE)	-0.06	-.10	[-.16, -.05]	.03	-3.52	.002	7.23*
Basic geometry	0.01	.07	[.01, .13]	.02	2.33	.076	6.58*
Sex[a]	-0.28	-.13	[-.19, -.08]	.02	-4.53	.000	4.91*
Approximate arithmetic	0.00	.01	[-.06, .07]	.01	0.30	.816	3.84*
Non-sym. mag. comp.	-0.02	-.03	[-.08, .03]	.00	-0.92	.646	0.30
Grade level	-0.02	-.01	[-.07, .04]	.00	-0.45	.771	0.23

Note: *B*: unstandardized regression weight; *β:* standardized regression weight; *L-CI:* lower boundary (2.5%); *U-CI:* upper boundary (97.5%); *RW:* raw relative weight (within rounding error raw weights will sum to R^2); *t:* t-value measures the size of the effect relative to the variation in sample data; *p:* p-value; *RS-RW:* relative weight rescaled as a percentage of predicted variance in the criterion variable attributed to each predictor (within rounding error rescaled weights sum to 100 %). [a] code female = -1, male = 1. * significantly different from a random variable.

[10] Please note that we did not additionally analyze our data separately for grade levels because grade level was not a significant predictor in our multiple regression and relative weight analysis. Moreover, all interaction predictors reflecting differential influences of the predictor variable for different grade levels were also not significant. This indicates that the influence of basic numerical skills on fraction understanding did not differ significantly across grade levels.

6.4 Discussion

In the present study, we investigated the influence of secondary school students' basic numerical skills on their fraction understanding. To evaluate our hypothesis that those basic numerical skills reflecting the understanding of number magnitude and basic arithmetic operations should be particularly relevant to fraction understanding (cf. Ye et al., 2016) in secondary school students and to explore the relative influence of other basic numerical skills such as conceptual knowledge or basic geometry we chose a two-step approach: in a first step, we identified significant predictors of fraction understanding using multiple regression. In a second step we then evaluated relative importance of individual basic numerical skills using relative weight analysis (cf. Johnson, 2000) to determine the relative contribution of each identified predictor to the explained variance and to uncover ulterior predictors that multiple regression analyses missed to detect. In the first step, our approach revealed that *multiplication* was the most important predictor of fraction understanding. Moreover, *general cognitive ability*, *subtraction*, *conceptual knowledge*, *number line estimation* and *sex* significantly predicted fraction understanding in the sense that better performance on these basic numerical abilities and female sex was associated with better performance in the fraction understanding test. In addition to these significant predictors, relative weight analyses indicated considerable influences of *addition*, *basic geometry* as well as *approximate arithmetic*. These findings are largely in line with previous findings of longitudinal studies: whole number magnitude knowledge (i.e., number line estimation) and whole number arithmetic knowledge (i.e., multiplication, subtraction and conceptual knowledge of arithmetic) were observed to be important predictors of fraction understanding (e.g., Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013; Ye et al., 2016). In the following, contributions of significant predictors as revealed by the multiple regression analysis, but also of potentially relevant predictors as indicated by the relative weight analysis will be discussed in more detail.

In our study, the most important predictor of fraction understanding, as identified consistently in regression and relative weight analysis, was *multiplication*, with better multiplication performance predicting better fraction understanding test performance. This seems reasonable as a large part of the fraction understanding test required fraction arithmetic problems with single-digit numerators and denominators, for which multiplication is the key operation. For fraction addition and fraction subtraction, multiplication procedures are required to find the common denominator and/or extend the numerators. Moreover, for multiplication or division of fractions it is necessary to multiply numerators and denominators with each other. Therefore, fluency with multiplication facts should help students to solve fraction problems independently of task type (i.e., magnitude comparison or a fraction arithmetic; cf. Hecht, Close, & Santisi, 2003; Seethaler, Fuchs, Star, & Bryant, 2011; Ye et al., 2016). Apart from that, (single-digit) multiplication is assumed to be solved by arithmetic fact retrieval (e.g., De Visscher & Noël, 2014; De Visscher, Noël, & De Smedt, 2016) which is fast, efficient and less effortful than any other procedural strategies based on magnitude manipulations.

Another important (and consistently observed) predictor of fraction understanding was *general cognitive ability*: better general cognitive ability predicted better fraction understanding. General cognitive ability may be defined as a general mental capability in problem solving, abstract thinking, reasoning, planning, and comprehension of novel problems, but also learning from experience (e.g., Cattell, 1963; Gottfredson, 1997; Horn & Cattell, 1966). To assess general cognitive ability, we used two subtests of the CFT-20-R (Weiß et al., 2006) reflecting fluid intelligence. Fluid intelligence is assumed independent of experience and previously acquired knowledge. According to Cattell (1963; 1966) it should also not be influenced by educational level and other environmental factors. Both, the subtest *continuation of sequences* as well as the subtest *completion of matrices* capture the ability to recognize rules and relationships. Transferred to fractions it is also important to recognize and then apply correct strategies and rules (e.g., Braithwaite, Leib, Siegler, & McMullen, 2019; Braithwaite, Pyke, & Siegler, 2017). As such, the ability to systematically solve problems on a more abstract level might be useful for operating on fractions. Unfortunately, most students tend to apply arithmetic strategies incorrectly when it comes to fractions (e.g., Hecht, 1998; Newton, Willard, & Teufel, 2014) – primarily as a result of wrongly generalizing knowledge on whole number arithmetic to fraction arithmetic (e.g., Braithwaite et al., 2019). Additionally, in the curriculum of Baden-Württemberg fractions are introduced at the end of 6^{th} grade, but hardly used afterwards in math classes and fraction problems are often not a common part of the daily math routine in schools resulting in a crucial lack of practice for solving fraction problems. Therefore, general cognitive ability may be an important predictor of fraction understanding test performance.

Moreover, *subtraction* was identified as a significant and relevant predictor of fraction understanding in both regression and relative weight analysis. Better subtraction performance was associated with better fraction understanding test performance. This may indicate that, in addition to arithmetic fact retrieval, processes of magnitude manipulation also play an important role in dealing with fractions, because subtraction is considered to be the arithmetic operation relying most on magnitude manipulations (Berteletti, Man, & Booth, 2015; Linsen, Verschaffel, Reynvoet, & De Smedt, 2014, 2015). As mentioned above, a considerable part of the fraction understanding test involved fraction calculations. The two most common operations used while calculating with fractions are multiplication (as described above) and addition. However, *addition* was not a significant predictor of fraction understanding in the regression analysis. One reason may be that subtraction and addition performance were highly correlated and variability was slightly more pronounced for subtraction as compared to addition performance (see Tables 6.1 and 6.2). Moreover, as already mentioned above subtraction is assumed to draw on magnitude manipulations more strongly than addition. Therefore, it is unlikely that addition may explain unique variance beyond that already captured by subtraction. This argument is corroborated by the relative weight analysis, in which addition explained a relevant proportion of the variance in fraction understanding performance.

Additionally, students' *conceptual knowledge* on numbers was a significant predictor of their fraction understanding. Better performance on the conceptual knowledge test was associated with better fraction understanding test performance. For this test, students had to correctly reason about relationships between numerical/arithmetic operations without actually solving the arithmetic problems by performing the relevant computations. Therefore, conceptual understanding of the reciprocal relationships between different arithmetic operations was necessary to correctly solve the task. Such understanding is important as recent research indicated that students often tend to rely on their procedural mathematical knowledge without really understanding which arithmetic procedure is the correct one and why it is the correct one (e.g., Lortie-Forgues, Tian, & Siegler, 2015). For the case of fractions, this may easily lead to an erroneous application of arithmetic procedures. Interestingly, resulting strategy errors (e.g., wrongly applying procedures for whole numbers) are more common than execution errors (e.g., Braithwaite et al., 2017;

Siegler & Pyke, 2013). Therefore, better conceptual knowledge may enable students to apply arithmetic procedures correctly because they have good understanding of their relationships across arithmetic procedures.

Furthermore, better *number line estimation* performance predicted better performance in the fraction understanding test. Number line estimation is considered to assess students' representation of number magnitude (e.g., Siegler & Opfer, 2003). For successful numerical development, it is essential to understand the concept that all numbers represent numerical magnitudes that are aligned in ascending order on a mental number line. According to Siegler (2016; see also Siegler & Braithwaite, 2017; Siegler & Lortie-Forgues, 2014), the mental number line is a dynamic structure that is capable of representing all kinds numerical magnitudes (i.e., whole numbers, negative numbers and rational numbers). Additionally, magnitude understanding reflects a universal characteristic that applies to all kinds of real numbers (cf. Siegler & Braithwaite, 2017). In this vein, several studies indicated that individual differences in whole number magnitude understanding as assessed by the number line estimation task predicted later differences in fraction magnitude knowledge (e.g., Bailey et al., 2014; Jordan et al., 2013; Ye et al., 2016). Therefore, representing the magnitudes of whole numbers adequately on a mental number line seems to be a building block for successful understanding of fraction magnitudes later on. This is in line with previous findings showing that whole number magnitude knowledge is indeed a building block of later fraction understanding (e.g., Siegler, Thompson, & Schneider, 2011). Moreover, Siegler and Braithwaite (2017) also argue that fundamental understanding of number magnitude is essential to understand arithmetic procedures. This is in line with our results as large parts of our fraction understanding test involved fraction arithmetic problems and better whole number magnitude understanding predicted better performance in fraction arithmetic problems.

Finally, *sex* was a significant predictor in our fraction understanding test with females performing better than males. This finding was expected because fraction understanding and in particular fraction arithmetic requires successful application of procedures or strategies and in general, females are known to be more successful in applying learned arithmetic procedures than males (Gallagher, 1998; Gallagher et al., 2000; Kessel & Linn, 1996).

However, when a predictor variable in a multiple regression analysis is not significant, this does not necessarily mean that it has no influence on the dependent variable. Therefore, we conducted relative weight analyses. The latter revealed that *addition*, *basic geometry* as well as *approximate arithmetic* additionally contributed substantially to R^2 and should also be considered as relevant predictors. However, as these three predictors were correlated with other predictors and therefore shared parts of their variance with other predictors it is likely that they were not able to become significant predictors in the multiple regression analysis because they may not have provided significant incremental information. As mentioned above, *addition* performance was highly correlated with subtraction performance, which was included in the final regression model. *Basic geometry* correlated significantly with number line estimation indicating that poorer performance in the number line estimation task was associated with poorer performance in the basic geometry task. Previous research showed that spatial skills, which would be needed for successful performance in basic geometry, also play a crucial role in magnitude representations as reflected by number line estimation. For instance, Gunderson, Ramirez, Beilock, & Levine (2012) argued that early spatial skills foster the development of magnitude knowledge in supporting the construction of a mental number line. Finally, *approximate arithmetic* was correlated significantly with subtraction and addition. This task requires confident and fast manipulation of magnitudes by relying on either addition or subtraction. Therefore, it is unlikely that performance on approximate arithmetic would explain unique variance over and above of what had already been explained by subtraction and/or addition.

6.5 Limitations and Perspectives

In summary, we observed that students' fraction understanding was predicted significantly by their basic numerical skills. In particular, performance in multiplication, subtraction, conceptual knowledge and number line estimation were identified as significant predictors of children's fraction understanding test performance beyond influences of sex and general cognitive ability. However, in contrast to previous studies (e.g., Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013), our study is based on cross-sectional data and thus does not provide any information about developmental changes in fraction understanding. Nevertheless, grade level was no significant predictor, neither in the multiple regression analysis nor in the relative weight analysis. Indicating that performance in our fraction understanding test was not predicted significantly by the grade level of participants. This is well in line with previous findings that fractions are difficult to understand and handle for students, adults and even math teachers (Ma, 1999; Smith, Solomon, & Carey, 2005; Vamvakoussi & Vosniadou, 2004, 2010b). Furthermore, our results fit nicely with that of prior longitudinal studies showing that whole number arithmetic (i.e., multiplication and subtraction) and number line estimation are important predictors for fraction understanding (e.g., Bailey et al., 2014).

Thus, our results indicate that specific basic numerical skills acquired in primary school influence and predict performance on more complex mathematical concepts in secondary school in general and fraction understanding in particular. In turn, strengthening whole number magnitude understanding and whole number arithmetic should provide children with a broader fundament for fraction understanding.

Acknowledgements

This research was funded and is part of the research program „Netzwerk Bildungsforschung" (Educational Research Network) of the Baden-Württemberg Stiftung provided to Katherina Lambert and was additionally partly funded by the LEAD Graduate School & Research Network [GSC1028], a project of the Excellence Initiative of the German federal and state governments. Silke M. Wortha was funded by the LEAD Graduate School & Research Network. Korbinian Moeller and Katharina Lambert are members of the LEAD Graduate School & Research Network.

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Appendix

A6.1 Item examples for the different subtests and the given instructions of the basic numerical skills test.

addition

Calculate as fast as you can.

14 + 3 = ☐

subtraction

Calculate as fast as you can.

7 – 5 = ☐

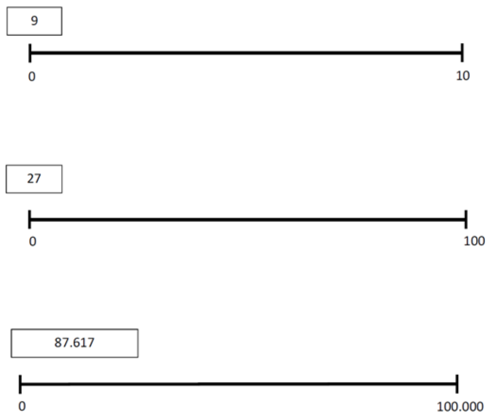
multiplication

Calculate as fast as you can.

2 • 4 = ☐

number line estimation

Mark the position of the number in the box on the number line with a vertical line.



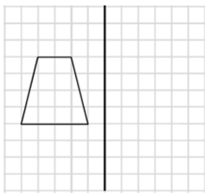
arithmetic estimation

Which number is closer to the correct result? Decide as fast as possible. Mark the correct answer.

18 + 35 = 74 / 58

basic geometry

Mirror the given figure as quickly and accurately as possible (without a ruler or any other auxiliaries) on the mirror axis.



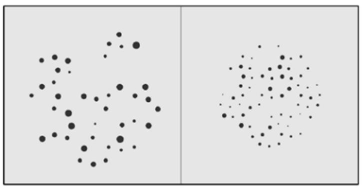
conceptual knowledge

Does the first task help you to solve the second task? Mark the correct answer. (DO NOT solve the task).

A1	168 : 24 = 7	yes
A2	6 · 24 =	no

non-symbolic magnitude comparison

In which box are more dots? Mark the corresponding box.



A6.2 Descriptive Statistics of all variables of interest distinguished by grade levels (i.e., 5th grade to 11th grade).

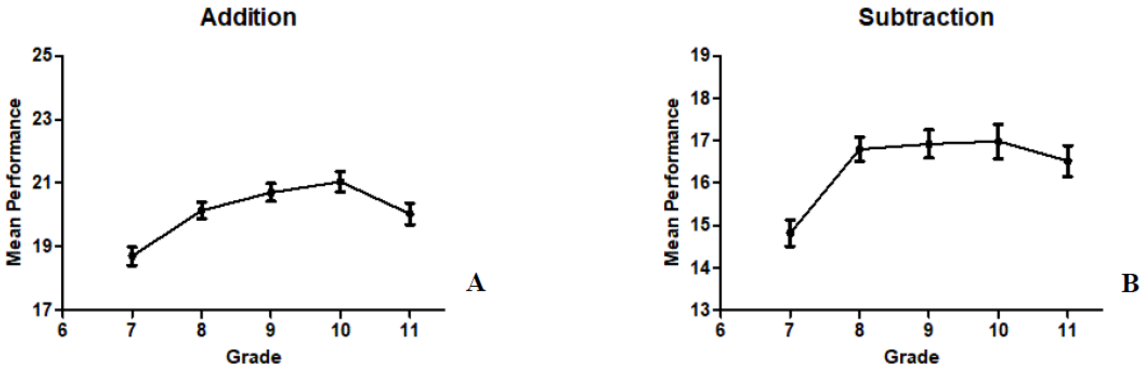


Figure A6.1: Students' mean performance for addition **(A)** and subtraction **(B)** scales distinguished by grade levels (7th grade *N* = 200, 8th grade *N* = 215, 9th grade *N* = 210, 10th grade *N* = 136, 11th grade *N* = 178).

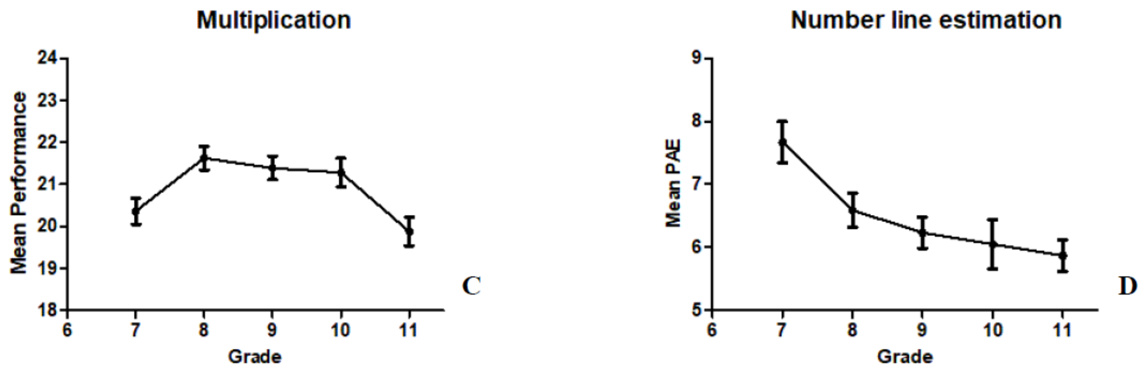


Figure A6.2: Students' mean performance for multiplication scale **(C)** and students' mean percentage absolute estimation error for number line estimation scale **(D)** distinguished by grade levels (7th grade *N* = 200, 8th grade *N* = 215, 9th grade *N* = 210, 10th grade *N* = 136, 11th grade *N* = 178).

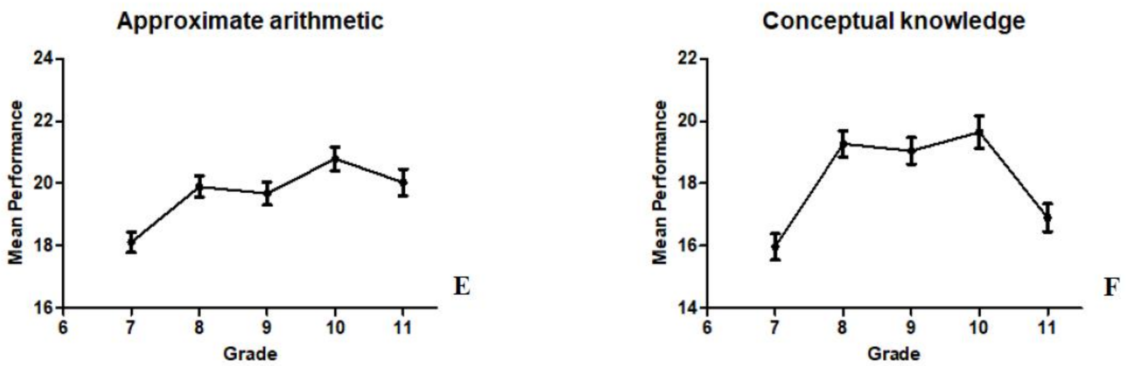


Figure A6.3: Students' mean performance for approximate arithmetic **(E)** and conceptual knowledge **(F)** scales distinguished by grade levels (7th grade N = 200, 8th grade N = 215, 9th grade N = 210, 10th grade N = 136, 11th grade N = 178).

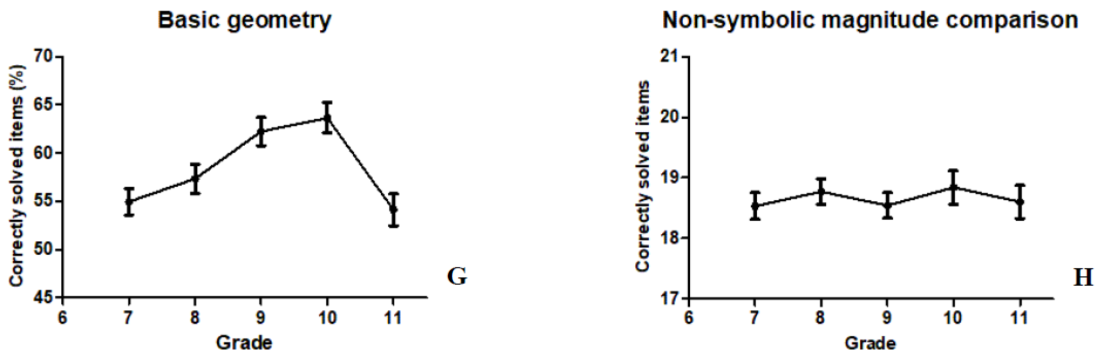


Figure A6.4: Students' performance for basic geometry **(G)** and non-symbolic magnitude comparison **(H)** scales distinguished by grade levels (7th grade N = 200, 8th grade N = 215, 9th grade N = 210, 10th grade N = 136, 11th grade N = 178).

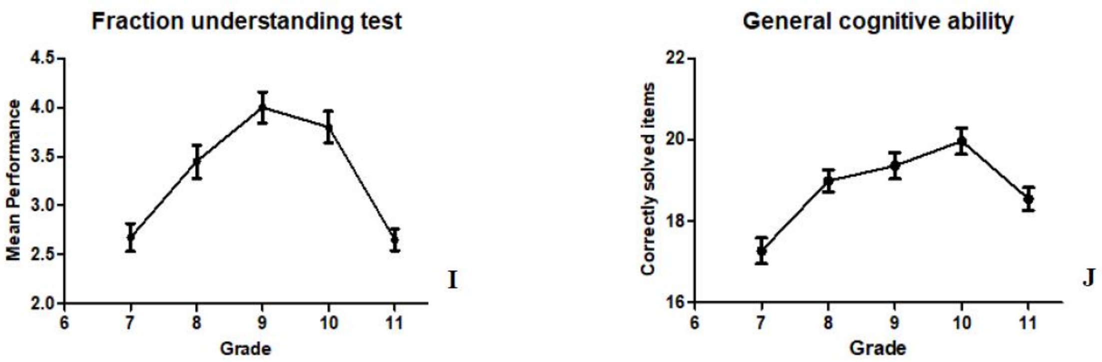


Figure A6.5: Students' mean performance for fraction understanding test **(I)** and performance for general cognitive ability **(J)** scales distinguished by grade levels (7th grade N = 200, 8th grade N = 215, 9th grade N = 210, 10th grade N = 136, 11th grade N = 178).

A6.3 Results of multiple regression analysis and relative weights analysis of basic numerical skills, general cognitive ability, grade level, sex and two-way interactions between grade level, sex with basic numerical skills and general cognitive ability.

	B	β	[*L-CI,U-CI*]	*RW*	*t*	*p*	*RS-RW* (%)
Criteria = Fraction understanding [multiple R^2= .35, *adj.* R^2 = .33, $F(29,909) = 17.08$, p <.001]							
Intercept	3.34	.00	[-.04, .08]	.00	52.91	.000	0
Multiplication	0.09	.18	[.11, .25]	.06	5.07	.000	17.41*
G. cognitive ability	0.08	.16	[.10, .23]	.06	4.88	.000	16.52*
Subtraction	0.07	.15	[.06, .23]	.05	3.52	.002	14.44*
Addition	0.04	.07	[-.01, .15]	.04	1.72	.249	11.29*
Conceptual knowledge	0.04	.12	[.05, .18]	.04	3.62	.002	10.60*
Number line estimation (PAE)	-0.06	-.10	[-.16, -.05]	.03	-3.52	.002	7.23*
Basic geometry	0.01	.07	[.01, .13]	.02	2.33	.076	6.58*
Sex[a]	-0.28	-.13	[-.19, -.08]	.02	-4.53	.000	4.91*
Approximate arithmetic	0.00	.01	[-.06, .07]	.01	0.30	.816	3.84*
Non-sym. mag. comp.	-0.02	-.03	[-.08, .03]	.00	-0.92	.646	0.30
Grade level × Multiplication	-0.10	-.10	[-.08, .05]	.00	-0.42	.771	0.41
Grade level × G. cognitive ability	-0.01	-.30	[-.01, .04]	.00	-0.85	.663	0.43
Grade level × Subtraction	-0.01	-.04	[-.13, .04]	.00	-0.98	.646	0.40

	B	β	[*L-CI,U-CI*]	*RW*	*t*	*p*	*RS-RW* (%)
Grade level × Addition	-0.01	-.03	[-.1, .06]	.00	-0.60	.771	0.41
Grade level × Conceptual knowledge	0.00	.02	[-.05, .08]	.00	0.50	.771	0.24
Grade level × Number line estimation	0.01	.04	[-.02, .1]	.00	1.30	.446	0.26
Grade level × Basic geometry	0.00	.01	[-.05, .08]	.00	0.39	.771	0.16
Grade level × Approximate arithmetic	0.01	.04	[-.03, .1]	.00	1.09	.594	0.48
Grade level × Non-sym. mag. comp.	0.00	.00	[-.06, .05]	.00	-0.04	.966	0.49
Multiplication × Sex[a]	-0.03	-.06	[-.13, .01]	.00	-1.69	.249	0.29
G. cognitive ability × Sex[a]	-0.01	-.01	[-.08, .05]	.00	-0.44	.771	0.86
Subtraction × Sex[a]	-0.01	-.03	[-.12, .05]	.00	-0.75	.708	0.46
Addition × Sex[a]	0.04	.08	[.00, .16]	.00	1.98	.159	0.22
Conceptual knowledge × Sex[a]	0.00	.00	[-.07, .06]	.00	-0.10	.951	0.05
Number line estimation × Sex[a]	0.01	.02	[-.04, .07]	.00	0.55	.771	0.77
Basic Geometry × Sex[a]	0.00	-.02	[-.08, .04]	.00	-0.72	.708	0.10
Approximate arithmetic × Sex[a]	-0.02	-.05	[-.12, .02]	.00	-1.53	.316	0.10
Non-sym. mag. comp. × Sex[a]	-0.02	-.03	[-.08, .03]	.00	-0.90	.646	0.06

	B	β	[*L-CI*,*U-CI*]	*RW*	*t*	*p*	*RS-RW* (%)

Note: *B*: unstandardized regression weight; *β:* standardized regression weight; *L-CI:* lower boundary (2.5%); *U-CI:* upper boundary (97.5%); *RW:* raw relative weight (within rounding error raw weights will sum to R^2); *t:* t-value measures the size of the effect relative to the variation in sample data; *p:* p-value; *RS-RW:* relative weight rescaled as a percentage of predicted variance in the criterion variable attributed to each predictor (within rounding error rescaled weights sum to 100 %). [a] code female = -1, male = 1. [*] significantly different from a random variable.

7. Study 3: Strategies for Comparing Negative Fractions[11]

Silke M. Wortha[1,2,3], Korbinian Moeller[2,4,5,6], Martin Keuler[7], Hans-Christoph Nuerk[2,7], & Julia Bahnmueller[2,5]

[1] Department of Neurology, University Medicine Greifswald, Greifswald, Germany

[2] LEAD Graduate School & Research Network, University of Tuebingen, Tuebingen, Germany

[3] Department of Psychiatry and Psychotherapy, University of Tuebingen, Tuebingen, Germany

[4] Leibniz-Institut für Wissensmedien, Tuebingen, Germany

[5] Centre for Mathematical Cognition, Loughborough University, United Kingdom

[6] Individual Development and Adaptive Education Center, Frankfurt am Main, Germany

[7] Department of Psychology, University of Tuebingen, Germany

[11] Please Note. This is an unpublished manuscript and therefore might not exactly replicate the final published version.

Abstract

Aspects of processing fractions and negative numbers were considered separately so far. However, the question remains open whether theoretical mechanisms postulated in multidigit number processing models for negative numbers and fractions (e.g., sign flip, whole number bias) also apply for the processing of negative fractions. A fraction magnitude comparison task was used to investigate fraction processing with differing polarities (i.e., positive (+ +) vs. negative (- -) vs. mixed polarity (+ - / - +)) and containing one common component (numerator vs. denominator). Additionally, eye-tracking data was recorded to further substantiate behavioral results.

Results indicated that, heterogeneous pairs (+-) were solved faster and more accurately than homogeneous pairs (++ and --) replicating the previously observed sign-shortcut strategy for natural numbers. Moreover, positive homogeneous fraction pairs were responded to faster than negative homogeneous pairs indicating added response costs of the sign flip.

Additionally, negative fraction pairs with common denominators were processed faster than fraction pairs with common numerators indicating added response costs of the denominator flip. Finally, processing costs were largest for negative homogeneous fraction pairs with common numerators indicating that both the denominator flip and the sign flip uniquely contribute to the complexity of negative fraction comparison. Interestingly, a small subgroup of participants managed to solve this complex comparison faster by applying a different and unexpected strategy. Results of eye-tracking data partially supported behavioral findings. Thus, results suggest that the denominator and sign flip mechanism postulated for positive fractions and negative integer numbers, respectively, generalize to negative fractions.

Keywords: fractions, negative numbers, negative fractions, strategies, eye-tracking

7.1 Introduction

Despite the recent increased interest in negative numbers and fractions, research on both topics is still relatively sparse compared to whole numbers. Crucially, however, research on both negative numbers and fractions has repeatedly shown that children seem to have problems in understanding the concept of the two number types compared to whole numbers (Bofferding, 2014; Bright, Behr, Post, & Wachsmuth, 1988; Fuson, 2012; Hartnett & Gelman, 1998; Smith, Solomon, & Carey, 2005; Vamvakoussi & Vosniadou, 2004, 2010; Young & Booth, 2015). In both cases, misconceptions seem to partially result from children's reliance on strategies and procedures learned and successfully applied for whole numbers. Such reliance on prior knowledge seems sensible as whole numbers represent critical elements based on which concepts of other numbers can be deduced and constructed (e.g., framework theory of conceptual change; Vosniadou, 1994, 2007).

Children first encounter, learn to manipulate, and understand whole numbers during their mathematical development and are only later introduced to negative numbers and fractions[12]. Additionally, their knowledge of the latter number sets is acquired by continuously expanding previously obtained number sets. Starting from natural numbers ($\mathbb{N}$; e.g., positive whole numbers) followed by integers ($\mathbb{Z}$; e.g., positive and negative whole numbers) and finally rational numbers ($\mathbb{Q}$; e.g., integers, fractions and decimals). Reflecting children's efforts to make sense of these new number sets, it seems obvious and plausible to apply knowledge of previously learned whole numbers to integers (i.e., negative numbers) and rational numbers (i.e., fractions; Bofferding, 2014; Vamvakoussi & Vosniadou, 2004, 2010).

On a conceptual level, negative numbers and fractions seem to have specific characteristics that make it especially difficult to apply knowledge about whole numbers. Both number types reflect abstract concepts that are difficult to grasp on a concrete level, which is not the case for whole numbers (Thompson & Saldanha, 2003; Varma & Schwartz, 2011). Additionally, both number types are characterized by an inverse relationship between their overall magnitude and the magnitude of their components. In the case of negative numbers, the overall magnitude of a negative number (e.g., -1 vs. -9) decreases as the magnitude of the integer increases (e.g., 1

[12] For an overview of the general curricula in mathematics in different countries, please see: http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/

and 9). For fractions, the larger the magnitude of the denominator, the smaller the fraction's overall magnitude (e.g., 1/2 vs. 1/9). This is in clear contrast to previously acquired whole number knowledge, which reflects that larger digits represent larger magnitudes (e.g., 1 vs. 9). These inverse relationships cause many problems for developing conceptual understanding of negative numbers (e.g., misconceptions while solving algebraic equations or solving subtractions with negative numbers; Bofferding & Wessman-Enzinger, 2017; Fagnant, Vlassis, & Crahay, 2005) as well as fractions (e.g., natural number bias; Alibali & Sidney, 2015; Ni & Zhou, 2005).

Furthermore, in both cases, the specific form of presentation (i.e., polarity sign for negative numbers and a quotient format for fractions) can lead to additional challenges. Regarding negative numbers, the polarity sign may lead to misconceptions as it carries at least three meanings: i) it can be interpreted as a subtraction sign (Vlassis, 2002), ii) as the sign for negativity corresponding to negative numbers (Vlassis, 2004), and iii) as a reflection of the opposite meaning for any given number x on a number line (Bofferding, 2019; i.e., -3 is the opposite of 3). In a similar vein, fractions can be interpreted in many ways. The three most common interpretations are quotients (i.e., the quotient interpretation of $\frac{3}{4}$ is 0.75), part-wholes (i.e., $\frac{3}{4}$ represents 3 parts out of 4), and operators (i.e., the fraction acts as a function $\frac{3}{4}x$). Thus, fractions take on different meanings depending on the context in which they are interpreted (e.g., meaning of a division in the quotient interpretation; Behr, Lesh, Post, & Silver, 1983; Kieren, 1993).

Finally, for negative numbers and fractions, the literature provides inconsistent evidence regarding their mental representation. For negative numbers, there is an ongoing debate on whether they are processed holistically (i.e., one integrated representation for polarity sign and integer) or componentially (i.e., separate representations for polarity sign and integer). Previous studies suggested that negative number processing depends on the way numbers are presented in the task at hand (e.g., Fischer, 2003; Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010; Ganor-Stern & Tzelgov, 2008; Shaki & Petrusic, 2005; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009). For instance, Ganor-Stern et al. (2010) showed that participants processed mixed-polarity pairs (e.g., + 6 and -7) during a magnitude comparison task holistically when the numbers were presented sequentially. However, participants relied on a componential representation when numbers were presented simultaneously.

Analog to negative numbers, there is evidence that fractions are either processed in a holistic (i.e., as an integrated entity, e.g., 1/2 = 0.5; see Ischebeck et al., 2009; Jacob & Nieder, 2009) or componential way (i.e., separate processing of numerator and denominator, e.g., Bonato, Fabbri, Umilta, & Zorzi, 2007; Huber, Moeller, & Nuerk, 2014; Kallai & Tzelgov, 2009). Again, the way fractions are processed seems to depend heavily on stimulus characteristics, type of task as well as on specific processing strategies (Faulkenberry & Pierce, 2011; Huber et al., 2014; Ischebeck, Weilharter, & Körner, 2016; Obersteiner & Tumpek, 2016; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Siegler, Thompson, & Schneider, 2011).

Critically, for both negative numbers and fractions, knowledge and correct application of strategies and procedures is essential for solving problems accurately (e.g., Bofferding, 2019; Booth, Newton, & Twiss-Garrity, 2014). For instance, Krajcsi & Igács (2010) observed that negative number processing relies on a strategy referred to as *sign shortcut* (see Figure 7.1 A). In their study, participants had to compare magnitudes of target numbers from -9 to +9 (except -5, 0, and +5) with reference numbers -5 and +5. Krajcsi & Igács (2010) found that pairs of numbers with different polarity signs (i.e., heterogeneous pairs + –/– +) were compared significantly faster than pairs with identical polarity signs regardless of the polarity sign of the reference number and numerical distance of the pairs. Based on this, the authors concluded that processing the magnitude of the numbers seems obsolete for pairs with different polarity signs because the correct decision can be based exclusively on the polarity signs (see also Tzelgov et al., 2009).

In addition to the sign shortcut strategy, the metaphor of a mirror mechanism was introduced for negative numbers (e.g., Krajcsi & Igács, 2010). For negative numbers, the mirror mechanism describes a two-step strategy by which the integers are first compared without considering the polarity sign. In the second step, the result of the comparison is then inverted. The mirror mechanism strategy can also be applied to fractions. In case of fractions with common numerators, the denominators are compared first, and afterward, the result is inverted. To avoid misunderstandings, we will refer to the mirror mechanism for negative numbers as sign flip and the mirror mechanism for fractions as denominator flip (see Figure 7 for an illustration).

Taken together, understanding negative numbers and fractions represents two important milestones during mathematical development of children and adolescent. Importantly, the correct application of specific strategies may make dealing with negative numbers and fractions more manageable.

The present study

So far, cognitive representations, strategies, and difficulties in processing negative numbers and fractions were investigated separately (e.g., Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2011; Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010; Huber, Moeller, & Nuerk, 2014; Ischebeck, Schocke, & Delazer, 2009; Shaki & Petrusic, 2005; Sprute & Temple, 2011; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009). The present study aimed at investigating the combination of these two types of numbers (i.e., negative fractions) for the first time.

We aimed to investigate cognitive processes underlying magnitude comparisons of fractions ranging from -1 to +1. To this end, four strategies were of particular interest for the current study: i) Sign-shortcut (strategy 1)**:** numbers with different polarity signs (e.g., + $\frac{1}{3}$ and - $\frac{2}{3}$) are compared by specifically considering polarity signs (see also Figure 7.1 A), ii) Sign flip (strategy 2): negative numbers (i.e., - $\frac{1}{3}$ and - $\frac{2}{3}$) are compared by only considering the absolute values and reversing the answer (see also Figure 7.1 B), iii) Denominator flip (strategy 3): positive fractions with common numerators (i.e., + $\frac{1}{3}$ and + $\frac{1}{7}$) are compared by specifically considering denominators and reversing the answer (see also Figure 7.1 C). Additionally, we were also interested in the combination of the sign and denominator flip strategies: negative fractions with common numerators (i.e., -$\frac{1}{3}$ and - $\frac{1}{7}$) are compared by executing the sign and the denominator flip consecutively (irrespective of order; see also Figure 7.1 D).

Expanding previous research findings, we aimed to evaluate whether a) strategies for negative number processing (i.e., sign shortcut and sign flip mechanism; Krajcsi & Igács, 2010) generalize to negative fraction processing.

Furthermore, we aimed at b) replicating the effect of a denominator flip for negative fractions (see Huber et al., 2014; Meert, Grégoire, & Noël, 2009).

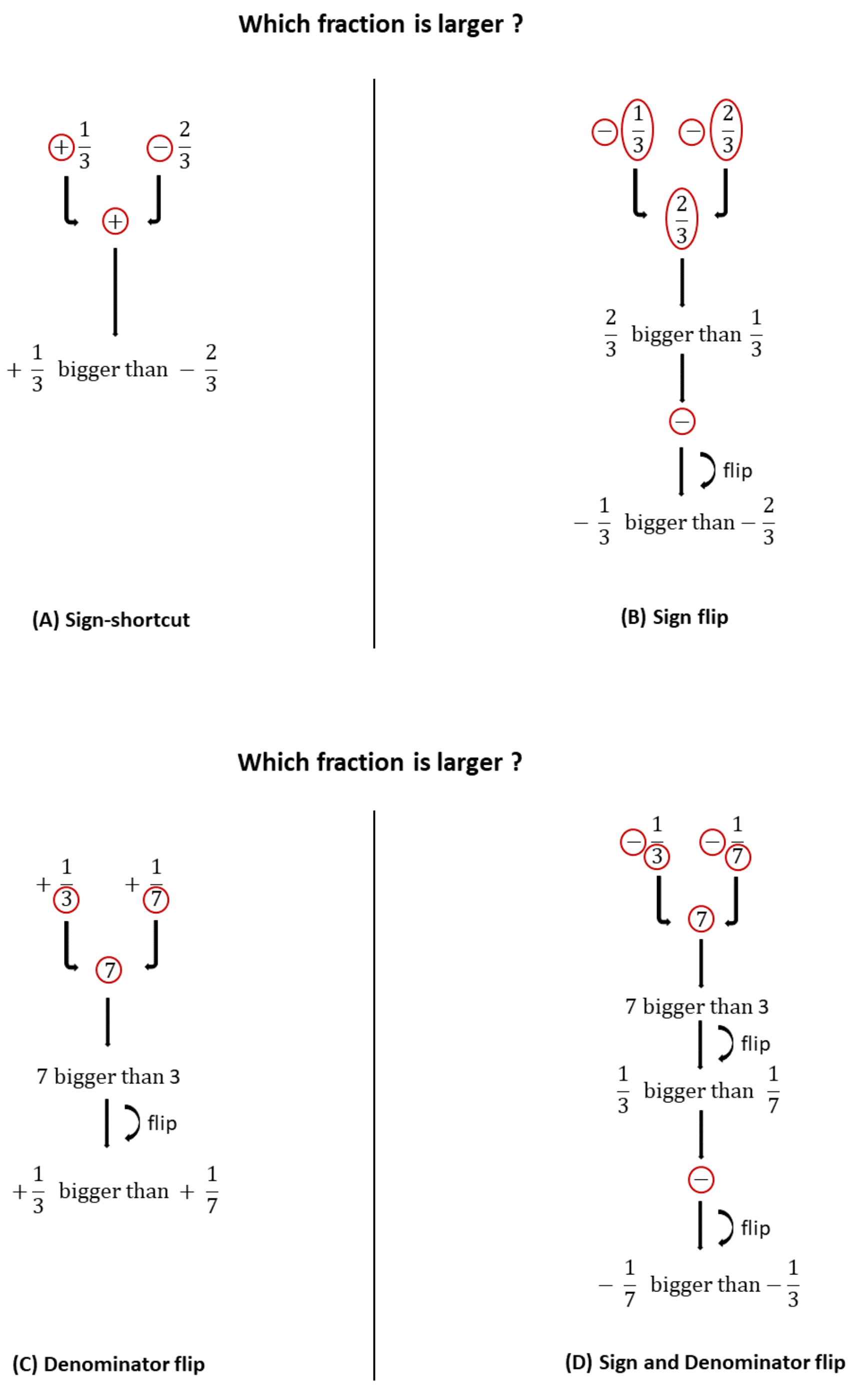


Figure 7.1: The different strategies of interest during the fraction magnitude comparison task. (A) Sign-shortcut: only the polarity signs are considered during the comparison. (B) Sign flip: The overall fraction magnitude is first considered. Then the answer is reversed. (C) Denominator flip: only the magnitudes of the denominators are compared. Then the answer is reversed. (D) Sign and Denominator flip: Both strategies are conducted one after the other.

For this purpose, we used a 2 (denominator- vs. numerator-relevant) x 2 (homogeneous vs. heterogeneous) x 2 (blocked vs. mixed presentation of homogeneous and hetereogeneous pairs) design. Fraction pairs used in the present study only differed concerning one component and thus either shared the numerator (i.e., the denominator is relevant) or the denominator (i.e., the numerator is relevant). We additionally manipulated the polarity of the fraction pairs: they consisted either of two positive fractions (i.e., homogenous pairs), two negative fractions (i.e., homogenous pairs), or a pair of fractions with one positive and one negative fraction (i.e., heterogenous pairs). Finally, the presentation format of the fraction comparison task was manipulated. The experiment consisted of a blocked (i.e., only one type of fraction comparison) and a mixed condition (i.e., different types of fraction comparisons) to account for predictability of the decision-relevant components of each type of fraction comparison. In addition to the speed and accuracy of responses, participant's eye movements were recorded.

Regarding the respective strategies used, we tested the following hypotheses with specific contrasts:

1. In line with the findings of Krajcsi & Igács (2010), we expected participants to use the sign-shortcut strategy resulting in significantly faster processing of heterogeneous fraction pairs (i.e., + – / – +) than homogeneous fraction pairs (i.e., – – or ++). Moreover, further evidence for componential processing should be found in participants' eye movements. Participants should spend proportionally more time fixating the polarity signs than on the other areas of interest (AOIs; i.e., the numerator and denominator).

2. In line with the sign flip strategy (see Krajcsi & Igács, 2010), we expected participants to be significantly faster when confronted with positive homogeneous numerator-relevant fraction pairs (++) than negative homogenous numerator-relevant fraction pairs (– –), because the sign flip strategy predicts additional costs for homogenous negative fraction pairs. Moreover, reflecting componential processing strategies, these additional costs should be reflected in proportionally more time spent on the numerators when comparing negative fraction pairs than positive fraction pairs.

3. Replicating the findings of Huber et al. (2014) and Meert et al. (2009), we expected to find the denominator flip strategy in positive fraction pairs: participants

should be significantly faster when processing positive homogenous numerator-relevant fraction pairs (++) than positive homogenous denominator-relevant fraction pairs, reflecting additional flipping costs for denominator-relevant fraction pairs. Furthermore, these additional costs should be reflected by proportionally more time spent fixating on the denominator for positive homogenous denominator-relevant fraction pairs compared to processing positive homogenous numerator-relevant fraction pairs.

4. Extending the findings of Huber et al. (2014) and Meert et al. (2009), we expected to find a combination of the sign- and denominator flip in negative fraction pairs (i.e., in trails were negative homogeneous denominator-relevant fraction pairs were compared). There are two ways to approach this analysis: i) comparing negative and positive denominator-relevant fraction pairs or ii) comparing negative numerator- and denominator-relevant fraction pairs. In particular, participants should need significantly more time processing negative denominator-relevant fraction pairs than positive denominator-relevant and negative numerator-relevant fraction pairs. Moreover, irrespective of processing order (i.e., polarity sing or fraction component) we expected participants to spend proportionally more reading time on the polarity sign and the denominators than the numerator, respectively.

5. Finally, the presentation format (i.e., blocked vs. mixed) of the fraction pairs should affect participants' performance. In particular, reaction times should be higher in mixed conditions as compared to blocked conditions because of additional switching costs during mixed conditions (Huber et al., 2014).

Table 7.1: Overview of the different processing strategies

strategy	relevant polarities in analysis	relevant fraction components in analysis	additional costs expected for	most time spend on
Sign-Shortcut	+ – / – + ++ / – –	numerator and denominator	++ / – – numerator- and denominator-relevant	polarity sign
Sign flip	++ / – –	numerator	– – numerator-relevant	numerator

Denominator flip	++	numerator and denominator	++ denominator-relevant	denominator
Sign & Denominator flip	++ / – –	denominator	– – denominator-relevant	denominator
	– –	numerator and denominator		denominator

7.2 Methods

Participants

Thirty volunteers from the University of Tübingen (24 female, 6 male) ranging in age from 18 to 51 years (*M* = 23.4 years, *SD* = 6.9 years) participated in the experiment. All participants reported to be right-handed and had normal or corrected to normal vision. Informed consent was obtained from all participants.

Stimuli

Stimuli consisted of 200 different fractions pairs with numerators comprising the numbers 1, 2, 3, 4, 5, and 7, and denominators comprising the numbers 3, 4, 5, 6, 7, 8, and 9. All generated fractions were proper fractions (i.e., smaller than 1) and could not be shortened. Polarity signs were presented for both negative and positive fractions. Furthermore, pairs always had one component (numerator or denominator, see Table 7.1) in common.

As illustrated in Table 7.1, there were six different types of fraction pairs manipulated according to *polarity* (two positive fractions [++] vs. two negative fractions [– –] vs. one positive and one negative fraction [+ –/– +]) and *relevant component* (numerator-relevant vs. denominator-relevant). For both numerator-relevant and denominator-relevant fraction pairs, respectively, there were 25 positive homogeneous (++) and 25 negative homogeneous fraction pairs (– –). Positive and negative homogeneous fraction pairs only differed from each other with respect to the polarity sign (i.e., the absolute numerical values were the same). For homogeneous fraction pairs, the position of the larger fraction (left vs. right) was counterbalanced. To generate heterogeneous fraction pairs, the polarity sign of the first fraction of the homogeneous fraction pairs was inverted. Thus, heterogeneous fraction types contained 50 stimuli with the positive fraction presented first in 25 fraction pairs, and the negative fraction presented first in another 25 pairs.

Fraction pairs were matched for numerator-relevant and denominator-relevant pairs with respect to absolute numerical distance between whole fractions, numerical distance between corresponding components, problem size of the whole fraction (i.e., (fraction 1 + fraction 2)/ 2) and problem size of the components [i.e., (numerator 1 + numerator 2)/ 2 or (denominator 1 + denominator 2)/ 2]. Some matching parameters for heterogeneous fraction pairs necessarily deviate from parameters of homogeneous fraction pairs: Overall distance is necessarily larger for heterogeneous pairs because the distance is larger from a positive to a negative fraction (or vice versa) than from a positive to another positive or a negative to another negative fraction. Additionally, overall problem size is necessarily smaller for heterogeneous pairs as one fraction is negative and therefore smaller. Furthermore, we assigned the polarity sign to the numerator for computing distances and problem sizes. Therefore, problem size of numerators of heterogeneous pairs is smaller than numerators of homogeneous pairs (see Table S7.1 for all stimuli; see Table 7.2 for all matching parameters).

Table 7.2: Matching parameters for the six stimulus categories.

example	polarity	relevant component	distance fraction	distance num	distance denom	PS fraction	PS num	PS denom
$+\frac{1}{3}$ $+\frac{2}{3}$	++	Num	0.15	0.00	2.60	0.40	2.72	7.06
$+\frac{1}{3}$ $+\frac{2}{3}$	++	Denom	0.37	2.56	0.00	0.40	2.68	6.92
$-\frac{1}{3}$ $-\frac{2}{3}$	– –	Num	0.15	0.00	2.60	-0.40	-2.72	7.06
$-\frac{1}{3}$ $-\frac{1}{7}$	– –	Denom	0.37	2.56	0.00	-0.40	-2.68	6.92
$+\frac{1}{3}$ $-\frac{2}{3}$	+ –/– +	Num	0.15	0.00	2.60	0.00	0.00	7.06
$+\frac{1}{5}$ $-\frac{1}{3}$	+ –/– +	Denom	0.37	2.56	0.00	0.00	0.00	6.92

Note: The negative algebraic sign was assigned to the numerator to calculate distances and problem sizes for heterogeneous number pairs (+ –). PS = problem size; num = numerator; denom = denominator.

Eye-Tracking

Eye movements were recorded using an Eye Link 1000 Eye Tracker (SR-Research, Kanata, Ontario, Canada). Employing a 9-point calibration at the beginning of the experiment and drift corrections before each trial, a spatial resolution of less than 0.5° visual angle was possible with this system.

Procedure

Stimuli and instructions were presented on a 19“ monitor (resolution: 1024 x 768 pixels, frame rate: 120 Hz). Viewing distance was approximately 60 cm. Stimuli were presented in white against a black background. The non-proportional font *Courier New* (size 48, bold) was used to ensure that all numbers had the same width. Fraction pairs were presented horizontally next to each other. The center of the left fraction was presented at x/y-coordinates 512/384 and the corresponding polarity sign at x/y-coordinates 458/384. The center of the right fraction was presented at x/y-coordinates 768/384 and the corresponding polarity sign at 714/384. Each trial was preceded by a fixation cross presented for 500 ms at the center of the screen.

The experiment was conducted in a single session in a quiet and dimmed room. Written instructions to identify the numerically larger fraction were presented on the monitor. To indicate that the right fraction was the larger one, participants had to press the right trigger button of a game controller with the right index finger and vice versa for the left fraction being the larger one. Participants were instructed to react as fast as possible while avoiding errors. Pairs of fractions were presented until a response was recorded. No feedback about the correctness of the response was given.

The experiment started with six independent practice trials (one trial per stimulus condition). All experimental stimuli were presented twice as we manipulated the presentation format (blocked vs. mixed). In the blocked condition, the six different types of fraction pairs were presented in six separate blocks. Stimulus order within these blocks was randomized. In the mixed condition, stimuli of all six stimulus conditions were presented in randomized order. Every 25 trials, participants had the chance to take a break irrespective of presentation format.

Order of presentation format and stimulus categories were pseudo-randomized across participants. Half of the participants started with the blocked, the other half with the mixed presentation format. Within the blocked condition, half of the participants started with the numerator-relevant trials; the other half started with the denominator-relevant trials. Finally, the order of polarity (++ vs. – – vs. +–/–+) was counterbalanced across participants. The experiment took approximately 45 minutes.

Analysis

Behavioral analysis

Analyses of reaction times (RT), error rates (ER), and eye movements were performed using R (R Development Core Team, 2019). To directly test our hypotheses, we fitted five specific linear mixed-effects models using ‘lmer’ from the “lme4” R package (Bates, Mächler, Bolker, & Walker, 2014) for RT and eye-tracking data, respectively.

For analyzing the eye-tracking data, the proportion of reading time in the relevant area of interest (AOI; i.e., *signs*, *numerators,* or *denominators*) compared to all relevant AOIs (i.e., *signs*, *numerators,* and *denominators*) for only correctly solved items was considered. Reading time was defined as the total amount of time a participant fixated an AOI. Interest areas around each digit and polarity sign were defined with a height of 100 pixels and width of 45 pixels for the digits, and a height of 75 pixels and width of 60 pixels for the polarity sign. As we were only interested in differences in the number of fixations for polarity signs, numerators, and denominators, we collapsed the data over both fractions of a pair, not differentiating between the left and right fraction.

Additionally, for ER data analyses, we conducted five generalized linear effect models (GLME) to address the four specific strategies, again applying the R package lme4 (Bates et al., 2015). ERs were analyzed using GLMEs with a binomial error distribution and logit as the link function. To increase readability, log odds are also given in percent error. To facilitate reading flow of the current paper and because results of RT and error rates largely overlapped, we report the results of the analyses of error rates in the supplementary material (S7.2).

For all conducted models, we used the ‘summary’ function of the “lmerTest” R package (Kuznetsova, Brockhoff, & Christensen, 2018) to provide *p*-values. Post hoc analyses were run using the R package lsmeans (Lenth & Lenth, 2018). To account for multiple testing, *p*-values were corrected using the false discovery rate (Benjamini & Hochberg, 1995). Additionally, for reaction times and eye-tracking analysis, summary statistics were extracted via the ‘analyze’ function of the “psycho” R package (Makowski, 2018).

RT data trimming

Only correctly solved trials were considered for analyses. Participants who committed more than 33% errors (50% was guessing rate) in one experimental condition were excluded from further analyses (this affected 6 participants). RTs smaller than 200ms were also excluded. After inspection, RTs were log-transformed to correct for their right-skewed distribution (as suggested by Ratcliff, 1993). Additionally, a model-based trimming procedure was applied to remove further outliers. In particular, a linear mixed-effects model on log-transformed RTs (logRT) and *presentation format* (blocked vs. mixed), *relevant component* (numerator-relevant vs. denominator-relevant), *polarity* (+ + vs. – – vs. + –/– +), as well as the respective two- and three-way interactions as fixed effects and a random effect for participants (random intercept as well as random slopes for presentation, component, polarity, and the respective interactions) was run. Data were trimmed by z-standardizing residuals of this linear mixed-effects model and excluding all logRTs with residuals deviating more than ±3 SD from the estimated mean (Baayen & Milin, 2010). Data trimming (i.e., reaction times < 200 ms and +/- 3 SDs) resulted in a loss of 0.08% of data. Results of analyses on reaction times are given in both logRT and plain RT to increase readability.

7.3 Results[13]

Strategy 1: Sign-shortcut

The sign-shortcut strategy reflects that heterogenous fraction pairs (+ –/– +) should be compared significantly faster than homogenous fraction pairs (++/– –). Thus, the vital role of the polarity sign for this strategy should also be reflected by proportionally longer times spend fixating the polarity signs than on other areas of interest. To investigate whether participants were using the sign-shortcut strategy, all fraction pairs (i.e., heterogenous and homogeneous and numerator- and denominator-relevant) were considered for analyses.

Reaction times

A linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed), *component* (numerator-relevant vs. denominator-relevant), and *polarity* (homogenous vs. heterogenous) and a random intercept to account for participants' individual differences was run on logRTs. The model explained a significant proportion of variance of logRT (R^2 = 81.58%; fixed effects: R^2 = 70.46%). Moreover, the fixed effects of presentation format [β = 0.67, *SE* = 0.07, $t(264)$ = 10.20, $p < .001$], component [β = - 0.22, *SE* = 0.07, $t(264)$ = - 3.32, $p < .01$], and polarity [β = - 0.67, *SE* = 0.07, $t(264)$ = - 10.14, $p < .001$] were significant. RTs were smaller in the blocked [*M* = 6.78 (883ms), *SE* = 0.04] as compared to the mixed condition (*M* = 7.34 (1540ms), *SE* = 0.04). Moreover, heterogeneous fraction pairs (*M* = 6.61 (745ms), *SE* = 0.05) were solved fastest, followed by positive homogeneous pairs (*M* = 7.22 (1365ms), *SE* = 0.05) and again followed by negative homogeneous pairs (*M* = 7.35 (1561ms), *SE* = 0.05). Pairwise post hoc comparisons indicated that RTs significantly differed between + – vs. – – ($t(275)$ = 22.02, $p < .001$) and + – vs. + + ($t(275)$ = - 18.03, $p < .001$) and + + vs. – – ($t(275)$ = 3.99, $p < .001$). Finally, numerator-relevant fractions pairs were compared faster (*M* = 6.98 (1071ms), *SE* = 0.04) than denominator-relevant fraction pairs (*M* = 7.15 (1269ms), *SE* = 0.04).

Furthermore, the two-way interaction of *presentation format* and *polarity* was significant [β = - 0.33, SE = 0.09, $t(264)$ = - 3.50, $p < .001$]. We further inspected the

[13] Please note that in a second analysis, we additionally included the holistic distance effect as variable of interest for each strategy separately. However, for the sake of readability and since this analysis revealed consistently no significant distance effect across all strategies the analyses are not reported in this manuscript.

interaction by running pairwise post hoc comparisons for the blocked and the mixed condition separately. We found that in the blocked condition RTs were significantly faster for heterogenous pairs (*M* = 6.44 (623ms), *SE* = 0.06) than for positive and negative homogenous pairs (*M* = 6.92 (1008ms), *SE* = 0.06 vs. *M* = 7.00 (1095ms), SE = 0.06). Polarity significantly differed between + – vs. – – ($t(118) = 10.64$, $p < .001$) and + – vs. + + ($t(118) = - 9.08$, $p < .001$). The comparison between ++ and – – was not significant ($t(118) = 1.56$, $p = .121$). For the mixed condition, RTs were again faster for heterogenous pairs (*M* = 6.79 (889ms), *SE* = 0.05) than for positive and negative homogenous pairs (*M* = 7.52 (1847ms), *SE* = 0.05 vs. *M* = 7.71 (2223ms), SE = 0.05). Polarity significantly differed between + – vs. – – ($t(118) = 23.10$, $p < .001$), + – vs. + + ($t(118) = - 18.42$, $p < .001$) and + + vs. – – ($t(118) = 4.67$, $p < .001$).

Finally, the two-way interaction *component* and *polarity* was significant [β = 0.21, *SE* = 0.09, $t(264) = 2.23$, $p < .05$]. We further inspected the interaction by running pairwise post hoc comparisons for denominator- and numerator-relevant fraction pairs separately. We found that for denominator-relevant fraction pairs RTs were faster for heterogenous pairs (*M* = 6.61 (745ms), *SE* = 0.06) than for positive and negative homogenous pairs (*M* = 7.38 (1606ms), *SE* = 0.06 vs. *M* = 7.44 (1709ms), *SE* = 0.06). Polarity significantly differed between + – vs. – – ($t(118) = 10.41$, $p < .001$) and + – vs. + + ($t(118) = - 9.63$, $p < .001$). The comparison between ++ and – – was not significant ($t(118) = 0.78$, $p = .435$). For numerator-relevant fraction pairs RTs were again lower for heterogenous pairs (*M* = 6.61 (744ms), *SE* = 0.06) than for positive and negative homogenous pairs (*M* = 7.06 (1160ms), *SE* = 0.06 vs. *M* = 7.26 (1425ms), *SE* = 0.06). Polarity significantly differed between + – vs. – – ($t(118) = 8.15$, $p < .001$), + – vs. + + ($t(118) = - 5.57$, $p < .001$) and + + vs. – – ($t(118) = 2.58$, $p < .05$).

Eye Tracking

A linear mixed effect model with *presentation format* (blocked vs. mixed), *component* (numerator-relevant vs. denominator-relevant), and *polarity* (++ vs. – – vs. + –/– +) as fixed factors and a random intercept to account for participants' individual differences was run on the relative total reading time on the AOI *polarity signs*. The model explained a significant proportion of variance (R^2 = 50.73%; fixed effects: R^2 = 22.90%). Moreover, the fixed effect of polarity was significant [β = 0.28, *SE* = 0.04, $t(264) = 6.97$, $p < .001$]. In heterogeneous fraction pairs the polarity sign was fixated relatively longer with 47.6% (*SE* = 2.6%) reading time compared to 31.6 % (*SE* = 2.6%) in positive homogenous pairs and 29.0% (*SE* = 2.6%) in negative homogenous pairs. Pairwise post hoc comparisons indicated that relative total reading time on the polarity sign significantly differed between + – vs. – – ($t(275) = -9.03$, $p < .001$) and + – vs. + + ($t(275) = 7.77$, $p < .001$). The comparison between ++ and – – was not significant ($t(275) = -1.26$, $p = .207$).

Additionally, the interaction of *presentation format* and *polarity* was significant [$\beta = -0.15$, *SE* = 0.06, $t(264) = -2.65$, $p < .01$]. We further inspected the interaction by running pairwise post hoc comparisons for the blocked and the mixed condition separately. We found that the reading time on the polarity sign in the blocked condition was higher for heterogenous pairs than for negative and positive homogenous pairs [51.5% (*SE* = 0.03%) vs. 27.2% (*SE* = 0.03%) vs. 26.1% (*SE* = 0.03%)]. More specifically, the reading time for the blocked condition significantly differed between + – vs. – – ($t(118) = -7.60$, $p < .001$) and + – vs. + + ($t(118) = 7.93$, $p < .001$). The comparison between ++ and – – was not significant ($t(118) = 0.34$, $p = .738$). Finally, for the mixed condition, we again found that reading time on the polarity sign was higher for heterogenous pairs than for positive and negative homogenous pairs [43.8% (*SE* = 0.03%) vs. 37.1% (*SE* = 0.03%) vs. 30.8% (*SE* = 0.03%)]. Moreover, reading time for the mixed condition significantly differed between + – vs. – – ($t(118) = -6.77$, $p < .001$), + – vs. + + ($t(118) = 3.49$, $p < .01$) and + + vs. – – ($t(118) = -3.28$, $p < .01$). Neither the fixed effects of presentation format nor component nor the interactions *presentation format* and *component* as well as *polarity* and *component* were significant [all $t \leq 1.89$, all $p_s > .05$].

Taken together, both RT and eye movement data provide strong evidence for a sign-shortcut strategy when comparing fractions. This applies to both blocked and mixed presentation of item categories.

Strategy 2: Sign flip

The sign flip strategy would be indicated when positive homogenous fraction pairs (++) were processed faster than negative homogenous fraction pairs (– –). The reasoning behind this assumption is that negative homogenous fraction pairs are processed in two steps: first by only comparing the magnitudes of the fractions independent of their polarity and second by then considering the negative sign. This two-step process should be reflected by increased reaction times in negative compared to positive homogenous fraction pairs as well as proportionally longer reading times spend on the numerator for negative fraction pairs than positive fraction pairs. To specifically investigate whether participants used the sign flip strategy, only homogenous numerator-relevant fraction pairs were considered for analyses.

Reaction times

A linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed) and *polarity* (i.e., homogenous pairs only ++ vs. – –) and a random intercept to account for participants' individual differences was run on logRTs. The model explained a significant proportion of variance in RT (R^2 = 71.75%; fixed effects: R^2 = 57.23%). The fixed effect of presentation format [β = - 0.37, *SE* = 0.04, $t(72)$ = - 10.37, $p < .001$] was significant. RTs were smaller in the blocked (*M* = 6.82 (920ms), *SE* = 0.05) as compared to the mixed condition (*M* = 7.49 (1797ms), *SE* = 0.05). Additionally, the fixed effect of polarity was significant [β = - 0.21, *SE* = 0.05, $t(72)$ = - 4.05, $p < .001$]. Participants answered faster when comparing positive homogenous pairs (*M* = 7.06 (1160ms), *SE* = 0.05) than negative homogenous pairs (*M* = 7.26 (1425ms), *SE* = 0.05). The interaction of presentation format and polarity was not significant [$t(72) = 1.48$, $p > .05$].

Eye Tracking

A linear mixed effect model with the fixed effects *presentation format* (blocked vs. mixed), and *polarity* (++ vs. – –) was run on the proportion of total reading time on the AOI *numerator.* The model explained a significant proportion of variance (R^2 = 52.77%; fixed effects: R^2 = 13.25%). Only the fixed effect of presentation format was significant [β = - 0.14, *SE* = 0.05, $t(72)$ = - 3.01, $p < .01$] with 46.1% (*SE* = 3.8%) of reading time on numerators in blocked trials vs. 30.3 % (*SE* = 3.8%) in mixed trials. Neither the fixed effect of polarity nor the interaction of presentation format and polarity was significant [all $t \leq - 0.57$, all $p_s > .05$].

To investigate whether the sign flip strategy might influence reading time on the polarity sign, a second linear mixed effect model with the same fixed and random factors was run on the relative total reading time on the AOIs *polarity signs* and showed no significant effects [all $t \leq 1.29$, all $p_s > .05$].

Taken together, only RT data provided evidence for a sign flip strategy when comparing fractions whereas our hypothesis for eye tracking data was not supported by the data.

Strategy 3: Denominator flip

When participants use the denominator flip strategy, they compare denominators of fraction pairs in two steps. First, the magnitudes of the denominators are compared. Subsequently, the answer is reversed due to the inverse relationship between the overall fraction magnitude and the magnitude of the denominator (e.g., denominators with greater magnitudes are the fractions with overall smaller magnitude: $\frac{1}{3}$ vs. $\frac{1}{7}$). Thus, reaction times are expected to be larger when comparing denominator-relevant fraction pairs compared to numerator-relevant fraction pairs. Additionally, participants should spend proportionally more reading time on the denominator than on other AOIs. To evaluate whether participants used the denominator flip strategy, only positive homogenous numerator- and denominator-relevant fraction pairs were considered for analyses.

Reaction times

A linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed) and *component* (numerator-relevant vs. denominator-relevant) and a random intercept to account for participants' individual differences was run to analyze logRTs. The model explained a significant proportion of variance in RT (R^2 = 76.92%; fixed effects: R^2 = 61.35%). Moreover, the fixed effects of *presentation format* [β = 0.62, *SE* = 0.06, *t*(72) = 10.08, *p* < .001] and *component* [β = - 0.31, *SE* = 0.06, *t*(72) = - 5.14, *p* < .001] was significant. RTs were faster in blocked (*M* = 6.92 (1008ms), *SE* = 0.05) as compared to mixed condition (*M* = 7.52 (1847ms), *SE* = 0.05). Additionally, numerator-relevant pairs were solved faster (*M* = 7.06 (1160ms), *SE* = 0.05) than denominator-relevant pairs (*M* = 7.38 (1606ms), *SE* = 0.05). The two-way interaction presentation format and component was not significant [*t*(72) = - 0.25, *p* > .05].

Eye Tracking

A linear mixed effect model with *presentation format* (blocked vs. mixed) and *component* (numerator-relevant vs. denominator-relevant) as fixed factors and a random intercept to account for participants' individual differences was run on the relative total reading time on the AOIs *denominators*. The model explained a significant proportion of variance (R^2 = 59.70%; fixed effects: R^2 = 31.95%). The fixed effect of presentation format [β = 0.62, *SE* = 0.06, $t(72)$ = 10.08, p < .001] was significant, indicating more (44.3%, *SE* = 3.8%) reading time on denominators in blocked as compared to mixed presentation (38.1%, *SE* = 3.8%). Additionally, the fixed effect of component was significant [β = 0.62, *SE* = 0.06, $t(72)$ = 10.08, p < .001]. This indicated more (53%, *SE* = 3.8%) reading time on denominators in denominator-relevant trials as compared to numerator-relevant trials (29.4%, *SE* = 3.8%). Furthermore, the interaction of *presentation format* and *component* was significant [β = 0.62, *SE* = 0.06, $t(72)$ = 10.08, p < .001]. We further inspected the interaction by running pairwise post hoc comparisons among conditions. We found that reading time for numerator-relevant fraction pairs during the mixed condition was significantly higher than during the blocked condition [34.7% (*SE* = 0.05%) vs. 24% (*SE* = 0.05%), p < .05]. Whereas for denominator-relevant fraction pairs reading time was significantly higher for blocked conditions than mixed condition [64.5% (*SE* = 0.05%) vs. 41.5% (*SE* = 0.05%), p < .001]. Therefore, the fixed effects of presentation format and component should not be interpreted.

Taken together, both RT and eye movement data provided clear evidence for a denominator flip strategy when comparing negative fractions. This held true for both blocked and mixed presentation format.

Strategy 4: Combination of sign and denominator flip

We ran two separate analysis to investigate whether participants were using a combination of the sign and denominator flip during the most challenging fraction comparisons (i.e., negative homogenous denominator-relevant fraction pairs). First, only homogenous (++ and – –) denominator-relevant fraction pairs were considered to isolate the sign flip strategy. Second, only negative homogenous denominator- and numerator-relevant fraction pairs were considered to isolate the denominator flip strategy. Participants RTs should be the longest when comparing negative denominator-relevant fraction pairs in case both strategies were used consecutively (see Table S7.2 for mean RTs for all conditions). Moreover, irrespective of processing order (i.e., polarity sing or fraction component) we expected participants to spend proportionally more reading time on the denominators than the other AOIs when comparing homogeneous denominator-relevant fraction pairs and more time on the polarity sign when comparing negative homogenous fraction pairs.

Reaction times

A first linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed) and *polarity* (++ vs. – –) and a random intercept to account for participants' individual differences was run on homogeneous denominator-relevant fraction pairs. The model explained a significant proportion of variance in RT (R^2 = 68.37%; fixed effects: R^2 = 47.50%). The fixed effect of presentation format [β = 0.67, *SE* = 0.08, $t(72) = 8.76$, $p < .001$] was significant. RTs were shorter in the blocked (*M* = 7.09 (1201ms), *SE* = 0.06) compared to the mixed condition (*M* = 7.73 (2286ms), *SE* = 0.06). Neither the fixed effect of polarity nor the interaction of presentation and polarity was significant [all $t \leq -0.46$, all $p_s > .05$].

Second, a linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed) and *component* (i.e., numerator- or denominator-relevant) and a random intercept to account for participants' individual differences was run on negative fraction pairs. The model explained a significant proportion of variance in RT (R^2 = 64.58%; fixed effects: R^2 = 49.97%). Both fixed effects of presentation format [β = 0.67, *SE* = 0.09, $t(72) = 7.51$, $p < .001$] and component [β = - 0.22, *SE* = 0.09, $t(72) = -2.45$, $p < .05$] were significant. RTs were significantly smaller in the blocked (*M* = 7.00ms (1095ms), *SE* = 0.06ms) compared to the mixed condition (*M* = 7.71ms (2223ms), *SE* = 0.06ms). Additionally, numerator-relevant pairs were solved significantly faster (*M* = 7.26ms (1425ms), *SE* = 0.06) than denominator-relevant pairs (*M* = 7.44ms (1709ms), *SE* = 0.06). The interaction of presentation and component was not significant [$t(72) = 0.58$, $p > .05$].

Eye Tracking

Similar to the reaction time analysis outlined above, two separate linear mixed effect models were conducted. First, a linear mixed effect model with the fixed factors *presentation format* and *polarity* and a random intercept to account for participants' individual differences was run on the proportion of total reading time on the AOI *denominators*. The model explained a significant proportion of variance (R^2 = 47.26%; fixed effects: R^2 = 14.84%). Only the fixed effect of presentation format was significant [β = - 0.14, *SE* = 0.05, $t(72) = -2.66$, $p < .01$], indicating more (62%, *SE* =

4%) reading time on denominators in blocked as compared to mixed presentation (43.6%, *SE* = 4%). The fixed effect of polarity and the interaction presentation format and polarity were not significant [all $t \leq 0.98$, all $p_s > .05$].

The second linear mixed effect model discerned the factors *presentation format* and *component* on the proportion of total reading time on the AOI *polarity signs*. The model explained a significant proportion of variance (R^2 = 50.87%; fixed effects: R^2 = 3.61%). The fixed effects of presentation format [β = 0.07, SE = 0.03, $t(72) = 2.11$, $p < .05$] and component were significant [β = 0.08, SE = 0.03, $t(72) = 2.20$, $p < .05$]. Participants spend more (30.8%, SE = 3%) reading time on the sign in mixed as compared to blocked presentation (27.2%, *SE* = 3%). Moreover, participants spend significantly more (30.9%, *SE* = 3%) reading time on the sign in numerator-relevant as compared to denominator-relevant trials (27%, *SE* = 3%). The interaction presentation format and component was not significant [$t(72) = -1.52$, $p > .05$].

Additional Exploratory Analysis

One notable discovery was that for the most complex condition (i.e., – – denominator-relevant), a subgroup of six participants managed to solve these trials roughly as fast as the other less complex conditions. In an exploratory, descriptive analysis, we examined the proportion of reading times on all relevant AOIs to identify specific, unpredicted strategies used in this subsample. Comparisons of mean values suggested that this subgroup of participants spent larger proportions of reading time inspecting denominators (*M* = 71.6%, *SE* = 6.3% vs. 54.5%, *SE* = 4.4%) and lesser proportions of reading time inspecting the polarity signs (*M* = 18.1% vs. *M* = 27.6%, *SE* = 3.9%). This exploratory analysis revealed a difference in strategy use between these participants and the rest of the sample suggesting that these participants might have applied a strategy that we did not consider beforehand.

For the two final analyses, we excluded these six participants which seemed to have used a different strategy to investigate whether there is evidence for the use of the sign and denominator flip strategy in the rest of the sample. First, to investigate the sign-flip strategy in homogeneous denominator-relevant fraction pairs, we re-ran the linear mixed effect model with the fixed factors *presentation format* (blocked vs. mixed) and *polarity* (++ vs. – –) and a random intercept to account for participants' individual differences. And indeed, results showed a significant fixed effect of polarity, indicating increased reaction times for negative homogenous denominator-relevant fraction pairs than positive denominator-relevant fraction pairs (*M* = 7.49 (1798ms), *SE* = 0.05 vs. *M* = 7.39 (1612ms), *SE* = 0.05).

In the second analysis we again investigated the use of the denominator flip strategy in the remaining 18 participants and found a significant fixed effect of component, indicating increased reaction times for negative denominator-relevant fraction pairs than positive denominator-relevant fraction pairs (*M* = 7.49 (1797ms), *SE* = 0.04 vs. *M* = 7.31 (1489ms), *SE* = 0.04).

Taken together, analyzing the combined use of the sign and denominator flip strategy yielded unexpected findings: a first analysis with all participants did not confirm a combined use of these strategies for negative denominator-relevant fraction pairs. However, two additional exploratory analysis on eye tracking and reaction time data revealed not only that one subgroup of participants used a different strategy, but also that the rest of the participants seemed to have used a combination of the sign and denominator flip strategy while solving negative homogenous fraction comparisons.

7.4 Discussion

The aim of the present study was to investigate strategies applied when processing negative fractions. In a magnitude comparison task homogeneous (++/– –) or heterogeneous (+ –/–+) numerator- or denominator-relevant fraction pairs were presented. Additionally, fraction pairs were presented either in a mixed or a blocked presentation format. Participants were asked to select the numerically larger of the two fractions, whereby reaction times, error rates (for results of the error rates please see S7.2), and the number of fixations on the defined AOIs were evaluated. In particular, we investigated four different strategies: i) sign shortcut, ii) sign flip (i.e., mirror mechanism for negative numbers), iii) denominator flip (i.e., mirror mechanism for fractions), as well as iv) a combined sign flip and denominator flip strategy. Furthermore, a potential modulation through block vs mixed presentation format was also investigated. In the following, the results concerning these four different strategies are discussed separately.

Sign Shortcut Strategy

The sign shortcut strategy reflects that comparisons of numbers with heterogeneous signs are processed faster than comparisons of number with homogenous signs (Krajcsi & Igács, 2010). Krajcsi & Igács, (2010) argued that in heterogeneous number pairs the exclusive consideration of the sign is sufficient for the decision in a magnitude comparison task. Additionally, their assumption was endorsed by a missing distance effect. The distance effect reflects an inverse relationship between numerical difference of two to be compared numbers and performance during the comparison (Moyer & Landauer, 1967): The closer the numbers on the number line, the more difficult the comparison. This becomes evident by increased reaction times and error rates. Thus, the missing distance effect while applying the sign shortcut strategy corroborates that the magnitudes of the numbers are not considered for solving the task.

In the present study, we investigated whether the sign shortcut strategy generalizes to the case of fraction comparison. And indeed, consistent evidence for the application of this strategy was found for both blocked and mixed presentation formats as indicated by faster reaction times for heterogeneous as compared to homogeneous fraction pairs.

While the comparison of heterogeneous and homogeneous fraction types clearly indicate that the sign shortcut strategy is used, the comparison of numerator- and denominator-relevant heterogeneous fraction pairs further suggests that participants do not seem to consider/be influenced by the (magnitudes of) other fraction components when a sign shortcut is a sufficient strategy. In particular, if the magnitudes of the fraction components do not play a role when employing the sign shortcut strategy, then the manipulation of the relevant fraction component (i.e., numerator- and denominator-relevant) should not impact performance in heterogeneous fraction pairs. In fact, there was no evidence for such an interaction: participants responded equally fast when comparing heterogenous fractions pairs independent of the relevant component. And similarly, reading time on the polarity signs was significantly longer for heterogenous fraction pairs than homogenous fraction pairs independent of the relevant component. Thus, while there are many examples indicating that the single components of multi-symbol numbers (cf. Huber et al., 2016) are processed even in case they are irrelevant for the task at hand, the current study may suggest that in case of (negative) fraction processing, parallel processing of all fraction components might be less automatically achieved or more successfully suppressed. One might speculate that participants seem to disregard any fraction-related information if not required for the task at hand.

Sign flip

The sign flip (cf., mirror mechanism; Krajcsi & Igács, 2010) refers to the observation that negative numbers are processed by first comparing the magnitude of the respective numbers without considering the negative polarity signs. Only in a second step the negative polarity sign is considered and the answer from step one is reversed. Krajcsi & Igács (2010), for instance, demonstrated that the sign flip strategy resulted in increased response times for negative homogenous number pairs compared to positive homogenous number pairs. Generalizing results by Krajacsi & Ignács' (2010) observed for whole number comparison to fraction comparison, participants in the present study answered significantly faster when comparing positive fraction pairs than when comparing negative fraction pairs. When inspecting mean reaction times between positive and negative fraction pairs (M = 7.06 (1160ms) vs. M = 7.26 (1425ms)), we found a considerable difference in RT of over 250ms. This difference might reflect the additional cost of reversing the answer while comparing negative numerator-relevant fraction pairs. Interestingly, ER and eye tracking results did not support our hypothesis. In both cases only the fixed effect of presentation was significant. Blocked conditions were less error prone and had proportionally more fixations on numerators than during mixed conditions. However, inspecting mean values of accuracy revealed descriptively more errors for negative numerator-relevant fraction pairs than for positive numerator-relevant fraction pairs (96.08% accuracy vs. 97.17 % accuracy). We additionally inspected proportional fixation duration for numerators when comparing negative and positive fraction pairs. We found that participants were descriptively fixating longer on numerators while comparing negative fraction pairs than positive fraction pairs (40,6% proportional reading time on the AOI numerator vs. 35.9% proportional reading time on the AOI numerator compared to all AOIs). Additionally, we ran a second analysis on fixation duration on the AOI signs when comparing negative and positive numerator-relevant fraction pairs. This analysis revealed no significant effect. Participants were fixating descriptively longer positive numerator-relevant fraction pairs than negative numerator relevant fraction pairs (34.8% proportional reading time vs. 30.9%

proportional reading time). This indicates, that for the reversion of the initial magnitude comparison participants need to fixate longer on the numerator but not necessarily on the sign. This might reflect additional needed working memory capacity for negative fraction pairs as the magnitudes of the numerators need to be memorized longer to give the correct answer. Whereas for positive fraction pairs this is not needed, and the positive sign gives the additional important information to not reverse the answer. According to Krajcsi & Igács (2010) the mirror mechanism (in our study renamed to sign flip strategy) represents a support system that transfers negative numbers to the mental number line, which in general is a metaphor for the nature of the number magnitude representation for positive numbers. In western cultures on this mental number line the magnitudes increase from left to right (Göbel, Shaki, & Fischer, 2011). Thus, this support system for negative numbers is necessary, since the phylogenetically old analogue magnitude system can only process positive numbers (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999).

Therefore, our results support the use of the sign flip strategy for negative homogenous numerator-relevant fraction pairs.

Denominator flip

A denominator flip might be used in case that homogeneous fractions with common numerator, but different denominators need to be compared. In a first step, the denominators of given fractions are compared as if they were natural numbers. In a second step the answer is reversed as the smaller denominator belongs to the larger fraction. In accordance with the findings of Huber et al. (2014) and Meert et al. (2009), using the denominator flip while comparing two fractions should result in longer reaction times as well as higher error rates and a larger number of fixations for denominator-relevant fraction pairs compared to numerator-relevant fraction pairs.

To investigate the use of the denominator flip strategy we focused on positive homogenous numerator- and denominator-relevant fraction pairs. Both RT and eye fixation results clearly showed that participants were relying on the denominator flip strategy while solving positive homogenous denominator-relevant fraction comparisons. Eye-tracking results examining the proportion of reading time on the AOI *denominators* additionally supported our hypothesis: the interaction presentation and component was significant. During blocked conditions proportional reading time on denominators was significantly higher for denominator-relevant fraction pairs than numerator-relevant fraction pairs. Interestingly, during mixed conditions proportional reading time on denominators was significantly higher for numerator-relevant fraction pairs than denominator-relevant fraction pairs. One possible reason might be that during mixed conditions a prediction of the next task to come is not given. Therefore, a basic reaction can always be to focus on the numerator first, since the magnitude of the numerator is easier to interpret in relation to the overall magnitude of the fraction than the magnitude of the denominator. However, for ERs we could only find a significant fixed effect of presentation format, but not a significant fixed effect of component.

According to Huber et al., (2014), the denominator flip strategy is a reflection of the inverse relationship between the magnitude of the denominator and the overall magnitude of the fraction in contrast to the corresponding relationship between the size of the numerator and the overall fraction magnitude. When the larger denominator is identified, the answer must be still reversed to correctly identify the larger fraction. This additional cognitive effort, which is not necessary for numerator-relevant fraction pairs, can be considered as the reason for longer reaction times, higher error rates, and larger number of fixations for denominator-relevant fraction pairs during blocked conditions.

Sign and denominator flip

To investigate the possible combined application of sign and denominator flip during negative denominator-relevant fraction comparisons a two-step analysis approach was used. First, to analyze the use of the sign flip strategy we only included homogenous (++ and – –) denominator-relevant fraction pairs in the analysis. Second, to analyze the use of the denominator flip strategy we only included negative homogenous numerator- and denominator-relevant fraction pairs.

For RTs, ERs and eye tracking data the first analysis revealed a significant fixed effect of presentation format. Participants responded faster and less error prone during blocked conditions than mixed conditions. Moreover, proportional reading time on denominators was higher during blocked conditions than mixed conditions. Additionally, for ERs the fixed effect of polarity was significant, indicating that participants made less errors for positive denominator-relevant fraction pairs than negative denominator-relevant fraction pairs. Surprisingly, neither for RT analysis nor eye tracking analysis the fixed effect of polarity was significant. This means that there was no difference between reaction times of positive of negative denominator-relevant fraction comparisons. Thus, there is no proof for the use of a sign flip strategy for negative denominator-relevant fraction comparisons.

The second analysis revealed again a significant fixed effect of presentation format for RTs, ERs as well as eye tracking data. Participants responded faster and less error prone during blocked conditions than mixed conditions. Moreover, proportional reading time on the AOI signs was higher during mixed conditions than blocked conditions. For this analysis we found also a significant fixed effect of component for all three data types. Numerator-relevant fractions pairs were compared faster and less errors were made during these comparisons than for denominator-relevant fraction pairs. Furthermore, eye fixation behaviour revealed that participants spend proportionally more time on the signs for numerator-relevant fraction pairs than denominator-relevant fraction pairs compared to all AOIs.

Inspecting the mean values of the participants for the comparison of negative denominator-relevant fraction pairs revealed that a subgroup of participants managed to solve the comparisons of negative denominator-relevant fraction pairs way faster than the rest of the participants. Moreover, they were roughly as fast as in other less complex comparisons. Thus, in an exploratory descriptive analysis we decided to examine the proportion of reading times on all relevant AOIs (i.e., denominators and signs) to find indications for different strategy use in these participants compared to the rest of the participants. Interestingly, this analysis revealed that these participants spend larger proportions of time inspecting denominators and lesser proportion of time inspecting signs. This indicates that there is a difference in strategy use between these participants and the rest of participants. Indicating, that these participants were spontaneously able to apply a different strategy by realizing that this most difficult comparison could actually be correctly solved in only comparing the magnitudes of the denominators. Moreover, a reversion of the answer for applying the denominator flip strategy and a second reversion for applying the sign flip strategy is not necessary (see Figure 7.2 B).

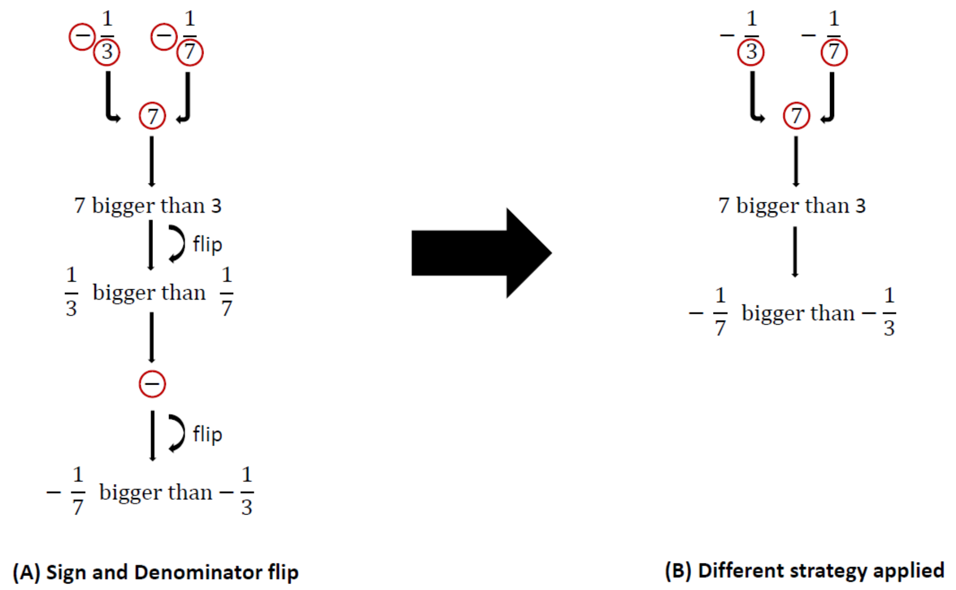


Figure 7.2: Graphical comparison of **(A)** Sign and Denominator flip: Both strategies are conducted one after the other and **(B)** The different strategy: A subgroup of the participants managed to find a simpler strategy to correctly solve negative denominator-relevant fraction pairs by realizing that they only had to compare the magnitudes of the denominators without reversing the answer and without considering the negative sign.

This leads to extraordinary short reaction times while solving this task. Furthermore, the different processing strategy of this subgroup might be the reason that we could not find the use of the sign flip strategy in the first reaction time and eye tracking analysis. This is further corroborated by the fact that the second eye-tracking analysis revealed that participants spend more time fixating the sign during numerator-relevant comparisons than denominator-relevant comparisons as the information of the sign is redundant for solving the task when applying the different strategy.

Finally, we excluded the six participants which used a different strategy to see whether the rest of the participants were applying the combination of the sign flip and denominator strategy. In the first analysis we again investigated the use of the sign flip strategy in these participants and found a significant fixed effect of polarity, indicating increased reaction times for negative homogenous denominator-relevant comparisons than positive denominator-relevant comparisons (*M* = 7.49 (1798ms), *SE* = 0.05 vs. *M* = 7.39 (1612ms), *SE* = 0.05). In the second analysis we again investigated the use of the denominator flip strategy in these participants and found a significant fixed effect of component, indicating increased reaction times for negative denominator-relevant fraction pairs than positive denominator-relevant fraction pairs (*M* = 7.49 (1797ms), *SE* = 0.04 vs. *M* = 7.31 (1489ms), *SE* = 0.04). Therefore, our results confirm the combined use of the sign and denominator flip strategy while solving negative homogenous fraction comparisons for the remaining participants that did not use a different strategy to solve the task.

7.5 Conclusion

Our results proofed, that processing mechanisms postulated for fractions and negative integers separately also apply for the processing of negative fractions. It needs to be further noted that we did not instruct participants to use the investigated strategies. We were able to substantiate the use of the sign shortcut, the sign flip, and the denominator flip strategy. Moreover, we could show that some participants applied a combination of the sign flip and denominator flip strategy for negative homogenous denominator-relevant fractions. The most interesting and surprising finding was the discovery that a small subgroup of participants managed to solve the most complex comparison (i.e., comparing negative homogenous denominator-relevant fraction pairs) by applying a different strategy. In realizing that they could solve the task by only comparing the magnitudes of both denominators participants processed these comparisons by using a task-related strategy shift.

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Acknowledgements

Silke M. Wortha was funded by the LEAD Graduate School & Research Network [GSC1028], a project of the Excellence Initiative of the German federal and state governments. Korbinian Moeller and Hans-Christoph Nuerk are members of the LEAD Graduate School & Research Network.

SUPPORTING ONLINE MATERIAL

Fraction pairs included in the study for all conditions

Table S7.1

denominator-relevant						numerator-relevant					
++		– –		+– / – +		++		– –		+– / – +	
+1/9	+1/8	-1/9	-1/8	-1/8	+1/9	+2/9	+1/9	-2/9	-1/9	+2/9	-1/9
+1/7	+1/9	-1/7	-1/9	-1/9	+1/7	+4/9	+1/9	-4/9	-1/9	+4/9	-1/9
+1/6	+1/9	-1/6	-1/9	-1/9	+1/6	+1/9	+5/9	-1/9	-5/9	+1/9	-5/9
+1/9	+1/5	-1/9	-1/5	-1/5	+1/9	+7/9	+1/9	-7/9	-1/9	+7/9	-1/9
+1/4	+1/9	-1/4	-1/9	-1/9	+1/4	+1/8	+3/8	-1/8	-3/8	+1/8	-3/8
+1/8	+1/7	-1/8	-1/7	-1/7	+1/8	+5/8	+1/8	-5/8	-1/8	+5/8	-1/8
+1/8	+1/5	-1/8	-1/5	-1/5	+1/8	+1/8	+7/8	-1/8	-7/8	+1/8	-7/8
+1/7	+1/5	-1/7	-1/5	-1/5	+1/7	+1/7	+2/7	-1/7	-2/7	+1/7	-2/7
+1/3	+1/7	-1/3	-1/7	-1/7	+1/3	+3/7	+1/7	-3/7	-1/7	+3/7	-1/7
+2/9	+2/7	-2/9	-2/7	-2/7	+1/9	+4/7	+1/7	-4/7	-1/7	+4/7	-1/7
+2/9	+2/5	-2/9	-2/5	-2/5	+1/9	+1/7	+5/7	-1/7	-5/7	+1/7	-5/7
+2/3	+2/9	-2/3	-2/9	-2/9	+1/3	+1/6	+5/6	-1/6	-5/6	+1/6	-5/6
+2/7	+2/5	-2/7	-2/5	-2/5	+1/7	+2/5	+1/5	-2/5	-1/5	+2/5	-1/5
+3/7	+3/8	-3/7	-3/8	-3/8	+3/7	+3/5	+1/5	-3/5	-1/5	+3/5	-1/5
+3/8	+3/5	-3/8	-3/5	-3/5	+3/8	+4/5	+1/5	-4/5	-1/5	+4/5	-1/5
+3/8	+3/4	-3/8	-3/4	-3/4	+3/8	+4/9	+2/9	-4/9	-2/9	+4/9	-2/9
+3/5	+3/7	-3/5	-3/7	-3/7	+3/5	+2/9	+5/9	-2/9	-5/9	+2/9	-5/9
+4/7	+4/9	-4/7	-4/9	-4/9	+4/7	+3/4	+1/4	-3/4	-1/4	+3/4	-1/4
+4/5	+4/9	-4/5	-4/9	-4/9	+4/5	+2/7	+3/7	-2/7	-3/7	+2/7	-3/7

+5/9	+5/8	-5/9	-5/8	-5/8	+5/9	+2/7	+4/7	-2/7	-4/7	+2/7	-4/7
+5/7	+5/9	-5/7	-5/9	-5/9	+5/7	+1/3	+2/3	-1/3	-2/3	+1/3	-2/3
+5/9	+5/6	-5/9	-5/6	-5/6	+5/9	+3/8	+5/8	-3/8	-5/8	+3/8	-5/8
+4/5	+4/7	-4/5	-4/7	-4/7	+4/5	+2/5	+3/5	-2/5	-3/5	+2/5	-3/5
+5/8	+5/7	-5/8	-5/7	-5/7	+5/8	+4/5	+2/5	-4/5	-2/5	+4/5	-2/5
+7/8	+7/9	-7/8	-7/9	-7/9	+7/8	+3/7	+5/7	-3/7	-5/7	+3/7	-5/7
				+1/8	-1/9					-2/9	+1/9
				+1/9	-1/7					-4/9	+1/9
				+1/9	-1/6					-1/9	+5/9
				+1/5	-1/9					-7/9	+1/9
				+1/9	-1/4					-1/8	+3/8
				+1/7	-1/8					-5/8	+1/8
				+1/5	-1/8					-1/8	+7/8
				+1/5	-1/7					-1/7	+2/7
				+1/7	-1/3					-3/7	+1/7
				+2/7	-2/9					-4/7	+1/7
				+2/5	-2/9					-1/7	+5/7
				+2/9	-2/3					-1/6	+5/6
				+2/5	-2/7					-2/5	+1/5
				+3/8	-3/7					-3/5	+1/5
				+3/5	-3/8					-4/5	+1/5
				+3/4	-3/8					-4/9	+2/9
				+3/7	-3/5					-2/9	+5/9
				+4/9	-4/7					-3/4	+1/4

	+4/9	-4/5	-2/7	+3/7
	+5/8	-5/9	-2/7	+4/7
	+5/9	-5/7	-1/3	+2/3
	+5/6	-5/9	-3/8	+5/8
	+4/7	-4/5	-2/5	+3/5
	+5/7	-5/8	-4/5	+2/5
	+7/9	-7/8	-3/7	+5/7

S7.2 Error rates

Strategy 1: Sign-shortcut

A GLME with the fixed effects *presentation format*, *component* and *polarity* as well as the respective two- and three-way interactions was run. Moreover, we included a random intercept for participants in the GLME. All three fixed effects were significant (presentation format: $\chi^2(1) = 55.35$, $p < .001$, component: $\chi^2(1) = 16.54$, $p < .001$, and polarity: $\chi^2(2) = 127.63$, $p < .001$, respectively). Fewer errors were made in the blocked (*log odds* = -4.43 (1.2%), *SE* = 0.17) than in the mixed presentation format (*log odds* = -3.42 (3.2%), *SE* = 0.13). Moreover, fewer errors were made for numerator-relevant pairs (*log odds* = -4.13 (1.6%), *SE* = 0.16) than for denominator-relevant pairs (*log odds* = -3.72 (2.4%), *SE* = 0.15). Finally, fewest errors were made for heterogeneous fraction pairs (*log odds* = -5.17 (0.6%), *SE* = 0.21), followed by positive homogeneous (*log odds* = -3.56 (2.8%), *SE* = 0.16) again followed by negative homogeneous fraction pairs (*log odds* = -3.05 (4.5%), *SE* = 0.14; all $p < .01$). Pairwise post hoc comparisons among the fixed effect polarity indicated that error rates significantly differed between + – vs. – – (z = 9.82, $p < 0.001$) and + – vs. + + (z = - 7.02, $p < 0.001$) and + + vs. – – (z = 3.11, $p < 0.01$). In contrast to RT results, none of the interactions reached significance ($\chi^2(2) \leq 2.90$, all $p_s > 0.05$).

Strategy 2: Sign flip

A GLME with the fixed effects *presentation format*, *polarity*, the interaction of *presentation format* x *polarity,* and *a random intercept for participants* was run on errors. The fixed effect of presentation format was significant ($\chi^2(1) = 18.70$, $p < .001$)

indicating that fewer errors were made in the blocked (*log odds* = -4.22 (1.5%), *SE* = 0.27) than in the mixed presentation format (*log odds* = -3.13 (4.2%), *SE* = 0.20). In contrast to RT results, neither the fixed effect of polarity nor the interaction of presentation format x polarity was significant ($\chi^2(2) \leq 2.27$, all $p_s > 0.05$).

Strategy 3: Denominator flip

A GLME with the fixed effects *presentation format*, *component*, the interaction of *presentation format* x *component*, and a random intercept for participants was run on errors. The fixed effect of presentation format was significant ($\chi^2(1) = 20.24$, $p < .001$) indicating that fewer errors were made in the blocked (*log odds* = -4.13 (1.6%), *SE* = 0.25) than in the mixed presentation format (*log odds* = -3.00 (4.7%), *SE* = 0.17). Neither the fixed effect of component nor the interaction of presentation format x component was significant ($\chi^2(2) \leq 2.26$, all $p_s > 0.05$).

Strategy 4: Combination of sign and denominator flip

A first GLME with the fixed effects *presentation format*, and *polarity* was run on errors. Random intercepts for participants were included in the GLME. Both fixed effects of presentation format and polarity were significant ($\chi^2(1) = 51.84$, $p < .001$ and $\chi^2(1) = 15.05$, $p < .001$, respectively). Fewer errors were made in the blocked (*log odds* = -3.86 (2.1%), *SE* = 0.18) than in the mixed presentation format (*log odds* = -2.74 (6.1%), *SE* = 0.14). Furthermore, fewer errors were made for positive homogeneous, denominator-relevant pairs (*log odds* = -3.58 (2.7%), *SE* = 0.17) than for negative homogeneous, denominator-relevant pairs (*log odds* = -3.02 (4.6%), *SE* = 0.15). The interaction of presentation format x polarity was not significant ($\chi^2(1) = 0.02$, $p = .882$).

A second GLME with the fixed effects *presentation format*, and *component* was run on errors. A random intercept for participants was also included in the GLME. Both the fixed effects of presentation format and component were significant ($\chi^2(1) = 31.56$, $p < .001$ and $\chi^2(1) = 16.18$, $p < .001$, respectively). Fewer errors were made in the blocked (*log odds* = -3.60 (2.6%), *SE* = 0.21) than in the mixed presentation format (*log odds* = -2.50 (7.6%), *SE* = 0.15). Furthermore, fewer errors were made for negative homogeneous, numerator-relevant fraction pairs (*log odds* = -3.43 (3.1%), *SE* = 0.20) than for negative homogeneous, denominator-relevant fraction pairs (*log odds* = -2.68 (6.4%), *SE* = 0.16). The interaction of presentation format x component was not significant ($\chi^2(1) < 0.01$, $p = .951$).

Mean RTs for all conditions

Table S7.2

Presentation format	*Relevant component*	*Polarity*	*RT (in ms)*	*logRT*
b	N	+– / – +	663.62	6.43
m	N	+– / – +	963.68	6.79
b	D	+– / – +	676.47	6.44
m	D	+– / – +	980.89	6.79
b	N	++	948.67	6.76
m	N	++	1654.86	7.35
b	D	+ +	1380.55	7.07
m	D	++	2315.11	7.68
b	N	– –	1172.46	6.89
m	N	– –	2200.67	7.62
b	D	– –	1477.22	7.11
m	D	– –	2551.39	7.76

Note: *Presentation format*: b = block, m = mixed; *Relevant component*: D = denominator-relevant, N = numerator-relevant; *Polarity:* – – = negative homogenous, ++ = positive homogenous, +– / – + = heterogenous

Studies of Section 3:

Motivational and Affective Predictors of Fraction Processing

Study 4: Ninaus, M., Kiili, K., Wortha, S. M., & Moeller, K. (2021). Motivationsprofile bei Verwendung eines Lernspiels zur Messung des Bruchverständnisses in der Schule - Eine latente Profilanalyse. *Psychologie in Erziehung und Unterricht*, *68*(1), 42-57.

Study 5: Klein, E.*, Bieck, S. M.*, Bloechle, J., Huber, S., Bahnmueller, J., Willmes, K., & Moeller, K. (2019). Anticipation of difficult tasks: neural correlates of negative emotions and emotion regulation. *Behavioral and Brain Functions*, *15*(1), 1-13.

* Equal contribution

8. Study 4: Motivationsprofile bei Verwendung eines Lernspiels zur Messung des Bruchverständnisses in der Schule - Eine latente Profilanalyse[14]

Manuel Ninaus[1,2,5], Kristian Kiili[3], Silke M. Wortha[1,2], Korbinian Moeller.[1,2,4]

[1] Leibniz-Institut für Wissensmedien, Tübingen, Deutschland

[2] LEAD Graduate School & Research Network, Eberhard Karls Universität, Tübingen, Deutschland

[3] Tampere University, Tampere, Finnland

[4] Psychologisches Institut, Eberhard Karls Universität, Tübingen, Deutschland

[5] Institut für Psychologie, Universität Innsbruck, Innsbruck, Österreich

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Zusammenfassung

Lernspiele gewinnen auch in der Schule zunehmend an Bedeutung als motivationsförderliche Lehr- und Lernmethode. In einer Feldstudie mit 256 Schülerinnen und Schülern der siebten Schulstufe wurde daher untersucht i) inwiefern sich grundlegende Effekte der Forschung zu numerischer Kognition mit einem digitalen Lernspiel zur Messung des Verständnisses von Brüchen replizieren lassen und ii) ob sich spezifische Motivationsprofile bei der Benutzung des Lernspiels identifizieren lassen. Die beobachtete spezifische Assoziation der Leistung im Lernspiel mit Mathematiknoten als auch der aus der Grundlagenforschung bekannte Distanzeffekt belegen die Validität des Lernspiels. Mittels latenter Profilanalyse wurden drei Gruppen von Schülerinnen und Schülern identifiziert, die sich hinsichtlich selbst- und fremdbestimmter Motivation sowie dem wahrgenommen positiven Affekt während des Spielens unterschieden. Erwartungsgemäß verbrachten selbstregulierte Schülerinnen und Schülern die meiste Zeit mit dem Spiel und hatten das positivste Spielerleben. Diese Ergebnisse spezifizieren die motivationalen Möglichkeiten (digitaler) Lernspiele im Schulunterricht.

Stichwörter: Lernspiele, Mathematik, Motivation, Brüche, Bruchverständnis

Summary

Educational games are becoming increasingly important to foster motivation in schools. In a field study with 256 seventh-grade students, we evaluated whether a digital learning game for assessing fraction understanding allows i) replication of fundamental effects of numerical cognition research and ii) the evaluation of motivation profiles when using the game. Validity of the learning game was demonstrated by replicating both the specific association of performance in the game and mathematics school grades as well as the numerical distance effect. Using latent profile analysis we identified three groups of students who differed in terms of self- and externally determined motivation as well as their perceived positive affect during playing the game. As expected, self-regulated pupils spent more time playing the game and reported most positive player experience. In sum, these results specify the motivational possibilities of (digital) games in school.

Keywords: game-based learning, mathematics, motivation, fractions, fraction understanding

In den letzten Jahren hat der Einsatz von spielbasierten Lernansätzen bzw. Lernspielen im Bildungskontext erheblich zugenommen (für eine Übersicht siehe Boyle et al., 2016). Dies ist nicht unbegründet, da aktuelle Metanalysen darauf hinweisen, dass spielbasierte Lernansätze konventionellen überlegen sein können (z.B. Sailer & Homner, 2019; Wouters, van Nimwegen, van Oostendorp & van der Spek, 2013). Neben der Verbesserung der Lernleistung deuten die Ergebnisse darauf hin, dass auch motivationale Variablen wie Einstellungen, Arbeitsmoral und intrinsische Motivation durch spielbasierte Ansätze erhöht werden können (für eine Meta-Analyse siehe z.B. Sailer & Homner, 2019). Daher ist es wenig verwunderlich, dass immer öfter versucht wird, sich den motivationalen Anreiz von Spielen für Lernzwecke zu Nutze zu machen um Lernerfolge zu verbessern (z.B. Erhel & Jamet, 2013; Ninaus, 2017). Empirische Studien dazu sind im schulischen Kontext jedoch bislang kaum vorhanden.

Im Bereich des Mathematiklernens scheinen Lernspiele im besonderen Maße effektiv zu sein (z.B. Kiili, Moeller & Ninaus, 2018; für eine Meta-Analyse siehe Wouters et al., 2013). In der aktuellen Feldstudie verwendeten wir daher ein digitales Lernspiel zur Verbesserung des Verständnisses von Bruchzahlen. In kleineren experimentellen Studien wurde dieses Lernspiel bereits erfolgreich als Mess- (Ninaus, Kiili, McMullen & Moeller, 2017) und Trainingsinstrument (Kiili, Moeller, et al., 2018) eingesetzt und grundlegende Effekte der Forschung zu numerischer Kognition konnten repliziert werden. In der aktuellen Studie sollen für das Lernspiel unter realen Schulbedingungen motivationale Profile einer größeren Stichprobe von Schülerinnen und Schüler mit einem personenzentrierten Ansatz untersucht werden.

Im Folgenden führen wir daher zuerst in die Inhaltsdomäne des verwendeten Lernspiels und relevante motivationale Theorien und Ergebnisse ein, bevor wir die Fragestellungen der aktuellen Studie genauer erläutern.

8.1 Theoretischer Hintergrund

Bruchzahlen

Brüche gelten als ein anspruchsvolles Thema im Mathematikunterricht (National Mathematics Advisory Panel, 2008). Das Verstehen von und der korrekte Umgang mit Brüchen sind jedoch wesentliche mathematische Kompetenzen und korrelieren entsprechend hoch mit aktueller (z.B. Booth & Newton, 2012; Kiili, Moeller, et al., 2018) aber auch zukünftiger Mathematikleistung (z.B. Bailey, Hoard, Nugent & Geary, 2012; Booth & Newton, 2012). Damit scheint das Veständnis von Brüchen mit entscheidend zu sein für das Erlernen weiterer mathematischer Inhalte.

Eines der größten Probleme für Schülerinnen und Schüler ist das Verständnis der numerischen Größe von Brüchen (im englischen Original magnitude understanding, z.B. Siegler, Fazio, Bailey & Zhou, 2013).

Das Konzept des mentalen Zahlenstrahls ist dabei eine häufig benutzte Metapher für die mentale Repräsentation von Zahlengröße. Dementsprechend wird die sogenannte Zahlenstrahlaufgabe (im englischen Original number line estimation task) häufig zur Messung und Förderung des Größenverständnisses von (Bruch)Zahlen benutzt (z.B. Kiili, Moeller, et al., 2018) und auch in vielen aktuellen Schulbüchern zur Einführung von Bruchzahlen eingesetzt (Padberg & Wartha, 2017). In dieser Aufgabe soll die Position einer Zielzahl (z.B. 1/4) auf einem Zahlenstrahl (z.B. von 0 bis 1) bestimmt werden (z.B. Siegler & Opfer, 2003). Die Performanz in dieser Aufgabe ist mit aktueller und zukünftiger Mathematikleistung korreliert (z.B. Booth & Siegler, 2006). Dies verdeutlicht die Bedeutung des Verständnisses von Zahlengröße für numerische Entwicklung (vgl. Siegler, 2016).

Aktuelle Studien zeigten, dass die mentale Repräsentation von Zahlengröße von Schülerinnen und Schülern durch das Training mit der Zahlenstrahlaufgabe verbessert werden kann (z.B. Kiili, Moeller, et al., 2018; Schneider & Stern, 2010). In den meisten Studien wurden jedoch konventionelle computer-basierte oder Papier-Bleistift Versionen der Zahlenstrahlaufgabe zum Training oder der Messung des Größenverständnisses benutzt. In den letzten Jahren wurden jedoch zunehmend neue und innovative Methoden zur Umsetzung der Zahlenstrahlaufgabe erprobt [z.B. verkörperlichte (Fischer, Dackermann, Cress, Nuerk & Moeller, 2014), oder spielbasierte Implementationen (z.B. Fazio, Kennedy & Siegler, 2016)].

Kinder entwickeln ein initiales Verständnis von ganzen Zahlen als zählbare Einheiten, bevor sie im Schulunterricht das Konzept von Brüchen erlernen müssen. Daher greifen sie bei der Verarbeitung von Brüchen auf dieses initiale Verständnis von ganzen Zahlen zurück und versuchen dies bei rationalen Zahlen -- wie etwa Brüchen -- anzuwenden (DeWolf & Vosniadou, 2015; Stafylidou & Vosniadou, 2004). Missverständnisse oder falsche Vorstellungen über Brüche entstehen daher oft auf Grund der fehlerhaften Annahme, dass Eigenschaften von ganzen Zahlen auf Brüche übertragen werden können (Padberg & Wartha, 2017). Laut DeWolf und Vosniadou (2015) neigen Kinder entsprechend dazu Nenner und Zähler als zwei getrennte ganze Zahlen zu behandeln, anstatt ihre Beziehung zueinander zu betrachten. Aufgrund dieser fehlerhaften Konzeptualisierung mancher Schülerinnen und Schüler (z.B. Gómez & Dartnell, 2019) schließen sie dann oftmals fälschlicherweise, dass die numerische Größe eines Bruchs zunimmt, wenn entweder der Nenner oder der Zähler größer wird [z.B. 2/5 (0,4) > 3/8 (0,375), obwohl 2 < 3 und 5 < 8].

Neben der Zahlenstrahlaufgabe wird Größenverständnis von Brüchen auch mit Größenvergleichsaufgaben erfasst (im englischen Original magnitude comparison task) in der Probanden entscheiden müssen, welcher von zwei Brüchen numerisch größer ist: 2/8 (0,25) oder 4/5 (0,8) (z.B. Padberg & Wartha, 2017). Entsprechend wird diese Aufgabe auch verwendet, um das Größenverständnis von (Bruch)Zahlen zu fördern bzw. Misskonzeptionen in Bezug auf Brüche zu identifizieren. Aus der Grundlagenforschung ist bekannt, dass der bei Größenvergleichsaufgaben der so genannte Distanzeffekt auf eine erfolgreiche Repräsentation von Bruchgröße hinweist [z.B. Schneider & Siegler, 2010; d.h. längere und fehleranfälligere Antworten beim Vergleich von Zahlen mit kleinerer numerischer Distanz, z.B. 2/5 (0,4) vs. 3/8 (0,375) im Vergleich zu 1/5 (0,2) vs. 3/4 (0,75)].

Dementsprechend wurden im vorliegenden Lernspiel zur Förderung des Größenverständnisses von Bruchzahlen (für eine Übersicht siehe Kiili, Koskinen & Ninaus, 2019) Aufgabenmechaniken der Zahlenstrahlaufgabe und Größenvergleichsaufgaben als Spielmechaniken implementiert.

Motivation - Selbstbestimmungstheorie

Der wohl am häufigsten angeführte Grund Spiele im Bildungskontext zu nutzen ist deren motivationaler Anreiz (z.B. Garris, Ahlers & Driskell, 2002; Wouters et al., 2013). Die sog. Selbstbestimmungstheorie (Deci & Ryan, 2000; Ryan & Deci, 2000) postuliert, dass der intrinsisch motivationale Anreiz von Spielen dadurch erklärt werden kann, dass Spiele grundlegende psychologische Bedürfnisse von sozialem Bezug, Autonomie und Kompetenz befriedigen (Przybylski, Rigby & Ryan, 2010). Die Befriedigung dieser Bedürfnisse führt nicht nur zu erhöhter intrinsischer Motivation, sondern bewegt Menschen auch dazu, jene befriedigende Aktivität weiterhin auszuführen (z.B. Ryan, Rigby, & Przybylski, 2006). Dementsprechend weisen Studien darauf hin, dass im Bereich von Mathematik Motivation, Interesse aber auch Freude an Mathematik bedeutsam für die Beschäftigung mit Mathematik sind (z.B. Hannula et al., 2016; Schiepe-Tiska & Schmidtner, 2013). Im Vergleich zu anderen OECD Ländern scheint die Freude an Mathematik bei Schülerinnen und Schülern in Deutschland jedoch eher gering ausgeprägt zu sein. Die Mehrheit der Jugendlichen berichtete in der Studie von Schiepe-Tiska und Schmidtner (2013) wenig bis keine Freude bzw. Interesse an Mathematik. Spielbasierte Lernansätze könnten, vor dem Hintergrund, dass Freude nicht nur die Lernbereitschaft, sondern auch die Beschäftigung mit Mathematik fördert, hier einen wichtigen Beitrag leisten. Für eine spielbasierte Lernumgebung würde dies bedeuten, dass die Lernenden mehr Spaß bzw. Freude an der Anwendung haben (z.B. Ninaus et al., 2019), sich länger mit den Lerninhalten beschäftigen und dadurch bessere Lernleistungen erzielen können (z.B. Kiili, Lindstedt & Ninaus, 2018; Kiili, Ojansuu, Lindstedt & Ninaus, 2018).

In erster Linie konzeptualisiert die Selbstbestimmungstheorie (Deci & Ryan, 2000; Ryan & Deci, 2000) Motivation entlang eines Kontinuums von Selbst- zu Fremdbestimmung, d.h. von intrinsischer Motivation, die auftritt, wenn Individuen Aktivitäten aus Freude an dieser ausführen, zu extrinsischer Motivation, die auftritt, wenn Aktivitäten v.a. auf Grund instrumenteller Gründe ausgeführt werden.

Bei intrinsischer Motivation handelt es sich um eine autonome und selbstgesteuerte Form von Motivation. Wenn Schülerinnen und Schüler u.a. aus Neugierde, persönlichem Interesse oder Freude lernen, ist dies mit wahrgenommener psychologischer Freiheit und internalen Kontrollüberzeugungen assoziiert (Vansteenkiste, Lens, De Witte, De Witte & Deci, 2004).

Aber auch wenn kein persönliches Interesse vorhanden ist, kann Lernen selbstbestimmt erlebt werden, wenn das zu Lernende als persönlich relevant bewertet wird. Diese sogenannte identifizierte Regulation tritt daher auf, wenn Personen entscheiden, dass eine bestimmte Aktivität von persönlicher Bedeutung ist um beispielsweise zukünftige Ziele zu erreichen (Vansteenkiste, Sierens, Soenens, Luyckx & Lens, 2009).

Intrinsische Motivation und identifizierte Regulation gelten als selbstbestimmte Typen der Motivation. Viele Studien weisen auf die positiven Folgen selbstbestimmter Motivation hin, wie zum Beispiel erhöhtes psychologisches Wohlbefinden (z.B. Levesque, Zuehlke, Stanek & Ryan, 2004), erhöhter Einsatz und Durchhaltevermögen (z.B. Hardre & Reeve, 2003; Ryan & Connell, 1989), sowie verbesserte kognitive Verarbeitung (z.B. Vansteenkiste, Simons, Lens, Soenens & Matos, 2005).

Externale Regulation ist eine nicht internalisierte Form der extrinsischen Motivation am unteren Ende des Selbst- bzw. Fremdbestimmungs-Kontinuums, die auftritt, wenn Aktivitäten auf Grund von externalen Belohnungen, Bestrafungen oder Erwartungen ausgeführt werden. Schließlich beschreibt die Selbstbestimmungstheorie die sogenannte Amotivation, die auftritt, wenn Personen Aktivitäten ausführen, ohne deren Sinn zu kennen oder diese erst gar nicht ausgeführt werden (Deci & Ryan, 2000; Ryan & Deci, 2000). In unterschiedlichen Studien wurde fremdgesteuerte Motivation mit negativen Folgen wie verringerter Konzentration (z.B. Vansteenkiste, Zhou, Lens & Soenens, 2005), oberflächlicherer kognitiver Verarbeitung (z.B. Vansteenkiste, Simons, et al., 2005), und erhöhtem Dropout (z.B. Soenens & Vansteenkiste, 2005) assoziiert.

Diese Ergebnisse und theoretischen Annahmen weisen deutlich auf die Vorteile selbstgesteuerter gegenüber fremdgesteuerter Motivation hin und betonen die hohe Bedeutung der Erfassung von unterschiedlichen Motivationstypen bei Schülerinnen und Schülern. Im Rahmen der Selbstbestimmungstheorie werden zwei Typen der Messung von Motivation unterschieden (Wang et al., 2017). Die gebräuchlichste Art ist die Verwendung von Fragebögen [z.B. „Situational Motivation Scale", SIMS; (Guay, Vallerand & Blanchard, 2000) oder „Intrinsic Motivation Inventory" (McAuley et al., 1989)]. Insbesondere im Bereich des spielbasierten Lernens wird zudem der positive Affekt beim Spiel [z.B. mit dem Game Experience

Questionnaire - GEQ (IJsselsteijn, de Kort & Poels, 2013) bzw. dessen Subskala „positiver Affekt“ (für eine Übersicht siehe Mekler, Bopp, Tuch & Opwis, 2014)]. Die zweite Art Motivation zu erheben ist die Messung von behavioralen Motivationsindikatoren, wie zum Beispiel die verbrachte Zeit mit einer Lernanwendung (z.B. Prins, Dovis, Ponsioen, ten Brink & van der Oord, 2011), oder die Anzahl an bearbeiteten Aufgaben (z.B. Mekler, Brühlmann, Tuch & Opwis, 2017). Die Verwendung von Fragebogendaten ohne dazugehörige behaviorale Indikatoren wird meist als Limitation vieler Studien genannt (z.B. Schwinger, Steinmayr & Spinath, 2012), da nur deren Kombination eine Untersuchung des Zusammenhangs zwischen Selbstbericht und Verhalten ermöglicht. Dementsprechend wurden in der aktuellen Feldstudie beide Arten der Messung von (intrinsischer) Motivation berücksichtigt.

Aktuelle Studie und Fragestellungen

Die aktuelle Studie verfolgte die folgenden zwei Fragestellungen:

1. Können grundlegende Effekte der Forschung zu numerischer Kognition in einer Feldstudie mit Schülerinnen und Schülern der siebten Schulstufe unter Verwendung eines digitalen Lernspiels zur Messung des Bruchverständnisses repliziert werden?
2. Welche motivationalen Profile können bei der Nutzung des Lernspiels von Schülerinnen und Schülern identifiziert werden und wie stehen diese in Zusammenhang mit dem Nutzungsausmaß und dem Spielerlebnis?

Im Rahmen der ersten Fragestellung sollte sich die beobachtete hohe Relevanz des Verständnisses von Bruchzahlen für die allgemeine Mathematikleistung (z.B. Booth & Newton, 2012; Kiili, Moeller, et al., 2018) durch eine spezifische Assoziation der Leistung im verwendeten Lernspiel mit den Schulnoten in Mathematik, nicht aber in einem unrelatierten Schulfach ausdrücken (Hypothese 1a). Zudem sollte sich bei Schülerinnen und Schülern der siebten Klassenstufe, die bereits im formalen Mathematikunterricht mit Brüchen konfrontiert waren, ein Distanzeffekt bei Größenvergleichsaufgaben zeigen (Hypothese 1b; z.B. Schneider & Siegler, 2010).

Zur Beantwortung der zweiten Fragestellung wurden die motivationalen Profile der Schülerinnen und Schüler mittels der personenzentrierten Methode der Latenten

Profilanalyse untersucht. Entsprechend der Selbstbestimmungstheorie sollten zumindest zwei unterschiedliche Gruppen von Schülerinnen und Schülern identifizierbar sein, welche vorrangig selbstgesteuerte vs. vorrangig fremdgesteuerte Motivationsprofile zeigen sollten (Hypothese 2a). Aufgrund der bisherigen Befundlage sollten vor allem selbstregulierte Schülerinnen und Schüler mehr Zeit in das Spiel investieren und eine positivere Spielerfahrung berichten als Schülerinnen und Schüler mit vor allem fremdregulierter Motivation (Hypothese 2b).

8.2 Methode

Die Daten dieser Studie wurden in einem großen Forschungsprojekt an deutschen Gymnasien unter der Leitung des Hector-Instituts für Empirische Bildungsforschung und dem Leibniz-Institut für Wissensmedien Tübingen erhoben. In diesem Projekt soll unter anderem untersucht werden, wie motivationale und kognitive Merkmale von Schülerinnen und Schülern zu einer lernförderlichen Nutzung von Tablets im Unterricht beitragen. Für die vorliegende Studie wurde im Rahmen dieses Projekts der Einsatz der spielbasierten Lernanwendung „Semideus“ für Tablets im Feld erprobt.

Stichprobe

Insgesamt nahmen 510 Schülerinnen und Schüler der siebten Jahrgangsstufe an der Studie teil. Das Einverständnis der Eltern wurde vor Beginn der Studie eingeholt. Die Beantwortung der aktuellen Fragestellungen beruht auf Daten von n = 256, 106 Schülerinnen und 97 Schülern (M_{Alter} = 12.64 Jahre; SD = 0.87 Jahre; 51/53 Teilnehmende ohne Altersangabe/Geschlechtsangabe), die eine Version von Semideus spielten, die sowohl Zahlenstrahl- als auch Größenvergleichsaufgaben beinhaltete. Die anderen Schülerinnen und Schüler bearbeiteten eine andere Version des Spiels und wurden daher nicht in der aktuellen Analyse berücksichtigt.

Beschreibung der Intervention und Ablauf

Semideus ist eine Spiele-Engine, womit unterschiedliche Spielversionen zur Förderung des Verständnisses von rationalen Zahlen erstellt werden können (vgl.

Kiili, Moeller, et al., 2018; Ninaus, Kiili, et al., 2017; Ninaus, Moeller, McMullen & Kiili, 2017). Für die vorliegende Studie wurde das Spiel SemideusDE konfiguriert und für iPads frei verfügbar gemacht (https://itunes.apple.com/de/app/semideus-de/id1363119927).

Die Grundmechanik des Spiels basiert auf einem Zahlenstrahl, der als begehbare Plattform implementiert ist. Spielerinnen und Spieler steuern den Charakter Semideus, der versucht Goldmünzen wiederzufinden, die gestohlen und entlang des Weges versteckt wurden indem sie das Tablet nach links oder rechts neigen. Für die aktuelle Studie wurden Zahlenstrahl- und Größenvergleichsaufgaben im Spiel implementiert. In den Zahlenstrahlaufgaben mussten Spielerinnen und Spieler die genaue Position der vergrabenen Goldmünzen identifizieren und ausgraben. Die Position der Goldmünzen wurde über die Zielzahl im jeweiligen Trial dargestellt (z.B. 7/9; siehe Abbildung 8.1 links). In der Größenvergleichsaufgabe mussten zwei Brüche, die auf zwei Steinen geschrieben standen, hinsichtlich ihrer numerischen Größe verglichen werden (siehe Abbildung 8.1 rechts) indem die Steine hinsichtlich ihrer numerischen Größe in aufsteigender Reihenfolge von links nach rechts angeordnet wurden. Die jeweilige Position auf dem Zahlenstrahl spielte dabei keine Rolle. In beiden Aufgaben erhielten Schülerinnen und Schüler positives/negatives Feedback für korrekte/falsche Antworten.

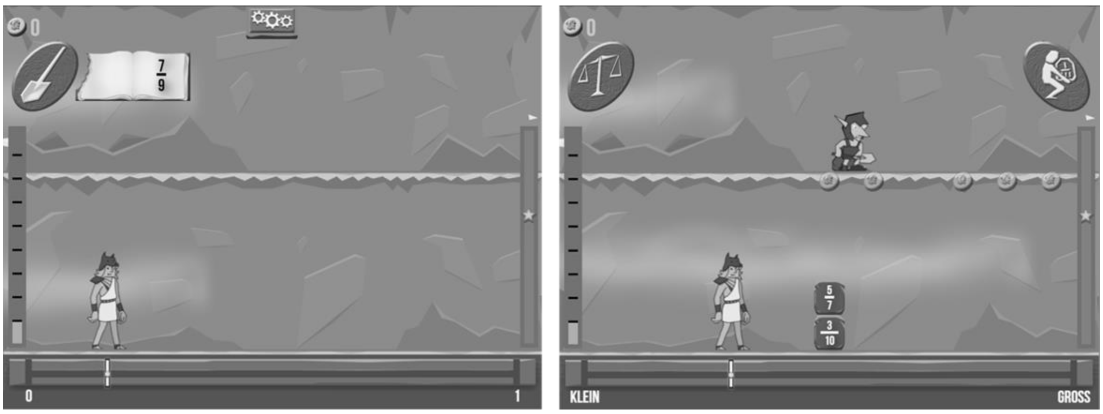

Figure 8.1: Beispiele für Zahlenstrahlaufgabe (links) und Größenvergleichsaufgabe (rechts).

SemideusDE sollte im Rahmen des Mathematikunterrichts in 5 aufeinanderfolgenden Wochen für jeweils 10 Minuten verwendet werden. Die Lehrerinnen und Lehrer wurden lediglich instruiert das Spiel im Mathematikunterricht wie gefordert zu verwenden. Dazu wurde den Lehrerinnen und Lehrern das Spiel vorgestellt und sie erhielten schriftliche Instruktionen zum Spiel. Eine Woche nach der letzten Spieleinheit wurden subjektive Erfahrungen mit dem Spiel mit Hilfe von unterschiedlichen Fragebögen erhoben[15] (siehe unten). Diese wurden für die vorliegende Studie mit den Spielinteraktionsdaten zusammengeführt.

Material

Insgesamt wurden 40 Level mit Zahlenstrahl- und 40 Level mit Größenvergleichsaufgaben erstellt, die sukzessive in alternierender Reihenfolge gespielt werden sollten. Jedes Level beinhaltete 8 Zahlenstrahl- bzw. Größenvergleichsaufgaben. Insgesamt wurden alle 45 möglichen Brüche im Zahlenraum zwischen 0 und 1 verwendet (d.h. mit Nennern von 2, z.B. 1/2, bis 10, z.B. 9/10). Aus diesen 45 Brüchen wurden 160 Größenvergleichsausgaben erstellt die jeweils zwei Mal dargeboten wurden (1/5 vs. 7/10 bzw. 7/10 vs. 1/5). Die numerischen Distanzen zwischen den Brüchen lagen zwischen 0.01 und 0.8 ($M_{Distanz}$ = 0.38; SD = 0.22). Die Reihenfolge der Items in den Zahlenstrahl- und Größenvergleichsausgaben war randomisiert. Bei der Planung und dem Design der Studie wurde die Anzahl an unterschiedlichen Aufgaben priorisiert, da Schülerinnen und Schüler das Spiel über einen längeren Zeitraum nutzen sollten. Aus diesem Grund war es nicht möglich die Aufgaben hinsichtlich kongruenter (d.h. Zähler 1 > Zähler 2 und Nenner 1 > Nenner 2; Bruch 1 > Bruch 2, z.B. 3/8 < 5/9) und inkongruenter Größenvergleiche (Zähler 1 > Zähler 2 und Nenner 1 > Nenner 2; Bruch 2 > Bruch 1, z.B. 3/8 < 2/5) im Sinne des sogenannten whole number bias (vgl. Ni & Zhou, 2005) zu balancieren.

Die Verwendung von SemideusDE ermöglichte die Aufzeichnung zahlreicher Spielinteraktionsdaten. Zur Beantwortung der vorliegenden Fragestellungen wurden die folgenden berücksichtigt: i) Genauigkeit der Schätzung in Zahlenstrahlaufgaben in Prozent (1 -- |geschätzte Position -- korrekte Position| * 100; z.B. für den Bruch 2/5: 1 -- |0,35-0,4| * 100 = 95,00%); ii) Leistung [(Anzahl an korrekten Antworten / Anzahl an Antworten) * 100] und Antwortzeit bei Größenvergleichsausgaben; iii)

[15] Im Rahmen des groß angelegten Forschungsprojekts an deutschen Gymnasien wurden noch weitere kognitive (z.B. Intelligenz, Lesegeschwindigkeit, etc.) und personenspezifische Variablen (z.B. Tabletnutzung in der Schule, familiärer Hintergrund, etc.) erhoben. Diese Daten sind jedoch für die Beantwortung der aktuellen Fragestellungen nicht relevant und werden daher nicht berücksichtigt.

Anzahl der gespielten Level und Anzahl der gespielten Tage im Interventionszeitraum und danach (bis 6 Monate nach Beendigung der Intervention).

Zur Erfassung der Motivation wurde die „Situational Motivation Scale" (SIMS; Guay et al., 2000) ins Deutsche übersetzt und vorgegeben. Diese umfasst 16 Aussagen, die auf einer Likert-Skala von 1 („trifft gar nicht zu") -- 7 („trifft voll zu") hinsichtlich der Gründe wieso SemideusDE gespielt wurde, beantwortet werden sollten. Die Skala beinhaltet 4 Subskalen mit folgenden Werten für die interne Konsistenz in der aktuellen Stichprobe: (i) intrinsische Motivation (α = .87; 4 Items z.B. „Weil ich denke, dass es eine interessante Aktivität ist."), (ii) identifizierte Regulation (α = .82; 4 Items z.B. „Ich spiele es zu meinem eigenen Besten."), (iii) externale Regulation (α = .71; 4 Items z.B. „Weil Semideus spielen etwas ist was ich tun muss") und (iv) Nicht-Regulation/Amotivation (α = .75; 4 Items z.B. „Es gibt vielleicht gute Gründe Semideus zu spielen, aber ich persönlich sehe keine").

Zur Erfassung des allgemeinem Spielerlebnisses wurde das Core-Modul des „Game Experience Questionnaire" (GEQ; IJsselsteijn, de Kort & Poels, 2013) ins Deutsche übersetzt und vorgegeben. Dieses umfasst 33 Aussagen, die auf einer Likert-Skala von 1 („überhaupt nicht") -- 5 („sehr") beantwortet werden sollen. Der Fragebogen besteht aus 7 Subskalen: (i) Kompetenz (α = .73; 5 Items, z.B.: „Ich habe die Ziele des Spiels schnell erreicht."), (ii) Sensorische und imaginative Immersion (α = .84; 6 Items, z.B.: „Ich habe mich fantasievoll und einfallsreich gefühlt."), (iii) Flow (α = .84; 5 Items, z.B.: „Während des Spielens habe ich alles um mich herum vergessen."), (iv) Anspannung/Ärger (α = .69; 3 Items, z.B.: „Ich war genervt"), (v) Herausforderung (α = .65; 5 Items, z.B.: „Ich habe mich herausgefordert gefühlt."), (vi) negativer Affekt (α = .79; 4 Items, z.B.: „Ich habe vom Spielen schlechte Laune bekommen.") und (vii) positiver Affekt (α = .90; 5 Items, z.B.: „Es hat mir Spaß gemacht."). Für die aktuelle Studie wurde nur die Subskala „positiver Affekt" verwendet, da diese am besten die Freude am Spiel erfasst (vgl. Mekler et al., 2014) und somit relevant für die Motivation der Schülerinnen und Schüler sein sollte.

Im Rahmen des groß angelegten Forschungsprojektes konnten wir zudem für einen Großteil der Schülerinnen und Schüler auf die Schulnoten unterschiedlicher Unterrichtsfächer im letzten Zeugnis zugreifen, welche mittels Selbstauskunft der Schülerinnen und Schüler erfasst wurden. Die Schulnoten wurden auf einer 21-stufigen Skala erfasst (Schulnote = Skalenwert; 1=1; 1--=2; 1--2=3; 2+=4; 2=5; 2--=6;

2--3=7; 3+=8; 3=9; 3--=10; 3--4=11; 4+=12; 4=13; 4--=14; 4--5=15; 5+=16; 5=17; 5--=18; 5--6=19; 6+=20; 6=21), wobei höhere Zahlenwerte schlechtere Schulleistung im jeweiligen Unterrichtsfach widerspiegeln. Für die vorliegende Studie verwendeten wir die Noten in den Fächern Deutsch und Mathematik.

Analyse

In einem ersten Schritt wurden zur Untersuchung des Zusammenhangs zwischen der Leistung in den Zahlenstrahl- bzw. Größenvergleichsaufgaben und den Schulnoten Rangkorrelationen nach Spearman gerechnet, um den Einfluss von Ausreißern zu minimieren. Zusätzlich wurde Steigers Z-Test verwendet, um Unterschiede zwischen den Korrelationen der Spielleistung und den Noten in Deutsch bzw. Mathematik zu überprüfen. Die Schulnoten in Mathematik und Deutsch waren jedoch nur für 166 Schülerinnen und Schüler verfügbar.

Zur Bestimmung des Distanzeffekts bei den Größenvergleichsaufgaben wurde eine lineare Regressionsanalyse durchgeführt mit Antwortzeit als abhängiger Variable und numerischer Distanz als Prädiktor.

Zur Identifikation von möglichen Subgruppen bzw. latenten Profilen mit ähnlichen Motivations- und Leistungsprofilen wurden in einem zweiten Schritt latente Profilanalysen (vgl. Hickendorff, Edelsbrunner, McMullen, Schneider & Trezise, 2018; mit dem Paket „mclust“, (Scrucca, Fop, Murphy & Raftery, 2016), in R durchgeführt. Die Klassifizierung der latenten Profile erfolgte auf Basis der berichteten Motivationstypen der Schülerinnen und Schüler anhand der vier Subskalen des SIMS und der Anzahl der gespielten Level als objektiven behavioralen Motivationsindex. Zusätzlich wurde noch die Subskala „positiver Affekt“ des GEQ als Indikator für Freude am Spiel und die Leistung in Zahlenstrahlaufgaben und Größenvergleichsaufgaben als Leistungsindikatoren mit berücksichtigt.

Bei der latenten Profilanalyse wurde anhand des Bayes‘schen Informationskriterien (BIC) das passendste Gaussian Mixture Modell für die Daten identifiziert, wobei verschiedene Clusteranzahlen k und verschiedene Modellklassen (Charakteristika wie Verteilung, Volumen, Orientierung und Form) berücksichtigt wurden und der am wenigsten negative BIC-Wert das am besten geeignete Modell beschreibt. Hierfür wurden die Daten z-standardisiert und nur vollständige Datensätze berücksichtigt (d.h. Daten von 218 Schülerinnen und Schülern). Für das Ergebnis wurde anschließend mit dem Integrated Completed Likelihood Kriterium noch ein weiterer Modellfit Index berücksichtigt, der zusätzlich die Überlappung der Cluster stärker berücksichtigt (Bertoletti, Friel & Rastelli, 2015). Schließlich wurde ein Bootstrap Likelihood Ratio Test (BLRT) mit den Standardeinstellungen des „mclust“ Pakets durchgeführt (999 Bootstrap-Replikationen; non-parametrisches Bootstrapping), um den Modellfit zwischen Modellen mit k-1 und k Clustern zu vergleichen.

8.3 Ergebnisse

Replikation basisnumerischer Befunde

Spielleistung und Schulnoten

Hypothese 1a: Es zeigte sich ein signifikant negativer Zusammenhang zwischen der Leistung in der Zahlenstrahlaufgabe (Genauigkeit der Schätzung) und der Schulnote in Mathematik (rho=--.45, p<.001, siehe Abbildung 8.2A). Zudem zeigte sich ein signifikant negativer Zusammenhang zwischen der Größen vergleichsleistung (Anzahl an korrekten Antworten / Anzahl an Antworten) und der Schulnote in Mathematik (rho=--.16, p<.05, siehe Abbildung 8.2 B). Das heißt, dass gute Leistung im Spiel mit besseren Mathematiknoten assoziiert war.

Zudem zeigte sich ein signifikanter Zusammenhang zwischen Schätzleistung (rho=--.19, p<.05, siehe Abbildung 8.2 C) bzw. Größenvergleichsleistung (rho=--.21, p<.01, siehe Abbildung 8.2 D) und der Schulnote in Deutsch. Jedoch war der Zusammenhang zwischen der Leistung in der Zahlenstrahlaufgabe und der Mathematiknote in signifikant größer als der Zusammenhangmit der Deutschnote (Z= --3.32, p < .001). Der Vergleich der Zusammenhänge der Größenvergleichsleistung mit den Schulnoten in Mathematik bzw. Deutsch ergab keinen signifikanten Unterschied (Z= 0.60, p = .55).

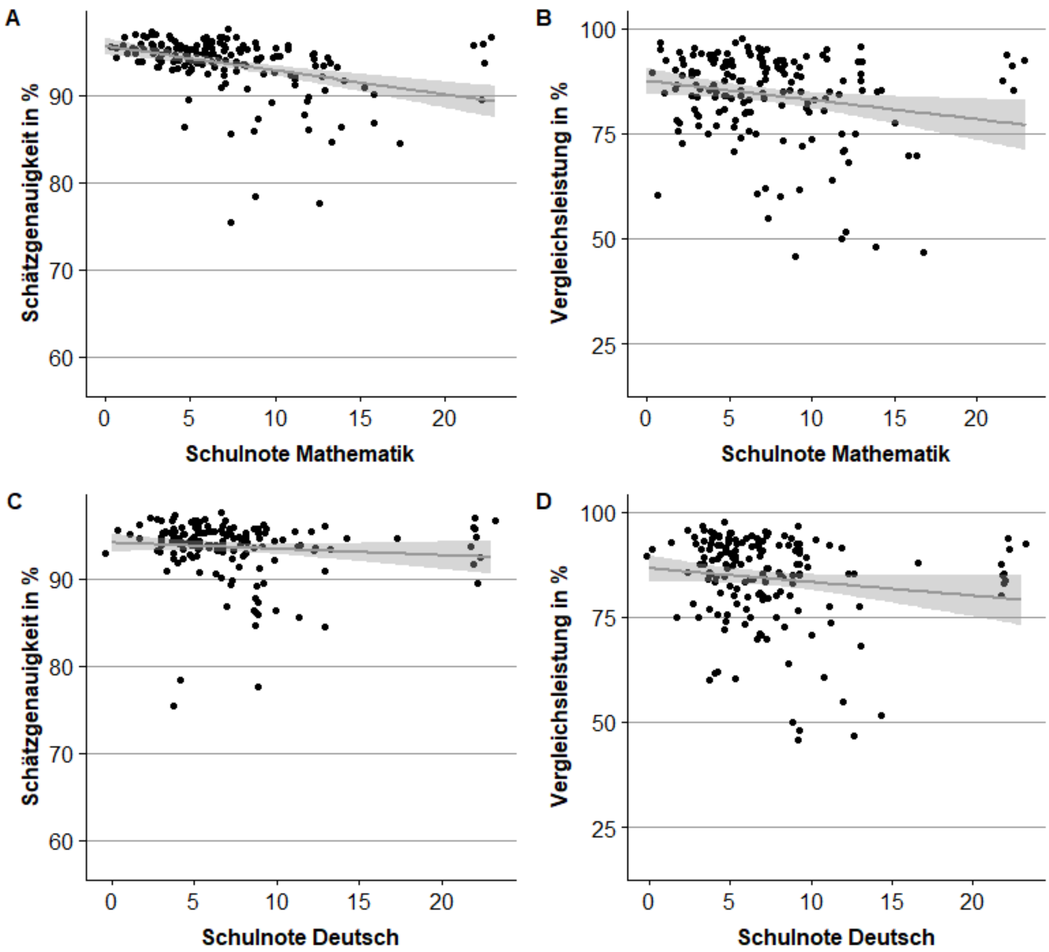

Figure 8.2: Streudiagramme; A: Korrelation zwischen Leistung in der Zahlenstrahlaufgabe und der Schulnote in Mathematik; B: Korrelation zwischen Leistung in der Vergleichsaufgabe und der Schulnote in Mathematik; C: Korrelation zwischen Leistung in der Zahlenstrahlaufgabe und der Schulnote in Deutsch; D: Korrelation zwischen Leistung in der Vergleichsaufgabe und der Schulnote in Deutsch; Die grau schattierten Bereiche zeigen 95%-Konfidenzintervalle der Korrelationen.

Distanzeffekt

Hypothese 1b: Die Ergebnisse der Regressionsanalyse weisen darauf hin, dass die Antworten mit größer werdender numerischer Distanz signifikant schneller wurden [β=--0.24; p<.01; F(1,137)=8.14; p<.01; korrigiertes R^2=0.05).

Motivation -- behaviorale und subjektive Indikatoren

Deskriptive Ergebnisse -- Spielausmaß

Die Anzahl an gespielten Leveln und an wie vielen Tagen die Schülerinnen und Schüler gespielt haben dienten als behaviorale Motivationsindikatoren. Während des Interventionszeitraums wurden durchschnittlich 47.92 Level (SE=2.29) gespielt (siehe Abbildung 8.3 A), was in etwa 384 Aufgaben entspricht, an durchschnittlich 6.36

unterschiedlichen Tagen (SE=0.21; (siehe Abbildung 8.3 C). Nach der Intervention -- Zeitraum vom ersten Tag nach der Intervention bis 6 Monate danach -- wurden immer noch durchschnittlich 4.60 Level (SE=0.82; siehe Abbildung 8.3 B) an durchschnittlich 0.85 Tagen (SE=0.07; siehe Abbildung 8.3 D) gespielt.

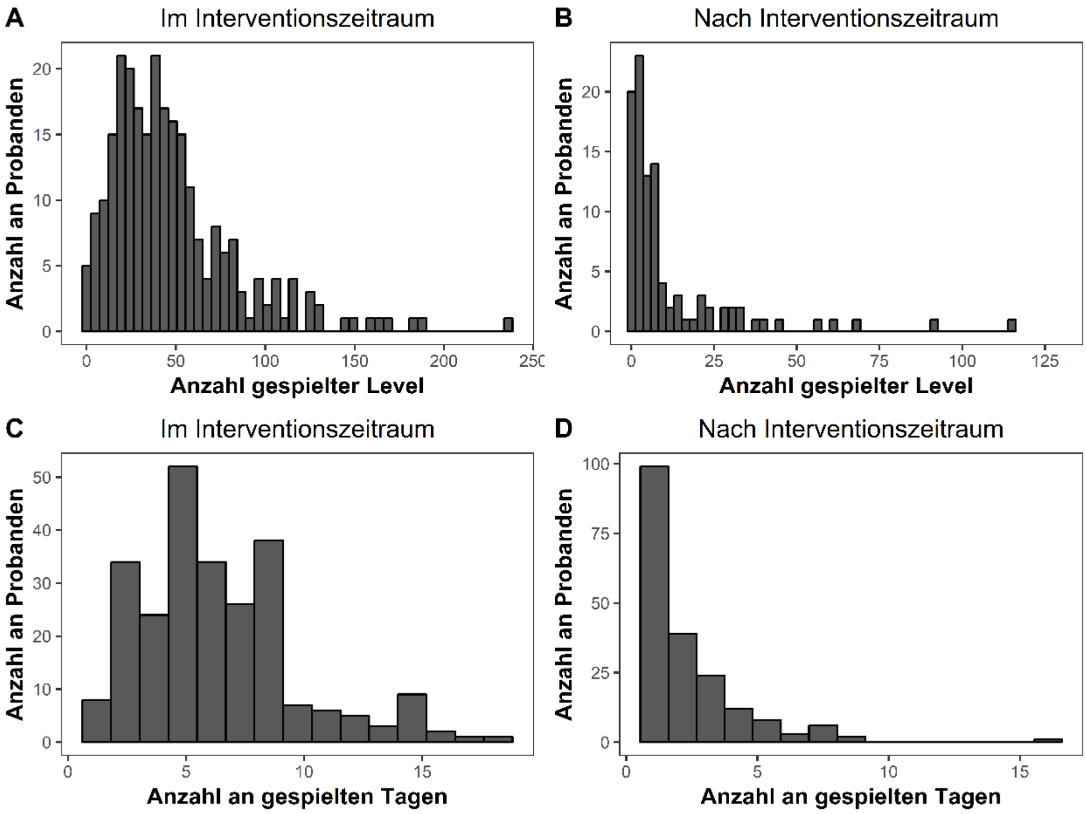

Figure 8.3: Deskriptive Darstellung von Anzahl der gespielten Level und der Anzahl an gespielten Tagen im Interventionszeitraum (A & C) bis 6 Monate danach (B & D; Darstellung für Schülerinnen und Schüler die mindestens ein Level bzw. einen Tag spielten).

Profilanalysen

Hypothese 2a: Tabelle 8.1 gibt einen Überblick über die Fitindizes BIC und ICL der 3 besten Profilösungen der latenten Profilanalyse. Sowohl BIC als auch ICL weisen darauf hin, dass eine Lösung mit 3 Profilen, ellipsenförmigen Clustern und gleicher Orientierung (VVE) anzustreben ist.

Table 8.1: BIC und ICL Fitindizes für die besten 3 Lösungen der latenten Profilanalyse; VVE = ellipsenförmigen Cluster mit gleicher Orientierung; EVE = ellipsenförmige Cluster mit gleichem Volumen und gleicher Orientierung.

Beste BIC Lösungen			
Form und Anzahl an Clustern	VVE, 3	VVE, 4	EVE, 3
BIC	-4160.45	-4205.42	-4206.19
BIC diff	0.00	-44.97	-45.74
Beste ICL Lösungen			
Form und Anzahl an Clustern	VVE, 3	EVE, 3	VVE, 4
ICL	-4217.73	-4244.63	-4259.97
ICL diff	0.00	-26.90	-42.24

Der Bootstrap Likelihood Ratio Test zeigte, dass die Hinzunahme eines weiteren Profils/Clusters keine signifikante Verbesserung ergab (3 vs. 4 Cluster: likelihood-ratio=46.33; p=.09). Daher wurde die 3-Clusterlösung als finale Lösung angenommen (finales Modell: log-likelihood=--1870.77; n=215; df =78). Die z-standardisierten Mittelwerte der aufgenommen Variablen in den jeweiligen Profilgruppen sind in Tabelle 8.2 dargestellt und in Abbildung 8.4 graphisch veranschaulicht.

Table 8.2: z-standardisierte Mittelwerte aller Variablen und Anzahl an Personen in den 3 latenten Profilen. GEQ_posAffekt = Subskala positiver Affekt des GEQ; SIMS-Amotivation = Subskala Amotivation des SIMS; SIMS-exReg = Subskala externale Regulation des SIMS; SIMS-idReg = Subskala identifizierte Regulation des SIMS; SIMS-intrMot = Subskala intrinsische Motivation des SIMS.

Latente Profile									
	n	Schätz-genauigkeit	Größen-vergleichs-leistung	Anzahl gespielter Level	GEQ-posAffekt	SIMS-amotivation	SIMS-exReg	SIMS-idReg	SIMS-intrMot
1	91	0.37	0.53	-0.32	-0.32	0.39	0.48	-0.51	-0.38
2	71	0.32	0.13	0.80	0.21	-0.23	-0.37	0.42	0.26
3	53	-0.89	-1.05	-0.40	0.14	-0.22	-0.28	0.26	0.26

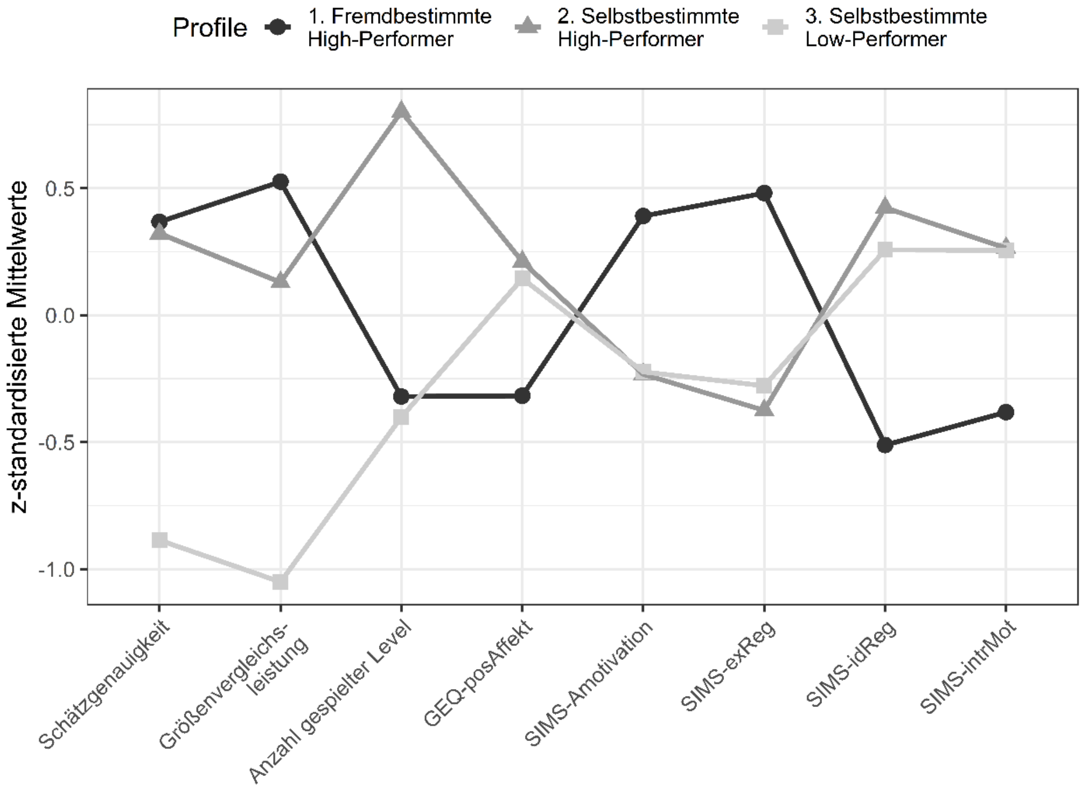

Figure 8.4: Ausprägung der unterschiedlichen Variablen in den 3 unterschiedlichen Profilen die durch die latente Profilanalyse festgestellt wurden. GEQ_posAffekt = Subskala positiver Affekt des GEQ; SIMS-Amotivation = Subskala Amotivation des SIMS; SIMS-exReg = Subskala externale Regulation des SIMS; SIMS-idReg = Subskala identifizierte Regulation des SIMS; SIMS-intrMot = Subskala intrinsische Motivation des SIMS.

Hypothese 2b: Schülerinnen und Schüler, die dem ersten Profil zugeordnet wurden, zeigten im Vergleich zur restlichen Stichprobe überdurchschnittliche Leistungen im Spiel (Schätzgenauigkeit und Größenvergleichsleistung), spielten mitunter jedoch die wenigsten Level, berichteten unterdurchschnittlich positiven Affekt während des Spielens und nahmen ihre Handlung des Spielens als überdurchschnittlich fremdbestimmt wahr (externale Regulation und Amotivation). Sie stellten somit eine Gruppe von fremdbestimmten High-Performern dar.

Schülerinnen und Schüler im zweiten Profil waren ebenso im Vergleich zur restlichen Stichprobe überdurchschnittlich gut im Spiel, spielten deutlich die meisten Level, berichteten überdurchschnittlich positiven Affekt während des Spielens und nahmen ihre Handlung des Spielens als überdurchschnittlich selbstbestimmt wahr (identifizierte Regulation und intrinsische Motivation). Sie stellten damit eine Gruppe selbstbestimmter High-Performer dar.

Die Schülerinnen und Schüler im dritten Profil erbrachten im Vergleich die schlechtesten Leistungen im Spiel, spielten jedoch ähnlich viele Level wie fremdbestimmte High Performer (Profil 1). Zudem berichteten sie im Vergleich zur restlichen Stichprobe einen überdurchschnittlich hohen positiven Affekt und nahmen ihre Handlung des Spielens als überdurchschnittlich selbstbestimmt wahr. Dies entspricht einer Gruppe von selbstbestimmten Low-Performern.

8.4 Diskussion

In der vorliegenden Feldstudie konnten (i) grundlegende Effekt aus der Forschung zur numerischen Kognition mit einem digitalen Lernspiel zur Messung des Bruchverständnisses im Feld repliziert werden und (ii) drei unterschiedliche Motivationsprofile von Schülerinnen und Schülern in der Verwendung des Lernspiels identifiziert werden. Im Folgenden werden diese Ergebnisse eingehender diskutiert.

Replikation basisnumerischer Effekte

Um die Validität als auch Anwendbarkeit von digitalen Lernspielen im Schulunterricht zu demonstrieren sollten in der aktuellen Studie grundlegende Effekte aus der numerischen Kognition repliziert werden. Es zeigte sich, dass sowohl die Schätzgenauigkeit in den Zahlenstrahlaufgaben als auch die Größenvergleichsleistung im Spiel signifikant mit der aktuellen Schulnote in Mathematik korrelierten. Dabei ist wichtig zu berücksichtigen, dass der Zusammenhang zwischen der Schätzgenauigkeit in Zahlenstrahlaufgaben im Spiel und der aktuellen Schulnote in Mathematik signifikant größer war als der Zusammenhang mit der aktuellen Schulnote in Deutsch. Dieses Ergebnis legt nahe, dass sich dieser Zusammenhang nicht auf allgemein bessere Schulleistungen zurückführen lässt. Jedoch muss angemerkt werden, dass sich kein signifikanter Unterschied zwischen den Zusammenhängen der Größenvergleichsleistung im Spiel und den aktuellen Schulnoten in Mathematik bzw. Deutsch ergab. Dies könnte darauf hinweisen, dass die Leistung in der Zahlenstrahlaufgabe ein besserer und spezifischerer Indikator für allgemeine Leistung im Mathematikunterricht ist als die Leistung in der Größenvergleichsaufgabe. Die Ergebnisse replizieren und substantiieren damit die schon in früheren Studien berichtete hohe Relevanz des Verständnisses von Brüchen für die allgemeine Mathematikleistung (z.B. Booth &

Newton, 2012; Kiili, Moeller, et al., 2018). Bei der Interpretation der aktuellen Ergebnisse sollte jedoch berücksichtigt werden, dass die Erfassung der Mathematikkompetenz mittels Selbstauskunft der Schülerinnen und Schüler hinsichtlich der von ihnen erreichten Schulnoten kein standardisiertes Testverfahren darstellt -- die Verwendung von Schulnoten für derartige Analysen ist in der Literatur jedoch nicht unüblich (Kiili, Moeller, et al., 2018; Schneider, Grabner & Paetsch, 2009; Torbeyns, Schneider, Xin & Siegler, 2014). Zudem wurde in einer Meta-Analyse (Schneider, Merz, et al., 2018), die den Zusammenhang zwischen Mathematikkompetenz und Leistung in der Zahlenstrahlaufgabe untersuchte, kein signifikanter Unterschied zwischen unterschiedlichen Maßen der Mathematikkompetenz (d.h., Schulnoten, standardisierte Mathematiktests, Arithmetik, etc.) beobachtet. Darüber hinaus ermöglichte dieser Zugang ein kosten- und zeiteffizientes Vorgehen -- sehr relevante Punkte in groß angelegten Schultestungen -- und die Möglichkeit Leistung unterschiedlicher Schulfächer zu berücksichtigen, um differentielle Aussagen zu ermöglichen.

Des Weiteren konnte der Distanzeffekt für die Größenvergleichsaufgaben repliziert werden. Die Schülerinnen und Schüler zeigten bei Vergleichen von Zahlen mit kleiner numerischer Distanz längere Reaktionszeiten als bei Vergleichen mit großer numerischer Distanz (z.B. Schneider & Siegler, 2010). Dieser Effekt weist darauf hin, dass ein erfolgreiches Verständnis von Bruchzahlen gegeben zu sein scheint und entspricht unseren Erwartungen für Schülerinnen und Schüler der siebten Klasse.

Die erfolgreiche Replikation dieser grundlegenden Effekte spricht für die interne Validität und Anwendbarkeit des verwendeten digitalen Lernspiels und betont die Bedeutung der Berücksichtigung von Befunden aus der Grundlagenforschung für die Entwicklung von neuen und innovativen Lernansätzen. Ein weiterer wichtiger Aspekt, welcher bei der Akzeptanz und Implementation von neuen Lernansätzen berücksichtigt werden muss, ist die Motivation der Schülerinnen und Schüler.

Motivation -- Selbst- vs. fremdbestimmte Motivationsprofile

Der motivationale Anreiz ist wohl einer der häufigsten Gründe Lernspiele in den Unterricht zu integrieren (z.B. Garris et al., 2002; Wouters et al., 2013). In der aktuellen Studie wurde die Motivation der Schülerinnen und Schüler auf zwei unterschiedliche Ebenen untersucht. Es wurden Motivationstypen mit einem Fragebogen erhoben sowie behaviorale Motivationsindikatoren herangezogen, um die Übereinstimmung von Selbstbericht und tatsächlich gezeigtem Verhalten zu untersuchen.

Die deskriptive Analyse der behavioralen Motivationsindikatoren deutete auf eine hohe Akzeptanz des Spiels seitens der Schülerinnen und Schüler hin. Während des Interventionszeitraums absolvierten diese eine beträchtliche Anzahl von durchschnittlich rund 384 Aufgaben. Zudem spielten die Schülerinnen und Schüler an durchschnittlich 6.36 unterschiedlichen Tagen, und damit an mehr als den fünf in der Studie vorgesehenen Trainingstagen. Dies scheint besonders indikativ für den hohen motivationalen Anreiz des Spiels und dessen Akzeptanz, wie auch die Tatsache, dass selbst nach Beendigung der Intervention das Spiel von den Schülerinnen und Schülern weiterhin benutzt wurde.

Um die motivationalen Beweggründe der Schülerinnen und Schüler besser zu verstehen wurde eine latente Profilanalyse durchgeführt. Mit dieser personenzentrierten Analyse konnten 3 unterschiedliche Profile von Schülerinnen und Schülern identifiziert werden. Wie erwartet unterschieden die Profile deutlich zwischen den Motivationstypen und entsprechen hinsichtlich der eindeutigen Unterscheidbarkeit von selbst- und fremdbestimmter Motivation den Ergebnissen vorangegangener Studien (z.B. Deci & Ryan, 2000; Schwinger et al., 2012). Schülerinnen und Schüler des ersten Profils bzw. der ersten Gruppe -- sogenannte fremdbestimmte High-Performer -- waren vor allem fremdbestimmt motiviert und eher demotiviert. Hingegen waren Schülerinnen und Schüler der zweiten (selbstbestimmte High-Performer) und dritten Gruppe (selbstbestimmte Low-Performer) identifiziert reguliert und intrinsisch motiviert. Es zeigte sich zudem, dass damit die Mehrheit der Schülerinnen und Schüler der aktuellen Stichprobe selbstbestimmt motiviert waren.

Bei deskriptiver Betrachtung scheint der wahrgenommene positive Affekt während des Spiels wichtig für die Unterscheidung von fremd- oder selbstbestimmter Motivation zu sein. Dies entspricht den Ergebnissen früherer Studien, die einen positiven Zusammenhang zwischen selbstbestimmter Motivation und positiven Spielerleben berichten (z.B. Kiili, Lindstedt, et al., 2018; Kiili, Ojansuu, et al., 2018). Interessanterweise ließ sich dies nicht auf die Leistung im Spiel zurückführen, da auch selbstbestimmte Low-Performer im Vergleich zur restlichen Stichprobe überdurchschnittlich positives Spielerleben berichteten. Das heißt, dass, obwohl selbstbestimmte Low-Performer entsprechend ihrer unterdurchschnittlichen Leistung relativ häufig negatives Feedback im Spiel erhielten -- zumindest im Vergleich zu den restlichen Schülern und Schülerinnen -- sich dies nicht negativ auf ihre selbstbestimmte Motivation und ihren Spaß am Spiel auswirkte. Es muss jedoch darauf hingewiesen werden, dass die Ergebnisse der Profilanalyse immer hinsichtlich der aktuellen Stichprobe von Schülerinnen und Schülern von deutschen Gymnasien interpretiert werden müssen; d.h. im Vergleich zu Stichproben aus anderen Schultypen (z.B. Hauptschule) könnten die Low-Performer der aktuellen Stichprobe gute Leistungen erbracht haben.

Nichtsdestotrotz stellt sich die Frage, welche weiteren Faktoren positives Spielerleben begünstigen. Neuere Erkenntnisse legen nahe, dass individuelle Präferenzen für unterschiedliche Spielelemente dabei eine Rolle spielen könnten (z.B. Jia, Liu, Yu & Voida, 2017; Tondello et al., 2016). Zudem hat sich gezeigt, dass die Präferenz von Schülerinnen und Schülern digitale Spiele im Schulunterricht zu verwenden außerdem von Faktoren wie der Vorerfahrung mit und der wahrgenommenen Nützlichkeit von digitalen Spielen beeinflusst wird (Bourgonjon, Valcke, Soetaert & Schellens, 2010).

Hinsichtlich der Anzahl an gespielten Leveln zeigte sich ein ähnlich erwartungskonformes Bild. Fremdbestimmte High-Performer absolvierten mitunter die wenigsten Level. Selbstgesteuerte High-Performer hingegen spielten die meisten Level, was für erhöhten Einsatz und Durchhaltevermögen bei Personen mit selbstgesteuerter Motivation spricht (siehe z.B. Hardre & Reeve, 2003; Ryan & Connell, 1989). Es muss jedoch berücksichtigt werden, dass selbstgesteuerte Low-Performer nicht mehr Level absolvierten als fremdbestimmte High-Performer. Dies könnte darauf hinweisen, dass die Menge an positivem Feedback in Kombination mit selbstbestimmten Formen der Motivation eine optimale Konstellation für freiwillig erbrachten Einsatz im schulischen Kontext darstellt. Entsprechend sollten Lehrerinnen und Lehrer selbstbestimmte Motivation fördern, um Schülerinnen und Schüler dazu zu bewegen sich freiwillig mehr mit mathematischen Inhalten zu beschäftigen. Dies entspricht auch Erkenntnissen von Wang et al. (2017), dass Schülerinnen und Schüler mit selbstbestimmter Motivation über die Hausaufgaben hinaus mehr Aufwand und Zeit in Mathematik investierten.

8.5 Zusammenfassung und Ausblick

Die Ergebnisse zeigten, dass mit dem vorliegenden digitalen Lernspiel zur Messung des Bruchverständnisses grundlegende Effekte der numerischen Kognition repliziert werden konnten. So war die Leistung im Spiel mit den Schulnoten in Mathematik assoziiert und es wurde der Distanzeffekt bei Größenvergleichsaufgaben beobachtet. Außerdem konnten drei unterschiedliche Motivationsprofile bei Schülerinnen und Schülern identifiziert werden, die sich vor allem hinsichtlich selbst- und fremdbestimmter Motivation sowie dem positiven Affekt während des Spielens unterschieden. Wie erwartet verbrachten selbstregulierte Schülerinnen und Schüler mehr Zeit mit dem Spiel und berichteten positiveres Spielerleben.

Die aktuelle Studie demonstriert damit sowohl die Validität von digitalen Lernspielen und deren Anwendbarkeit im Unterricht als auch die hohe Akzeptanz von digitalen Lernspielen seitens der Schülerinnen und Schüler. Es soll jedoch erwähnt werden, dass es durchaus Schülerinnen und Schüler gab, die weniger positive Erfahrungen mit dem Spiel schilderten. Aus praktischer Sicht weisen die Ergebnisse darauf hin, dass (digitale) Lernspiele dazu führen können, dass sich Schülerinnen und Schüler über das geforderte Maß hinaus mit mathematischen Inhalten beschäftigen. Zudem zeigten Ergebnisse aus anderen Studien, dass Lernspiele wie Semideus konventionelle Lehr- und Lernmethoden sinnvoll ergänzen können (z.B. Kiili, Moeller, et al., 2018).

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9. Study 5: Anticipation of difficult tasks – neural correlates of negative emotions and emotion regulation[16]

Elise Klein[1,2,&] , Silke M. Bieck[1,2,&, *], Johannes Bloechle[3], Stefan Huber[1], Julia Bahnmueller[1,2,4], Klaus Willmes[5], & Korbinian Moeller[1,2,4]

[1] Neuro-cognitive plasticity lab, Leibniz-Institut für Wissensmedien, Tuebingen, Germany

[2] LEAD Graduate School and Research Network, University of Tuebingen, Germany

[3] Department of Psychiatry and Psychotherapy, University of Tuebingen, Germany

[4] Department of Psychology, University of Tuebingen, Germany

[5] Department of Neurology, University Hospital, RWTH Aachen University, Aachen, Germany

[&] Elise Klein and Silke M. Bieck contributed equally to this work and should be considered shared first authors

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Abstract

Background: Difficult cognitive tasks are often associated with negative feelings. This can be already the case for the mere anticipation of having to do a difficult task. For the case of difficult math tasks, it was recently suggested that such a negative emotional response may be exclusive to highly math-anxious individuals. However, it is also conceivable that negative emotional responses simply reflect that math is perceived as difficult. Here we investigated whether non-math-anxious individuals also experience negative emotional responses when anticipating to do difficult math tasks.

Methods: We compared brain activation following the presentation of a numerical cue indicating either difficult or easy upcoming proportion magnitude comparison tasks.

Results: Comparable to previous results for highly math-anxious individuals we observed a network associated with negative emotions to be activated in non-math-anxious individuals when facing cues indicating a difficult upcoming task. Importantly, however, math anxiety scores did not predict the neural response. Furthermore, we observed activation in areas associated with processes of cognitive control areas such as anterior cingulate cortex, which were suggested to play a key role in emotion regulation.

Conclusion: Activation in the emotion processing network was observed when anticipating an upcoming difficult (math) task. However, this activation was not predicted by individual' degree of math anxiety. Therefore, we suggest that negative emotional responses to difficult math tasks might be a rather common reaction not specific to math-anxious individuals. Whether or not this initial negative response impairs math performance, however, might depend on the ability to regulate those emotions effectively.

Word count: 250

Key words: task difficulty, difficult math, cognition and emotion, emotion regulation, fMRI

9.1 Introduction

Research repeatedly showed that doing difficult tasks is often associated with negative feelings such as high arousal, stress or anxiety. In particular, these negative feelings seem to be more pronounced for difficult as compared to more easy tasks (see [1] for a review) and can negatively affect cognitive performance on a variety of tasks (e.g., reading, attention, etc. [2–4]). In this context, difficulty is typically characterized by the demand for attentional, cognitive, etc. resources needed to master the task at hand. This way, a difficult (i.e., more demanding) task in turn leads to compromised performance because arousal, stress and anxiety consume cognitive resources (e.g., [5]). This may even end in a vicious circle where the individual perceives demanding tasks as threatening, which then leads to more anxiety. Anxiety is typically defined as a negative emotional state that occurs in situations in which the level of perceived demands to the individual is experienced as outweighing her/his resources to complete the task at hand [6].

The relationship between task difficulty and performance is reflected by cortical activation during task performance. One assumption is that anxious individuals worry more about a demanding and potentially threatening task and how to cope with it. As a result, these anxious participants try to employ strategies to reduce effects of anxiety to master the task, which is reflected by enhanced activation in amygdala and reduced recruitment of areas associated with cognitive control and inhibition such as dorsolateral prefrontal cortex (DLPFC; see [7], for a review). In contrast, non-anxious participants might also feel anxious about a difficult upcoming task but then, however, successfully employ processes of emotion-regulation (e.g., [8]) to reappraise negative feelings in unemotional terms, which may be reflected by increased activation in these areas associated with cognitive control.

Interestingly, negative feelings are also often reported in the context of mathematical tasks (e.g., [7–9]). For instance, dealing with numbers was shown to induce intense negative emotions and stress so reliably that mental arithmetic is one of the most commonly used tasks to induce stress in laboratory settings (e.g., [10–13]). Increased stress levels may, for instance, be reflected by increases in heart rate and blood pressure (e.g., [14,15]). It is possible that these strong negative reactions to mathematical tasks reflect a specific anxiety associated with numerical tasks (i.e., math anxiety, e.g. [9]). It is, however, also conceivable that the negative reactions simply reflect the fact that math is perceived as difficult. Interestingly, more difficult numerical tasks seem to elicit stronger negative emotions and physiological responses than easier numerical tasks (e.g., [16,17]; see also [8] for neural correlates). Therefore, probably both, difficult tasks as well as numerical tasks are associated with negative feelings in general, so that the strongest emotional and physiological response to math should be observed when participants have to solve difficult mathematical tasks [8] in particular.

As mentioned above, it is possible that strong negative reactions to mathematical tasks simply reflect that math is perceived difficult. Nevertheless, there is a rich body of literature on math anxiety, showing that if such negative emotional responses to math or the anticipation of having to do math cannot be regulated or compensated, performance in numerical tasks is significantly reduced in a wide variety of everyday life and academic situations. In this case, affected individuals are classified as math-anxious (e.g., [18,19]; for a review see [20]). Math-anxious individuals were observed repeatedly to perform poorly in tasks which involve numerical information, while their performance in other general reasoning tasks is not affected and typical (e.g., [21,22]).

In order to deepen our understanding of underlying mechanisms leading to decreased numerical information processing in the context of difficult math tasks, research on the neuro-cognitive underpinnings of negative emotional reactions while doing math is highly relevant. However, it was suggested that a negative emotional response to mathematic is only observed in high math-anxious participants [8,9]. Moreover, because emotional responses to math are strongest when participants have to solve difficult mathematical tasks, Lyons and Beilock [8,9] evaluated neural activation patterns in response to difficult tasks from participants performing both, difficult and easy numerical and non-numerical tasks. Importantly, the authors evaluated neural activation during the actual completion of the task [8] as well as during the mere anticipation of having to do the respective numerical task, this means, following presentation of a cue indicating the nature of the upcoming task (i.e., difficult vs. easy [8,9]). The authors observed that already after the presentation of a cue indicating a difficult upcoming numerical task and thus before the respective task has to be performed, brain regions associated with the processing of negative emotions [8] and even pain [9] were activated. The network for negative emotions comprised bilateral hippocampus and (pre)frontal areas [8], but not the amygdala, which is typically activated in emotion processing [23], whereas the pain network included the insula and middle cingulate cortex [8].

However, Lyons and Beilock [8,9] reported that math-anxious participants, who showed typical and thus unimpaired performance in math tasks, activated not only neural networks associated with the processing of negative emotions and pain, but also a network associated with cognitive control processes. This cognitive control network was argued to be involved in regulating negative feelings of fear, despair or pain by reappraisal and involved dorsolateral prefrontal cortex (DLPFC) and anterior cingulate cortices (ACC). Importantly, areas forming this cognitive control network also seem to play a key role in emotion regulation more generally (e.g., [24]; for a review see [25]).

Interestingly, neural networks for processing negative emotions and pain as well as for evincing cognitive control were shown to be activated already when participants anticipated the upcoming numerical task. This indicates that emotional effects associated with math can already be observed when investigating the time interval between cue presentation and the beginning of the actual task [8,9] and these emotional effects are thus not confounded by neural activation associated with actually performing the task. In case emotion regulation is successful, the task at hand may then be performed with all available cognitive resources [21] so that performance should not be impaired [8].

However, it is important to note that the latter assumptions of Lyons and Beilock (i.e., no impairment of performance with successful emotion regulation) are based on the comparison of highly math-anxious with low math-anxious participants (i.e., participants scoring in the upper vs. in the lower quintile in a math anxiety screening test [8] and on a comparison within highly math-anxious participants [9]. According to the authors, the network associated with processing negative emotions should only be observed in highly math-anxious participants because low math anxious persons “[...] do not have a negative emotional response in anticipation of math that requires reinterpreting" ([8], p. 2108). If this assumption was true, one should neither be able to observe activation within this network for processing negative emotions in non-math-anxious participants when anticipating a difficult upcoming numerical task, nor should non-math-anxious participants express negative feelings when asked after task completion (e.g., in a questionnaire). Additionally, they should not show activation in the cognitive control network subserving the regulation of emotions.

However, there is evidence showing that i) negative emotional responses in anticipation of a difficult task seem to be more like a general tendency rather than an exception (cf. [26] for a review) and ii) that these negative emotions can be successfully regulated using reappraisal [27] and mechanisms of cognitive control [24,28]. According to the process model of emotion regulation, reappraisal is an early emotional control process and provides one of the most effective means to diminish the negative emotions associated with an aversive event [24,27,29]. When the regulation of the initial emotional response is successful, task performance will be better, as demonstrated by Lyons and Beilock [8,9] for the case of highly math-anxious participants. Therefore, we hypothesize that non-math-anxious participants should not necessarily be characterized by the absence of activation in the network associated with processing negative emotions. Instead non-math-anxious participants should be characterized by the presence of (sufficient) activation in networks subserving cognitive control/ emotion regulation during number processing – an interplay which was shown by Lyons and Beilock [9] for highly math-anxious participants with normal math performance.

The present study

In this study, we aimed at investigating neural activation of non-math anxious participants during the anticipation of difficult and easy numerical tasks. We used a numerical task because math is typically perceived as difficult and demanding. In particular, we employed a comparison task on relative magnitudes (i.e., fractions and proportions) and investigated neural activation in response to cues indicating these upcoming tasks. We used magnitude comparison of proportions, because fractions and proportions are difficult enough to elicit emotional responses in non-math-anxious participants. At the same time, proportions are also well suited for manipulating task difficulty. To avoid that observed effects are driven by notation-specific processes, we used both symbolic and non-symbolic proportions. This is important because we wanted to evaluate rather general cognitive processing mechanisms. Therefore, we did not expect to observe specific IPS activation, because the IPS is generally assumed to subserve the processing of number magnitude in a notation independent manner [30–33]. As it was found that magnitude comparison of decimals is easier than magnitude comparison of fractions [34], we used fraction comparison as the more difficult and decimal comparison as the easier condition in symbolic notation. For non-symbolic notation, we used proportions visualized by dot patterns (i.e., the relation of blue to yellow dots, see also Fig 1) as the more difficult and pie charts as the easier condition according to behavioral pilot data.

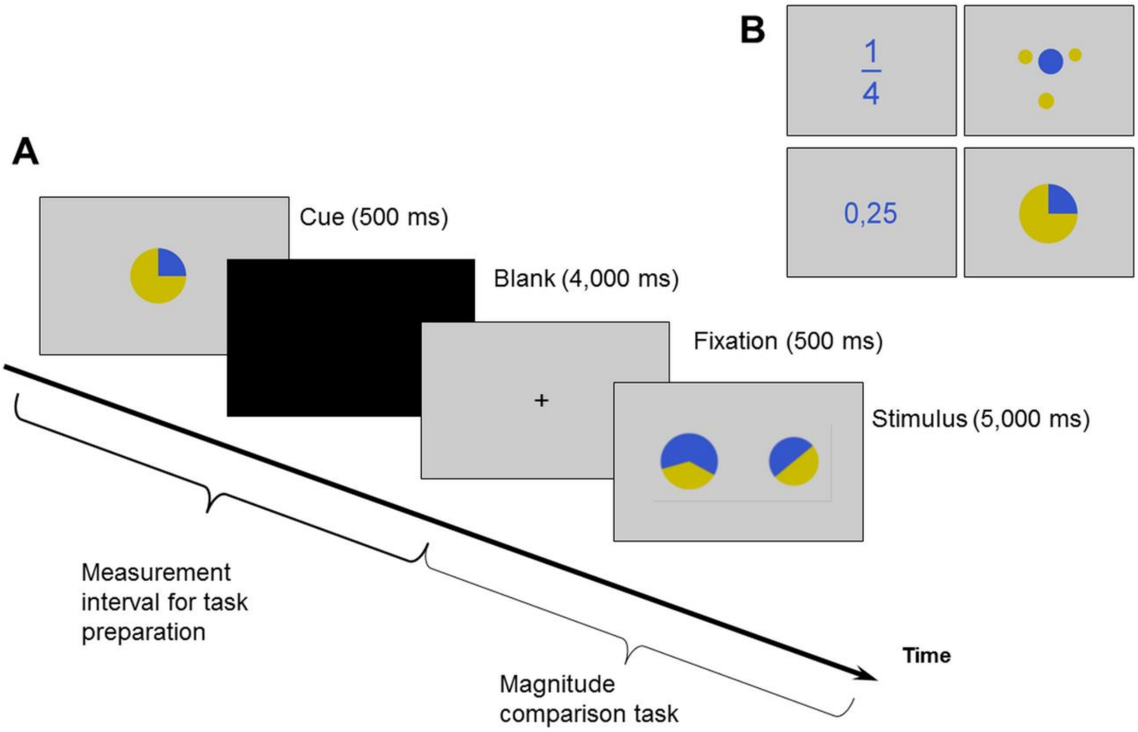

Figure 9.1: Experimental design. (A) Illustration of the experimental procedure at the beginning of each block (i.e., one out of five trials). (B) Presented cues in all notation formats.

Moreover, to complement neural activation data with a subjective measure of emotional responses and to determine whether these responses were actually negative or positive, participants also answered a stress appraisal questionnaire after completing the tasks. We hypothesized that negative emotional responses to the anticipation of doing difficult math are present in all individuals irrespective of their math-anxiety level. In this case, it would be less important how strong this emotional response is rather than how well it can be regulated.

Thus, we hypothesized that the more difficult tasks should elicit stronger negative emotional responses in our non-math-anxious participants both subjectively (as shown in a questionnaire) and objectively (as shown by neural activations during anticipation) independent of presentation notation. Since we evaluated non-math-anxious participants, we also expected concurrent activation in a network associated with cognitive control reflecting emotion regulation. In order to determine the influence of math anxiety values subjectively assessed with the Abbreviated Math Anxiety Scale (AMAS; [35]) on activation patterns, the activation predicted by math anxiety values was modelled separately.

In addition to this hypothesis on notation-independent processes of emotion regulation, we also expected specific differences in activation patterns for symbolic digital and non-symbolic cues. In particular, we expected to find activation in areas associated with the identification of Arabic digits such as the visual number form (VNF) following symbolic cues (i.e., fractions, decimals). As the VNF is supposed to be automatically involved in the processing of visual numerals [36,37], whenever one visually perceives symbolic numerical stimuli.

9.2 Methods and Materials

Participants

25 right-handed adult volunteers (13 female, mean age = 23.2 years; SD = 2.99 years) participated in the study. Written informed consent was obtained from all participants. The experiment was approved by the local Ethics Committee of the Medical Faculty of the University of Tuebingen (274/2013BO2). All participants reported normal or corrected to normal vision and no previous history of neurological or psychiatric disorders. In particular, neurological and psychiatric disorders were assessed using both, a detailed self-assessment questionnaire and an diagnostic questionnaire, which was completed by a specifically trained and certified MR investigator to rule out anxiety disorders according to DSM-5 [38]. Moreover, participants were not math anxious according to the Abbreviated Math Anxiety Scale (AMAS; [35]). Only participants who were not taking any medications other than oral contraceptives were included in the study.

Stimuli and Design

In the magnitude comparison task, participants had to decide which of two presented proportions was the larger one. We used four different presentation notations of proportions: fractions, decimals, pie charts, and dot patterns (see Fig 1A for an example). Each block of magnitude comparison tasks was preceded by a cue indicating the respective proportion notation to be expected on the next five trials. The cue always was the proportion 1/4 shown in the different notations in the center of the screen against a grey background (see Fig 1B). A total of 24 items entered the cues analysis. Importantly, we evaluated only the neural response following cue presentation, but not the activation during the actual magnitude comparison.

For the magnitude comparison tasks, we constructed 30 items for each of the four presentation notations. Proportions were presented in pairs with the magnitude of the first proportion ranging from 0.13 to 0.86 and the second proportion ranging from 0.22 to 0.89. Absolute distances between proportions ranged from 0.02 to 0.22.

Before and after completing the proportion comparison task within the scanner, participants were asked to rate their anticipated feelings regarding the four presentation notations using an adapted version of the Stress Appraisal Measure [39,40] provided as paper and pencil questionnaire. In this Stress Appraisal Measure, six items (three items each) assessed challenge (positive emotional valence) and threat (negative emotional valence) participants experienced related to the task (e.g., challenge, positive: “I think I can master these tasks”; threat, negative: “I am afraid of not being able to solve the tasks”). The idea is that individuals assess whether they can master a difficult task by weighing the perceived task demands (e.g., task effort) against their perceived resources (e.g., skills; [41,42]). A task is considered as a threat when their task demands seem to outweigh their resources or as a challenge when their resources seem to match or exceed task demands [42,43].

Procedure

Before entering the scanner, participants completed the Stress Appraisal Measure. Then participants were put into the scanner and stimuli were projected on a screen above their head. Participants viewed the stimuli through a mirror mounted on the head coil of the scanner. Fractions as well as decimals were presented in blue (RGB-values: 53, 85, 204; font type: Arial; font size: 80) against a grey background

(RGB-values: 204, 204, 204). Pie charts were drawn by dividing circles into two pie segments, one depicted in blue (same blue as for fractions) and the other in yellow (RGB-values: 203, 187, 0; see Fig 1B) against the same grey background color. Dot patterns were colored according to the fractions they denoted using the same colors as for the pie charts.

Head movements were prevented by using foam pads. To familiarize participants with the task, all volunteers were given the opportunity to practice on several items of each condition preceded by the respective cues before starting the actual experiment. None of these practice items was repeated during the critical measurement.

At the beginning of each block a cue was presented for 500ms that indicated the respective proportion notation to be expected on the next five trials. Subsequently, a black screen was presented for 4,000ms. To investigate processes associated with handling inherent numerical features of and affective associations with the numerical cues, we chose a design, in which the presentation of a cue was followed by a long-time interval (4,000ms) with no further visual input until the cued task was actually presented. As pointed out by Brass and von Cramon [44], it might be difficult to otherwise isolate task preparation from task execution using neuroimaging methods (see also [8,9]). While the time needed to prepare for a task may be very short, the hemodynamic response is comparably slow (i.e., peaking at about 6 seconds post stimulus, cf. [45]). This can possibly lead to an overlap of the hemodynamic responses for the cue and target period.

After 4,000ms, comparison trials were presented starting with a black fixation cross on a grey background for 500ms, followed by the presentation of a proportion comparison stimulus for up to 5,000ms (see Fig 1A for an illustration of the procedure at the beginning of a block). Participants had to respond within this time frame by pressing one of two MRI compatible response buttons with either their left (indicating left proportion larger) or right thumb (indicating right proportion larger). Proportion comparison items were presented in six blocks of 5 items each. After one block was completed, the next block was introduced by the next cue. We focused our analyses of neural activation observed during cue presentation and the following 4,000ms of a blank screen. Activation during the actual comparison of proportions was not considered in the present study.

(f)MRI acquisition

MRI data were acquired using a 3T Siemens Magnetom TrioTim MRI system (Siemens AG, Erlangen, Germany). A high resolution T1- weighted anatomical scan (TR = 2300 s, matrix = 256 × 256, 176 slices, voxel size = 1.0 × 1.0 × 1.0 mm^3; FOV = 256 mm^2, TE = 2.92 ms; flip angle = 8°) was collected at the end of the experimental session. All functional measurements covered the whole brain using standard echo-planarimaging sequences (TR = 2400 ms; TE = 30 ms; flip angle = 80°; FOV = 220 mm^2, 88 x 88 matrix; 42 slices, voxel size = 2.5 × 2.5 × 3.0 mm^3, gap = 10%). fMRI data was acquired in a single run. Total scanning time was approximately 20 minutes. We included pauses between blocks in which a black screen was presented for 6,000 ms.

Analysis

Behavioral analysis

Because the primary focus of the current study was on the neural correlates of processing numerical cues for task preparation activation, we only report the analysis of behavioral data with the view of a manipulation check for our 2 x 2 manipulation of task difficulty and notation.

We inspected response times (RT) and error rates (ER) to examine whether the difficulty of the presentation notations differed. Initial inspection of the behavioral data indicated that the distribution of response times was strongly skewed to the right, in particular for decimals (skewness: 2.608, SD=.456) and pies (skewness: 2.426, SD=.456). Therefore, we applied the inverse transformation converting response times into speed with measurement unit 1/sec to approach normal distribution [46]. This way larger values indicate faster speed, while smaller values indicate slower responses.

Speed and error rates were analyzed running (generalized) linear mixed models [(G)LME] to include random effects for both participants and items (e.g., [47]). In the GLME (for ER), we used the logit as the link function and assumed a binomial error distribution. We included the fixed effect of presentation notation (fractions, decimals, pie charts, and dot patterns) and random intercepts for participants as well as items (crossed random effects), and a random slope for presentation notation in the LME as well as the GLME.

In the analysis of speed, we considered correctly solved trials only. Moreover, we removed trials with absolute z-scaled residuals of the full model larger than 3. In total, the analysis of speed was based on 82.6% of all trials. Data from the appraisal questionnaire completed after the experiment on negative (‘threat’) as well as positive (“challenge”) emotions towards the four conditions were analyzed each by a 2 × 2 repeated measures ANOVA discerning the two factors task difficulty (difficult vs. easy) and notation (symbolic vs. non-symbolic).

Statistical analyses were run using R [48] and the R packages lme4 [49] and afex for executing the (G)LME [50]. *P*-values for fixed effects of LME were calculated running F-Tests using the Kenward-Roger approximation for degrees of freedom (e.g., [51]) and for GLME, we carried out likelihood ratio tests (LRT). Post-hoc tests were run relying on the R package lsmeans [52] and the Tukey HSD (honestly significant difference) method was used to adjust *p*-values for multiple comparisons. Plots were drawn using the R packages ggplot2 [53] and cowplot [54].

Imaging analysis

Imaging data analysis was performed using SPM12 (http://www.fil.ion.ucl.ac.uk/spm). Images were slice-time corrected, motion corrected, and realigned to each participant’s mean image. The mean image was co-registered with the anatomical whole-brain volume. Imaging data was then normalized into standard stereotaxic MNI space (Montreal Neurological Institute, McGill University, Montreal, Canada). Images were resampled every 2.5 mm using 4th degree spline interpolation to obtain isovoxels and then smoothed with a 6 mm full-width at half-maximum (FWHM) Gaussian kernel to accommodate inter-subject variation in brain anatomy and to increase signal-to-noise ratio in the images. Data were high-pass filtered (128s) to remove low-frequency noise components and corrected for autocorrelation assuming an AR(1) process.

In the first-level analysis, the onsets of the cues for the four presentation formats (i.e., fractions, decimals, pie charts, dot patterns) were entered as separate conditions in the GLM. Importantly, the neural response associated with the critical items was evaluated from the beginning of each cue presentation until the start of the fixation cross preceding the magnitude comparison task (duration of 4,500ms). Thus, activation during the actual comparison of proportions was not considered in the present analysis. Movement parameters estimated at the realignment stage of preprocessing were included as covariates of no interest. Motion parameters did not exceed 2.5 mm translation in total (i.e., they did not exceed voxel size) and a head rotation of 1.5 degree in pitch, roll, and yaw in total. Therefore, none of the participants had to be excluded from the analyses because of head movements. Brain activity was convolved over all experimental trials with the canonical hemodynamic response function (HRF) as implemented in SPM12 and its time and dispersion derivatives.

These contrast images then entered the second-level random-effects group analysis. The second-level analysis was realised using a flexible factorial design for repeated measures with difficulty (easy/difficult) and notation (symbolic/non-symbolic) as within-subject factors as well as math anxiety (AMAS score) as covariate. We evaluated both, main effects of difficulty and notation as well as the interaction between the two factors. Additionally, we evaluated the fMRI signal explained by low or high values of the covariate math anxiety.

The SPM Anatomy Toolbox [55], available for all published cytoarchitectonic maps (www.fz-juelich.de/ime/spm_anatomy_toolbox), was used for anatomical localization of effects where applicable. In areas not yet implemented, the anatomical automatic labelling tool (AAL) in SPM12 (http://www.cyceron.fr/web/aal anatomical_automatic_labeling.html) was applied. Activations were thresholded at an uncorrected p-value of < .001 at the voxel level with a cluster size of k = 10 voxels and were reported when they remained significant following family-wise error correction (FWE) at the cluster-level with $p_{\text{cluster-corr}} < .05$.

9.3 Results

9.3.1 Behavioral results

The (ordinal) interaction between notation and difficulty was significant for speed, $F(1, 24.45) = 78.96$, $p < .001$ (see Fig 2A) indicating that the effect of the factor difficulty was more pronounced for symbolic as compared to non-symbolic notation.

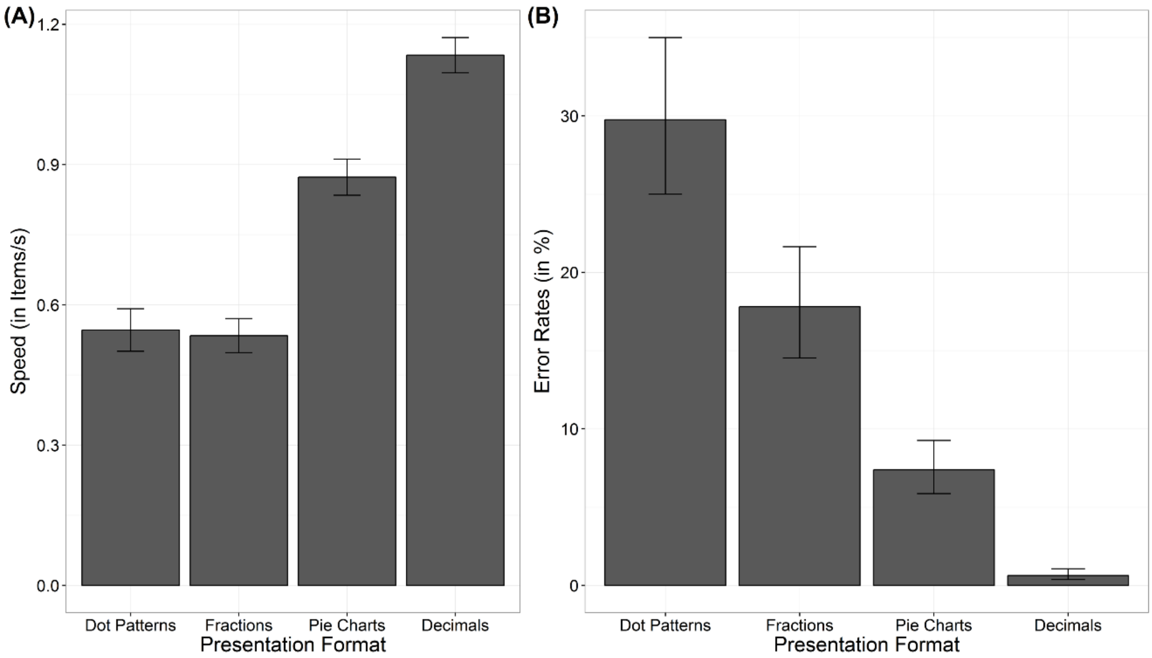

Figure 9.2: Behavioral data as manipulation check. (A) mean speed and (B) error rates in the four conditions (dot patterns, pie charts, fractions, and decimals). Error bars indicate one standard error of the mean.

We further inspected the interaction by running pairwise post-hoc comparisons among conditions. We found that speed differed significantly between these different presentation formats [dot patterns vs. pie charts: $t(28.70) = 8.65$, $p < .001$; dot patterns vs. decimals: $t(24.37) = 12.64$, $p < .001$; fractions vs. pie charts: $t(26.33) = 10.04$, $p < .001$; fractions vs. decimals: $t(29.71) = 18.08$, $p < .001$; pie charts vs. decimals: $t(28.53) = 9.48$, $p < .001$], except for dot patterns and fractions [$t(32.28) = 0.39$, $p = .980$]. Thus, decimals were compared fastest ($M = 1.34$ item/s, $SE = 0.04$ items/s), followed by pie charts ($M = 0.87$ item/s, $SE = 0.04$ items/s), whereas fractions ($M = 0.54$ item/s, $SE = 0.04$ items/s) and dot patterns ($M = 0.55$ item/s, $SE = 0.05$ items/s) were compared about equally fast. Moreover, main effects of notation, $F(1, 27.75) = 24.93$, $p < .001$, and difficulty, $F(1, 24.24) = 208.88$, $p < .001$, were significant. However, the main effect of notation should not be interpreted, because the (simple) main effect of notation was only present for easy tasks [pie charts vs. decimals: $t(28.53) = 9.48$, $p < .001$] (i.e., not for dot patterns vs. fractions [$t(32.28) = 0.39$, $p = .980$]). In contrast, there were significant differences between easy and difficult tasks for both symbolic [fractions vs. decimals: $t(29.71) = 18.08$, $p < .001$] and non-symbolic [dot patterns vs. pie charts: $t(28.70) = 8.65$, $p < .001$] notations. This indicated that overall, easier tasks were compared faster than more difficult tasks (easy: $M = 1.00$ item/s, $SE = 0.04$ items/s vs. difficult: $M = 0.54$ item/s, $SE = 0.04$ items/s).

We also observed a significant interaction between notation and difficulty for ER, $\chi^2(1) = 22.92$, $p < .001$. It indicated that - although all pairwise comparisons were significant (dot patterns vs. fractions: $z = 2.92$, $p = .019$; dot patterns vs. pie charts: $z = 8.33$, $p < .001$; dot patterns vs. decimals: $z = 8.09$, $p < .001$; fractions vs. pie charts: $z = 4.84$, $p < .001$; fractions vs. decimals: $z = 7.14$, $p < .001$; pie charts vs. decimals: $z = 4.95$, $p < .001$) – the difference between symbolic and non-symbolic notation was smaller for difficult as compared to easy comparisons (difficult: log odds = 0.57, $SE = 0.23$ vs. easy: log odds = 2.53, $SE = 0.51$). For reasons of readability, the following descriptions of results also incorporate ERs in percent. Accordingly, error rates for decimals were lowest (log odds: $M = -5.06$, $SE = 0.53$; 1%), followed by pie charts (log odds: $M = -2.53$, $SE = 0.25$; 7%) and fractions (log odds: $M = -1.53$, $SE = 0.24$; 18%) and highest for dot patterns (log odds: $M = -0.86$, $SE = 0.24$; 30%). Again, main effects of notation, $\chi^2(1) = 27.73$, $p < .001$, as well as difficulty, $\chi^2(1) = 46.32$, $p < .001$, were significant. The main effect of notation indicated that comparing symbolic proportions (log odds: $M = -3.30$, $SE = 0.32$; 4%) was less error prone than comparing non-symbolic proportions (log odds: $M = -1.29$, $SE = 0.22$; 16%). Furthermore, participants' error rates were lower in easier (log odds: $M = -3.80$, $SE = 0.32$; 2%) than in more difficult tasks (log odds: $M = -1.19$, $SE = 0.21$; 23%).

Results for Appraisal questionnaire

The ANOVA on *threat* (i.e., negative emotions) revealed main effects of both notation, $F(1, 24) = 21.45$, $p < .001$, and difficulty, $F(1, 24) = 31.93$, $p < .001$. The main effect of notation indicated that participants rated proportions presented in non-symbolic notations (i.e., dot patterns and pie charts) more negative (i.e., threatening, M = 7.86, SE = 0.55, 95 % CI = [6.74, 8.98]) than proportions presented symbolically (i.e., fractions and decimals, M = 6.00, SE = 0.55, 95 % CI = [4.88, 7.12]). The main effect of difficulty indicated that participants rated difficult proportions (i.e., fractions and dot patterns) as more negative (i.e., threatening, M = 8.66, SE = 0.59, 95 % CI =

[7.46, 9.86]) than easy proportions (i.e. pie charts and decimals, M = 5.20, SE = 0.59, 95 % CI = [4.00, 6.40]). The interaction between notation and difficulty was not significant, $F(1, 24) = 1.78$, $p = .60$.

The ANOVA on *challenge* (i.e., positive emotions) revealed a main effect of difficulty, $F(1, 24) = 16.43$, $p < .001$. The main effect of difficulty indicated that participants rated easy proportions more positive (i.e., challenging, pie charts and decimals, M = 4.73, SE = 0.24, 95 % CI = [4.24, 5.23]) than difficult proportions (i.e., fractions and dot patterns, M = 4.20, SE = 0.23, 95 % CI = [3.72, 4.68]). There was neither a main effect of notation, $F(1, 24) = 2.11$, $p = .16$, nor an interaction between notation and difficulty, $F(1, 24) < 1$, $p = .44$."

9.3.2 Imaging results

The *F*-contrast of the ANCOVA revealed no supra-threshold clusters for the interaction between difficulty (easy/difficult) and notation (symbolic/non-symbolic).

The analysis of the main effect of difficulty yielded the following results: Cues indicating a difficult upcoming task (dots, fractions) as compared to an easy task (pies, decimals) led to increased activation in a network including bilateral amygdala, bilateral ACC, bilateral hippocampus, left temporal gyrus and bilateral paracentral gyrus (Fig 3, Table 9.1). For the opposite contrast (cues for easy vs. difficult tasks) no supra-threshold clusters were observed.

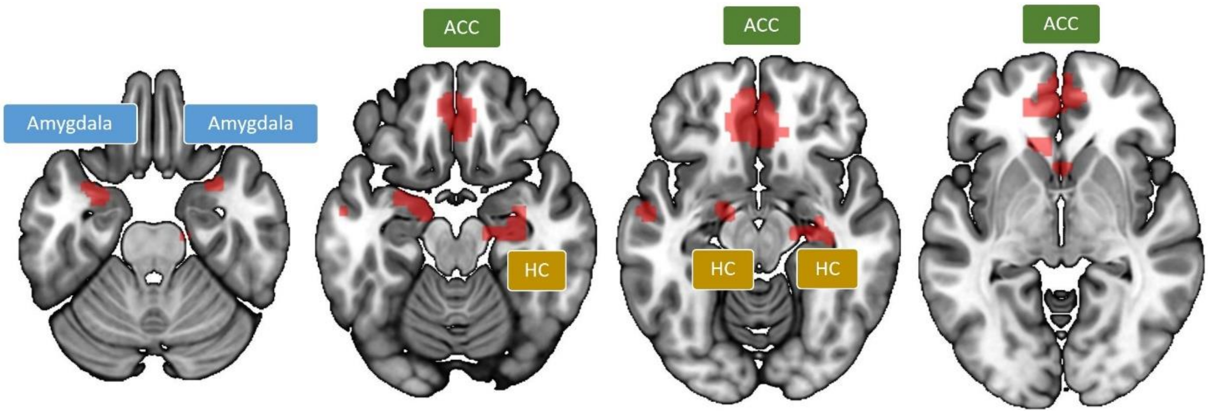

Figure 9.3: Negative emotional response to anticipating difficult math. Negative emotion network stronger associated with cues indicating upcoming difficult (including fractions and dots) than with cues indicating an easy proportion comparison task (involving pies and decimals). Abbreviations: ACC – anterior cingulate cortex; HC – hippocampus.

Table 9.1: Cortical regions more strongly activated in the conjunction of viewing at cues for upcoming dots and fractions (difficult conditions) compared to pies and decimals (easy conditions), controlled for math anxiety values as measured with the AMAS.

Contrast	Brain region	MNI (x, y, z)			Cluster size	t
Difficult vs. easy	LH anterior cingulate cortex	-5	43	-15	446	5.59
	RH anterior cingulate cortex	3	31	-15		4.64
	LH amygdala	-27	1	-23	63	4.61
	RH amygdala	28	6	-28	15	3.75
	RH hippocampus	31	-10	-18	66	4.13
	LH middle temporal gyrus	-57	-10	-13	15	3.92
	LH paracentral gyrus	-7	-40	73	46	4.50
	RH postcentral gyrus	13	-42	70	31	4.33

*$p_{cluster\text{-}corr}$ < .05 (k = 10 voxels); LH: left hemisphere; MNI: Montreal Neurological Institute coordinates; RH: right hemisphere; *: minor maximum; t = t-value.*

The main effect of notation revealed no supra-threshold activation differences between conditions when corrected for multiple comparisons. However, as we had the hypothesis that symbolic vs. non-symbolic cues should activate the VNF, we also did the analysis at an uncorrected *p*-value of .001 and found activation in the left inferior temporal gyrus at MNI coordinates -51, -55, -16 (t = 3.22).

Neither large nor small values of the covariate math anxiety (as reflected by participants' AMAS scores) explained any suprathreshold cluster of activation. This means that math anxiety scores did not explain variance of the fMRI signal during cue presentation. For the sake of completeness, simple effects for each of the four cues (dots, fractions, pies, decimals) are given in the supplementary materials (Figures S9.1 – S9.3 and Tables S9.1 – S9.3).

9.4 Discussion

The mere anticipation of doing difficult tasks is often associated with negative emotions (e.g., [5,56]). However, in the case of math such a negative emotional response was proposed to be only observed in highly math-anxious participants according to Lyons and Beilock [8]. In the present study, we aimed at evaluating i) that negative emotional responses in anticipation of a difficult math task is a general response, which can be seen in non-math-anxious individuals as well and ii) that it is the regulation of the initial emotional response which is crucial for later task performance.

Subjective and objective behavioral measures confirmed that our manipulation of difficulty was successful. Additionally, difficult items were subjectively rated as more negative (i.e., threatening), while easy items were rated as more positive (i.e., challenging). More specifically, our non-math-anxious participants explicitly indicated that more difficult comparisons of fractions and dot patterns were associated with more negative feelings. This further supports the notion that activation associated with the anticipation of difficult trials indeed reflects processes related to negative emotions.

In line with our hypothesis, imaging data revealed a common (sub-)network associated with the processing of negative emotions activated independently of notation (symbolic/non-symbolic) and math anxiety scores, but dependent on the degree of difficulty. In our non-math-anxious participants, cues indicating a more difficult upcoming task led to activation in a network comprising bilateral amygdala and hippocampus, which has been previously associated with the processing of negative emotions (e.g., [57,58]). In line with our hypothesis, these participants also revealed activation in areas associated with cognitive control such as anterior cingulate cortex (ACC). Networks of cognitive control have been suggested to be involved in emotion regulation during numerical tasks (e.g., [8,9]). Importantly, when examining whether fMRI signal was predicted by the degree of math anxiety (as reflected by participants' AMAS scores) in our non-math anxious sample, neither high nor low values of math anxiety explained any supratheshold cluster of activation. This means that math anxiety scores did not explain fMRI signal during cue presentation. In other words, while the activation observed in the emotion processing network seems to be associated with the anticipation of an upcoming difficult (math) task, it was clearly not associated with participants' degree of math anxiety.

Finally, in line with our expectations we only found specific activation of the VNF area following symbolic numerical cues, whereas IPS activation was not observed specifically for symbolic notation.

Affective responses associated with cues for difficult numerical tasks

Behavioral data indicated that magnitude comparison performance indeed differed significantly as a function of task difficulty. Comparisons of fractions and dot patterns were responded to slower and with more errors than comparisons of decimals and pies. Imaging data revealed that, when presented with cues indicating a difficult upcoming task, our non-math-anxious participants showed activation in a network associated with the processing of negative emotions comprising hippocampus and amygdala, while math anxiety scores did not modulate the fMRI signal. Lyons and Beilock [8] recently reported activation of a similar network comprising hippocampus in anticipation of doing math for individuals with high math anxiety and claimed that networks of emotion processing should not be observed to be active for non-math-anxious individuals. However, it has been shown in both human lesion and neuroimaging studies that the amygdala, which we additionally found active, plays a crucial role in classical fear conditioning and fear-potentiated startle [57,59]; for a review see [26]. In turn, amygdala and hippocampus were shown to work closely integrated in emotional responses (e.g., [58]). Therefore, it is likely that the network observed in the present study resembles a network for emotion processing. Involvement of networks associated with the processing of emotions and pain when anticipating doing difficult numerical tasks [9] was further substantiated by a conjunction analysis between cues indicating upcoming dot and fraction magnitude comparison tasks (see Table and Figure S9.1).

It needs to be acknowledged, however, that we only measured non-math-anxious participants. As such we cannot be sure that the networks identified in non-math-anxious participants when they anticipated a difficult math task are identical to those recently reported for high math-anxious participants [8,9]. However, math anxiety scores did not explain variance of the fMRI signal. Furthermore, the fact that we observed activation of a network typically associated with the processing of (negative) emotions suggests an emotional reaction of the present non-math-anxious participants anticipating the upcoming difficult math tasks. Additionally, it is important to note that these results do not reflect absolute activation of the network while participants anticipated the difficult math task but the relative stronger activation of these areas in anticipation of a difficult vs. an easier math task. Finally, the results on the neural level are substantiated by the results of the appraisal questionnaire that suggest that the respective emotions may have been negative indeed (see also [17] for physiological data).

Importantly, cues indicating difficult upcoming tasks also led to activation in bilateral anterior cingulate cortex. ACC is part of the cognitive control network, which is in turn suggested to play a key role in emotion regulation and the regulation of negative emotions via reappraisal (e.g., [24]); for a review see [25,60]. This suggests that in our non-math-anxious participants the initial negative emotional response seemed to be sufficiently regulated so that the participants were not identified as math anxious in the Abbreviated Math Anxiety Scale [35].

This idea of a counter play between initial negative emotional responses and emotion regulation is in line with recent work on math anxiety (e.g., [8,22]). The authors suggested that efficient regulation and control mechanisms of negative emotions before starting a task should increase math performance in highly math anxious participants. Therefore, Maloney and Beilock [22] suggested that training highly math anxious individuals in emotion regulation might limit negative effects of math anxiety on math performance. In turn, better emotion regulation may even lead to better math performance in highly math-anxious people. However, we suggest that negative emotions in anticipation of doing difficult math are to be expected in general, which does not only occur in highly math-anxious but also in non-math-anxious participants. This means that the (successful) regulation of an initial emotional response by means of cognitive control processes might be crucial for actual math performance. In case emotion regulation is sufficient, the task at hand can be performed with all available cognitive resources [21], so that performance does not have to be impaired even in highly math-anxious participants [8]. We suggest that successful emotion regulation accompanied by the experience of better performance may in turn reduce negative feelings such as fear in anticipation of doing math so that individuals with better cognitive control/ emotion regulation should be less likely to develop math anxiety. However, we wish to note that in the current study we did not directly assess emotion regulation or demonstrate a relationship between emotion regulation and task performance. Therefore, this interpretation has to remain speculative; nevertheless, this interpretation is in line with the idea already proposed by Lyons & Beilock [8,9] that it may not be the initial negative affective response, which is indicative of math anxiety, but the inability to regulate this response effectively, which in turn may lead to resource depletion and reduced math performance in those with high math anxiety.

Responses associated with cues containing Arabic digits

We observed no general effect of notation. However, only symbolic cues (i.e., fractions and decimals) led to significant number-specific activation in the posterior inferior temporal gyrus (pITG). This region was reported to selectively respond to visually presented numerals using intraoperative electrocorticography recordings [61] and fMRI [62]. Accordingly, the authors suggested that the visual number form might be represented in the bilateral inferior temporal gyri rather than the bilateral fusiform gyri as proposed by Dehaene and Cohen [36,63]. Recently, Daitch et al., [64] substantiated this claim for a subregion within the pITG selectively responding to numerals compared to morphologically similar stimuli using electrocorticography. Therefore, the present data are in line with the notion that pITG might indeed be involved in the processing of visual numerals, while the fusiform gyrus may be less selectively involved in the detection and early non-semantic higher order visual analysis of symbolic and non-symbolic patterns.

Interestingly and in contrast to pITG, IPS activation observed in the current study was neither specific for symbolic notation nor for task difficulty. This is consistent with the findings of Shi et al. [65], who reported IPS activation associated with the mere anticipation of numerical magnitude comparison without the actual presentation of numbers themselves. As regards notation-related effects, our results are also in line with the literature because the IPS is generally assumed to subserve a notation-independent representation of number magnitude [30–33].

Implications for the concept of math anxiety

Our results are fully consistent with the account proposed by Lyons and Beilock [8,9] that more difficult numerical tasks may elicit negative emotions or even associations with pain, which require counter regulation by processes of cognitive control so that individuals can keep their focus on accomplishing the task at hand. However, the present results indicate that this may not be exclusively the case for math-anxious individuals, as suggested by Lyons and Beilock [8,9]. Instead, these data imply that the account of initial (negative) emotional responses and the need for subsequent counter regulation of these responses generalizes also to the case of non-math-anxious participants. Thus, a neuronal signature of negative emotions in anticipation of doing difficult math seem to be a rather general response taking place in the human brain, even in individuals not diagnosed with math-anxiety. This would be in line with previous findings that individuals generally become more anxious when anticipating a relatively difficult task and thus require emotional regulation [56,66].

However, we agree with Lyons and Beilock [8,9] who question whether or not this initial negative response actually hinders math performance might depend on the ability to regulate these emotions. We suggest that successful regulation of negative emotions not only helps to solve the actual tasks (reflected by typical task performance), as proposed by Lyons and Beilock [8,9], but that, depending on cognitive predisposition and other factors such as social influences [22], successful cognitive regulation might also prevent the development of math anxiety. This might have been the case for our non-math-anxious participants. Therefore, our study supports the idea that it should be more effective to train math-anxious individuals in emotion regulation to foster cognitive control processes (see for example [20]) than to train mathematical tasks themselves, as previously proposed by Lyons and Beilock ([8]; see also [22]).

9.5 Limitations of the present study

It is important to note that there are some aspects that need to be considered when interpreting the results of the current study. First, we did not assess a non-mathematical control condition. Therefore, we cannot and do not want to make any claims on whether the effect observed is a general effect of difficulty or indeed specific to the anticipation of difficult math tasks. However, we would suggest that the idea of a negative emotional response in anticipation of doing difficult tasks might rather be a general mechanism instead of a mechanism specific to vulnerable individuals. Future studies are needed to decide whether this finding is limited to the case of mathematical tasks or not.

Furthermore, we evaluated non-math-anxious participants only. Therefore, we cannot tell for sure whether the observed activation patterns in anticipation of doing difficult (math) tasks are identical in low and highly math-anxious participants. However, we want to point out that the general neural response pattern we observed seems to indicate a (negative) emotional response in non-math-anxious participants as well as processes of emotion regulation. Therefore, we suggest that the overall response patterns seem to show at least similar reactions (negative emotions, emotion regulation) to the anticipation of difficult tasks as they have already been shown for highly math-anxious participants. As such, we agree with the account proposed by Lyons and Beilock [8] who described coupled processes of negative emotions with subsequent emotion regulation in anticipation of doing math. Nevertheless, we would like to suggest that this account may also generalize to the case of non-math-anxious individuals. Future studies would be desirable to evaluate this suggestion.

Finally, we wish to note that in the current study we did not directly assess emotion regulation or demonstrate a relationship between emotion regulation and task performance. Therefore, our interpretation that it is the ability to sufficiently regulate emotions that prevents the initial negative response to difficult math tasks from actually hindering math performance must remain speculative for the time being. Nevertheless, we wish to note that our interpretation is generally in line with ideas already proposed by Lyons & Beilock [8,9].

9.6 Conclusion

When anticipating a difficult upcoming task, non-math-anxious participants revealed activation within a network associated with the processing of negative emotions. However, whether or not this initial negative response actually hinders math performance seems to depend on the ability to sufficiently regulate these emotions. While the relevance of such emotion regulation for typical task performance has been suggested before for the case of highly math-anxious individuals, we propose to extend this account to the case of non-math-anxious individuals. We suggest that the observed pattern of neuronal responses on emotion processing and emotion regulation mechanisms seem to indicate a general mechanism rather than a mechanism specific to math-anxious individuals. As such successful emotion regulation might be a general prerequisite for cognitive performance when facing demanding numerical tasks.

Funding

This research was funded by the German Research Foundation (DFG) providing funding to Korbinian Moeller and Elise Klein (MO 2525/2-1), by a Margarete-von-Wrangell Fellowship appointed to Elise Klein (European Social Fund and the Ministry of Science, Research and the Arts Baden-Württemberg) as well as by the Leibniz-Competition Fund (SAW) providing funding for Elise Klein and supporting Stefan Huber, Johannes Bloechle and Julia Bahnmueller. Silke M. Bieck was funded by the LEAD Graduate School [GSC1028], a project of the Excellence Initiative of the German federal and state governments. Korbinian Moeller is principal investigator at the LEAD Graduate School and Research Network [GSC1028], a project of the Excellence Initiative of the German federal and state governments.

The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

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SUPPORTING ONLINE MATERIAL

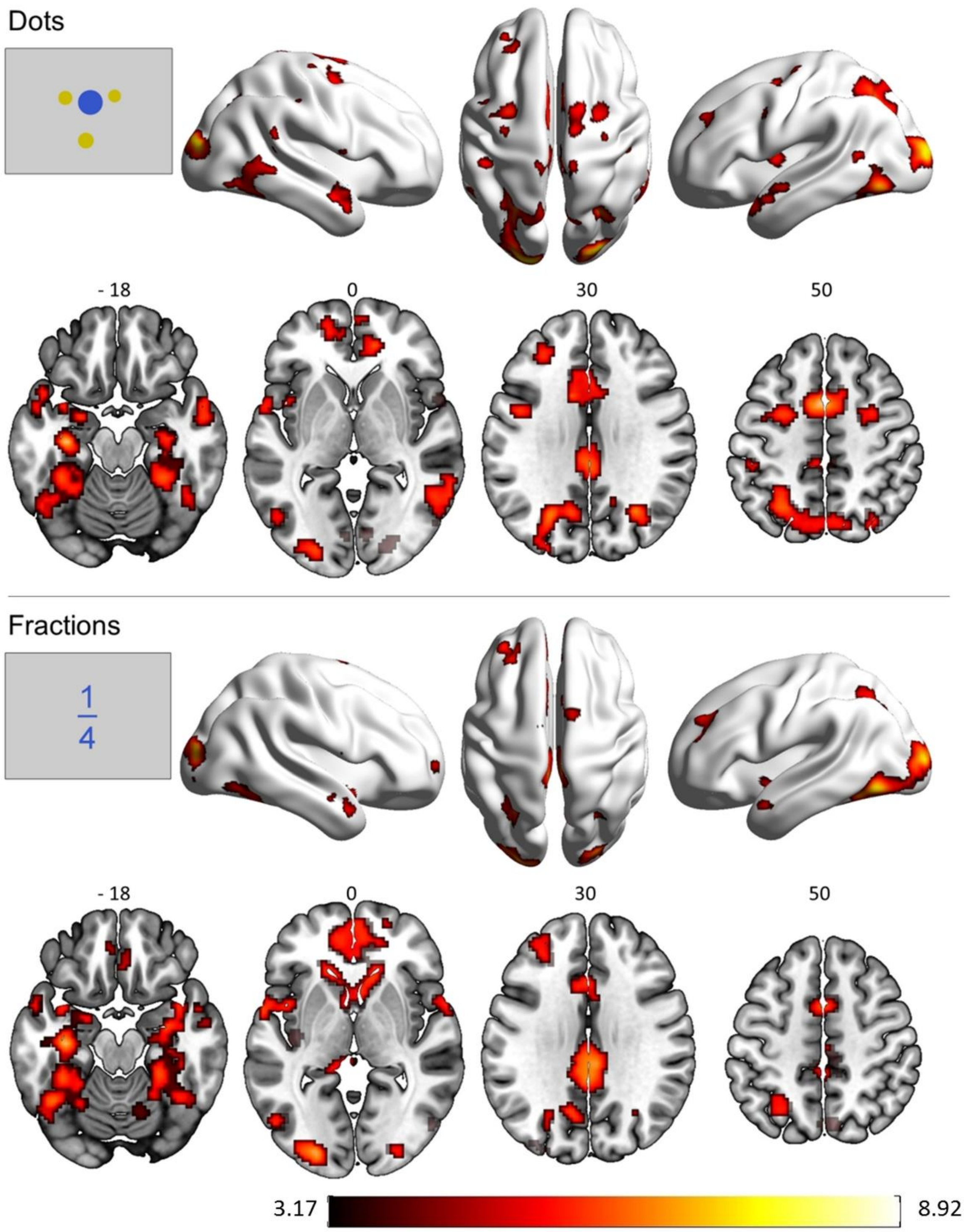


Figure S9.1 Preparation network associated with cues indicating an upcoming magnitude comparison task with either dots or fractions. The color bar indicates *t*-values ($p_{cluster\text{-}corr} < .05$, cluster size $k = 10$). A negative emotion network can be observed including amygdala, hippocampus, insula, and ACC as well as the preparation network (fusiform gyrus, IPS).

Table S9.1 Cortical regions more strongly activated when looking at cues indicating an upcoming dot or fraction magnitude comparison task compared to rest. $p_{cluster\text{-}corr} < .05$ (k = 10 voxels); LH: left hemisphere; MNI: Montreal Neurological Institute coordinates; RH: right hemisphere; t = t-value. * Minor maximum.

Contrast	Brain region	MNI (x, y, z)			Cluster size	*t*
Cue dots vs. baseline	LH amygdala	-30	-2	-30	24	4.47
	LH insula	-40	-2	8	46	4.25
	LH hippocampus	-35	-20	-18	48	6.45
	RH hippocampus	33	-17	-20	18	5.00
	RH anterior cingulate cortex	13	43	0	13	4.68
	LH intraparietal sulcus (hIP3)	-25	-65	43	311	5.48
	LH supplementary motor area	-2	6	53	559	6.71
	RH supplementary motor area*	11	6	45		5.73
	LH middle frontal gyrus	-27	-2	53	27	4.66
	LH middle frontal gyrus	-30	36	30	20	4.25
	RH middle temporal gyrus	66	-47	5	105	4.84
	RH middle temporal gyrus	58	-2	-15	27	4.48
	LH temporal pole	-47	11	-23	43	4.51
	LH middle temporal gyrus*	-45	3	-25		4.47
	RH fusiform gyrus	41	-42	-28	291	7.00
	LH fusiform gyrus	-37	-45	-18	58	4.64
	LH retrosplenial cortex	1	-35	30	103	4.88
	RH precuneus	8	-70	40	54	4.54
	LH inferior occipital gyrus	-47	-70	-13	110	6.24
	RH superior occipital gyrus	31	-70	40	110	5.59
	LH middle occipital gyrus	-20	-95	10	304	8.92
	RH middle occipital gyrus	28	-90	15	165	7.09
Cue fractions vs.	LH amygdala	-22	-5	-23	31	5.23
baseline	LH insula	-40	-15	3	11	4.42
	RH insula	41	6	-10	10	4.24
	LH hippocampus	-35	-20	-18	85	6.32
	RH hippocampus	31	-10	-18	15	4.64
	RH anterior cingulate cortex	3	40	-5	34	5.07
	LH intraparietal sulcus (hIP3)	-27	-63	40	77	5.09

RH supplementary motor area	1	3	55	287	5.45
LH supplementary motor area*	-7	13	45		4.86
RH caudate nucleus	8	18	5	535	5.49
LH middle frontal gyrus	-30	43	35	29	4.64
LH middle frontal gyrus	-35	48	23	17	4.59
LH superior temporal gyrus	-62	-22	13	23	4.64
LH superior temporal gyrus	-60	3	0	46	4.52
RH medial temporal pole	36	11	-33	18	5.21
LH post. inferior temporal gyrus	-45	-65	-10	260	6.61
RH post. inferior temporal gyrus	51	-67	-10	17	4.22
RH inferior temporal gyrus	43	-55	-13	17	4.20
RH fusiform gyrus	33	-40	-25	191	5.27
LH fusiform gyrus	-24	-82	-10	53	5.78
LH precuneus	-12	-65	33	92	4.67
LH retrosplenial cortex	-2	-40	25	274	5.92
LH middle occipital gyrus	-22	-95	8	255	7.65
RH middle occipital gyrus	28	-90	13	129	6.78

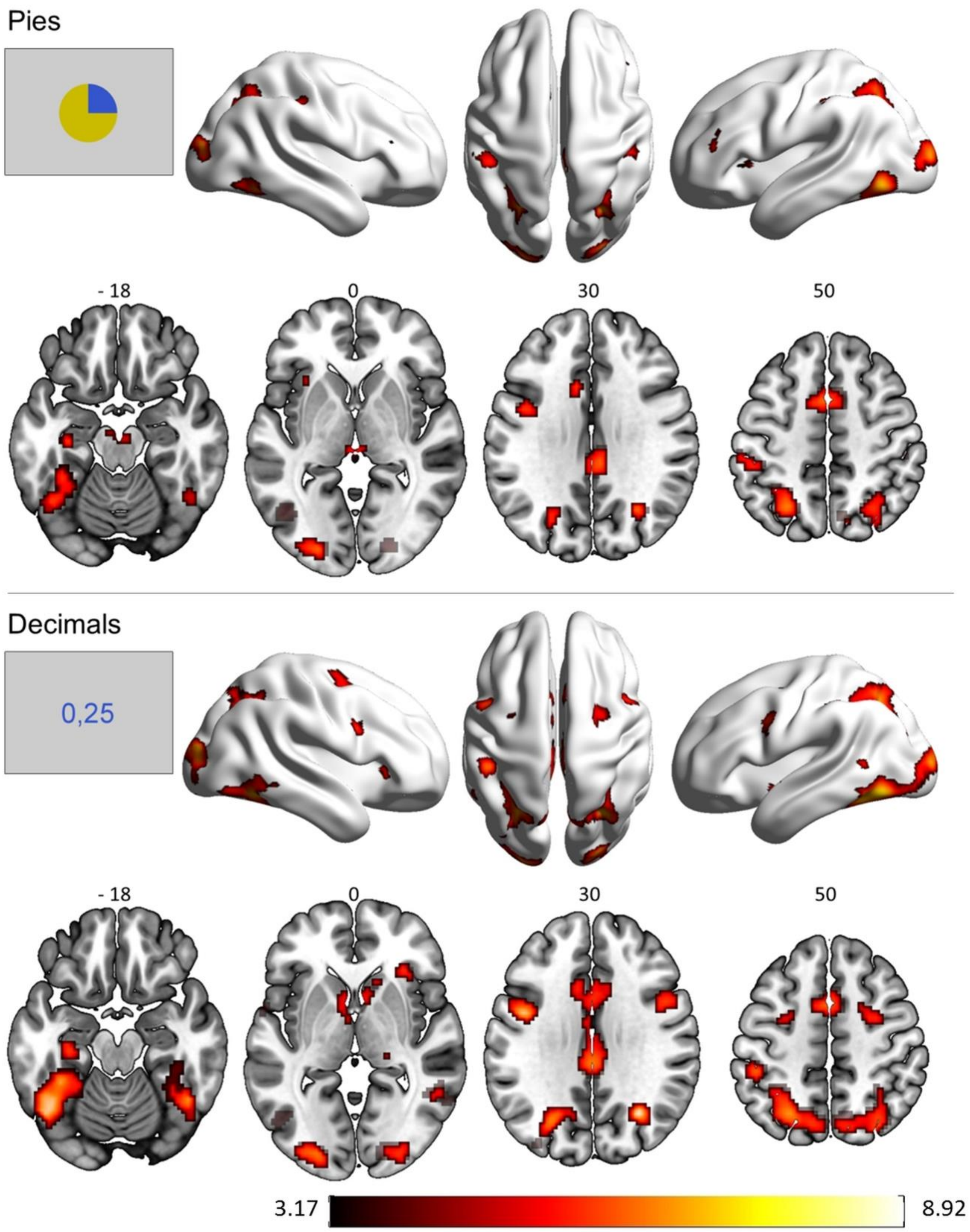


Figure S9.2 Preparation network for cues indicating an upcoming magnitude comparison task with either pies or decimals. The color bar indicates *t*-values ($p_{cluster\text{-}corr} < .05$, cluster size $k = 10$).

Table S9.2 Cortical regions more strongly activated when viewing at cues indicating an upcoming pie or decimal magnitude comparison task compared to rest. $p_{cluster\text{-}corr} < .05$ (k = 10 voxels); LH: left hemisphere; MNI: Montreal Neurological Institute coordinates; RH: right hemisphere; t = t-value. * Minor maximum.

Contrast	Brain region	MNI (x, y, z)			Cluster size	*t*
Cue pies vs. baseline	RH intraparietal sulcus (hIP3)	30	-56	45	246	6.46
	LH intraparietal sulcus (hIP2)	-48	-42	50	58	4.77
	LH superior parietal lobe (PSPL)	-22	-72	55	321	5.88
	LH supplementary motor area	-5	6	55	94	5.28
	LH middle cingulate cortex	-10	13	38	290	4.57
	LH inferior temporal gyrus	-47	-67	-10	158	6.33
	RH inferior temporal gyrus	46	-55	-10	94	5.68
	RH fusiform gyrus	33	-37	-25	109	5.14
	LH fusiform gyrus	-37	-45	-18	344	4.22
	RH retrosplenial cortex	6	-35	30	34	4.89
	LH middle occipital gyrus	-20	-95	10	377	7.37
	RH middle occipital gyrus	28	-90	15	133	6.36
Cues decimals vs. baseline	LH hippocampus	-32	-25	-18	10	4.43
	LH anterior cingulate cortex	1	11	30	17	4.14
	LH middle cingulate cortex	-10	13	38	33	5.23
	LH intraparietal sulcus (hIP3)	-25	-62	48	471	6.96
	LH intraparietal sulcus (hIP2)	-45	-39	43	66	5.48
	RH intraparietal sulcus (hIP3)	33	-52	43	494	4.29
	LH supplementary motor area	-2	6	53	67	4.91
	LH inferior frontal gyrus (44)	-45	1	30	97	6.19
	RH inferior frontal gyrus (45)	46	8	28	22	4.61
	RH middle frontal gyrus	31	-2	53	14	4.32
	LH post. inferior temporal gyrus	-47	-55	-15	322	6.56
	RH post. inferior temporal gyrus	46	-57	-13	291	6.41
	RH retrosplenial cortex	1	-34	28	93	5.25
	LH fusiform gyrus	-17	-90	-10	326	8.83
	RH fusiform gyrus	18	-90	-8	276	7.23
	LH middle occipital gyrus	-20	-95	8	1164	7.29
	LH inferior occipital gyrus	-47	-65	-15	446	7.32
	RH middle occipital gyrus	28	-90	13	408	7.23

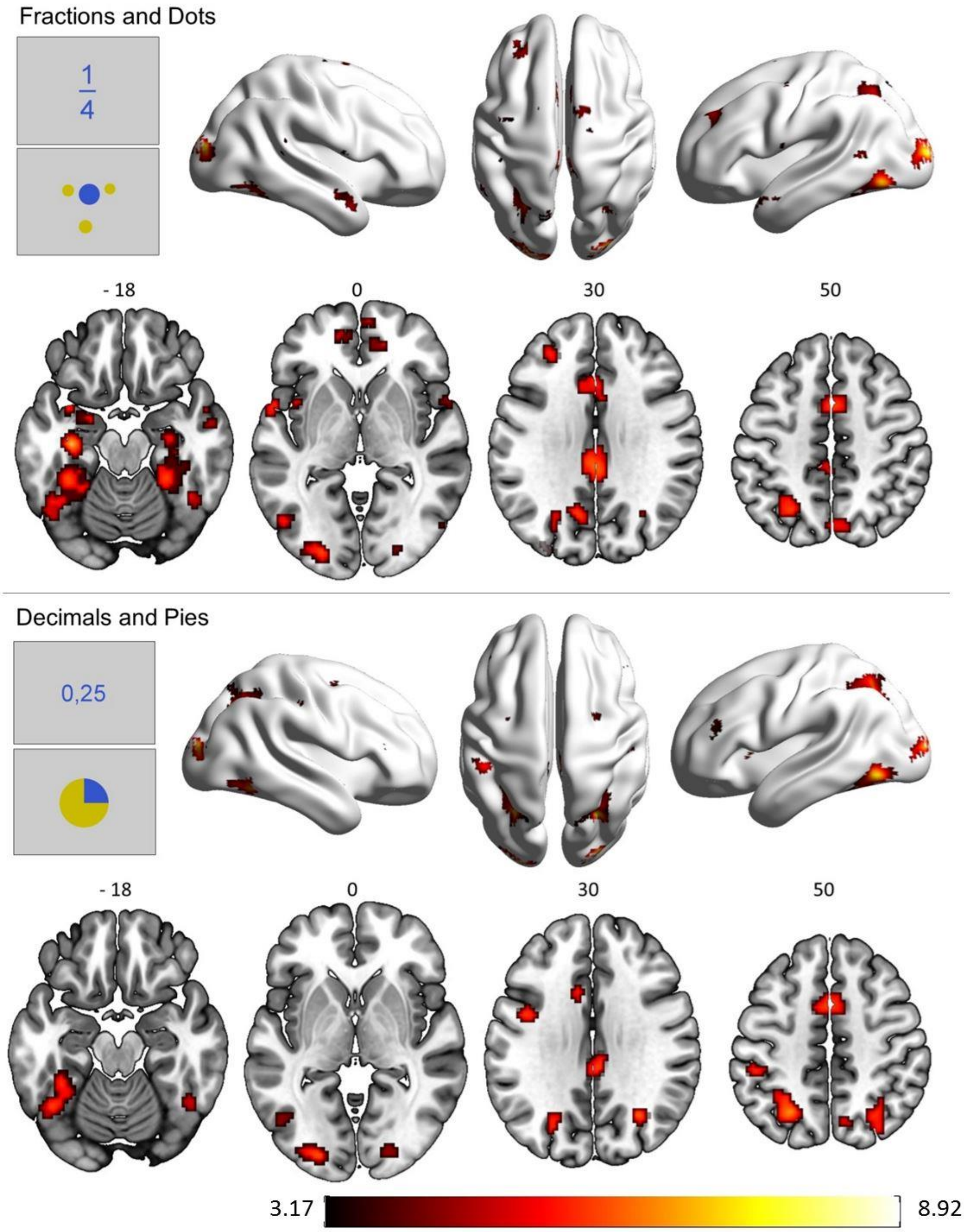


Figure S9.3 Conjunction of cues indicating a difficult (involving dots and fractions) or easy (involving pies and decimals) upcoming magnitude comparison task. The color bar indicates *t*-values ($p_{cluster\text{-}corr} < .05$, cluster size $k = 10$).

Table S9.3 Cortical regions more strongly activated in the conjunction of viewing at cues for upcoming dots and fractions compared to rest. $p_{cluster\text{-}corr} < .05$ ($k = 10$ voxels); LH: left hemisphere; MNI: Montreal Neurological Institute coordinates; RH: right hemisphere; t = t-value.

Contrast	Brain region	MNI (x, y, z)			Cluster size	*t*
Conjunction	LH amygdala	-30	-2	-30	84	5.40
cues dots and fractions	LH hippocampus	-35	-20	-18	195	6.32
	LH insula	-42	8	0	16	3.57
	RH insula	63	3	10	10	3.55
	RH anterior cingulate cortex	13	46	-3	14	4.20
	RH middle cingulate cortex	3	8	43	408	5.45
	LH intraparietal sulcus (hIP3)	-27	-65	40	429	5.09
	RH intraparietal sulcus (hIP3)	31	-60	45	31	3.90
	LH fusiform gyrus	-37	-42	-20	367	6.24
	RH fusiform gyrus	31	-42	-23	396	5.09
	RH lingual gyrus	21	-90	-8	77	4.87
	LH lingual gyrus	8	-82	-10	15	3.85
	RH retrosplenial cortex	1	-35	30	206	4.88
	RH supplementary motor area	13	1	68	104	3.64
	LH middle frontal gyrus	-30	41	33	64	4.18
	LH middle frontal gyrus	-32	48	23	15	3.91
	LH temporal pole	-55	6	-5	74	4.04
	RH temporal pole	58	8	-5	10	3.57
	RH medial temporal pole	38	8	-33	22	4.13
	LH superior temporal gyrus	-62	-25	13	23	3.84
	RH middle temporal gyrus	51	6	-25	19	3.74
	RH middle temporal gyrus	56	-5	-18	10	3.44
	LH superior medial gyrus	-15	51	0	11	3.44
	RH middle orbital gyrus	8	61	-3	11	3.49
	LH middle occipital gyrus	-22	-95	8	412	7.59
	RH middle occipital gyrus	28	-90	13	163	6.63
Conjunction	RH intraparietal sulcus (hIP3)	30	-65	42	221	6.46
cues pies and decimals	LH intraparietal sulcus (hIP3)	-25	-65	40	313	5.88
	LH intraparietal sulcus (hIP2)	-45	-35	43	98	4.75

LH fusiform gyrus	-37	-45	-18	325	6.33
RH fusiform gyrus	47	-58	-10	175	5.68
RH fusiform gyrus	33	-37	-25	55	3.86
RH lingual gyrus	21	-90	-8	95	5.14
LH supplementary motor area	-2	6	53	189	4.91
LH inferior frontal gyrus (44)	-42	1	28	35	4.38
RH retrosplenial cortex	3	-35	30	67	4.42
RH precuneus	11	-67	43	34	3.83
LH middle occipital gyrus	-20	-95	8	323	7.11
RH middle occipital gyrus	28	-90	15	121	6.36

PART III: DISCUSSION

General Discussion

Previous studies suggested that a lack of fraction knowledge limits academic and professional as well as general life prospects. The ability to handle and understand fractions is essential for children's mathematical development in secondary school (Lamon, 2020; Litwiller & Bright, 2002; NMAP, 2008). In particular, higher-order mathematics like algebra, but also the development of abstract mathematical thinking, logical reasoning, modeling, and pattern recognition have been associated with fraction knowledge (Booth & Newton, 2012; Booth et al., 2014; DeWolf et al., 2016; Empson, 1999; Empson & Levi, 2011; Empson et al., 2011; Wu, 2001). Moreover, in the light of the recent COVID-19 pandemic, this lack of understanding fractions in terms of relative frequencies has become even more apparent (Lau et al., 2021; Thompson et al., 2020). Over the last four decades, research on fraction processing has increased rapidly (for a selection of reviews on this topic, see Booth & Newton, 2012; Lortie-Forgues et al., 2015; Obersteiner, Dresler, et al., 2019; Siegler et al., 2013). One crucial development in this research area was Siegler et al.'s proposal of the integrated theory of numerical development (ITND; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). The ITND highlights magnitude processing as the key mechanism that integrates all numbers and is therefore also crucial for fraction understanding. Moreover, recent research on the improvement of fraction knowledge via NLE training seems to substantiate this theory (e.g., Barbieri, Rodrigues, Dyson, & Jordan, 2020; Dyson, Jordan, Rodrigues, Barbieri, & Rinne, 2020; Fazio, Kennedy, & Siegler, 2016; Gunderson, Hamdan, Hildebrand, & Bartek, 2019; Hamdan & Gunderson, 2017; Kiili, Moeller, & Ninaus, 2018; Schumacher et al., 2018; Sidney, Thompson, & Rivera, 2019). However, despite intense research in this area, the problems children, adults, and even teachers face when learning and dealing with rational numbers and especially fractions remain the same (Siegler & Lortie-Forgues, 2014).

For this reason, the present thesis aimed to evaluate different cognitive and non-cognitive predictors of fraction processing with the main goal of proposing a comprehensive framework of fraction processing. This framework builds on the core assumption of the ITND (i.e., magnitude processing as the central ability for fraction understanding) by evaluating the neural correlates of NLE training and expands it by integrating additional cognitive, motivational, and affective predictors important for the mastery of fractions. Thus, the framework aims to provide a more comprehensive picture of important predictors for fraction processing beyond magnitude processing and across domains.

Therefore, the general discussion is organized in four parts: First, I will discuss the major results of the five empirical studies with respect to the research question of this thesis. Second, I propose and discuss a comprehensive framework of fraction processing by extending the core assumptions of the ITND based on my empirical findings and findings in the literature presented in sections 2 and 3. In addition, I also integrate a preliminary model proposed by me, which depicts the temporal course of fraction processing into the framework. Third, I will present possible perspectives for future research that might help to validate the proposed framework. Finally, I discuss a selection of additional cognitive, non-cognitive and meta-cognitive predictors that should be considered in future studies and added to the proposed framework.

10. Discussion of the Empirical Findings

In the following, I will summarize the main results of the five empirical studies presented in my dissertation in light of the comprehensive framework of fraction processing that I will propose and discuss in subsequent sections 11 and 12.

10.1 Cognitive Predictors of Fraction Processing

The main focus and the major contribution of the ITND lies in the overarching and unifying role of magnitude for all numbers. The ITND postulates that magnitude is the common factor shared between all kinds of numbers (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). Therefore, the representation of number magnitude brings together all numbers by reference to the mental number line. The mental number line is a popular metaphor for the assumed mental representation of number magnitude: especially in western cultures, numbers seem to be represented spatially with their magnitudes increasing from left to right (Göbel, Shaki, & Fischer, 2011). Research on the beneficial role of NLE training on fraction magnitude processing substantiates these assumptions (Barbieri et al., 2020; Gunderson et al., 2019; Hamdan & Gunderson, 2017). However, given the complexity and difficulty of fraction processing and understanding compared to natural numbers, focusing only on magnitude processing as a core cognitive predictor of fraction understanding might not give a complete picture of all relevant factors that influence fraction processing. In the present work, the main assumption of the ITND and possible additional cognitive predictors, which are partially magnitude unrelated (i.e., independent of magnitude processing), were addressed in *Section 2*. Therefore, in the first three studies of my dissertation, I aimed to evaluate the core assumption of the ITND and extend this theory with additional cognitive predictors that might play a role for fraction processing.

In Study 1, I investigated changes in neuro-functional correlates of fraction magnitude processing following an intensive 5-day NLE training in adult participants. The aims of Study 2 and 3 were to investigate additional cognitive predictors that might be relevant for fraction processing as well as related and unrelated to magnitude processing. Therefore, in Study 2, I investigated the role of domain-specific numerical skills by using a comprehensive battery of basic numerical skills, not only assessing magnitude processing, to evaluate their relative importance for fraction processing. Finally, in Study 3, I used a fraction magnitude comparison task with positive and negative fractions to focus on the role of strategies for fraction processing and to evaluate which role magnitude processing actually plays in processing negative fractions.

10.1.1 Findings of Study 1: Neural correlates of a fraction training

Concerning the role of magnitude processing for fraction understanding and learning, Study 1 was able to substantiate the main assumption of the ITND that magnitude information is a crucial aspect thriving fraction processing even in well-educated adults that should be familiar with fractions (i.e., even two-digit fractions; Jordan et al., 2017; Mou et al., 2016; Siegler, 2016; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). Moreover, the ITND indicates that promoting fraction magnitude understanding via NLE training can foster conceptual understanding of fractions (see also Fazio & Siegler, 2011). Behavioral intervention studies have already shown that the NLE task is an important tool to improve fraction processing (Barbieri et al., 2020; Fazio, Kennedy, et al., 2016; Gersten et al., 2017; Gunderson et al., 2019; Hamdan & Gunderson, 2017; Kiili et al., 2018). Additionally, several recommendations for teachers have been made by policymakers and researchers to highlight the importance of number lines for fraction understanding (Fazio & Siegler, 2011; NMAP, 2008).

To the best of my knowledge, Study 1 is the first to show the effect of an NLE training on both the behavioral and the neural level. In addition to a behavioral improvement (i.e., fewer errors), the training led to an improvement of symbolic fraction processing as indicated by the numerical distance effect on a neural level. In particular, significant modulation of brain activation by numerical magnitude (as reflected by the distance effect) was found for the processing of symbolic fractions after but not before the training. Importantly, comparing the distance effect after the training to the distance effect before the training in the symbolic fraction magnitude comparison task revealed significantly increased activation differences in a bilateral fronto-parietal network centered around the intraparietal sulcus. Moreover, this training effect was observed to generalize to untrained items. This means that a transfer effect from trained to untrained items was elicited by the NLE training indicating that strengthening fraction magnitude processing for some fractions also seems to foster more general conceptual knowledge of fractions. Hence, the assumptions of the ITND regarding the prominent role of (fraction) magnitude processing for fraction understanding is supported by Study 1 and extended to numerate adults, complex fractions, and even untrained (novel) fractions. As such, this study further supports the importance of number line representations for fraction understanding.

10.1.2 Findings of Study 2: The role of magnitude-related and unrelated basic numerical skills

Even though the role of cognitive predictors has been extensively investigated in previous studies (see Table 2.1 of this dissertation for a summary of the main studies; e.g., Jordan, Resnick, et al., 2017), the ITND focuses exclusively on the unique role of magnitude for number processing in general and fraction processing in particular as well as the role of NLE representations for promoting conceptual fraction knowledge (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). Although the fundamental role of basic numerical skills for numerical development is well known, most of the studies focused on cognitive domain-general (e.g., working memory, attention, reading fluency) as well as domain-specific (e.g., number line estimation, arithmetic, number knowledge) predictors which involved also advanced mathematical skills like fraction or proportional reasoning measures (e.g., Bailey et al., 2014; Hansen et al., 2015; Jordan et al., 2013; Namkung & Fuchs, 2016). While the ITND predicts that fraction magnitude processing should be correlated with performance in fraction arithmetic problems, evidence on other basic numerical skills not primarily reflecting magnitude processing and their association with fraction processing is still patchy (Siegler et al., 2011).

Study 2 investigated the role of a comprehensive battery of basic numerical skills for fraction processing. Importantly, this battery included magnitude-related (e.g., addition, subtraction, number line estimation) and magnitude unrelated (e.g., multiplication, basic geometry, and conceptual knowledge about arithmetic) basic numerical skills. Study 2 revealed that number line estimation, subtraction, conceptual knowledge about arithmetic, and multiplication significantly predicted fraction understanding. This is largely in line with previous studies that showed that number line estimation with whole numbers (Bailey et al., 2014; Jordan et al., 2013; Resnick et al., 2016) and arithmetic with whole numbers (Bailey et al., 2014; Resnick et al., 2016) were important predictors of different measures of fraction understanding. While number line estimation, and subtraction are magnitude-related variables (Berteletti, Man, & Booth, 2015; Siegler & Opfer, 2003), the test on conceptual knowledge about arithmetic requires more general knowledge about relationships of arithmetic operators with each other. Thus, basic principles of arithmetic are tested that do not necessarily require magnitude processing (e.g., multiplication as the repeated addition of equal summands). Moreover, fact retrieval plays a crucial role when solving multiplication problems. Fact retrieval is associated with activation in the gyrus angularis (Grabner et al., 2009) and therefore might be more unrelated to magnitude processing than, for instance, solving subtraction problems.

Regarding the relative importance of the predictor variables for the fraction understanding task, Study 2 revealed that the most relevant predictor was *multiplication,* followed by *subtraction*, *conceptual knowledge*, and *number line estimation*. Furthermore, this analysis revealed that the basic numerical predictors *addition*, *basic geometry*, and *approximate arithmetic*, which were initially not found to be significant predictors in the multiple regression analysis, might still play a relevant role for fraction processing.

Interestingly, Study 2 also showed that number line estimation, which is considered as one of the classical magnitude measures, was not the most important predictor for fraction processing. In fact, the role of basic arithmetic operations like multiplication, subtraction, addition, and general knowledge about these operations were more important than just magnitude processing. Thus, although arithmetic processes are magnitude-related, this study can extend the ITND in showing that not only number line estimation, but also other basic numerical skills are important predictors for fraction processing. Moreover, Study 2 revealed that magnitude-related basic numerical skills and magnitude unrelated skills like conceptual knowledge about arithmetic, multiplication, and basic geometry are important predictors of fraction understanding. Therefore, Study 2 supports previous findings and assumptions of the ITND and extends the ITND in considering magnitude-related and unrelated basic numerical skills as important predictors for fraction processing.

10.1.3 Findings of Study 3: The importance of magnitude related and unrelated strategies

Although the predecessor version of the ITND points out that the development and successful use of strategies is critical for acquiring fraction knowledge (Siegler et al., 2011), this point received little attention in the current version of the ITND (Siegler & Lortie-Forgues, 2014). However, as fractions are unlikely to be processed automatically, the proper application of processing strategies seems critical. Moreover, almost all strategies that can be found in the literature are magnitude-related strategies (e.g., holistical or componential strategies). Thus, magnitude processing is highly interrelated with strategy use for fraction processing.

Moreover, the final report of the NMAP concludes that "*the most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percent, and negative fractions)*"(NMAP, 2008, p.18). However, although this was in 2008 still almost nothing is known about the processing of negative fractions. Nevertheless, the ITND claims that the unifying theme of all number types is magnitude knowledge, and that the role of magnitude is similar for positive and negative numbers.

To the best of my knowledge, Study 3 of this dissertation is the first study to investigate fraction magnitude processing for both negative and positive fractions. Concerning this point, Study 3 revealed that processing strategies of negative fractions for magnitude comparison tasks depend on the combination of fractions at hand (i.e., comparing only positive fraction pairs, comparing only negative fraction pairs, or comparing a positive with a negative fraction). Four different strategies were tested: i) *sign-shortcut strategy*, ii) *sign flip strategy*, iii) *denominator flip strategy*, and iv) *a combination of sign flip and denominator flip strategy*. Fraction pairs included in this study, where either heterogenous (i.e., one of the to-be compared fractions is positive and the other one is negative) or homogenous (i.e., both to-be compared fractions are positive or negative) as well as numerator-relevant (i.e., different numerators but identical denominators) or denominator-relevant (i.e., different denominators but identical numerators).

When applying the sign-shortcut strategy, magnitude processing should not be necessary. Whereas when applying the sign flip strategy, denominator flip strategy, and the combination of both, magnitude processing should be involved. Results indicated that the sign-shortcut strategy (Krajcsi & Igács, 2010) was employed when heterogenous numerator- and denominator-relevant fraction pairs (e.g.., comparing -1/8 and +1/9) were compared and relied only on the information of the sign. Thus, a decision can be made by only considering the sign without involving the magnitudes of the fractions. Accordingly, no significant holistic distance effect could be found for the sign-shortcut strategy.

The sign flip strategy (Krajcsi & Igács, 2010) was primarily applied when negative homogenous numerator-relevant (e.g., comparing -2/9 and -1/9) fraction pairs were compared. With this strategy, a decision can be made by only considering the absolute magnitudes of the numerators and reversing the answer. Again, no significant holistic distance effect could be found, indicating that participants did not process the magnitudes of both fractions holistically. Suggesting that participants processed the components separately and therefore it is more likely that a componential strategy was applied. Interestingly, eye-tracking data revealed that participants did not significantly spend more reading time on the numerator or the sign. Thus, only behavioral data provided evidence for the sign flip strategy.

The denominator flip strategy (Huber et al., 2014) was applied when positive homogenous denominator-relevant (i.e., comparing +1/8 and +1/9) fraction pairs were compared. When applying this strategy, a decision can be made by only considering the absolute magnitudes of the denominators and reversing the answer. Similar to the sign flip strategy, no significant holistic distance effect could be found.

Thus, participants did again not process the magnitudes of both fractions holistically. However, eye-tracking data revealed that participants spend more reading time on the denominators in denominator-relevant trails as compared to numerator-relevant trails. Thus, eye-tracking data indicated that participants might have used componential processing strategies.

Finally, a combination of sign flip and denominator flip strategy was investigated. This strategy can be applied when negative denominator-relevant fraction pairs are compared (e.g., comparing -1/8 and -1/9). Again, no significant holistic distance effect was found. Effects for eye-tacking data were mixed: while participants spend a significant amount of reading time one the sign, reading time on the denominator was not significant.

Hence, Study 3 suggested that the processing of (negative) fractions is mostly based on componential processing strategies and that a comparison of a negative and a positive fraction can be solved without applying a magnitude-related strategy but by only considering the sign. Therefore, Study 3 supports the extension of the ITND in considering magnitude-related and unrelated strategies as important predictors for fraction processing. Additionally, Study 3 underpins the assumption of the ITND that magnitude processing also plays an important role when comparing negative fractions.

10.2 Motivational and Affective Predictors of Fraction Processing

While the main focus of the ITND lies on magnitude processing, it does not consider that non-cognitive variables could also substantially influence fraction processing. Therefore, this study aims to extend the ITND with crucial non-cognitive predictors for fraction processing. For example, in education and learning, two common non-cognitive predictors are motivation and (negative) emotions. It is well known that both non-cognitive variables are crucial for academic success in general (Graziano et al., 2007; Linnenbrink & Pintrich, 2002) and also for mathematical achievement in particular (Hannula, 2006a, 2006b, 2015; Schukajlow et al., 2017). Moreover, considering that fractions are perceived as complex learning content (Lamon, 2020) motivation and (negative) emotions could explain proficiency with fractions in addition to and beyond cognitive predictors.

In the present work, the possible role of motivation and negative emotions for learning and, particularly for fraction learning, were evaluated in *Section 3*. Therefore, in the last two studies of my dissertation, I aimed to investigate the possible role of these two non-cognitive predictors and to extend the ITND by including motivational and affective predictors that might play a role in fraction processing.

In Study 4, different motivation profiles and their association with fraction understanding were examined to elaborate on the role of intrinsic and extrinsic motivation for fraction processing. For this, 256 7th grade school students played a computerized learning game for assessing fraction understanding over a period of 5 weeks. Finally, in Study 5, I investigated the role of negative emotions and emotion regulation for the anticipation of symbolic (i.e., fractions and decimals) and non-symbolic (i.e., pie charts and dot patterns) proportion processing, which differed in task difficulty.

10.2.1 Findings of Study 4: Motivation is important but not always sufficient

The role of motivation for learning, especially in academic settings, is well known. Motivation is a driving force that can either lead to task avoidance or devotion. Although Hecht & Vagi (2010) and Bailey et al. (2014) point out that motivation might be a factor of particular relevance for fraction achievement, little is known about the role of motivation for fraction learning and processing. One of the few studies investigating the role of teaching practices, motivation, and fraction achievement found that high student motivation was associated with better skills on fraction tasks (Stipek et al., 1998). However, as motivation is critical when engaging with complex mathematical content, it seems crucial to focus more on this relationship.

Thus, Study 4 of this dissertation aimed to investigate in more depth the role of motivation (intrinsic and extrinsic) for achievement in two computerized game-based fraction tasks (i.e., NLE and fraction magnitude comparison). To the best of my knowledge, this is the first study to examine motivation profiles for fraction achievement. For this, latent profiles based on students' reported motivation types (via a motivation questionnaire) and their performance in these fraction tasks were examined. Additionally, the number of levels and days students played were additional motivational indicators to examine the consistency of self-reported motivation and actual corresponding behavior. Three relevant profiles were identified:

Students of the first motivation profile showed above-average performances. Interestingly, they engaged less with the computerized game, as indicated by played levels, experienced below-average positive affect while playing the game, and perceived their game play as more externally determined. Thus, they were categorized into a profile of *externally determined high performers*.

Students of the second motivation profile also showed above-average performance. However, they engaged highly with the game, experienced above-average positive affect while playing the game, and perceived their gameplay as more self-determined. Therefore, they could be assigned to a profile of *self-determined high performers*.

Finally, students of the third motivation profile performed poorly but played a similar number of levels as students of the first profile. Interestingly, they experienced above-average positive affect and perceived their gameplay as above-average self-determined. Thus, they were categorized into a profile of *self-determined low performers*.

Hence, Study 4 suggests that motivation can play a role for performance in fraction tasks like the NLE and fraction magnitude comparison task. However, this is not always sufficient. As seen in the third student profile, these students were highly intrinsically motivated but performed the worst. Additionally, the first motivation profile shows that students were able to perform above average, although they experienced mainly extrinsic motivation and were actually rather demotivated. Only in the second profile, students perceived that high intrinsic motivation was corresponding with their task performance. Therefore, Study 4 suggests that motivation might play a vital role for students' performance and their achievement on fraction tasks. However, it might not have the same importance as other cognitive and non-cognitive predictors. Nevertheless, especially with difficult learning content, motivation could be decisive in engaging more with the learning content, which can strengthen conceptual understanding and leads to better performance. For this reason, Study 4 supports the extension of the ITND by considering motivation as a relevant predictor for fraction processing which plays a mediating role between engagement with the learning content and performance.

10.2.2 Findings of Study 5: The role of negative emotions and emotion regulation for fraction processing

Negative emotions are common factors in educational settings. They can have a detrimental impact on learning engagement and academic achievement (Pekrun & Linnenbrink-Garcia, 2012, 2014). Similar to motivation, emotions are driving forces. Thus, negative emotions can lead to impaired task performance and even avoidance of the task (Pekrun et al., 2002; Pekrun & Linnenbrink-Garcia, 2012). In this context, emotion regulation is also of particular relevance (Boekaerts & Pekrun, 2015; Bradley et al., 2010). Especially in school mathematics, math anxiety is a prominent effect where negative emotions impair performance on several math-related tasks (Ashcraft, 2002; Dowker et al., 2016). In connection with fractions, few studies showed that math anxiety negatively influences task performance on NLE, magnitude comparison, and procedural and conceptual knowledge tasks in adult math-anxious participants (Rayner et al., 2009; Sidney, Thalluri, et al., 2019). However, a more in-depth investigation on how negative emotions affect fraction processing and the role of emotion regulation for fraction processing is still missing.

In this context, the final study of this dissertation aimed to investigate the role of negative emotions and emotion regulation in non-math-anxious adult participants while anticipating proportion magnitude comparison tasks. Importantly, these tasks could be distinguished into difficult (i.e., fractions and dot patterns) and easy (i.e., decimals and pie charts) proportion magnitude comparison tasks. Additionally, a stress appraisal questionnaire was conducted to assess if participants experienced challenge (positive emotional valence), or threat (negative emotional valence) associated with the respective task.

The questionnaire revealed that participants experienced difficult proportions more negative than easy proportions. Moreover, participants experienced easy proportions more positively than difficult proportions. Additionally, neurofunctional data revealed that the anticipation of difficult upcoming tasks (i.e., fractions and dot patterns) lead to increased activation of a so-called negative emotion network (Phelps, 2004), including the bilateral amygdala, bilateral hippocampus, left temporal gyrus, and bilateral paracentral gyrus compared to the anticipation of easy (i.e., decimals and pie charts) upcoming tasks. Importantly, the bilateral anterior cingulate cortex (ACC) was also activated, and the covariate math anxiety did not yield to any suprathreshold cluster of activation. This indicates that math anxiety scores did not explain the variance of the fMRI signal during the anticipation of proportion magnitude comparison tasks.

Moreover, the ACC is known to be a key brain region involved in emotion regulation (Stevens, Hurley, & Taber, 2011). Thus, Study 5 indicated that non-math-anxious individuals experience more negative feelings in association with fractions and dot patterns, which are more complex compared to easier proportions like decimals and pie charts. Moreover, this study showed that, although participants were not math-anxious, their initial neural response to difficult proportion cues was an increased activation of brain regions which normally can be found active in math-anxious participants when engaging with a mathematical task (Lyons & Beilock, 2012). However, the additional activation of the ACC might lead to a regulation of this initial negative emotional response to difficult proportions. Which in turn might be crucial for task performance.

Therefore, Study 5 highlights the importance of negative emotions and emotion regulation for fraction processing. Even in non-math-anxious adult participants, fractions elicit an initial negative reaction which might impair proficiency with fractions if the ability to regulate this initial response is impaired. For this reason, Study 5 supports the extension of the ITND to negative emotions as an important non-cognitive predictor for fraction processing.

Taken together, I could show that a variety of cognitive and non-cognitive predictors play a crucial role for fraction processing and, therefore, proficiency with fractions. Study 1 supported the assumptions of the ITND regarding the prominent role of (fraction) magnitude processing and number line representations for fraction understanding and processing. Additionally, this assumption was extended to numerate adults, complex fractions, and even untrained (novel) fractions. Study 2 could show that magnitude-related and unrelated basic numerical skills are important predictors of fraction understanding. Therefore, this study supports previous findings and assumptions of the ITND and extends the ITND in considering magnitude-related and unrelated basic numerical skills as important predictors for fraction processing. Study 3 indicated that the processing of negative fractions is mostly based on componential processing strategies (i.e., a magnitude-related strategy) and that a comparison of a negative and a positive fraction can be solved without applying a magnitude-related strategy but by only considering the sign. Therefore, Study 3 shows that strategies play an important role for fraction processing and suggests that the ITND should be extended by considering magnitude-related and unrelated strategies as important predictors for fraction processing. Study 4 suggests that motivation can play a role for performance in fraction tasks. However, it seems that motivation is not always sufficient because some students where highly motivated but still performed poorly in fraction tasks. Nevertheless, this study indicates that the ITND should be extended with motivation as an important predictor. Finally, Study 5 investigated the importance of negative emotions and emotion regulation for fraction processing. This study could show that both play a role for fraction processing even in non-math-anxious adult participants. For this reason, Study 5 supports the extension of the ITND to negative emotions as an important non-cognitive predictor for fraction processing.

While the ITND is an influential theory for the development of number processing and especially fraction processing, it mainly focuses on the role of magnitude processing as the link between all number types. However, number processing and, in particular, fraction processing is a complex mechanism that involves many different processes. Therefore, as a first step, in the following section, I want to propose a comprehensive framework which aims to integrate different cognitive and non-cognitive predictors to better understand their complex role for fraction processing.

11. A comprehensive Framework for Fraction Processing

In the following, the major findings of the five empirical studies of this thesis are integrated into a *comprehensive framework of fraction processing* by considering both cognitive and non-cognitive predictors. In particular, I will build and evaluate the comprehensive framework in two steps: Since the main scope of this dissertation is on fraction processing, I will first introduce a tentative model focusing on the *temporal course of fraction magnitude processing*, which I have recently already proposed in another context (Wortha, Obersteiner, & Dresler, 2021). Then, in a second step, I will subsequently build a *comprehensive framework of fraction processing*. For this, I incorporated the main assumption of ITND (i.e., the role of magnitude processing) and extended this approach by including cognitive as well as non-cognitive (i.e., motivational and affective) predictors.

11.1 The temporal course of fraction processing

Successful fraction magnitude processing is the key ability underpinning fraction learning and understanding. Accordingly, the temporal course in which individuals process (symbolic) fraction magnitudes is an important aspect of this mechanism. Unfortunately, little is known about the temporal processes. One methodological possibility to investigate such chronological sequences is electroencephalography (EEG) because of its high temporal resolution (msec range; Nidal & Malik, 2014). With this method, it is possible to record event-related potentials (ERPs). ERPs are voltage fluctuations (positive or negative) that are time-locked to the onset of an event of interest (e.g., fraction magnitude processing). Some well-known ERPs are the N100, N200, P300 and N400. For instance, the N100 has a negative peak around 100 msec after stimulus onset and can be associated with matching the physical properties of the perceived stimulus with the previous stimulus (Sur & Sinha, 2009). The N200 is associated with conflict monitoring and has a negative peak around 200 msec after stimulus onset (Donkers & Van Boxtel, 2004). The P300 can reflect information processing and its integration into memory representation and its positive peak can be found most prominently 300 msec after stimulus onset (Polich, 2007). Finally, the N400 peaks around 400 msec after stimulus onset and is elicited during the semantic evaluation of the stimulus (Lau, Phillips, & Poeppel, 2008). While there are a couple of studies that investigated electrophysiological correlates of fraction magnitude processing (Barraza, Gómez, Oyarzún, & Dartnell, 2014; Fu, Li, Xu, & Zeng, 2020; Rivera & Soylu, 2018; Zhang, Wang, Lin, Ding, & Zhou, 2013; Zhang et al., 2012), there is yet no consensus on the temporal sequence in which this process occurs.

However, based on a temporal model of Dehaene (1996) for number processing and the EEG literature on fraction processing, I proposed a first preliminary model which highlights four chronological stages of fraction magnitude processing during a magnitude comparison task (see Fig. 11.1). Stage 1: identification (i.e., identifying visual stimulus), Stage 2: magnitude representation (depending on the experience of the individual; representing the numerator and denominator of both fractions separately and integrating the ratios to one magnitude representation for each fraction), Stage 3: comparison (i.e., comparing the magnitudes of both fractions), and Stage 4: response (i.e., deciding which fraction is the larger one). While this model needs to be evaluated empirically, previous evidence from the literature supports the model. For instance, Zhang et al. (2013) showed that the N100 was elicited during stimulus-specific identification of symbolic fractions and non-symbolic proportions. This may reflect Stage 1 of the proposed model. Moreover, Fu et al. (2020) and Zhang et al. (2012) showed that both the N200 and P300 are involved in the componential processing of fractions and inhibitory control to exceed the natural number bias, which could be caused by the individual components of the fractions. This would correspond to Stage 2 of the model. Finally, Barraza et al. (2014) found that the N400 was more pronounced during holistic processing of the fractions than componential processing. Which also corresponds well with Stage 2 but could also be an indicator of the initiation of Stage 3 (i.e., the actual fraction magnitude comparison) of the temporal model of fraction processing.

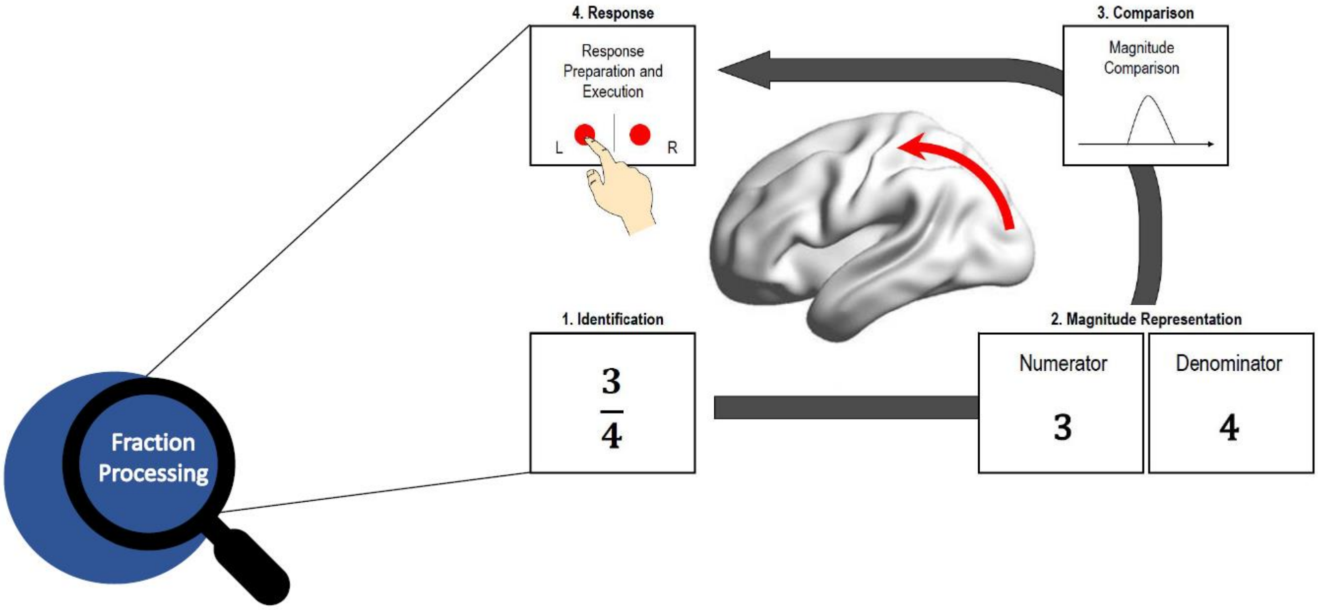


Figure 11.1: Proposed model for the temporal course of fraction magnitude processing. Adapted from (Wortha et al., 2021). Please note that for reasons of convenience, only one symbolic fraction is displayed. However, this model describes temporal processes during a fraction magnitude comparison task which always requires two fractions. Thus, the first two stages must be performed twice for each fraction, whereas in the third stage, both fraction magnitudes are compared to provide a response in Stage 4 to the question of which fraction is the larger one.

Errors can occur during each stage of the proposed model. However, Stage 2 is particularly prone to errors as integrating the two respective magnitudes from the numerator and denominator into one fraction magnitude representation is an effortful process. Moreover, this integration depends on individual strategic preferences (e.g., cross-multiplication, visualization, estimation, benchmark strategies) and is not fully understood yet. Therefore, it is of importance to investigate these and other possible stages of fraction magnitude processing to understand better the involved cognitive mechanisms and their temporal course. The framework proposed in the next subsection aims to identify possible relevant predictors of fraction processing, whereby I always refer to the already presented temporal course of fraction (magnitude) processing.

11.2 Integrating Cognitive and Non-Cognitive Predictors into a comprehensive Framework of Fraction Processing

In the following I will propose a comprehensive framework of fraction processing in three steps: First, I will explain the general structure of the framework to provide an overview about the different building blocks of each section (i.e., building blocks of the cognitive predictor section and the non-cognitive predictor section). These building blocks are based on the findings of my five empirical studies included in this dissertation. Additionally, I will further substantiate the connections between building blocks proposed in the first step by existing studies (see Fig. 11.2). In a second step, I will elaborate on connections between building blocks that I did not investigate empirically in the context of this thesis with existing studies from the fraction literature (see Fig. 11.3). Finally, for connections between building blocks where studies on fraction processing are still missing, I will provide evidence from studies involving natural numbers to show that these connections exist, at least for another type of number (see Fig. 11.4).

It is important to emphasize, however, that unlike other prominent models or theories of numerical processing and development such as the Triple Code Model (TCM; Dehaene, Piazza, Pinel, & Cohen, 2003) or the ITND (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011), the goal of this approach is not to explain how fraction processing occurs or develops in detail, but rather to provide an account of what factors might be important and influence fraction processing.

Step 1: The general structure of the comprehensive framework

This framework (see Fig. 11.2) consists of two areas and five distinct building blocks. The two areas are called *cognitive* and *non-cognitive predictors* which each include their respective building blocks. Thus, the building blocks *magnitude processing*, *basic numerical skills* and *strategies* are located within the area cognitive predictors. Moreover, the building blocks *motivation* and *(negative) emotions* are located within the area non-cognitive predictors. The predictors can be interrelated with each other (called connections in the following) or with fraction processing. Therefore, connections between building blocks indicate that some predictors are partially dependent on other predictors (i.e., some basic numerical skills are magnitude-related and depend on the building block magnitude).

Similar to the ITND, my starting point is *magnitude processing*. An overwhelming body of literature has proven that the core assumption of the ITND is indeed fundamental for fraction processing (Gersten et al., 2017; Gunderson et al., 2019; Hamdan & Gunderson, 2017; Liu, 2018; Obersteiner, Dresler, et al., 2019; Sidney, Thompson, & Rivera, 2019; Siegler & Lortie-Forgues, 2014). Moreover, although I aimed to evaluate the influences of different cognitive and non-cognitive predictors in the five studies of my dissertation, magnitude processing was always a common theme across these studies because magnitude is the shared core across all number types. Thus, magnitude processing is without a doubt the key feature that is fundamental for fraction processing. Additionally, it has a unique and substantial role for number processing in general (Leibovich, Katzin, Harel, & Henik, 2017; Siegler, 2016). Therefore, in my comprehensive framework, magnitude processing builds the basis of the whole framework (see Fig. 11.2, building block *magnitude processing*). Additionally, in Study 1 of my dissertation I was able to substantiate the main assumption of the ITND and provide behavioral as well as neuro-functional evidence for the importance of magnitude processing for fraction learning. Therefore, in my comprehensive framework the building block *magnitude processing* is connected with fraction processing (see Fig. 11.2; thick arrow 1). Moreover, as revealed by Study 2 and 3 of my dissertation magnitude processing also has direct links to other basic numerical skills (e.g., number line estimation, basic arithmetic, counting) and strategy choice (e.g., componential vs. holistic processing of fractions). Therefore, magnitude processing is connected with *basic numerical skills* (Fig. 11.2; thick arrow 2) and the building block *strategies* (Fig. 11.2; thick arrow 3).

Based on magnitude processing, the two other building blocks that indirectly influence fraction processing in terms of cognitive predictors are *basic numerical skills* and different *strategy* types of processing fractions. In addition, a variety of studies have shown that different domain-specific skills, which partially involved also basic numerical skills, were direct or indirect predictors of fraction processing (Bailey et al., 2014; Hansen et al., 2015; Hecht & Vagi, 2010; Jordan et al., 2013; Liu & Wong, 2020; Mou et al., 2016; Namkung & Fuchs, 2016; Namkung et al., 2018; Seethaler et al., 2011; Siegler & Pyke, 2013; Stelzer, Andrés, et al., 2019; Stelzer, Richard's, et al., 2019; Vukovic et al., 2014; Ye et al., 2016).

Most of the domain-specific skills examined in these studies were magnitude-related (e.g., number line estimation, arithmetic), which means that it was necessary to process numerical magnitude to master that skill. For instance, when performing basic arithmetic like subtraction (e.g., 4 - 2), it is necessary to process the minuend and the subtrahend. However, in Study 2 of this dissertation, I also showed that magnitude unrelated basic numerical skills such as basic geometry might play a role for fraction processing. Therefore, the building block basic numerical skills is also directly connected with fraction processing in my comprehensive framework (Fig. 11.2, thick arrow 2). Moreover, applying or recruiting a particular basic numerical skill may also be interpreted as a strategic choice to solve a particular task. For instance, to solve the basic arithmetic problem "6 + 9", individuals can either add the two numbers to solve this problem or solve by adding "6 + 9" and subsequently subtract 1 (i.e., 6 + 10 - 1= 6 + 9). To account for this in my tentative framework the building block *basic numerical skills* projects on the building block *strategies*.

Strategies play a prominent role in fraction processing. Compared to natural numbers, where magnitudes are typically processed automatically (Berch et al., 1999; Gebuis et al., 2009; Rubinsten & Henik, 2005; Siegler & Braithwaite, 2017), fraction processing is usually not a highly automated process but rather solved by strategic choices (Fazio, DeWolf, et al., 2016; Siegler et al., 2011). The two processing strategies typically observed are componential and holistic processing of fractions (Meert, Grégoire, & Noël, 2009). However, in Study 3 of this dissertation, I showed that there are also magnitude unrelated strategies (i.e., sign shortcut strategy) that allow coming to a correct decision in a magnitude comparison task without the necessity of processing the magnitudes of the two to be compared fractions. Therefore, the building block *strategies* is also directly connected with fraction processing in my comprehensive framework (Fig. 11.2, thick arrow 3). So far, the comprehensive framework considers that the three predictors of the cognitive domain *magnitude processing*, *basic numerical skills*, and *strategies* are related to each other and influence fraction processing directly or indirectly.

Motivation and *negative emotions* are considered explicitly in the non-cognitive section of the framework. To allow clear visibility of all arrows and for reasons of simplified presentation, the non-cognitive predictors appear twice in the framework above and below the cognitive predictors. Research on the role of non-cognitive motivational and affective predictors is still scarce. In Study 4 of my dissertation, I showed that different motivation profiles are associated with differences in performance in fraction magnitude tasks. Thus, the building block *motivation* is connected with magnitude processing (see Fig. 11.2, thick arrow 4). Finally, Study 5 of my dissertation suggests that negative emotions and emotion regulation might have an impact on fraction processing (see Fig. 11.2, thick arrow 5). Taken together with the five studies of my thesis and supported by further literature on fraction processing, I proposed a first tentative framework for fraction processing (see Fig. 11.2).

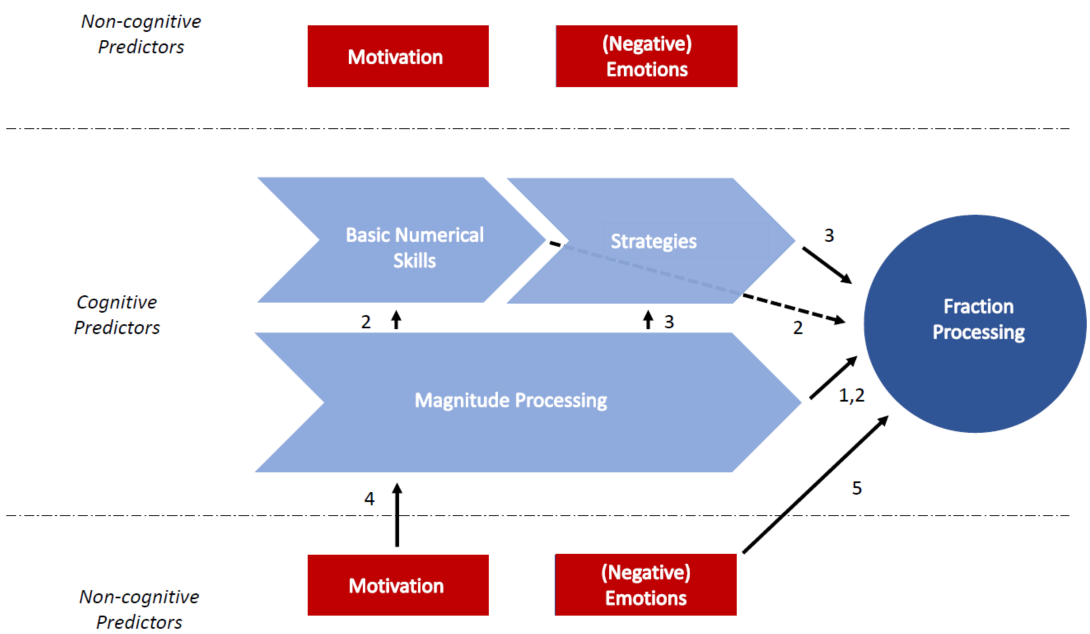


Figure 11.2: First step of the proposed comprehensive framework of fraction processing. Thick arrows: interrelations of the building blocks of this framework as evaluated by the five empirical studies of this dissertation. 1) Study 1 investigated the role of fraction magnitude processing via an NLE training for fraction processing. 2) Study 2 investigated basic numerical skills related to and unrelated to magnitude processing for fraction processing. 3) Study 3 investigated the role of magnitude-related and unrelated strategies for fraction processing. 4) Study 4 investigated the role of motivation by considering different motivation profiles, and 5) Study 5 examined the role of negative emotions and emotion regulation for fraction processing.

Step 2: Substantiating connections between building blocks by existing studies on fraction processing

In the second step, I will point to studies that underpin connections between building blocks that I could not investigate as a part of my thesis (thin lines, see Fig. 11.3). In particular, this concerns the role of negative emotions (i.e., math anxiety) for fraction magnitude processing (Sidney, Thalluri, et al., 2019) and the use of strategies to solve procedures (Rayner et al., 2009). In fact, Sidney, Thalluri, et al. (2019) showed that performance on both fraction magnitude comparison and NLE task was poorer in adult participants with higher math anxiety compared to low math-anxious participants. Therefore, the building blocks *(negative) emotions* and *magnitude processing* are connected in my comprehensive framework (see Fig. 11.3, thin arrow a). Additionally, Rayner, Pitsolantis, and Osana (2009) found that math anxiety negatively affected preservice teachers' performance on a procedural fraction knowledge test, which involved solving fraction arithmetic problems. This finding is depicted by the connection between *(negative) emotions* and *strategies* (thin arrow b).

Taken together, these studies support the proposed preliminary comprehensive framework and indicate that the role of negative emotions for fraction processing should not be underestimated and considered as an important building block that can influence the ability to process fractions dramatically.

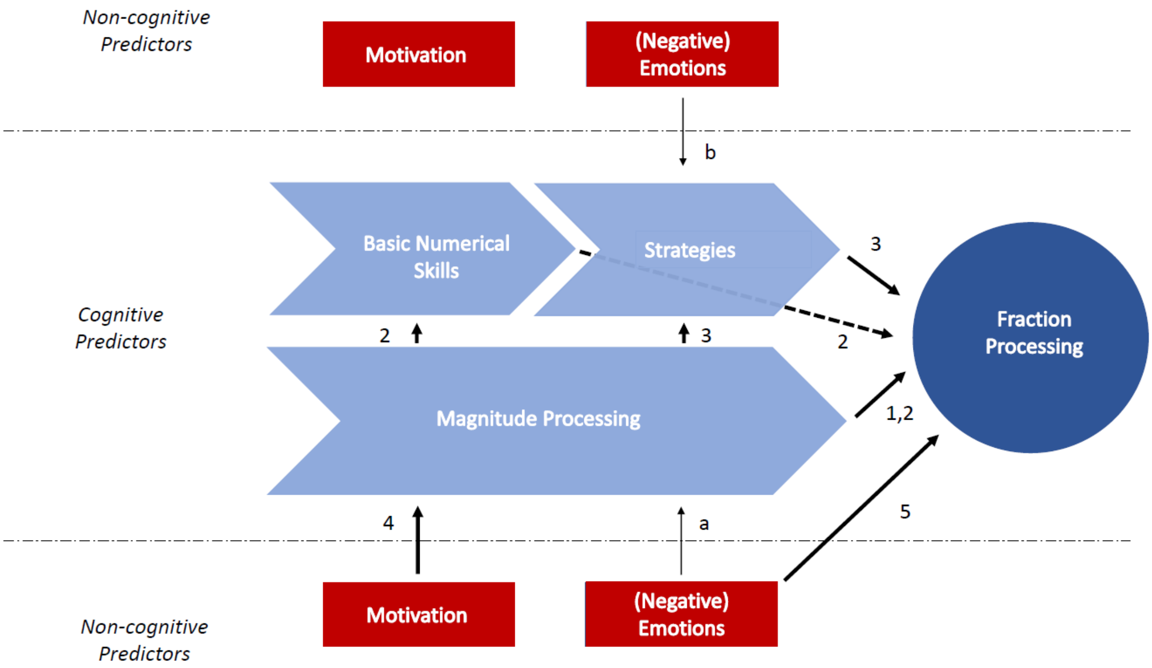


Figure 11.3: Second step of the proposed comprehensive framework of fraction processing. For the sake of readability only the new proposed connections are described. For an explanation on the connections evaluated by the studies of my dissertation (thick arrows 1-5), please see Fig. 11.2. Thin arrows: interrelations of the building blocks of this framework that were not investigated by this dissertation but can be evaluated with existing studies. a) Sidney, Thalluri, et al. (2019) and Rayner, Pitsolantis, and Osana (2009) could show that math anxiety affected performance in different fraction tasks like magnitude comparison, NLE, and conceptual knowledge. b) Rayner, Pitsolantis, and Osana (2009) also showed that math anxiety affected performance in a procedural knowledge task for fraction arithmetic.

Step 3: Evaluating missing connections between building blocks by existing studies on natural numbers

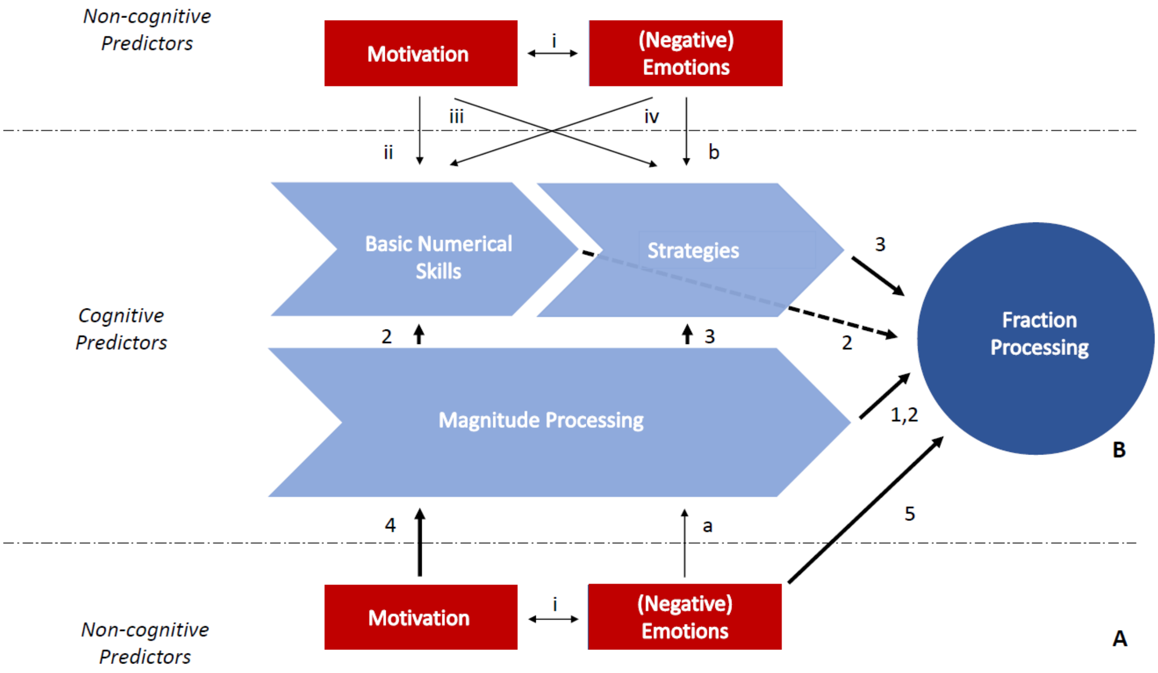
So far, this tentative comprehensive framework is based on findings provided by the five studies of my dissertation and further already existing studies. Additionally, the interrelations between building blocks and fraction processing that I could not investigate in the context of this dissertation have been evaluated by previous studies investigating fraction processing. In a final step, I want to give some examples for missing connections between building blocks that were neither part of my dissertation nor could be substantiated by already existing studies from the large body of fraction literature with the help of studies involving natural numbers (Fig. 11.4, grey lines). Although natural numbers are a different number type and there are many notable differences between natural numbers and fractions (see Section 1.2), it is reasonable to assume that the superordinate influence of motivation and negative emotion observed for natural number processing may be generalized to fraction processing.

In line with this argument, Wang, Shakeshaft, Schofield, & Malanchini (2018) observed eight distinct profiles which could be distinguished by different combinations of math anxiety and motivation levels. Additionally, these distinct profiles differed in ranges of mathematical achievement (example study for the connection between motivation and negative emotion in the comprehensive framework; see Fig 11.4, grey line i).

Moreover, in a longitudinal study, Mercader, Miranda, Presentación, Siegenthaler, & Rosel (2018) showed that motivation affected early numeracy skills and indirectly affected later mathematical achievement (example study for the connection between motivation and basic numerical skills in the comprehensive framework; see Fig 11.4, grey line ii). Furthermore, many studies have shown that motivation plays a crucial role in problem-solving, which can be defined as a strategic process in mathematics (see Schukajlow et al., 2017 for a review on the role of motivation on mathematics achievement and problem-solving; see Fig 11.4, grey line iii). Finally, an overwhelming body of literature shows that math anxiety affects various mathematical tasks and skills, including problem-solving abilities and basic numerical skills (see Maloney & Beilock, 2012 for a review as an example between negative emotions and basic numerical skills; see Fig 11.4, iv).

It must be pointed out that connections between some building blocks are more likely to be intertwined, but for the sake of simplicity, only one direction of influence is shown in the comprehensive framework. For instance, the building block negative emotion can have mutual effects with every other building block. In general, negative emotions like math anxiety lead to poorer task performance, which can lead to even more anxiety, resulting in a vicious circle that is hard to overcome.

Finally, in Fig.11.4 the complete proposed comprehensive framework of fraction processing is additionally combined with the tentative model of the temporal course of fraction processing (Fig. 11.4, B). This shows that fraction processing is a notoriously challenging process which can go wrong in many areas.



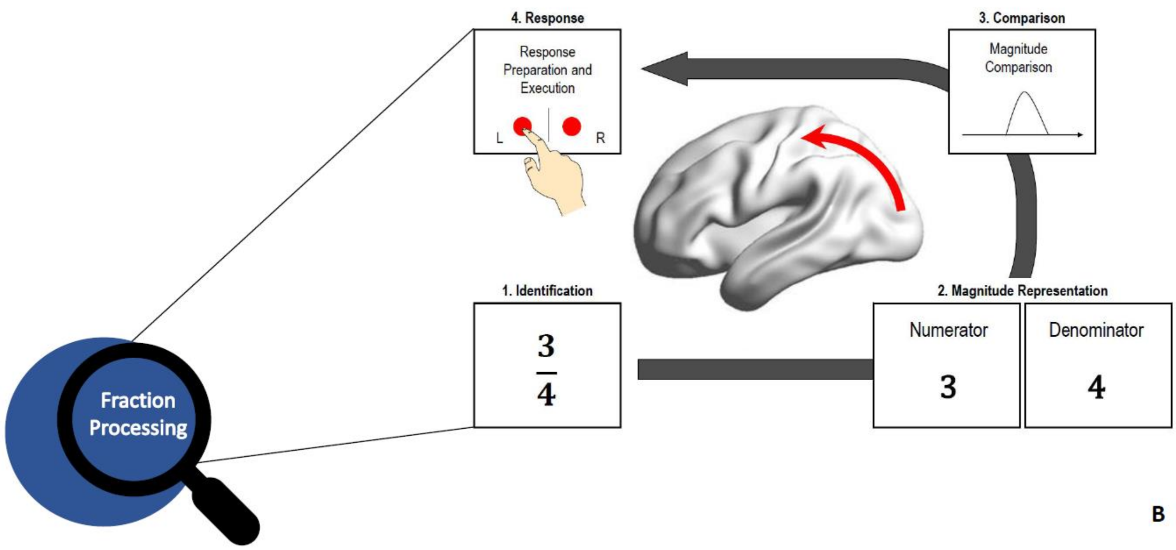


Figure 11.4: Third step of the proposed comprehensive framework of fraction processing. **A.** Proposed framework of fraction processing. **B.** Tentative model of the temporal course of fraction processing. For the sake of readability only the new proposed connections are described. For an explanation on the connections evaluated by the studies of my dissertation (thick arrows 1-5) and by additional studies of the literature on connections that were not investigated by my dissertation (thin arrows a + b), please see Fig. 11.2 and Fig. 11.3. Grey lines: interrelations of the building blocks of this framework that were neither evaluated in this dissertation nor by existing studies investigating fraction processing. i) Wang, Shakeshaft, Schofield, & Malanchini (2018) were able to show that there are different profiles distinguished by combinations of math anxiety and motivation levels. Furthermore, these profiles differ in mathematical achievement. ii) Mercader, Miranda, Presentación, Siegenthaler, & Rosel (2018) showed that motivation affected early numeracy skills. iii) Schukajlow et al., 2017 show that mtivation is important for motivation on mathematics achievement and problem-

solving iv) A large body of studies has shown that math anxiety affects performance on basic numerical skills. For a review, see Maloney & Beilock (2012).

12. Future Directions

The proposed framework of fraction processing may be a first approach to comprehensively describe the complex processes involved in fraction understanding and learning by integrating both cognitive and non-cognitive predictors involved in fraction processing. However, it should be noted that this framework is a preliminary theoretical proposal based mainly on the synthesis of the present results and the described literature. Thus, research is needed to evaluate the validity of the proposed framework in more depth. Furthermore, as one of the major contributions of this framework is the integration of non-cognitive predictors, I want to specifically focus on the connection between negative emotions and fraction processing. Therefore, in the following, I will propose some ideas of how to test this connection of the framework in more depth.

In particular, direct and indirect links, as well as developmental aspects between negative emotions and fraction processing, should be investigated more closely. Study 5 of this dissertation made a first attempt to shed more light on the role of negative emotions on fraction processing. However, to understand this role, different aspects need to be considered: i) differences and similarities between non-math anxious participants and math-anxious participants, ii) differences and similarities between typically developed participants and participants with mathematical disabilities, iii) the direct impact of negative emotions on achievement in fraction tasks (e.g., fraction magnitude comparison task), and iv) whether applying emotion regulation strategies or an intervention for emotion regulation is beneficial especially for math-anxious participants and participants with mathematical disabilities.

In doing so, it is not necessary to design a completely new study. But one might instead extend the study design of study 5 of this dissertation into a pre-post intervention study with different groups and an emotion regulation training as intervention. For instance, the study may include several groups, like one group consisting of math-anxious participants with no other mathematical disabilities, the second group consisting of participants with mathematical disabilities but without suffering from math anxiety, and a control group that is neither math-anxious nor suffers from mathematical disabilities. Including math-anxious participants and participants with mathematical disabilities at the same time might help to examine with more extend the possible impact of an emotion regulation training on task achievement. Math-anxious participants are known to be poorer in handling mathematical problems because they tend to avoid any situation that requires mathematical skills. However, math-anxious individuals do not suffer from cognitive impairments that would result in their inability to solve the task (Ashcraft, 2002; Dowker et al., 2016). In contrast, participants with mathematical disabilities like dyscalculia suffer from the inability to solve number-related tasks (Von Aster & Shalev, 2007).

Therefore, emotion regulation training should have a more pronounced positive impact on the math-anxious participants than on the other two groups in the experiment (i.e., mathematical disability and control group). This effect should also be detectable in neuro-functional measures in terms of a more pronounced activation of the ACC, especially in the math anxiety group from pre- to post-measurement. However, while previous studies suggested that dyscalculia and math anxiety are two disabilities that can usually be separated, it is known that children suffering from dyscalculia also can experience math anxiety as a result of their poor performance in math-related tasks (Devine, Hill, Carey, & Szűcs, 2018). Therefore, an emotion regulation intervention might also be helpful to improve the attitude towards mathematics in children with developmental dyscalculia. However, it is unlikely that an emotion regulation intervention would improve performance on mathematical problems in children with developmental dyscalculia. Since participants with dyscalculia suffer from dysfunctions especially involving the IPS (Price et al., 2007), it is rather unlikely that an intervention targeting the ACC will improve impairments originating from dysfunctions involving the IPS.

Additionally, another study might investigate the direct impact of negative emotions on achievement in a fraction magnitude comparison task. This might be possible by investigating performance on a fraction magnitude comparison task and employing an emotion regulation task before the fraction task. This emotion regulation task either triggers or reduces negative emotions or does not elicit any negative emotions (neutral condition; Morawetz, Mohr, Heekeren, & Bode, 2019). For this purpose, images are presented that evoke a negative reaction or that are not intended to evoke any emotion (so-called neutral images). The experiment would consist of three conditions: a) a negative condition without emotion regulation, b) a neutral condition without emotion regulation (which should not evoke any emotion and therefore emotion regulation is not necessary), and c) a negative condition with emotion regulation. In the negative and neutral condition without emotion regulation, participants would be presented either an image that evokes negative emotions or an image that does not evoke emotions and would be asked to experience any emerging emotion without actively manipulating them. In the negative condition with emotion regulation, participants are again shown images that evoke negative emotions and are asked to reduce the intensity of this initial negative reaction actively. This can be done by distancing themselves from the presented image while actively becoming an observer of the presented image. Subsequently, after each emotion regulation task, a fraction magnitude comparison task would be presented. If negative emotions affect fraction magnitude performance, participants should perform better after each negative condition with emotion regulation compared to the neutral condition without emotion regulation. In turn, participants should perform worse on a fraction magnitude comparison task after the negative condition without emotion regulation compared to the neutral condition without emotion regulation. Employing this study design in combination with fMRI measurements would also help further examine the role of emotion regulation and, in particular, ACC for fraction processing.

13. Additional relevant Predictors of Fraction Processing

Since a dissertation can usually only examine and illuminate a narrow part of a research area due to economic and time constraints, in the following section, I would first like to briefly introduce a selection of cognitive, non-cognitive, and meta-cognitive predictors that might also be of relevance for fraction processing. Following this introduction, I will extend my proposed *comprehensive framework of fraction processing* with these additional predictors derived from the literature.

13.1 Cognitive Predictors

Many cognitive abilities may play a role in rational number and especially fraction processing. However, I want to highlight executive functions as mental processes that might be particularly relevant for fraction processing. In general, executive functions are crucial for learning and development (e.g., Hughes, 2002). They are superordinate mental processes with which we can monitor our behavior (for a review, see Diamond, 2013). For instance, executive functions enable controlling emotions, attention, and pursuing goal-directed behavior (Cristofori, Cohen-Zimerman, & Grafman, 2019). In particular, executive functions include a) working memory, b) inhibitory control, and c) cognitive flexibility. In the following, I want to focus on working memory and inhibitory control as two executive functions that are assumed to be relevant for fraction processing.

13.1.1 Working Memory

Working memory enables the mental storage and manipulation of information over a short period of time. In educational research, cognitive load is particularly relevant (Sweller, Van Merrienboer, & Paas, 1998). Cognitive load theory suggests that working memory capacity is limited. This means that the amount of available working memory (e.g., the number of units simultaneously held, manipulated, and remembered) is a restricting factor for learning and problem-solving. Suggesting solving a task becomes more difficult if the task occupies more working memory capacity (e.g., De Jong, 2010).

A meta-analysis including 110 studies showed a significant correlation between working memory and mathematics (r = .35), with whole number arithmetic and word problems showing the strongest association. Moreover, the correlation between working memory and mathematics was stronger for children with mathematical difficulties. Additionally, this meta-analysis indicated a significant correlation of r = .30 between working memory and fraction processing in particular. However, the number of included correlations for fractions was relatively small compared to other included numerical variables (i.e., 26 correlations for fraction tasks vs. 143 for word problems), suggesting that the real effect might be more pronounced (Peng, Namkung, Barnes, & Sun, 2016).

The role of working memory for fraction problem-solving may be of particular relevance compared to natural numbers, especially for componential fraction processing. For instance, when solving a fraction magnitude comparison task with the help of componential processing, the following solution steps might be necessary: 1) separately processing numerators and denominators of the two fractions, 2) separately compute overall magnitudes from the bipartite structure of the two fractions by comparing numerators and denominators (please note that there are many different ways to achieve this step, for instance, one might be cross-multiplication), 3) compare both magnitudes for solving the task. Given the number of processing steps and the units that must be held and manipulated in working memory during such a task, it becomes evident that cognitive load might be a limiting factor when solving fraction tasks. However, so far, only a small number of studies have investigated the role of working memory for fraction processing either directly or indirectly (e.g., Fuchs et al., 2014; Hansen et al., 2015). For instance, Fuchs et al. (2014) could show that working memory is an important moderator of NLE fraction interventions focusing on fraction interpretations (e.g., part-whole interpretation and ordering, comparing and placing fractions on number lines). Moreover, Hansen et al. (2015) found that working memory was a significant predictor of 6^{th} graders' understanding of fraction concepts. Thus, more research is needed to examine whether working memory capacity might be crucial, especially in the early stages of fraction learning.

13.1.2 Inhibitory Control

Another aspect of executive functions that might be relevant for fraction processing is inhibitory control. Inhibitory control reflects the ability to suppress goal irrelevant stimuli and behavioral responses. Additionally, it is also essential for regulating and controlling emotions. In academic settings, inhibitory control was frequently reported to be predictive of mathematical achievement (e.g., Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006).

In the context of rational number processing, it is assumed that inhibitory control is needed to overcome the natural number bias. This means that the primary urge to solve a fraction problem by applying previously acquired knowledge about natural numbers and apply them to rational numbers needs to be overcome by inhibition. For instance, previous studies showed that inhibition processes are necessary to quickly and correctly solve fraction magnitude comparison tasks (Fu et al., 2020; Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015; Rossi, Vidal, Letang, Houdé, & Borst, 2019). Thus, it might be possible that the initial unconscious (brain) reaction to solve a fraction problem is trying to apply natural number knowledge (except for highly trained fractions like 1/2, 1/4, or 3/4). Additionally, with study 5 of this dissertation, I showed that fractions, as challenging learning content, trigger the initial neural activation of a negative emotion network, which might be regulated via inhibitory control through activation of the ACC. Therefore, future research should focus more on the role of inhibitory control for fraction processing, the relationship between inhibitory control and the natural number bias, and interventions that can foster inhibiting the natural number bias and negative emotions associated with rational numbers.

13.2 Non-Cognitive Predictors

In this dissertation, I have shown that non-cognitive predictors like motivation and negative emotions also play a role in fraction processing. Both are driving forces that can result in avoiding or embracing to deal with fractions. In educational research, however, other non-cognitive predictors also play a role as driving forces for learning and in the classroom. Two constructs of particular interest in mathematics that are known to influence learning and achievement are boredom and self-efficacy. In the following, I will introduce both constructs as two additional non-cognitive predictors that may be of interest for future research on fraction processing.

13.2.1 Boredom

Academic boredom is a common emotion in educational and school settings. It can be broadly defined as a negative experience involving low arousal and unpleasant feeling. Furthermore, boredom is classically categorized as a deactivating emotion which is more common than frustration, anxiety, and anger during learning (Ahmed, van der Werf, Minnaert, & Kuyper, 2010; Pekrun, 2006). For instance, Larson & Richards (1991) showed that around 36% of middle school students had experienced boredom in the classroom and 40% while doing their homework. It is known to have a negative impact on learning and achievement and can lead to school dropout (Bearden, Spencer, & Moracco, 1989; Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010).

Additionally, boredom can also have a negative effect on motivation (Pekrun et al., 2010). It can be caused by a perceived over- or under-challenge while dealing with the learning content. Moreover, little perceived importance of school tasks and content might also be a relevant cause for boredom (Pekrun, 2006). Given that the importance and relevance of rational numbers in general and fractions, in particular, is often doubted by students and adults, it might be possible that boredom plays a role for fraction processing (Padberg & Wartha, 2017). Additionally, perceived over-challenge while dealing with fractions could also lead to boredom. Finally, as already mentioned, boredom is one of the most common emotions in the classroom and during learning. Therefore, it might also be of relevance for fraction learning.

13.2.2 Self-efficacy

Academic self-efficacy describes learners' perceptions of their academic abilities and performance to achieve goals in school settings (Elias & MacDonald, 2007). It has strong links to performance, achievement, and learning across the lifespan (Honicke & Broadbent, 2016) as well as on school subjects like mathematics (Randhawa, Beamer, & Lundberg, 1993). Unfortunately, there is little research on self-efficacy and achievement in the content area of fractions. However, one of the few studies revealed gender differences in self-efficacy for a fraction task in 7 to 10 graders, with girls being less confident than boys to accurately solve the task, even though no performance differences were found between girls and boys (Ross, Scott, & Bruce, 2012). Moreover, a second study showed that self-efficacy beliefs on the ability to use multiple representations (e.g., number line and different area diagrams) for fraction concepts had a beneficial influence on achievement in fraction addition problems (Panaoura, Gagatsis, Deliyianni, & Elia, 2009). Thus, strengthening students' self-efficacy for fraction knowledge and concepts might be helpful for fraction learning and facilitate fraction processing. Moreover, self-efficacy seems to be a crucial factor for self-regulation, which will be described in the following subsection.

13.3 Predictors involving metacognition and self-regulation

Metacognition has become an increasingly popular topic in educational research. Simplified, it can be defined as *cognition about cognition* (Flavell, 1976). Thus, reflecting on one's own cognition. Including knowledge about how to use specific strategies for learning and problem-solving. In general, metacognition entails monitoring and control of cognitive processes (Nelson & Narens, 1994). Based on these estimations, students decide if, what, and which strategies they will learn (Ehrlinger, Johnson, Banner, Dunning, & Kruger, 2008; Koriat, 2012). These decisions are part of and build the basis for self-regulation and self-regulated learning (Schunk & Greene, 2018). Many studies across different age groups, content domains (including mathematics), and settings (e.g., school or laboratory experiments) have shown that especially self-regulation and self-regulated learning plays a crucial role for successful academic learning (Dignath, Buettner, & Langfeldt, 2008; Dignath & Büttner, 2008; Jansen, Van Leeuwen, Janssen, Jak, & Kester, 2019; Sitzmann & Ely, 2011).

However, research on fraction processing involving self-regulation or even self-regulated learning is still scarce. One recently developed intervention to improve fraction processing is the so-called self-regulated strategy intervention (SRSD). This intervention has already been successfully applied in teaching writing and reading (for two meta-analyses, see Graham & Harris, 2003; Sanders et al., 2019). SRSD aims to improve students' academic abilities by fostering self-regulation skills. In particular, students with emotional and learning disabilities can benefit from this intervention (Bak & Asaro-Saddler, 2013; Lienemann & Reid, 2006). The intervention consists of six stages which necessarily involve student-teacher interaction and teacher guidance by providing background knowledge (stage 1), strategy discussion (stage 2), strategy modeling (stage 3), memorize techniques for the strategy (stage 4), and strategy support (stage 5). During these stages, the teacher monitors the students' performance of the strategy. Ultimately, the student should be able to use the strategy independently with the help of different self-regulation skills (stage 6). In connection with fractions, SRSD has been successfully used as a teaching intervention (Ennis & Losinski, 2020; Hacker, Kiuhara, & Levin, 2019; Kiuhara et al., 2020; Losinski, Ennis, Sanders, & Wiseman, 2019; Losinski, Ennis, & Shaw, 2021; Wang et al., 2019). For instance, Losinski et al. (2021) used an SRSD intervention with a step-by-step strategy to improve fraction addition and subtraction with unlike denominators. This strategy consists of the following steps: i) identify the denominators, ii) identify multiples of the denominators, iii) find the least common multiple, iv) extend fractions with corresponding multiples two create two fractions with common denominators, and v) solve fraction subtraction or addition problem. The stages of the SRSD intervention subsequently provided the following lessons conducted by teachers: a) essential knowledge on the structure (numerator and denominator) and concepts (e.g., part-whole) of fractions, b) discussion of the strategy for fraction addition and subtraction with unlike denominators, c), and d) practice and memorize the strategy on different fraction problems and strengthening self-regulation by using a checklist with the different steps of the strategy, self-explanation and visual aids, e) teacher and students practice the strategy together, f) students apply the strategy independently and without help of the strategy checklist and visual aids. Studies targeting improvement of proficiency with fractions through SRSD interventions revealed pre-post improvements of participants in fraction addition and subtraction tasks with unlike denominators (Ennis & Losinski, 2019b; Losinski et al., 2021) as well as fraction knowledge (Hacker et al., 2019; Kiuhara et al., 2020). Thus, SRSD seems to be a promising intervention to improve fraction understanding and could also be helpful for typically developing children.

13.4 Extending the comprehensive Framework of Fraction Processing with additional Predictors

In this last subsection, I want to extend my comprehensive framework of fraction processing with the additional predictors mentioned in subsections 13.1 – 13.3. However, for the sake of readability, this extension will only focus on the connections between the primarily involved building blocks of my comprehensive framework (i.e., strategies, magnitude, and motivation) and the additional predictors that were previously identified. Moreover, the approach of this extension is to provide a picture of the negative and positive effects of each predictor on the respective building blocks and their indirect or direct impact on fraction processing. However, it is important to note that this extension is highly speculative as little to no research has been done in connection with the additional proposed predictors and fraction processing.

In the previous subsections, possible additional cognitive (i.e., working memory and inhibitory control), non-cognitive (i.e., boredom and self-efficacy), and metacognitive predictors (i.e., self-regulation) were identified. To capture the association of each predictor to their respective area faster, the predictors are color-coded. Similar to the comprehensive framework, cognitive predictors are depicted in blue, non-cognitive predictors are depicted in red, and self-regulation as the only metacognitive predictor is depicted in ocher. In the following, I will explain each identified connection and the influence of each additional predictor on previous building blocks and direct influences (if applicable) on fraction processing.

Working Memory: This variable is a cognitive predictor which could be added to the comprehensive framework. Higher working memory capacity should positively impact strategy use, and magnitude processing (e.g., Fuchs et al., 2014; Hansen et al., 2015) as both variables are highly interrelated. Especially when it comes to componential processing of fractions as the different numerators and denominators must be held in working memory until the fraction task at hand is solved. In turn, this should improve fraction processing. The more processing steps and units that must be held in working memory, the more important working memory capacity is to solve the task.

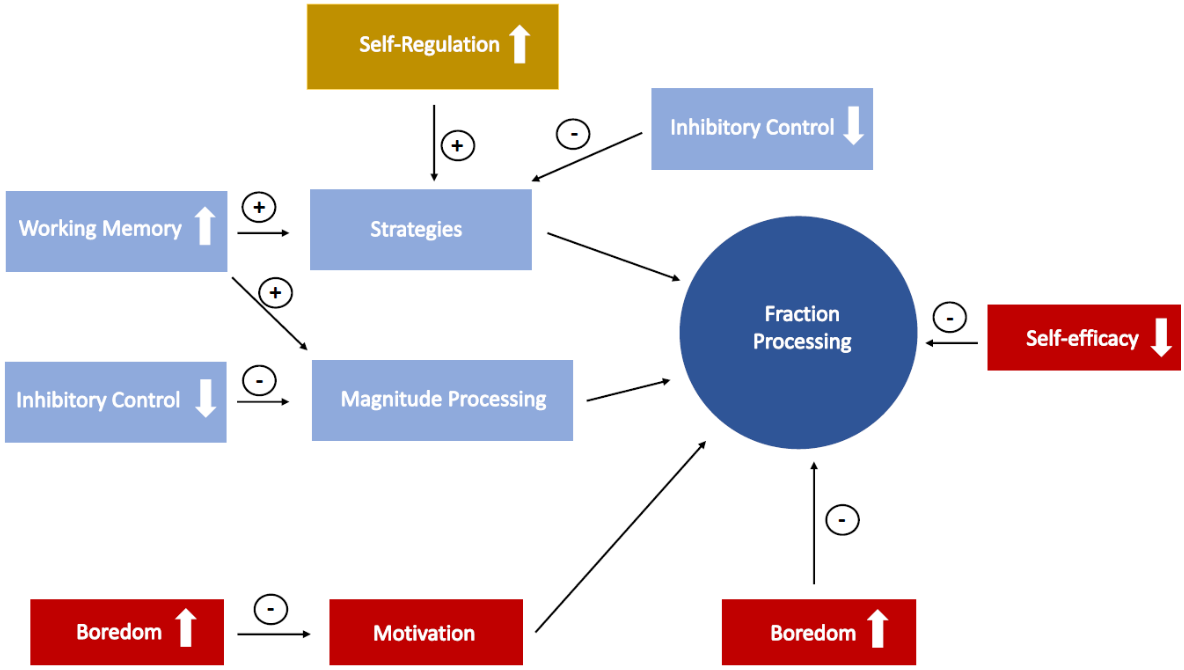


Figure 13.1: Overview of the extension of the proposed comprehensive framework of fraction processing. Red boxes: non-cognitive predictors. Blue boxes: cognitive predictors. Ocher box: metacognitive predictor. White arrow pointing upwards: increase. White arrow pointing downwards: decrease. Plus sign: positive effect. Minus sign: negative effect.

Inhibitory control: Inhibitory control is the second identified additional predictor that might be added to this framework. Inhibitory control might be relevant to overcome the natural number bias (e.g., Fu et al., 2020; Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015; Rossi, Vidal, Letang, Houdé, & Borst, 2019). Natural numbers are deeply rooted within ourselves, as they are the first number type individuals encounter in school, and natural numbers are dominant in occurrence compared to other number types. Therefore, similar to working memory, this predictor could influence strategy use and magnitude processing. For instance, a lack of inhibitory control could lead to wrongly rely on strategies that are usually applied to natural numbers (e.g., treating numerators and denominators as natural numbers and adding them separately while solving a fraction addition task).

Boredom: Boredom is an additional non-cognitive predictor that might be added to the comprehensive framework of fraction processing. Increased boredom is known to have a negative impact on motivation (Pekrun et al., 2010). This could lead to less motivation or even demotivation, which in turn might negatively affect fraction processing. Additionally, many students perceive fractions as less important, which can also cause boredom and less engagement with fractions. Thus, boredom could also have a direct negative effect on fraction processing.

Self-efficacy: Self-efficacy is the second identified non-cognitive predictor that might be added to the framework. Although there is little research on self-efficacy and fraction processing, it can be assumed that less self-efficacy negatively impacts fraction processing. If individuals do not perceive themselves as confident enough to solve a task, task performance could be affected because insecurities might lead to more inattention and errors during problem-solving.

Self-regulation: This predictor is the only metacognitive predictor that could be additionally added to the framework. In general, self-regulation has been identified to be important for learning. As shown by studies on SRSD interventions (Ennis & Losinski, 2020; Hacker, Kiuhara, & Levin, 2019; Kiuhara et al., 2020; Losinski, Ennis, Sanders, & Wiseman, 2019; Losinski, Ennis, & Shaw, 2021; Wang et al., 2019), increased self-regulation can positively impact strategy learning and, in turn, strategy use. Additionally, SRSD interventions foster student's self-regulation to improve their task performance.

Taken together, the suggested extensions of the previously proposed comprehensive framework of fraction processing give first insights into the interrelations and direct or indirect (positive or negative) influences of the predictors on fraction processing. However, the proposed extensions need to be supported by empirical studies as little to no research has been done to investigate the role of the respective predictors on fraction processing.

14. Conclusion

While the importance of the mastery of fractions for individuals' mathematical development is undeniable, the inability to handle and understand fractions still remains (Booth & Newton, 2012; Obersteiner, Dresler, et al., 2019). Therefore, the current dissertation aimed to identify important predictors that are essential to deal with fractions efficiently. To date, the ITND is the most influential developmental theory of number processing and, in particular, fraction processing (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). However, focusing only on magnitude processing might not give a complete picture of the involved mechanisms to fraction processing. To this end, a *comprehensive framework of fraction processing* was proposed by including cognitive and non-cognitive (i.e., motivational, and affective) predictors. This framework extends upon the ITND by building on the core assumption of the integrated theory that magnitude processing is the key ability for fraction understanding and processing. Specifically, it expands the ITND by integrating additional cognitive, motivational, and affective predictors important to master fractions.

To pursue this issue, I employed five empirical studies that investigated potential predictors of fraction processing. In particular, I was able to confirm the importance of magnitude processing and the relevance of number line representations for fraction processing by providing both behavioral and neuro-functional evidence (Study 1). Moreover, I was able to substantiate the importance of different magnitude-related and unrelated basic numerical skills as well as magnitude-related and unrelated strategies for fraction processing (Study 2 and 3). Further, I showed that motivation might be a relevant predictor of fraction processing (Study 4). Finally, I could underpin the importance of negative emotions and emotion regulation for fraction processing (Study 5).

Nevertheless, the comprehensive framework remains a tentative proposal based on the present findings and the existing literature on fraction processing. Future research is needed to evaluate the validity of the proposed framework. However, the comprehensive framework sheds more light on the complex mechanisms involved in fraction processing. Together with the presented tentative model of the temporal course of fraction processing and the additionally suggested extension of the comprehensive framework, a bigger picture is provided to answer the question on what are relevant factors of fraction processing and were are starting points to improve it. This provides the basis for future research and interventions that foster comprehensive fraction understanding and related processes.

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Acknowledgments

This dissertation and my accompanying adventure as a PhD student would not have been possible without the support and cooperation of many impressive people that I was lucky to meet.

First, I would like to thank my supervisor Korbinian Moeller for giving me the possibility to be part of one of the most special and enriching working environments. Thank you for your patience, uplifting spirit, and constant will to provide support. You really made science fun for me.

My next and equally significant thank goes to my second supervisor Thomas Dresler. Thank you for providing my “emotional education,” teaching me patience (as far as this was possible), for listening to me, and helping me to overcome many stumbling blocks and insecurities during my time as a PhD student.

Moreover, I would like to thank all my lovely colleagues from NG8 (also known as Ebene 5 3/4). Even in tough times, we were always able to count on each other. It was a pleasure to work with you all.

My special thanks go to Elise Klein for all the endless support and your friendship. I hope to see you soon in Paris! Another big thank goes to Manuel Ninaus and my office buddy Simon Greipl for the many shared laughs and the great support.

Thank you to my friends Katerina Tsarava and Roberta Barrocas for all the great times we had together in Tübingen and for all that is (hopefully) still to come. Another special thanks goes to my friend Vera Moliadze for understanding and supporting me for almost 10 years on a scientific and personal level.

Furthermore, I want to thank the LEAD Graduate School and Research Network for providing a rich interdisciplinary context and a variety of opportunities that enabled me to pursue my PhD.

Additionally, I would also like to thank Andreas Obersteiner for the many fruitful discussions about rational numbers and fraction processing.

Finally, and most importantly, I want to thank my husband Franz. No words can express how much you and your support mean to me. Thank you for being you and being by my side.